Gravitational Waves - Theory and Analysis

Lecture I

Sreekanth H

Course overview

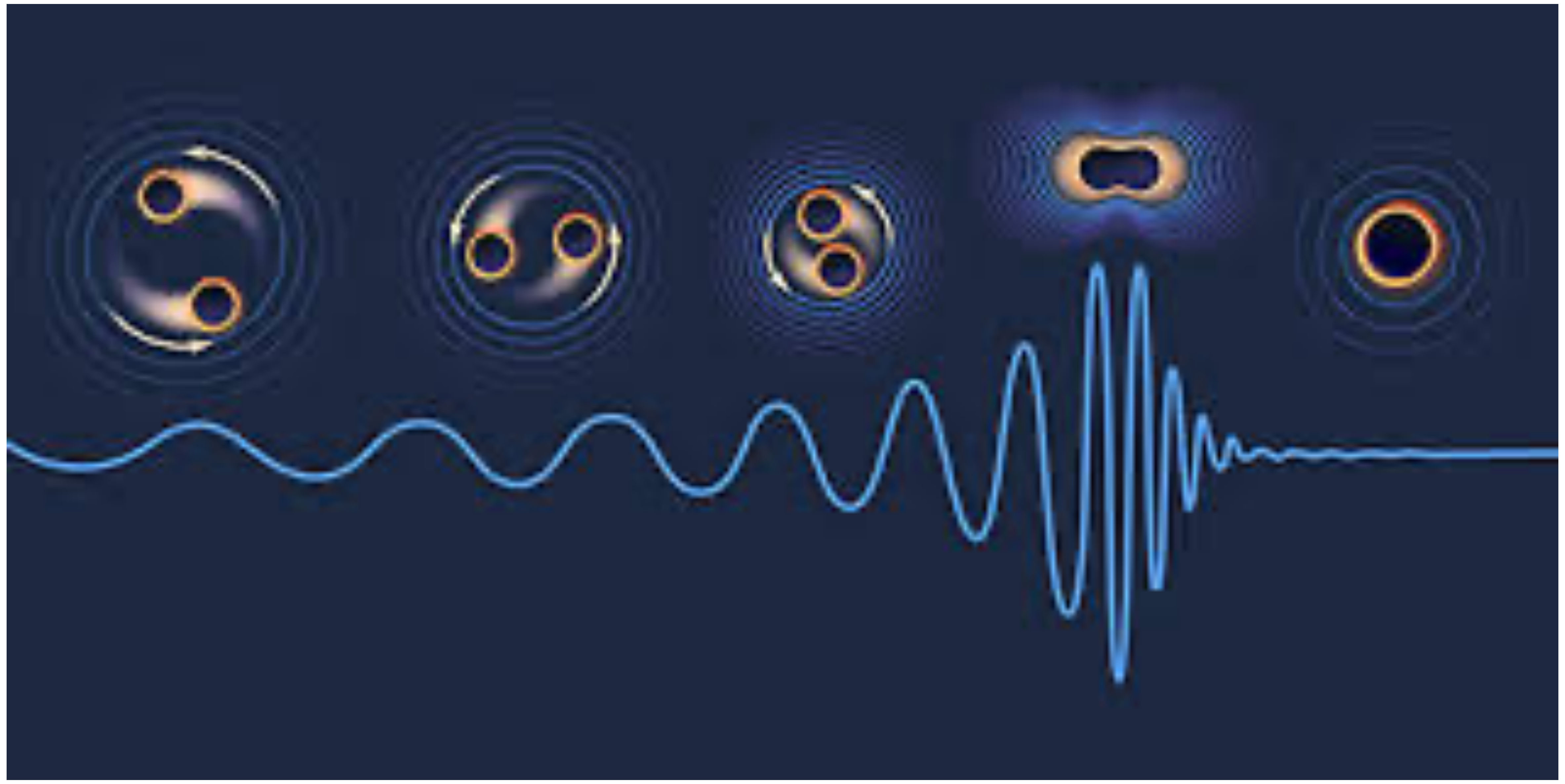
- Historical Introduction to gravitational waves (1 lecture)
- General Relativity (a warm up) (2 lectures)
- GW theory (2 lectures)
- GW sources & Detectors (1 lecture)
- Basics of GW data analysis (2 lectures)
- Applications (1 lecture)

Criteria for passing the course:

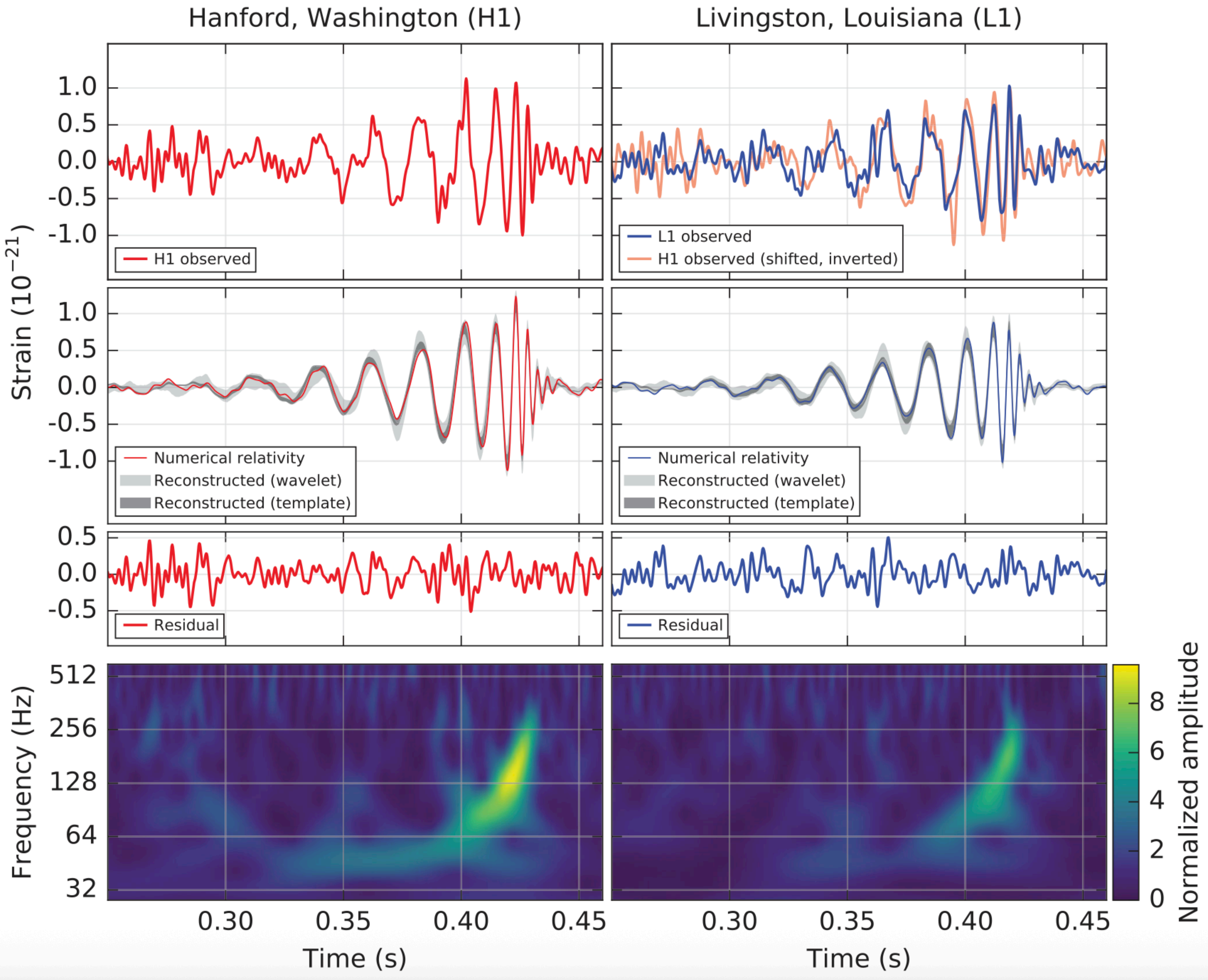
- 80% attendance
- Oral exam — You will be given a list of paper to choose from and we'll have an oral exam based on it.

Listening to the black holes for the first time - GW150914

at 11:50:45 am CET on 14 September 2015

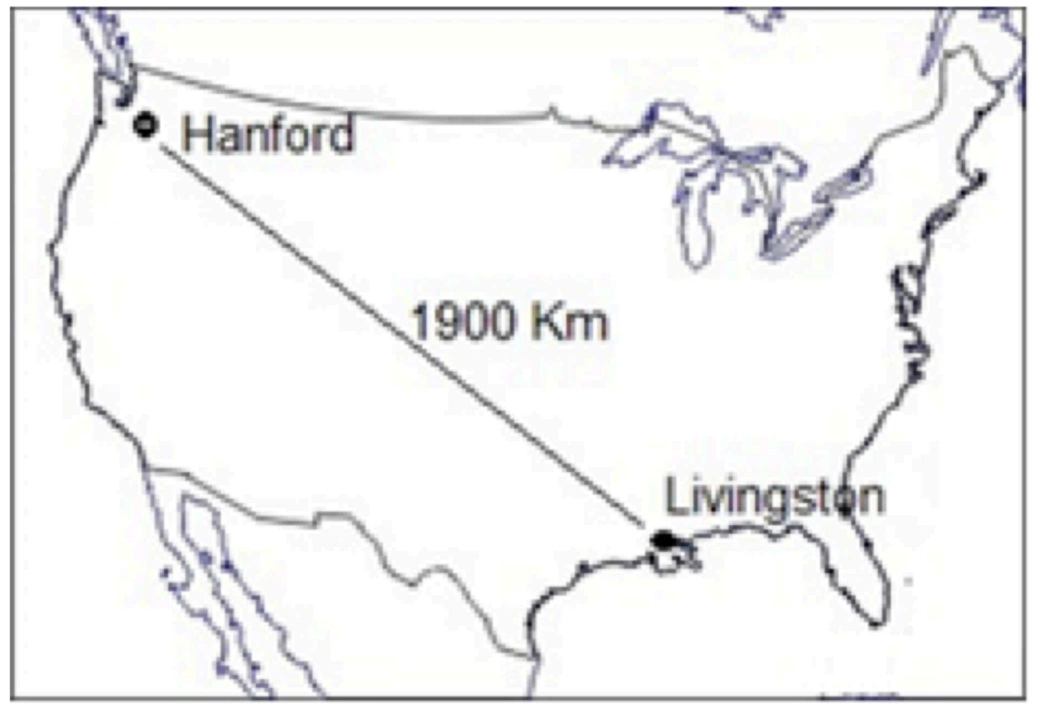


Phys. Rev. Lett. 116, 061102



This marked the birth of gravitational wave astronomy

Most important discovery in this century



GW150914

YY MM DD

60 years of experimental efforts and 100 years since its prediction we detected it.

Selected for a Viewpoint in *Physics*
PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

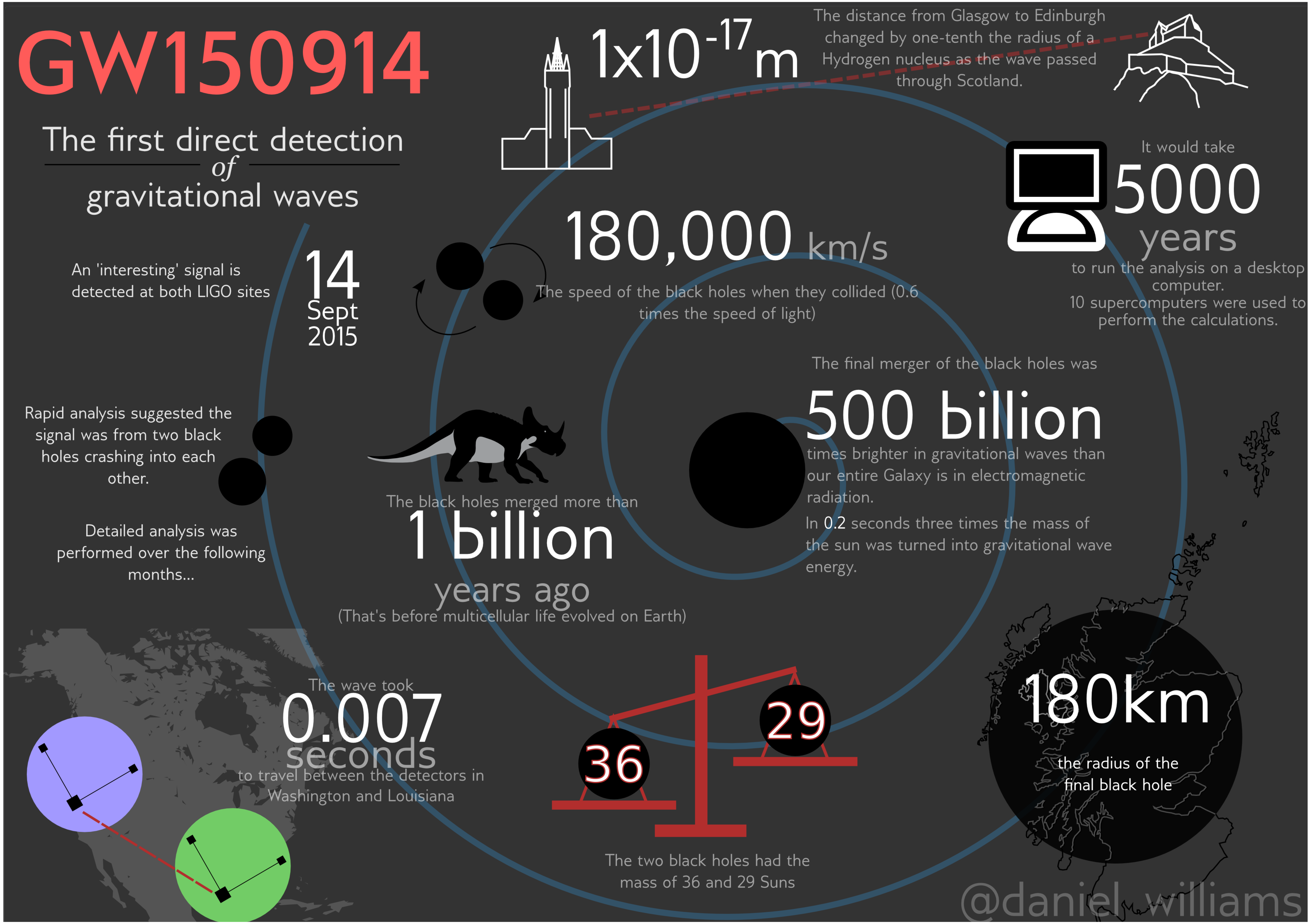
B. P. Abbott *et al.**

(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 21 January 2016; published 11 February 2016)

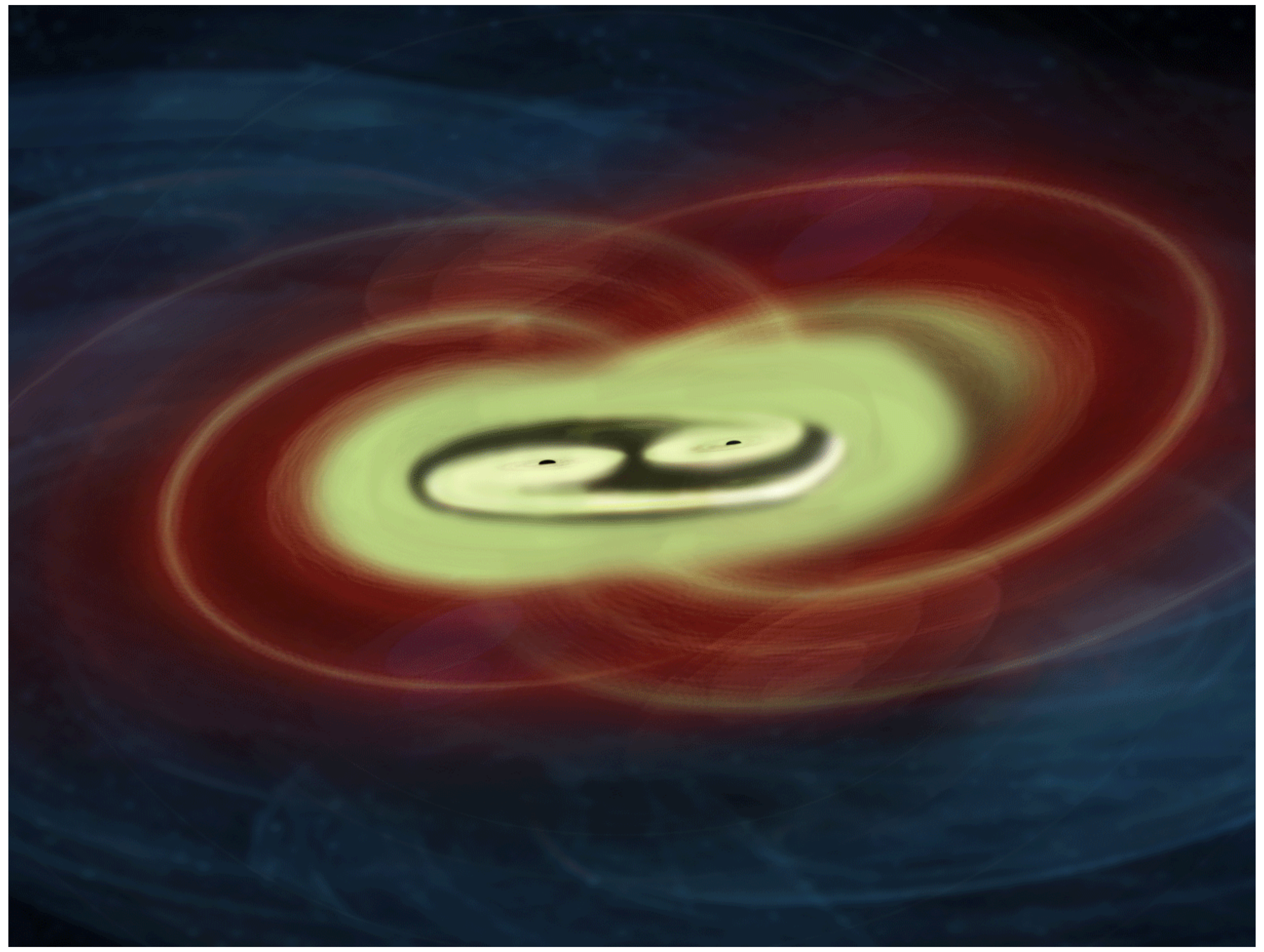
On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4}M_\odot$ and $29^{+4}_{-4}M_\odot$, and the final black hole mass is $62^{+4}_{-4}M_\odot$, with $3.0^{+0.5}_{-0.5}M_\odot c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

DOI: 10.1103/PhysRevLett.116.061102


GW150914
The first direct detection of gravitational waves
1x10^{-17}m
The distance from Glasgow to Edinburgh changed by one-tenth the radius of a Hydrogen nucleus as the wave passed through Scotland.
It would take
5000
years
to run the analysis on a desktop computer.
10 supercomputers were used to perform the calculations.
An 'interesting' signal is detected at both LIGO sites
14 Sept 2015
180,000 km/s
The speed of the black holes when they collided (0.6 times the speed of light)
The final merger of the black holes was
500 billion
times brighter in gravitational waves than our entire Galaxy is in electromagnetic radiation.
In 0.2 seconds three times the mass of the sun was turned into gravitational wave energy.
Rapid analysis suggested the signal was from two black holes crashing into each other.
Detailed analysis was performed over the following months...
The black holes merged more than
1 billion
years ago
(That's before multicellular life evolved on Earth)
The wave took
0.007
seconds
to travel between the detectors in Washington and Louisiana
36
29
The two black holes had the mass of 36 and 29 Suns
180km
the radius of the final black hole
@daniel_williams

Signal morphology

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

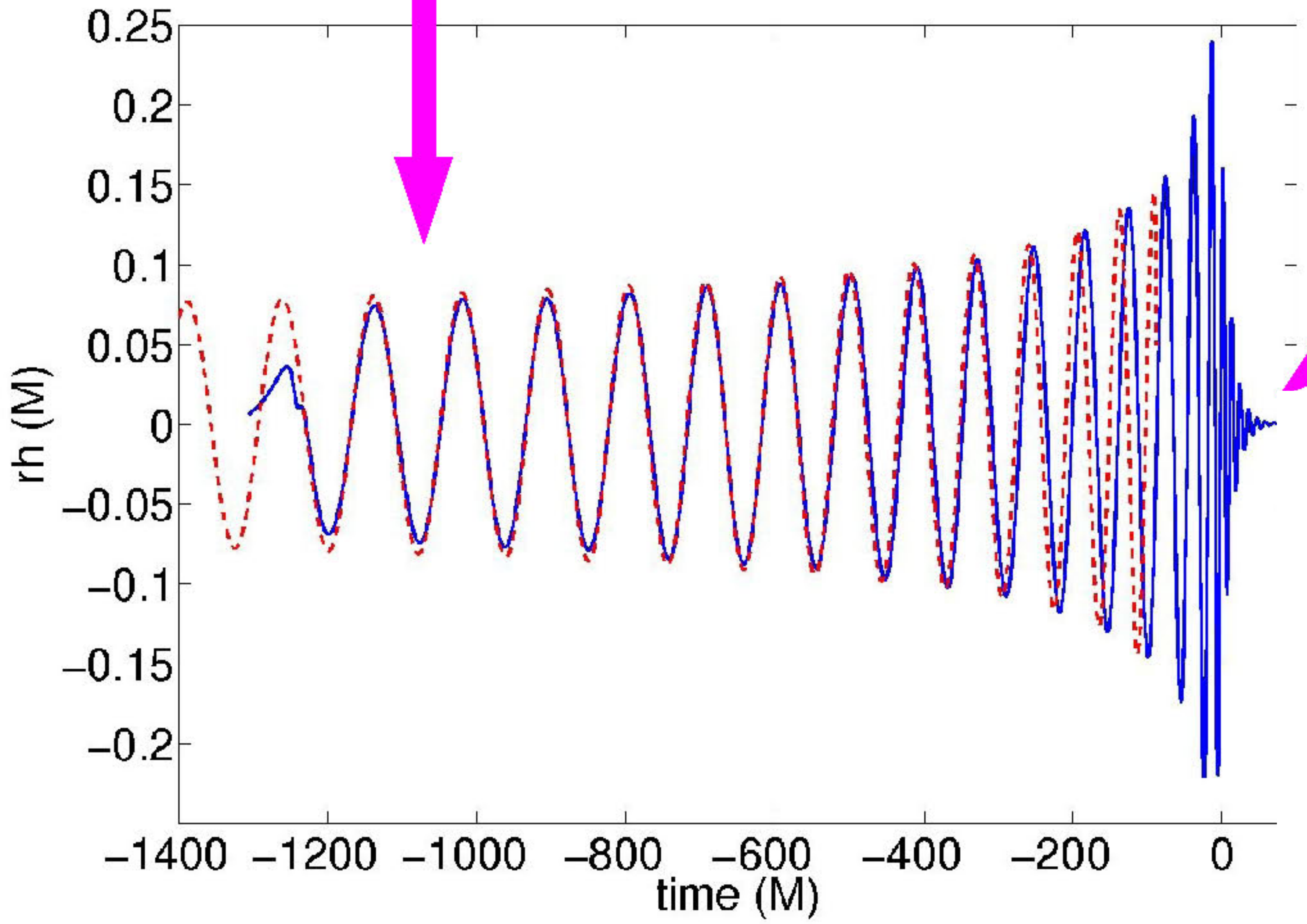


Credit: ESA

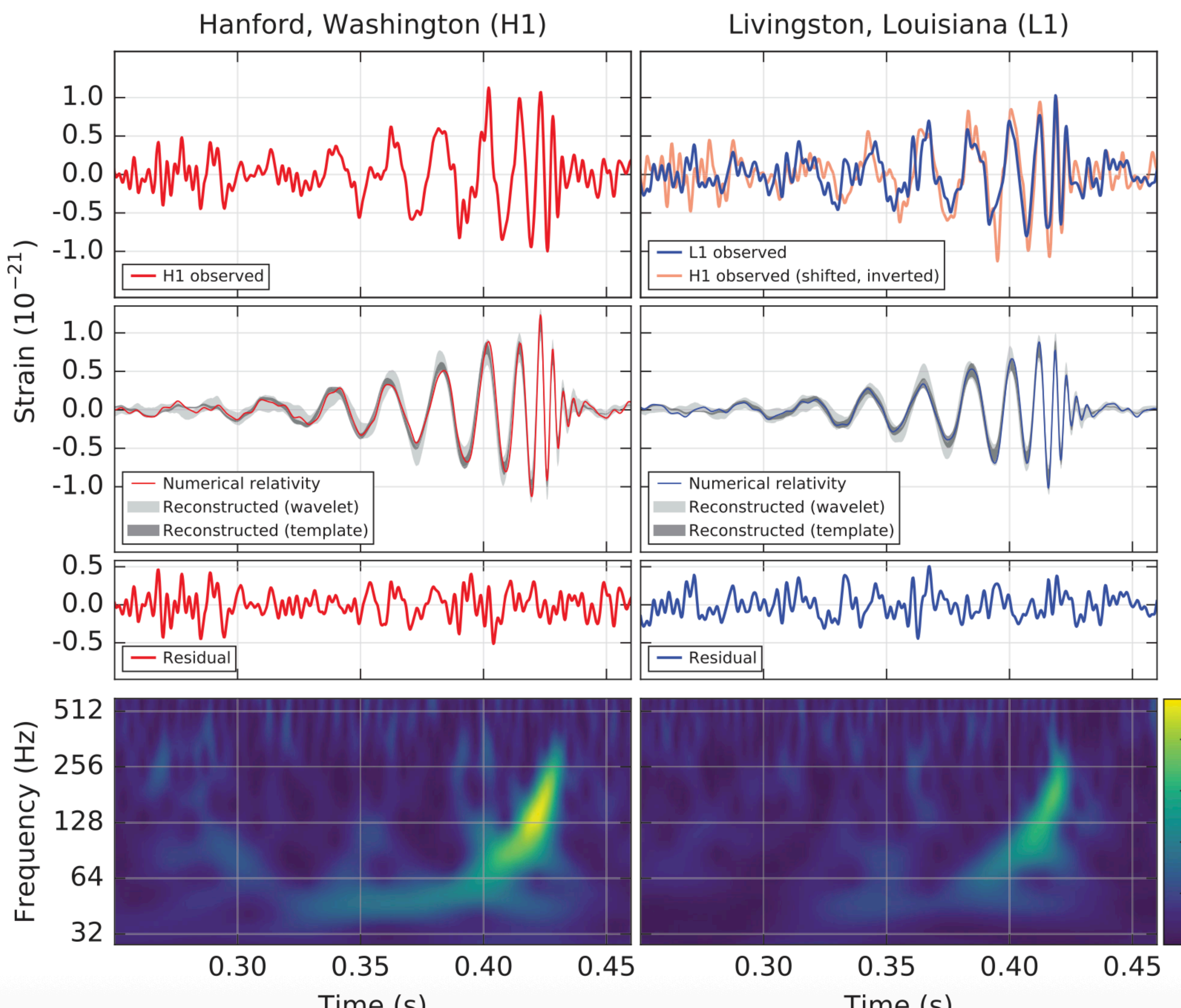
inspiralling phase
post-Newtonian theory

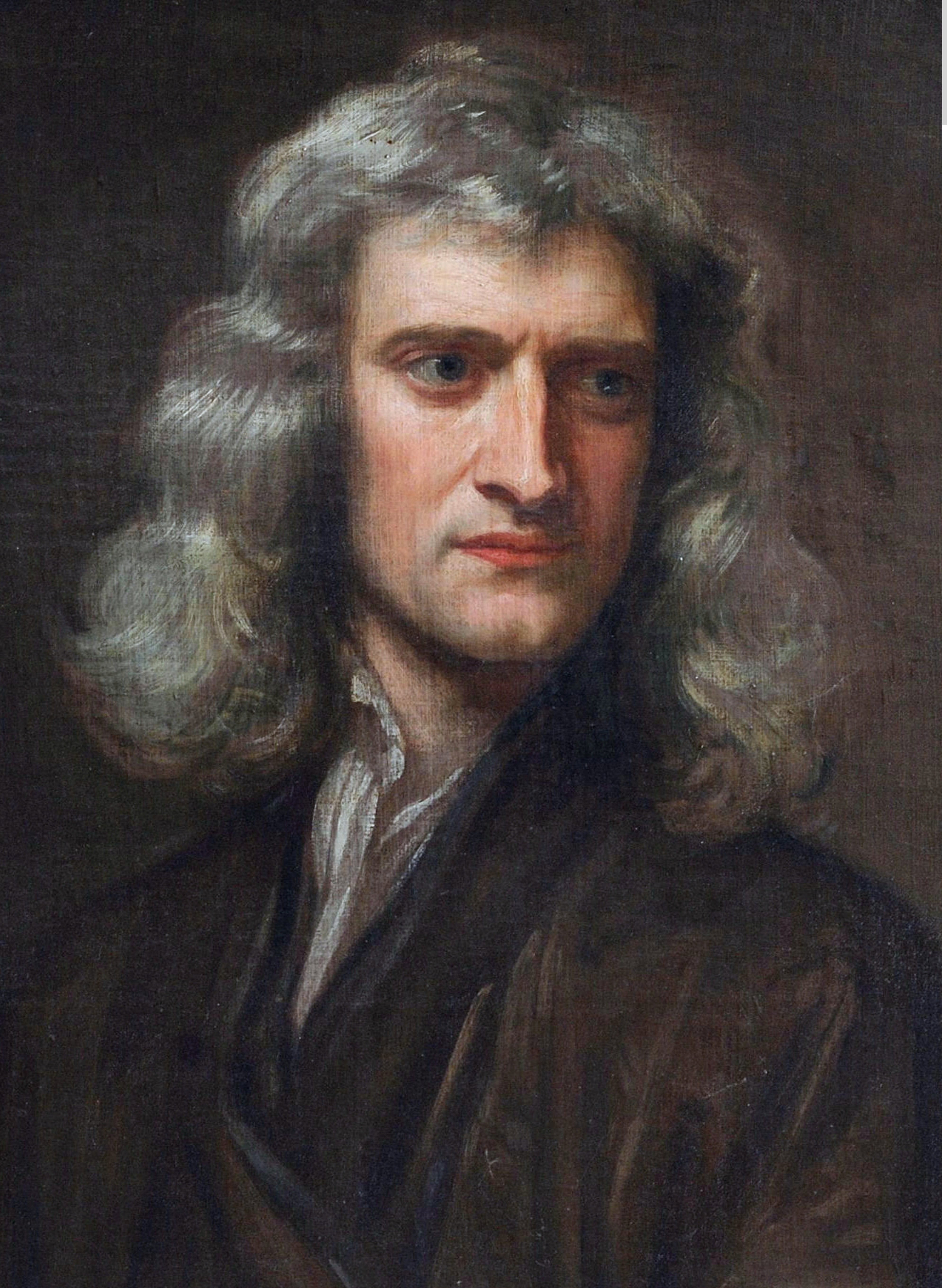
merger phase
numerical relativity

ringdown phase
perturbation theory



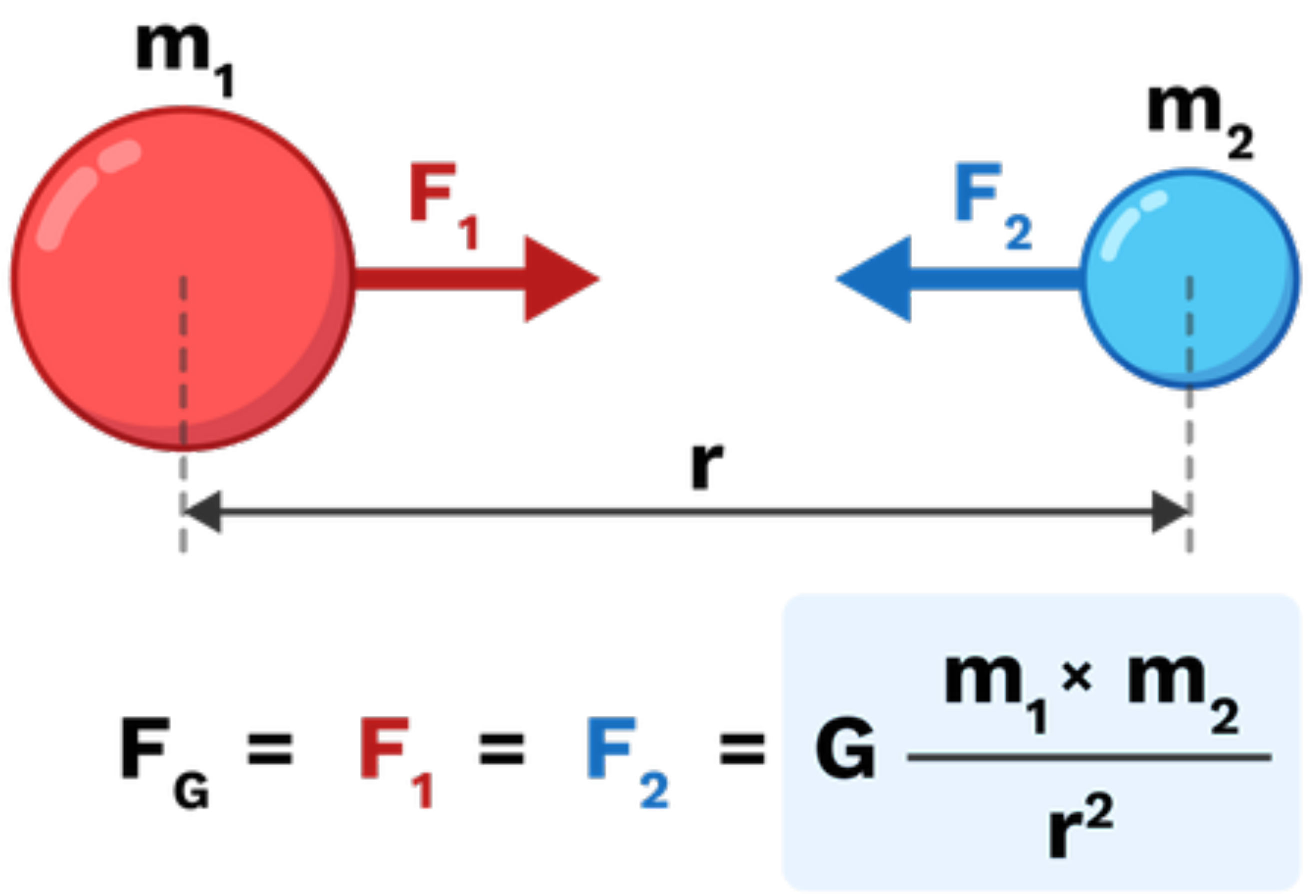
Credit: https://www.iap.fr/actualites/laune/2016/OndesGr/forme_onde_an.jpg





Credit : Godfrey Kneller's 1702 portrait of Isaac Newton

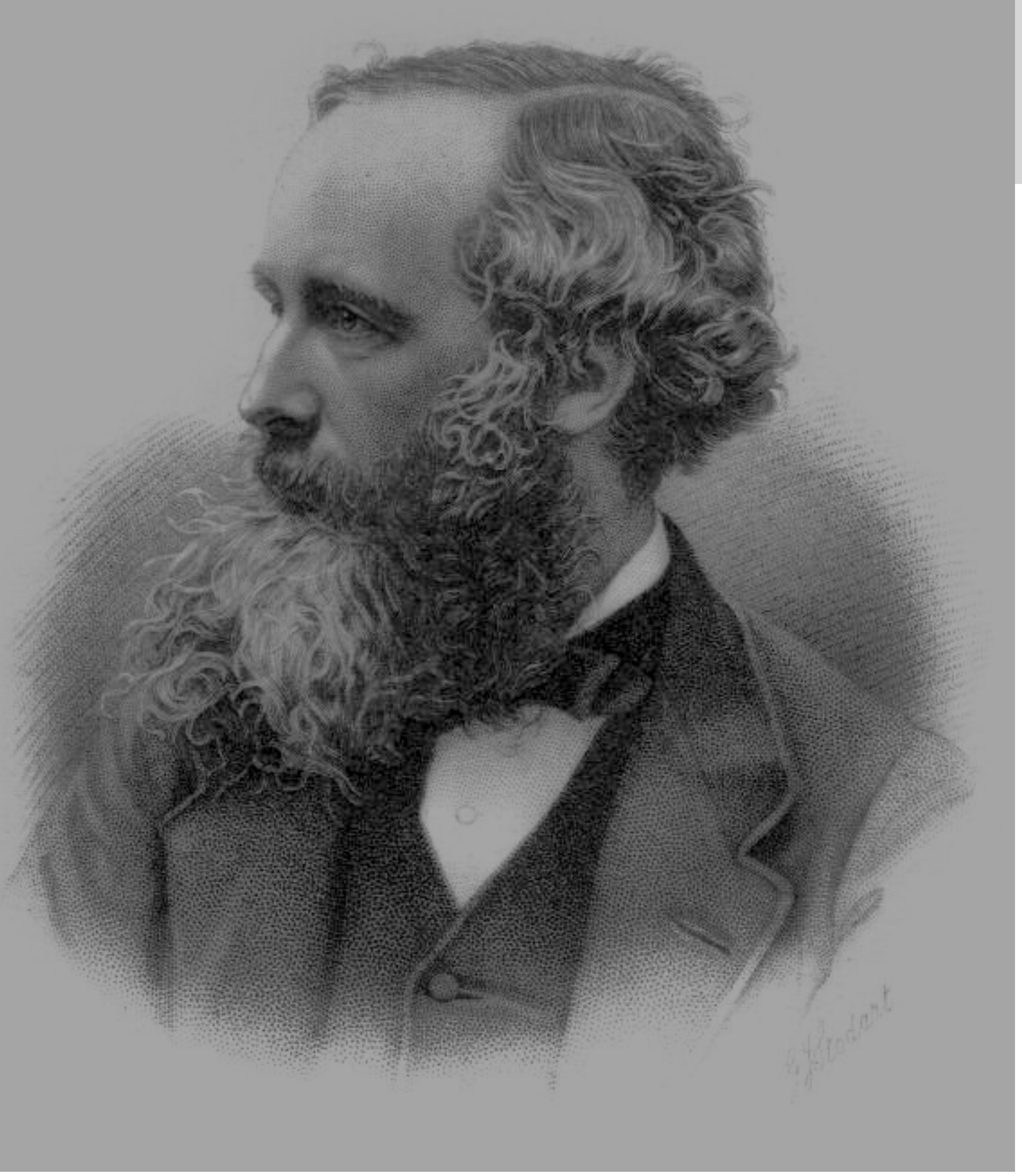
Newtonian theory of gravity



- Planetary motion
- Galaxy dynamics
- Stellar dynamics
- Everyday physics
- Large scale structure simulations

“Gravity is an instantaneous phenomenon”

So no wave phenomenon……



James Clark Maxwell

Propagation of waves in EM theory and relativity

- Maxwell unified Electricity and magnetism
- This lead to the discovery of electromagnetic waves

With Special Relativity (1905) Einstein figured out Electricity and Magnetism are the different manifestations of the same field

$$\nabla . E = \frac{\rho}{\epsilon_0} \quad (1) \quad \text{Gauss' law}$$

$$\nabla . B = 0 \quad (2) \quad \text{Magnetic monopoles}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (3) \quad \text{Faraday's law}$$

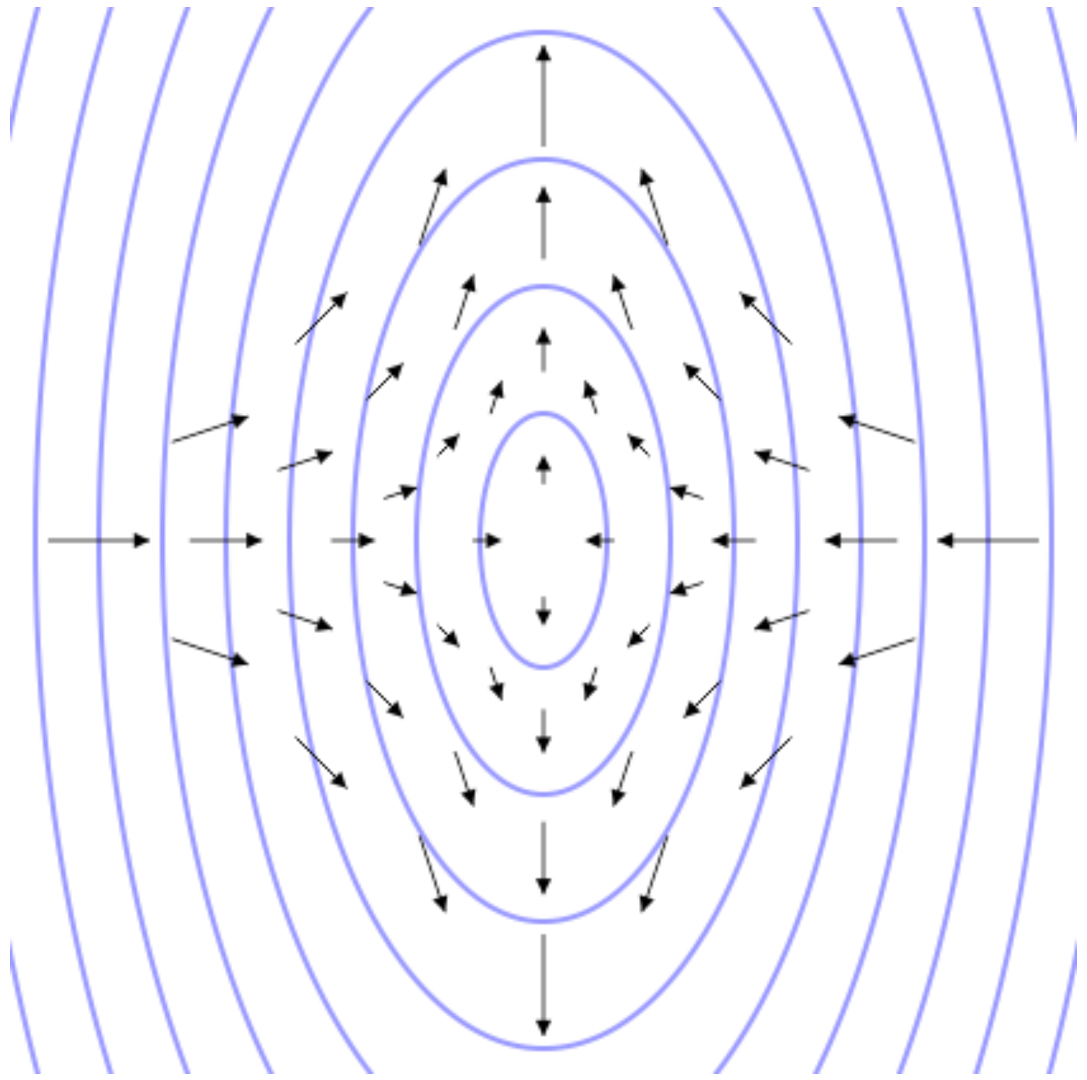
$$\nabla \times H = J + \frac{\partial D}{\partial t} \quad (4) \quad \text{Ampere-Maxwell law}$$

Highlights

- No medium required for propagation
- Travels with speed $c = 3 \times 10^8 ms^{-1}$

Einstein’s Quadrupolar formula

Set a universal speed limit



1905 -

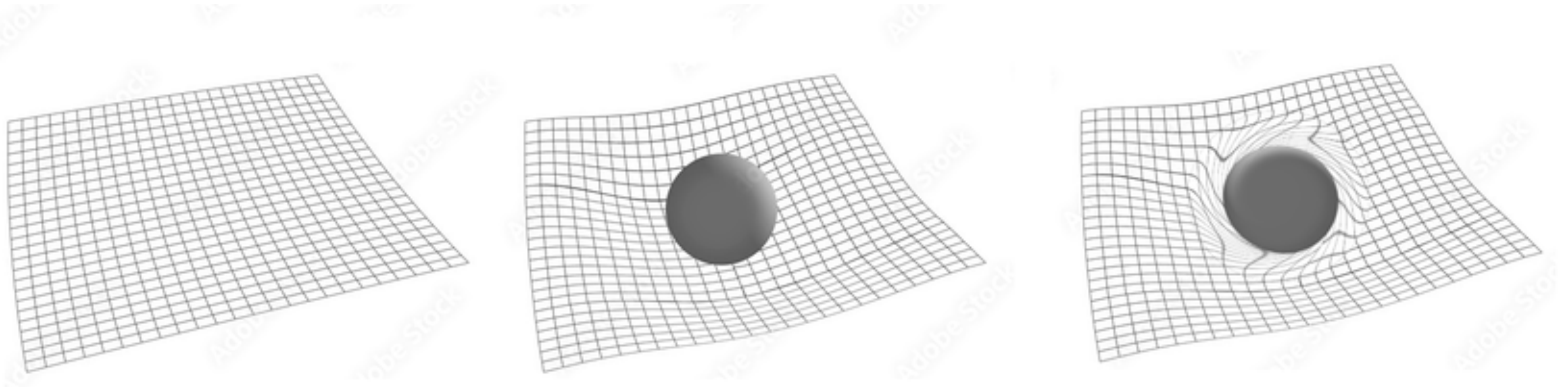
Einstein published **Special Relativity**

Henri Poincaré suggested that gravity propagates at the speed of light

1916 -

Einstein published **General Relativity**

He noticed that his new theory can have a wave like phenomenon like EM waves



Quadrupole formula

$$h = \frac{2G}{c^4}\frac{1}{r}\frac{\partial^2 Q}{\partial t^2}$$

<u>Einstein doubted its existence</u>

- **Is it physical ?**
- **May be a coordinate effect..**

Einstein's second biggest blunder

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

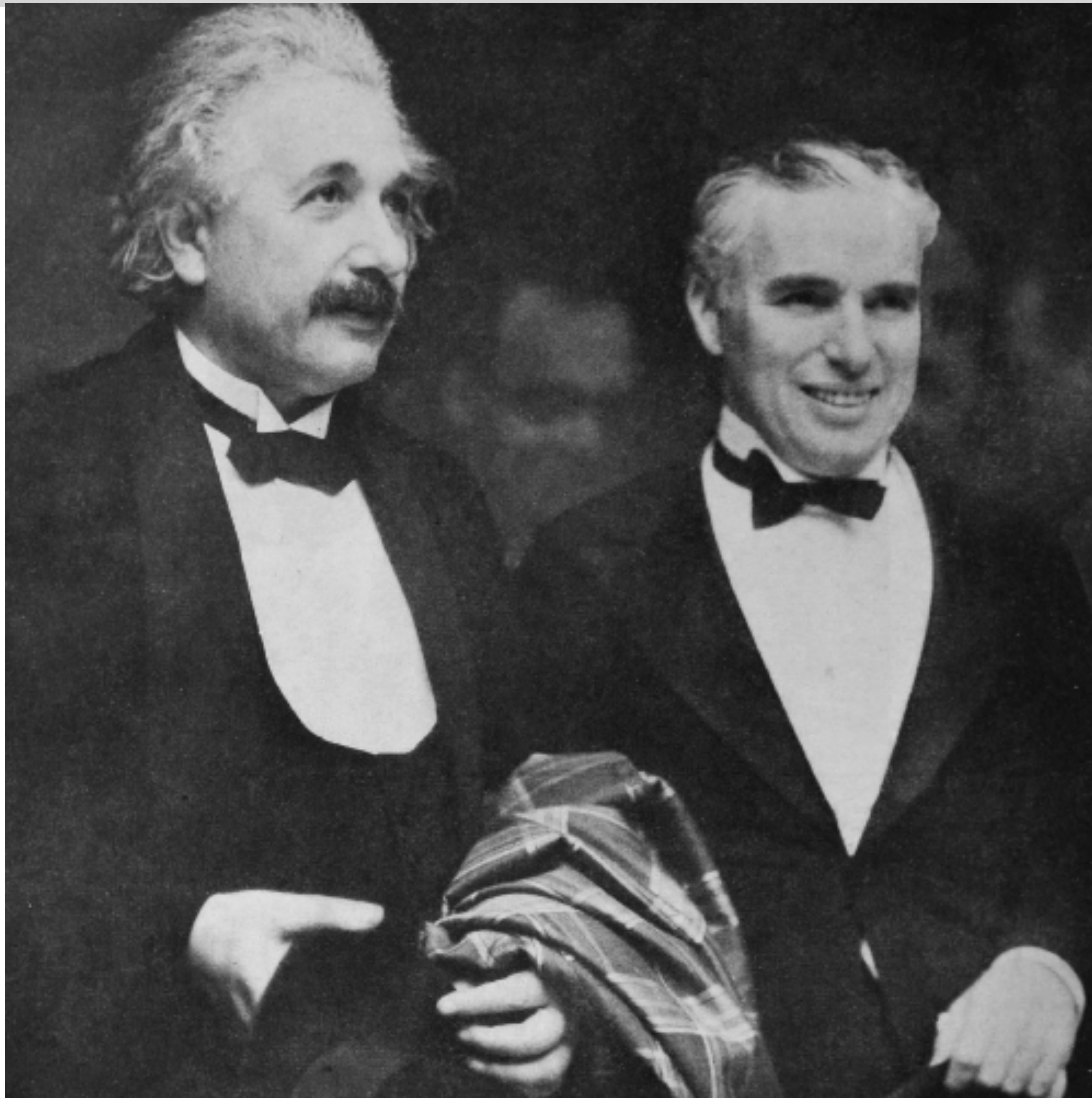
1916 - Einstein published **General Relativity and GW solution**

1932 - Arthur Eddington's paper on **GWs**

- **Three propagating solutions**

1936 - **Einstein and Nathan Rosen's PRL paper**

- **Gravitational waves do not exist**
- **Referee requested revision.**



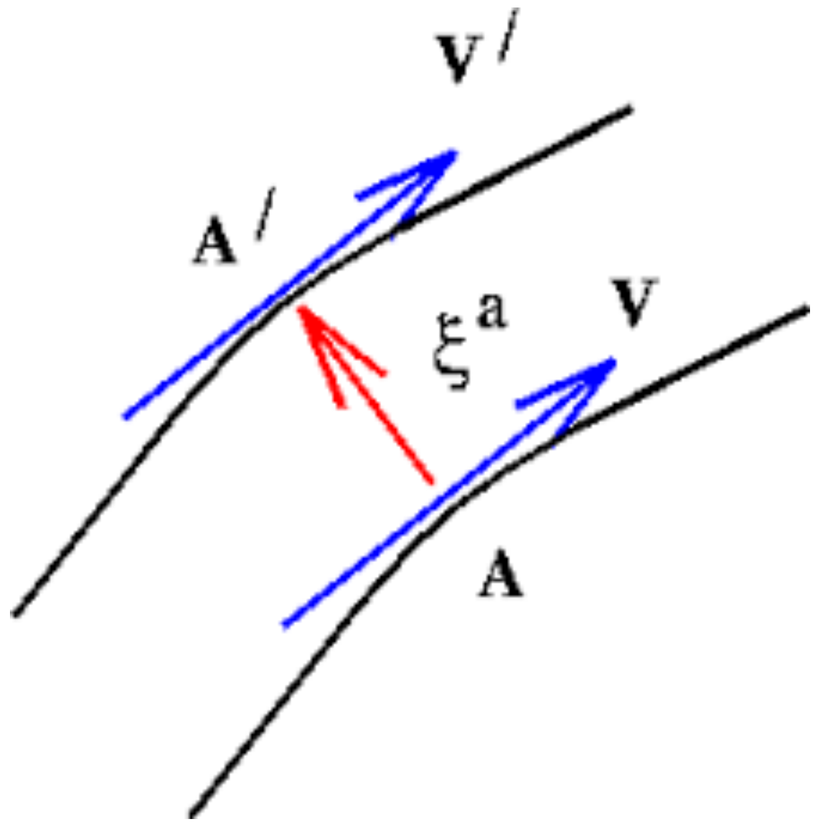
Sticky bead argument

Chapel-Hill conference (year 1957)

Felix Pirani

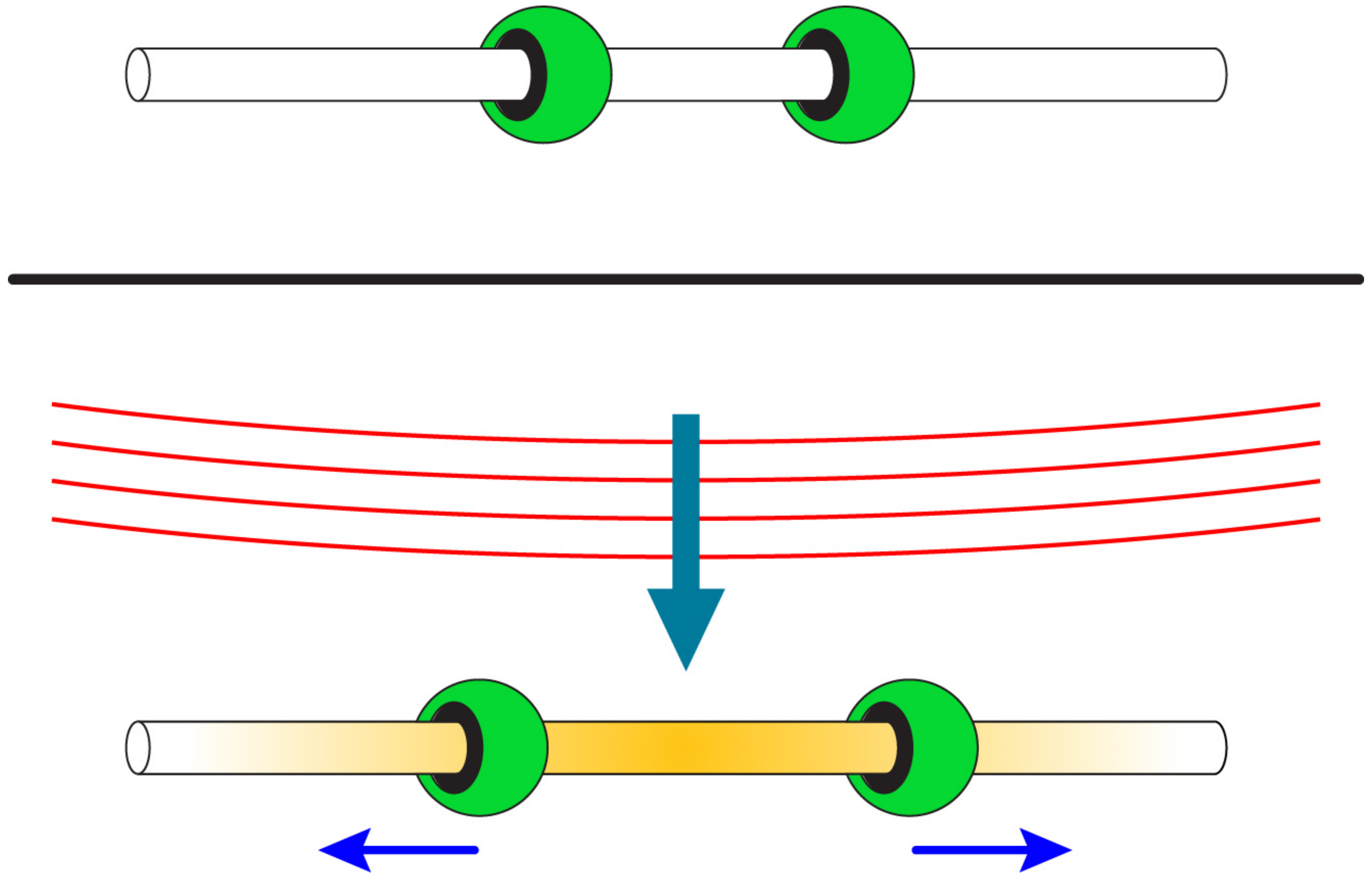
Felix Pirani had presented his latest research on the "***observational effects of passing gravitational waves on two test masses in space***"

$$\frac{d^2x^a}{ds^2}+\Gamma^a_{bc}\frac{dx^b}{ds}\frac{dx^c}{ds}=0$$

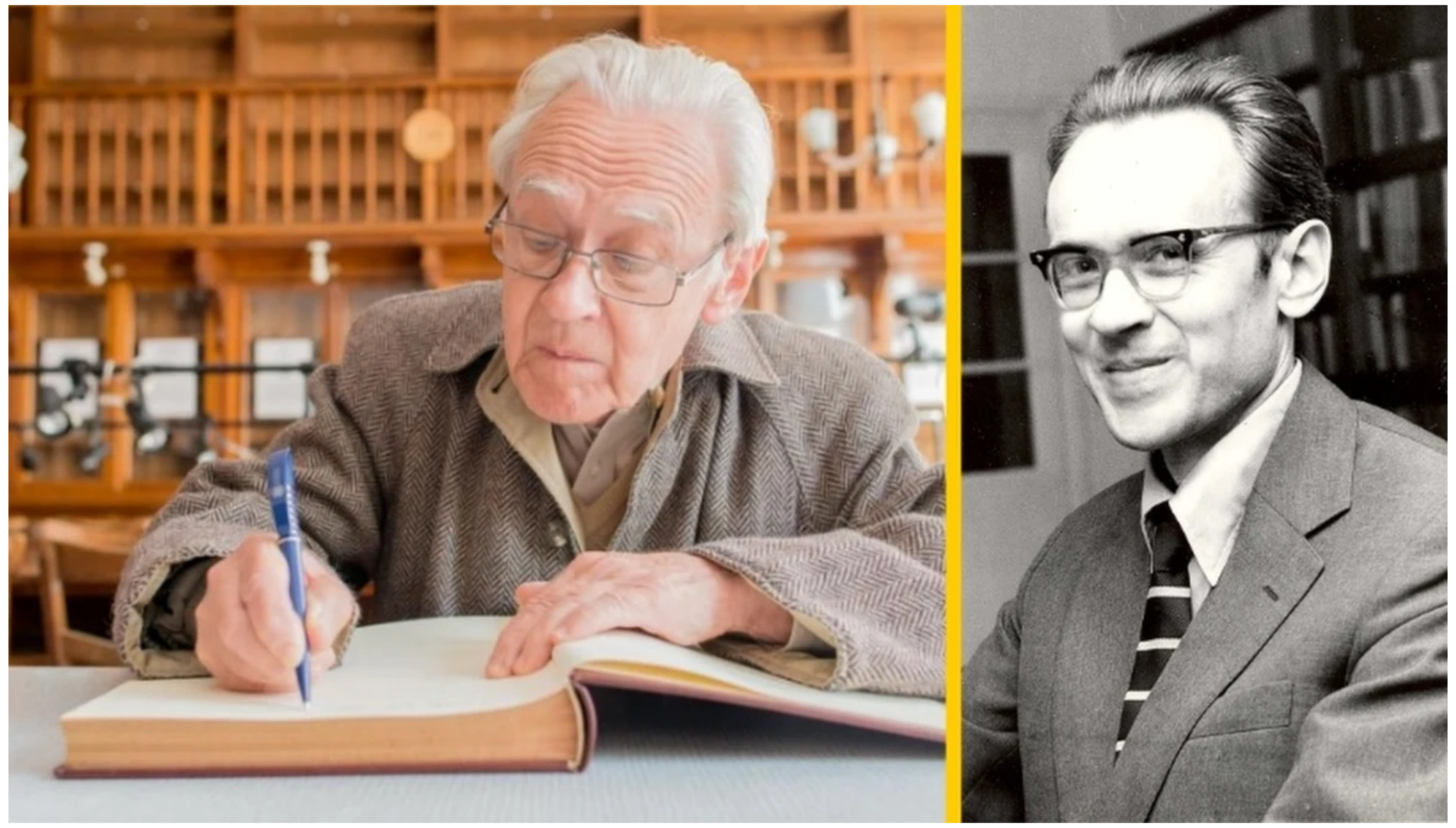


This was 18 months after Einstein passed away…..

Sticky bead on a rod



Andrzej Trautman’s contributions (1958)

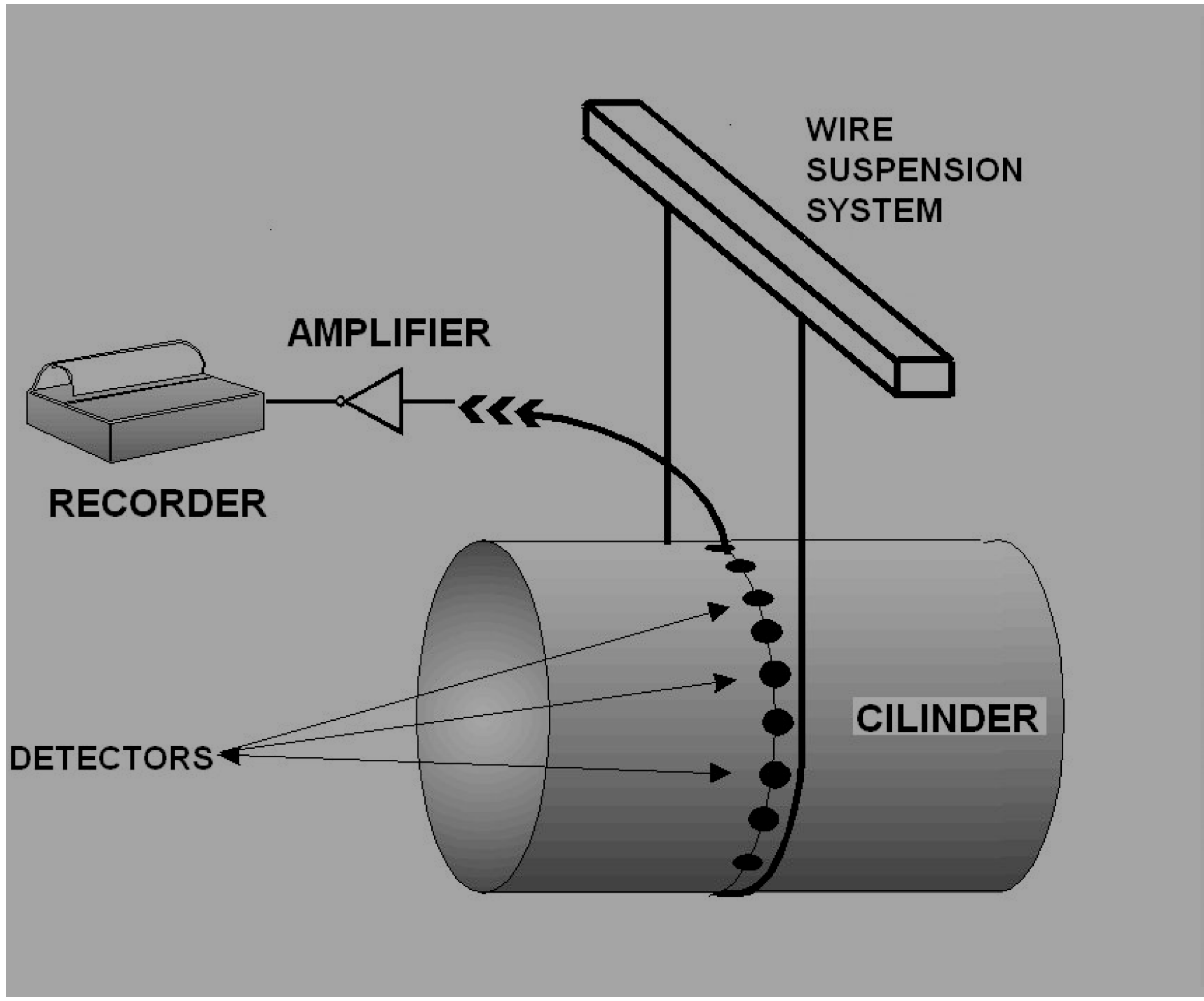


Andrzej Trautman

Trautman provided one of the first mathematically rigorous proofs that gravitational waves are real, radiative solutions of Einstein’s equations that carry energy to infinity.

- Helped settle the theoretical debate
- Convinced the community that gravitational waves are real
- GWs produce measurable physical effects
- Carry nonzero energy and momentum

Leopold Infeld, Jerzy Plebański (PhD supervisor) did not allow to defend his PhD thesis. But Dirac convinced him

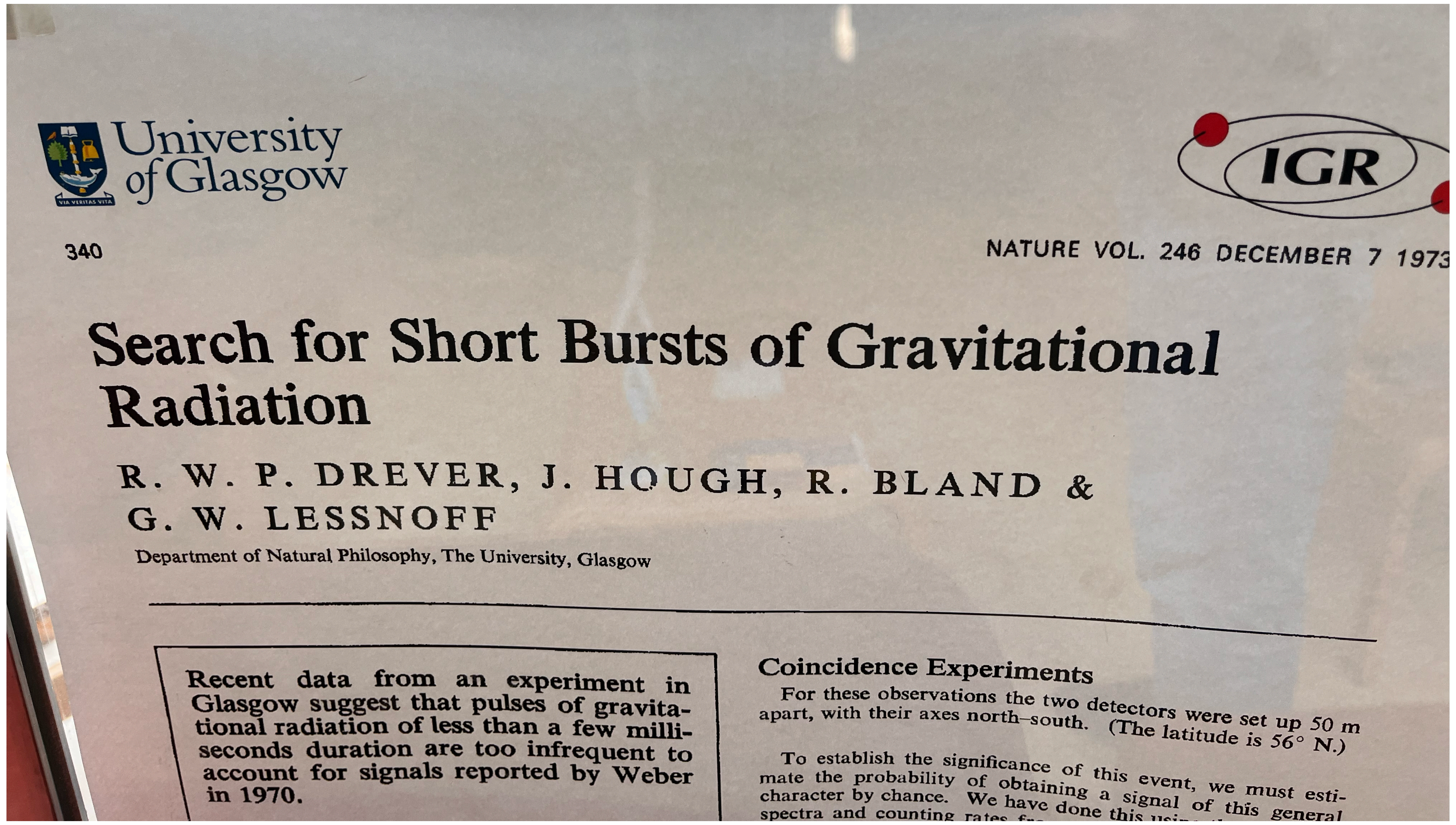


The detector was a **large aluminum cylinder**

- ~2 meters long
- ~1 meter diameter
- Weighed about **1.2 tons**
- It was suspended to isolate it from vibrations.
- Piezoelectric sensors were attached to measure tiny vibrations.

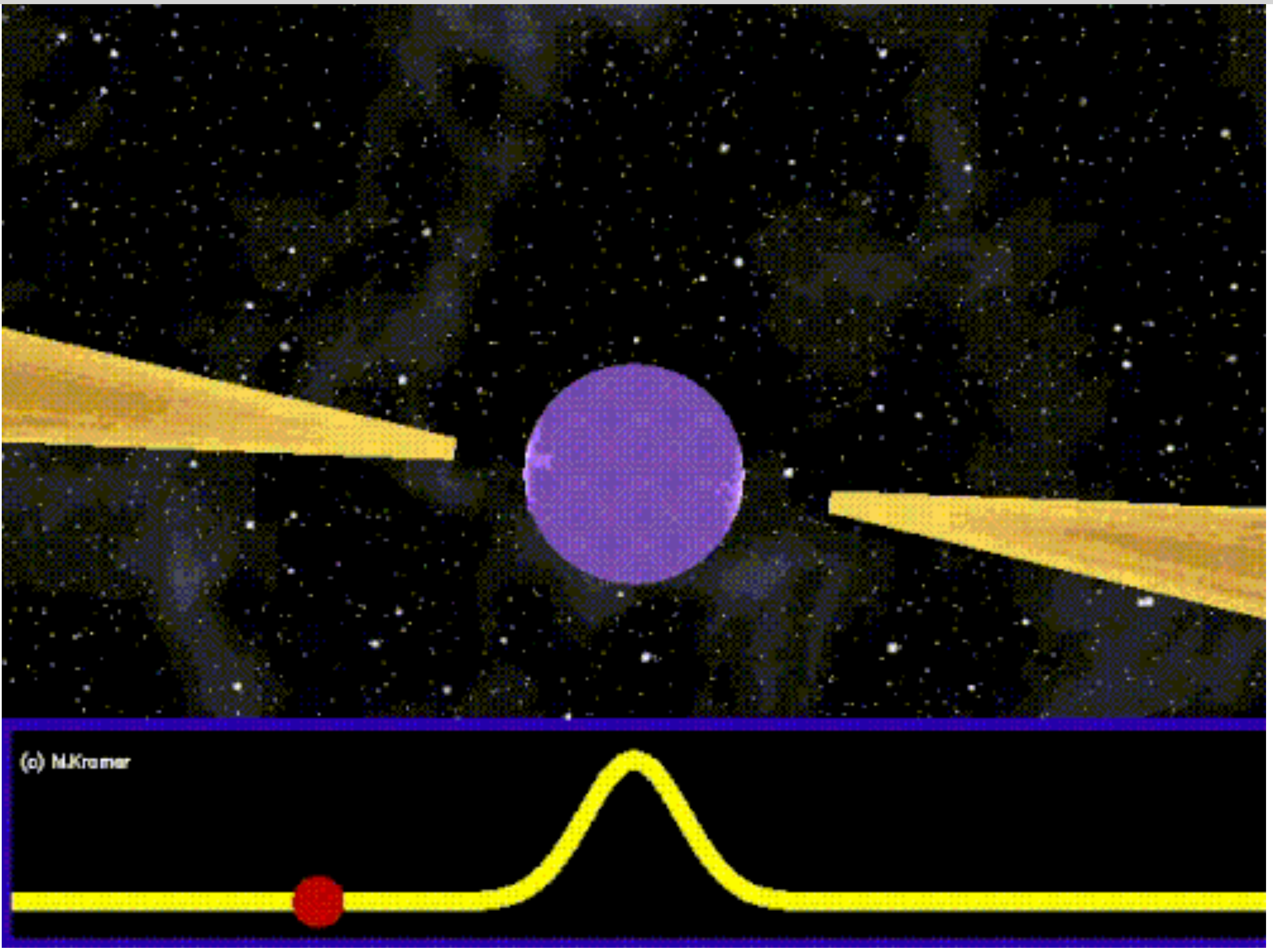
Joseph Weber's **bar detector** was the **first serious experimental attempt to detect gravitational waves**, built in the late 1950s and 1960s.

Weber bar detector (continued)



- **May such detectors were built**
- **But Weber's results could not be reproduced**
- **It created excitements and paved way for LIGO**

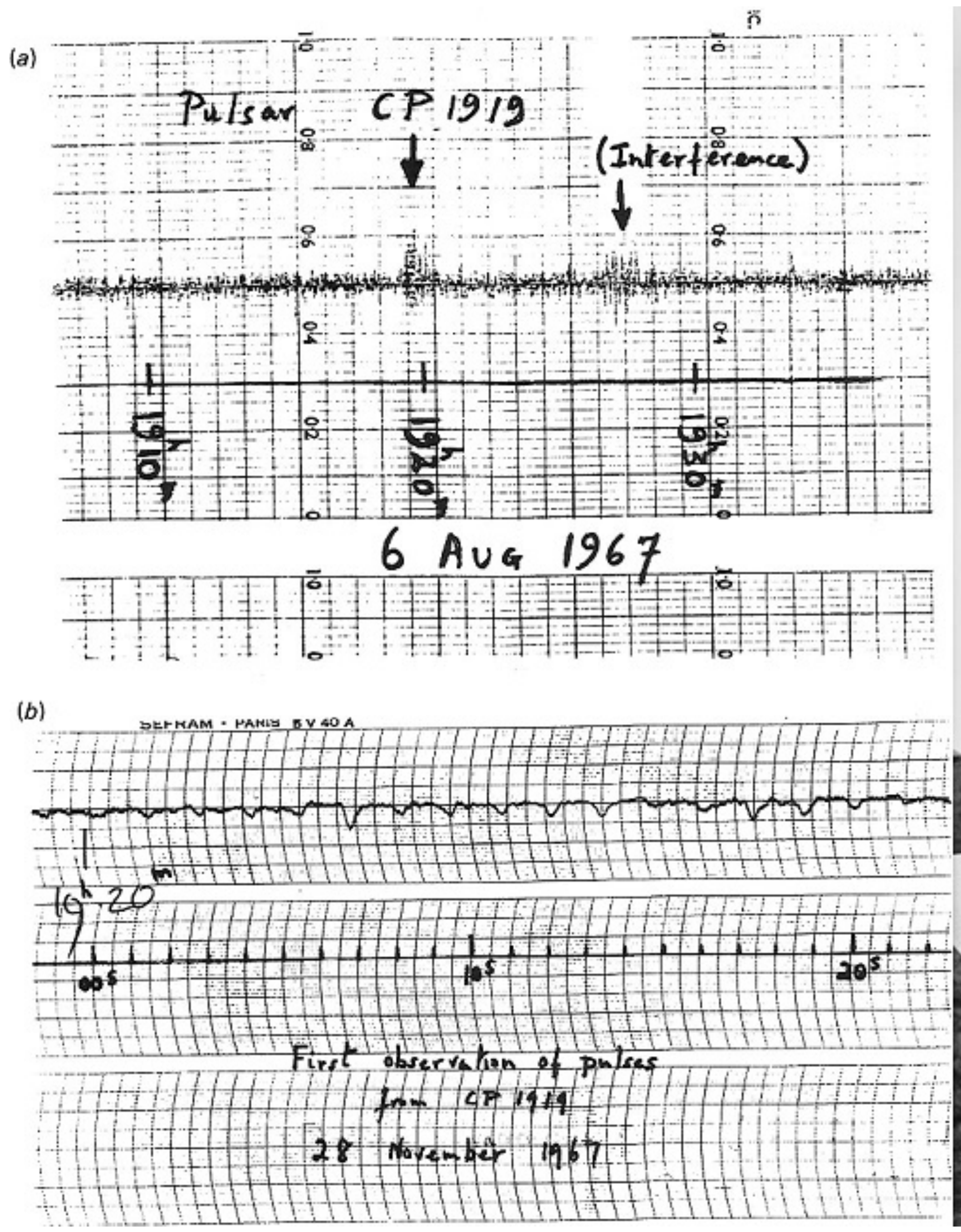
Discovery of Pulsars



Credits: Michael Kramer

PSR B0329+54

Pulsars are Neutron Stars (remnants of SN explosion)



PSR B1919+21

The first pulsar is know as Light Green Men (LGM)

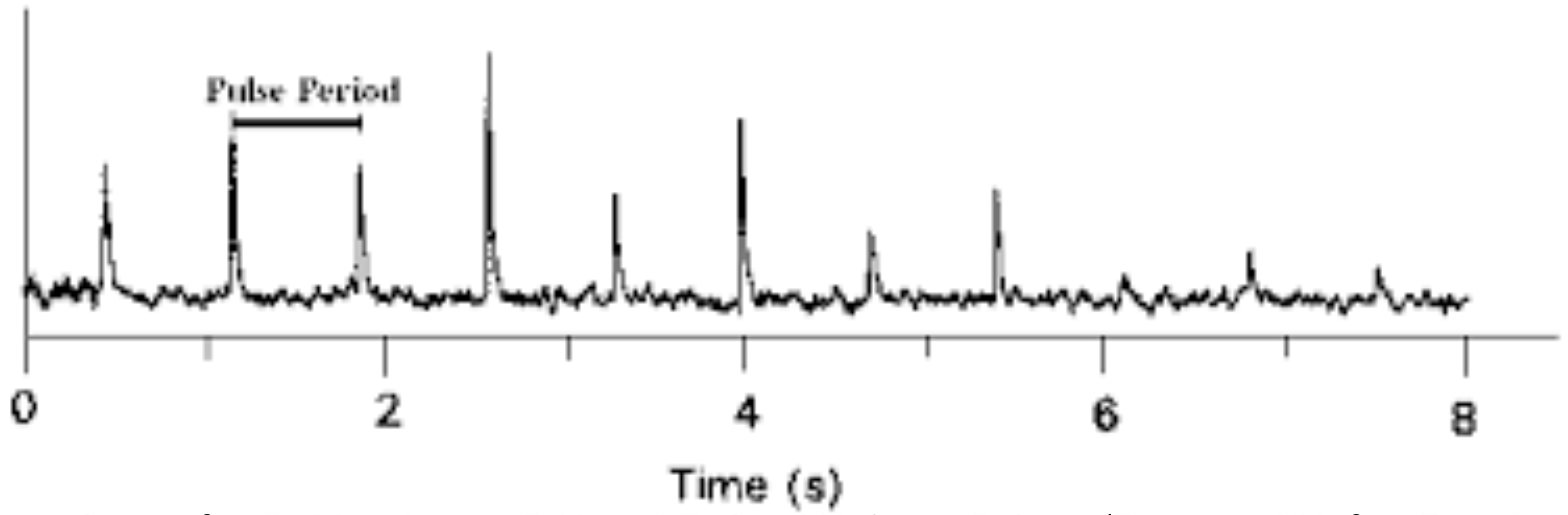
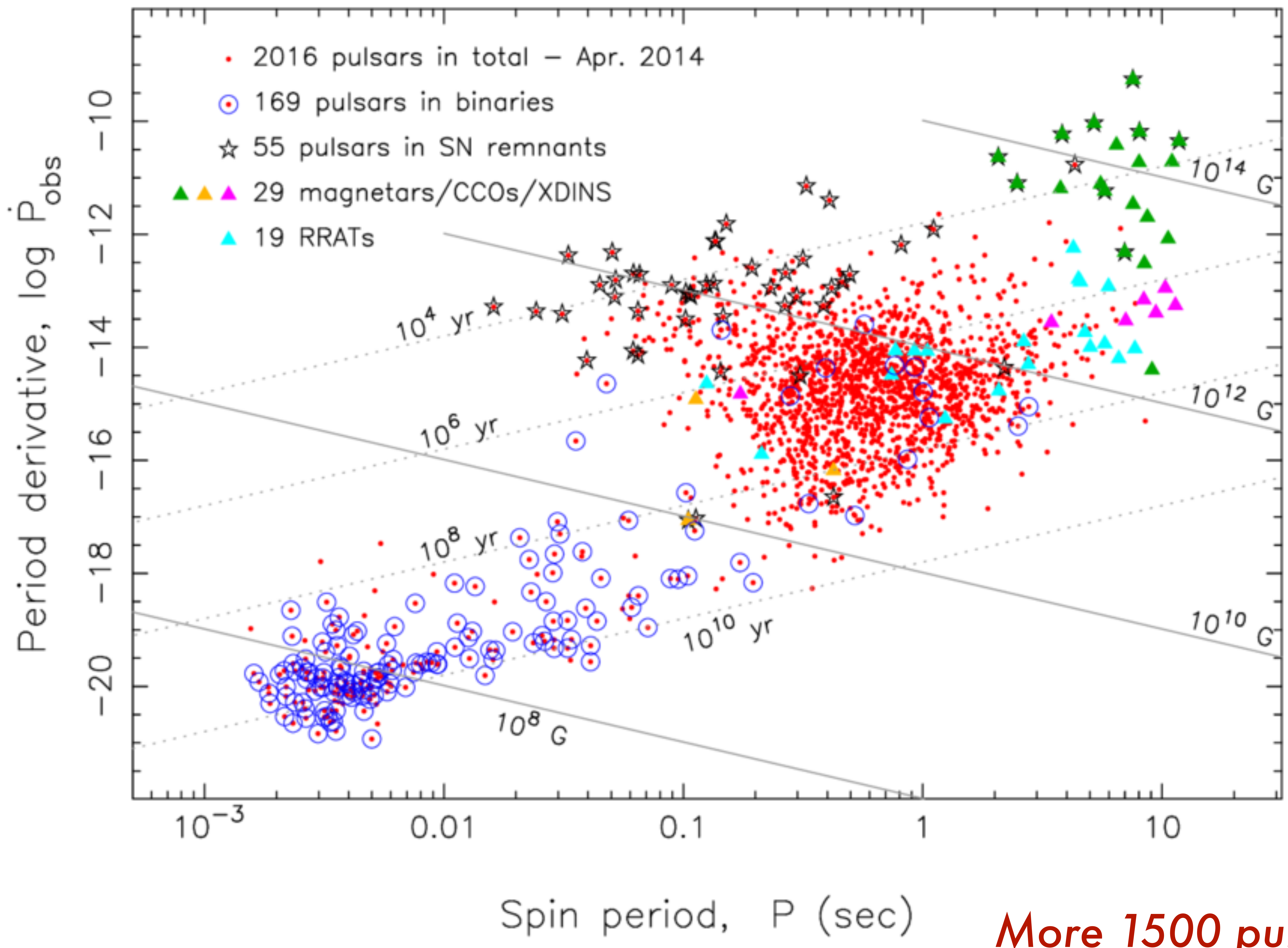


Image Credit: Manchester, R.N. and Taylor, J.H. from , Pulsars (Freeman WH, San Francisco, 1977)

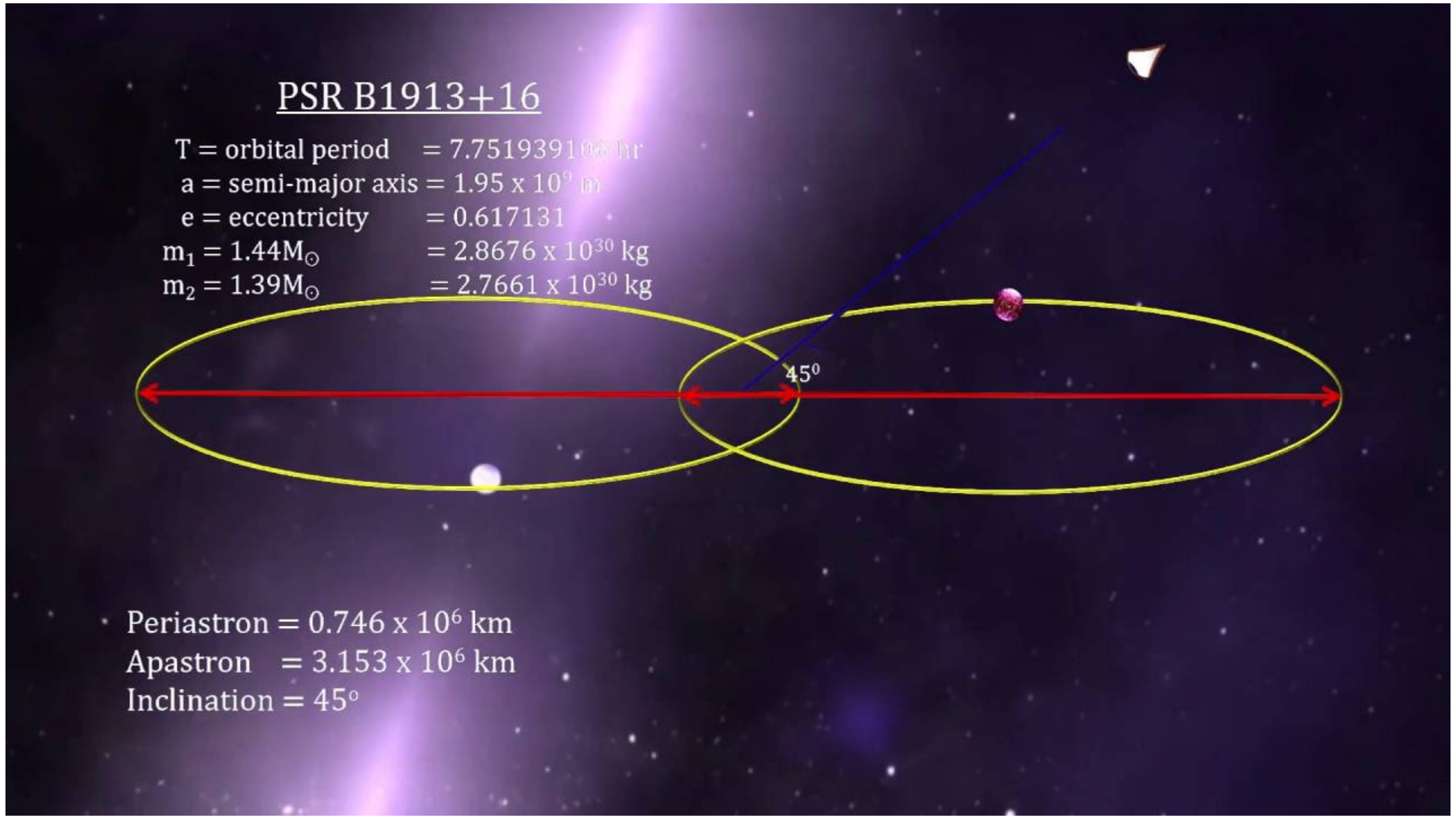
Pulsars



- *They are Neutron Stars*
- *They are highly magnetised*
- *They are born in SN explosions.*
- *They emit beams in Radio, X-rays and γ rays*
- *They are extremely precise clocks*
- *They can even be used as GPS*

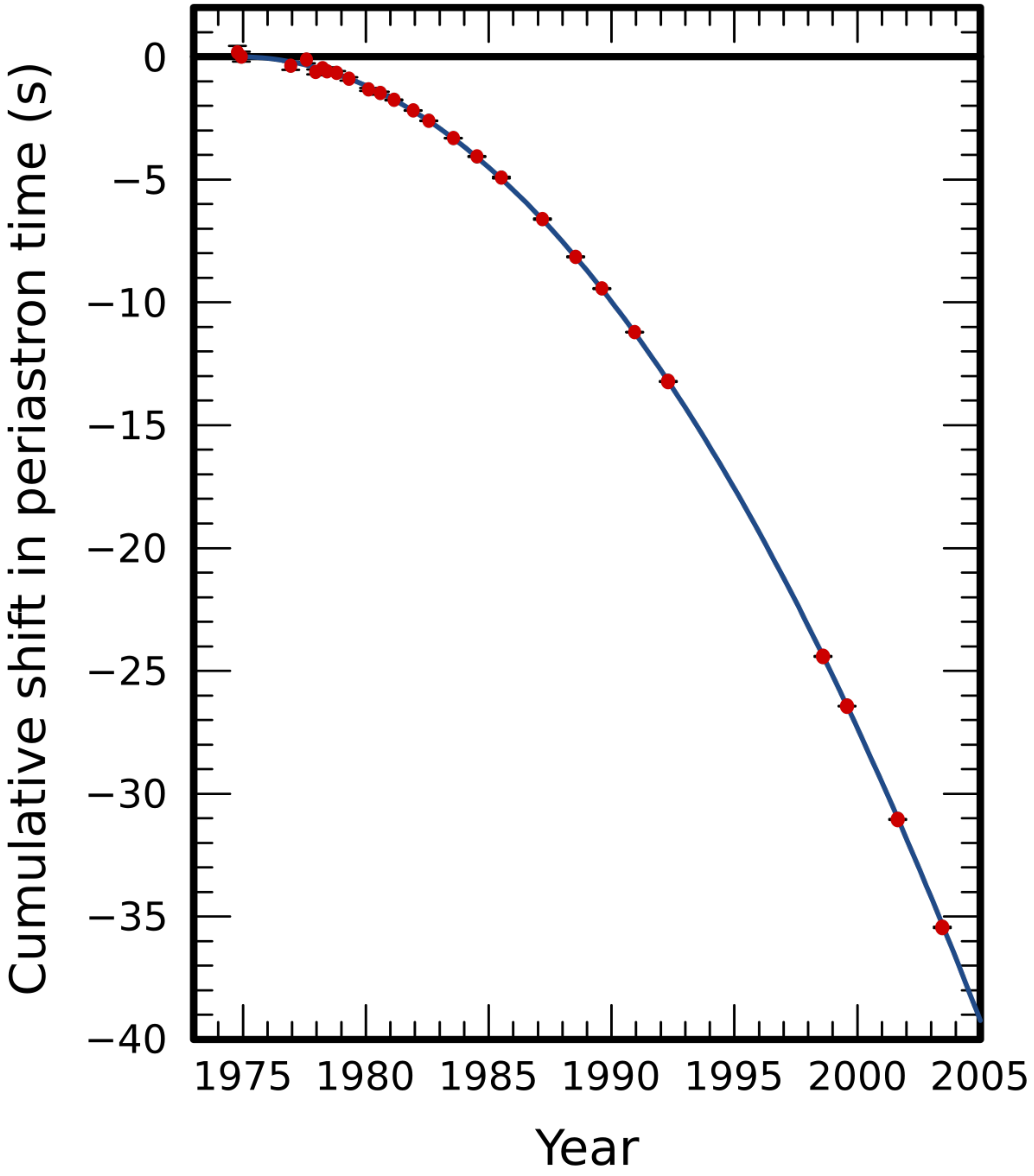
More 1500 pulsars are known now

Hulse Taylor Binary

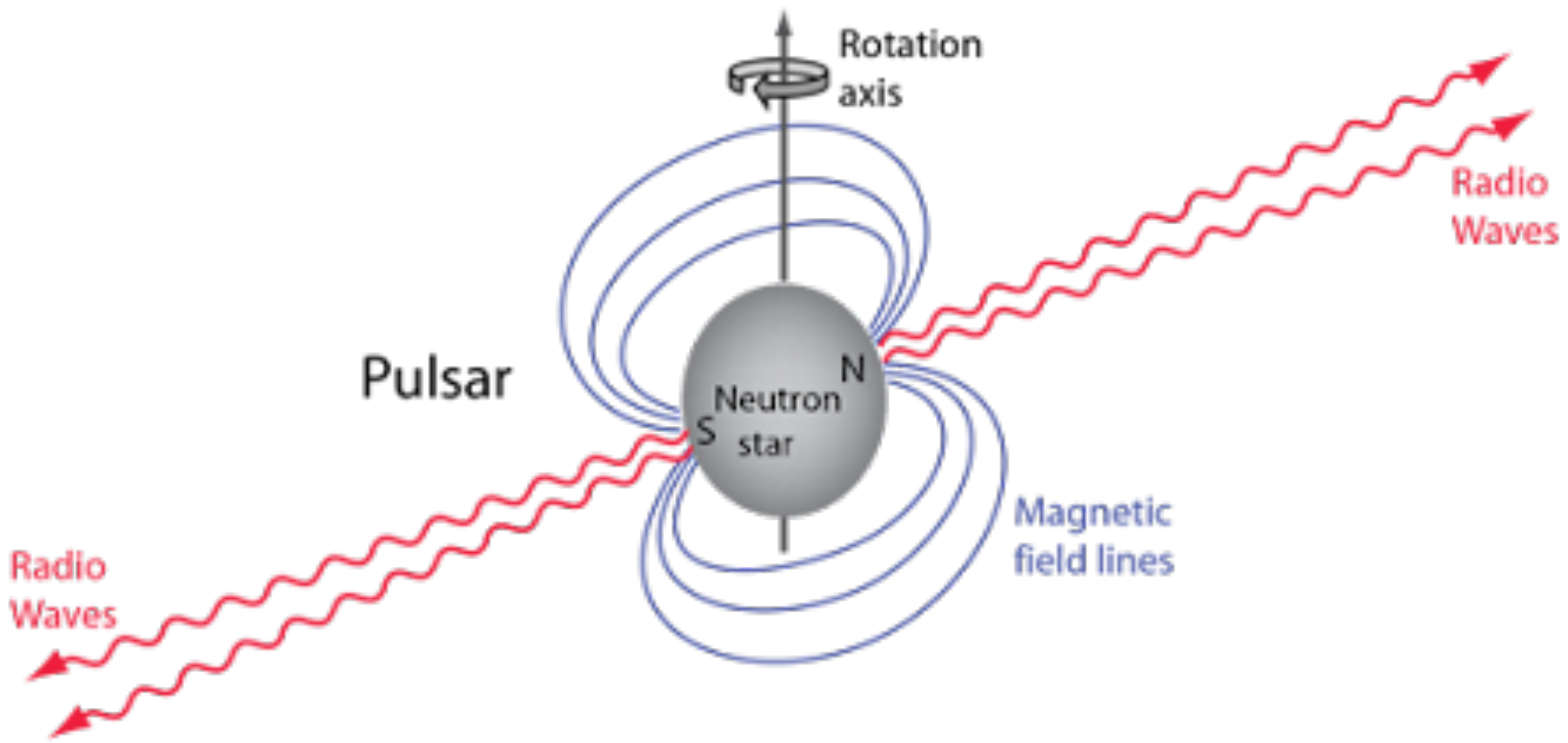


Hulse and Taylor

- First discovered binary pulsar system
- It consisted of a neutron star and a pulsar
- The period of orbital motion is $7.75 hr$
- They found that their period is decreasing.

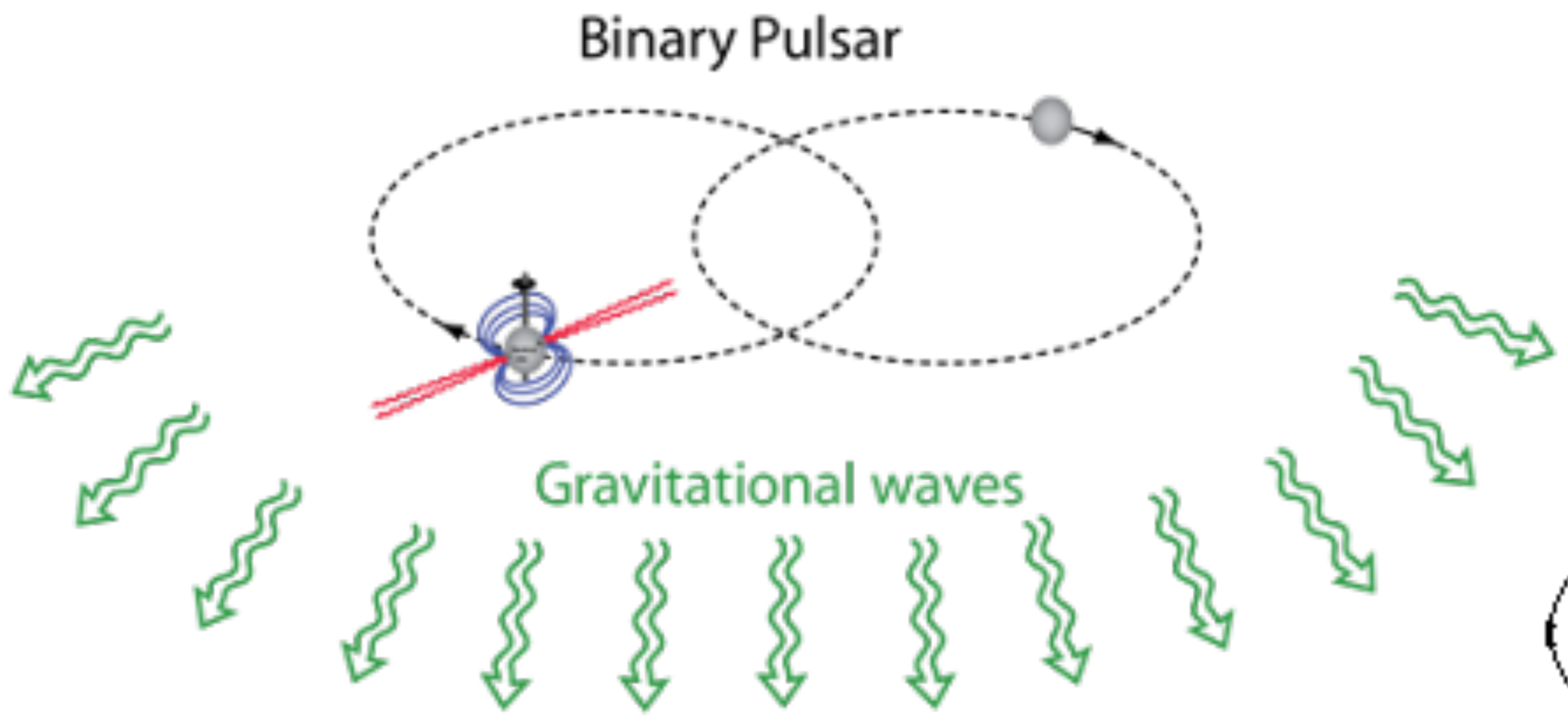


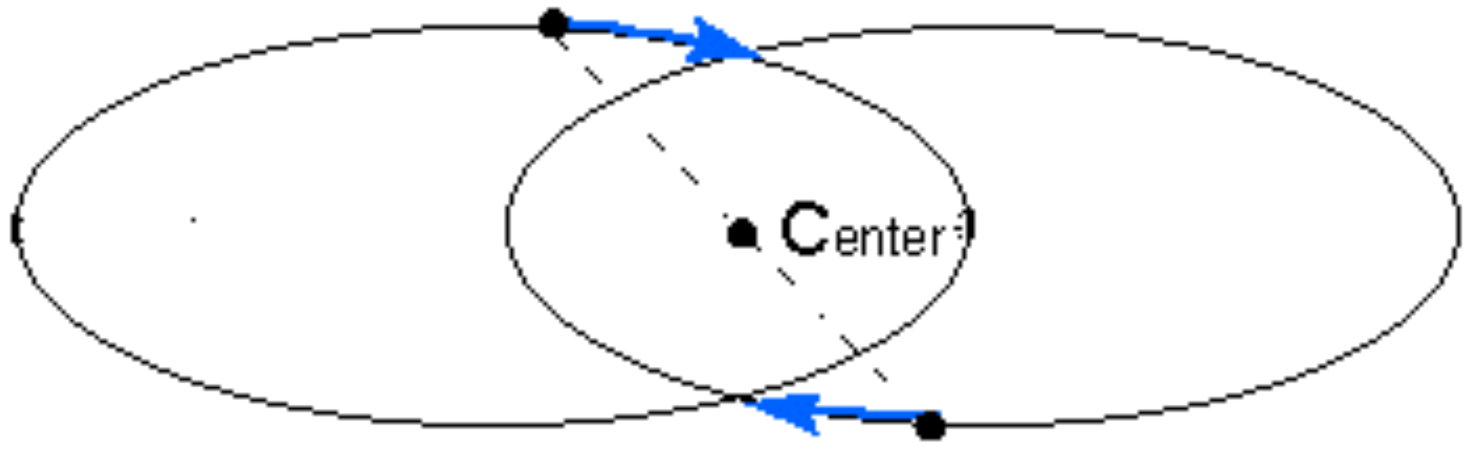
First indirect evidence for gravitational waves…..



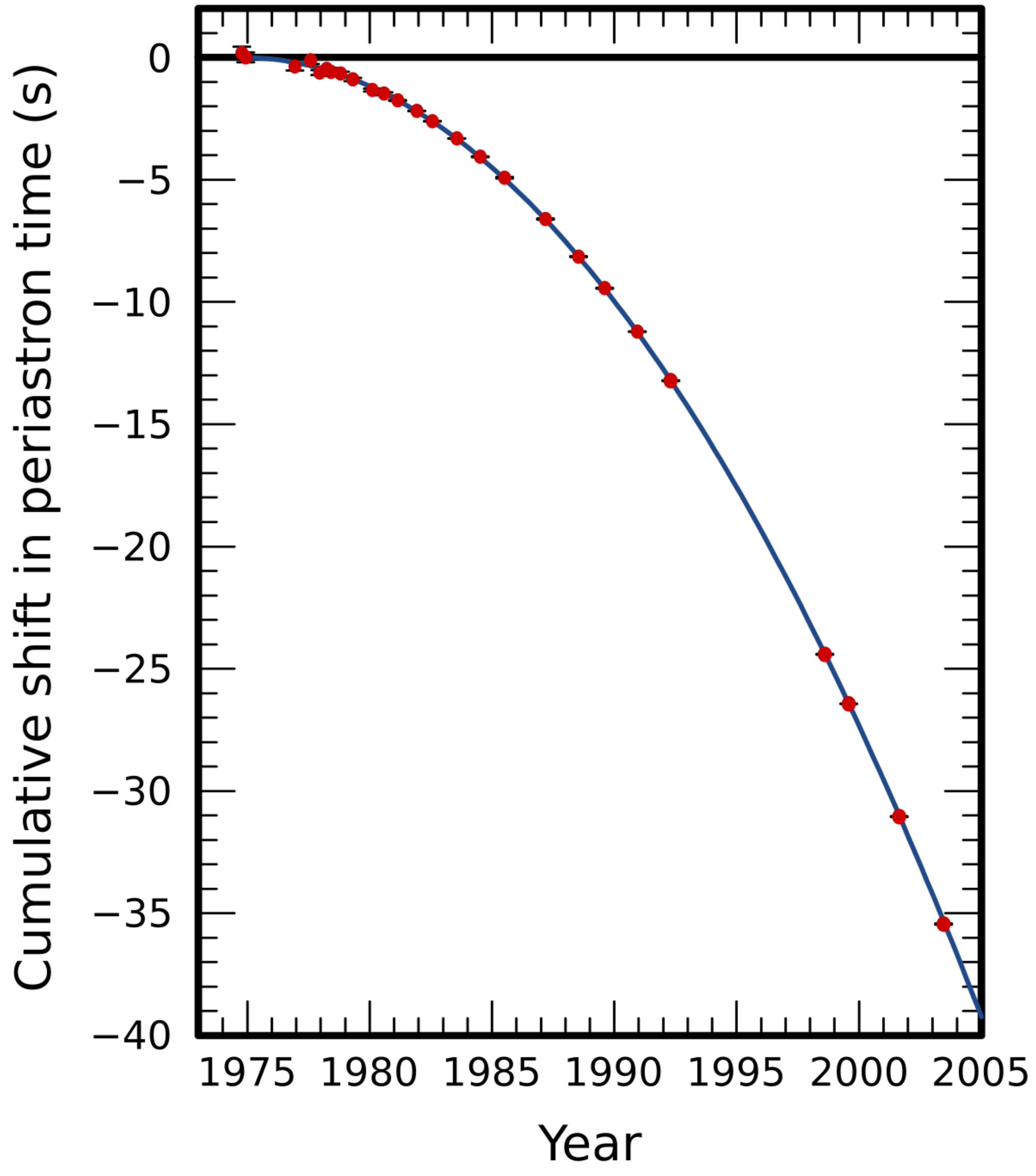
Energy lost in GWs

$$\left\langle \frac{dE}{dt} \right\rangle = -\frac{32}{5}\frac{G^4}{c^5}\frac{m_1^2 m_2^2 (m_1+m_2)}{a^5}\frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}}$$





Orbit of a Binary Pulsar



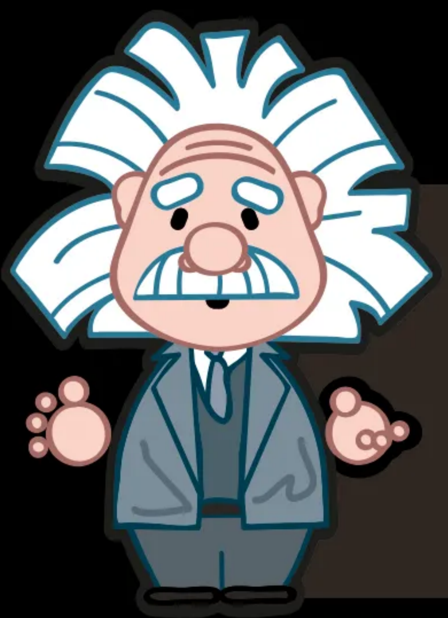
Fun fact:

Solar system generates 5000 W in GW due to orbital motion of the planets.

Infographics: Ben Gilliland/STFC

GRAVITATIONAL WAVES:

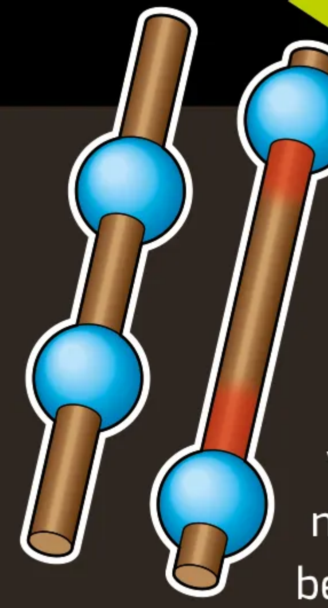
TIMELINE TO DISCOVERY



1916

Albert Einstein first proposes the existence of gravitational waves as part of his general theory of relativity. Many researchers doubt that they exist at all, believing them to be a mathematical quirk .

1957



Physicists Felix Pirani, Hermann Bondi, and Richard Feynman predict that gravitational waves might be detected by a 'sticky bead argument'. The idea being that if a gravitational wave passed through a stick with a bead on it, it would cause the bead to move back and forth and heat up both the bead and stick with the friction generated.

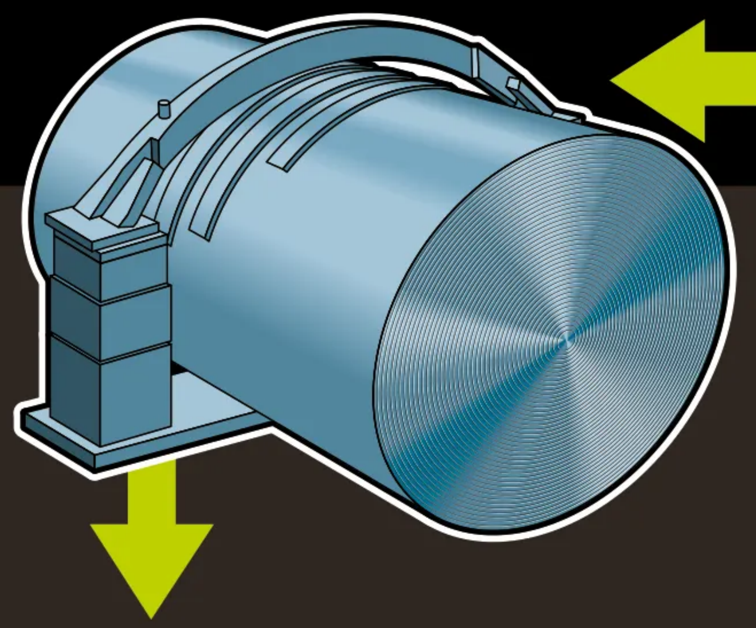
1962

Russian scientists ME Gertsenshtein and VI Pustovoit published a paper proposing 'interferometers' as a way to detect gravitational waves.

1967

Rainer Weiss (one of LIGO's co-founders) proposes a method that would use laser beams to measure the stretching and squashing of space caused by a passing gravitational wave. They were working independently of ME Gertsenshtein and VI Pustovoit, who proposed something similar in 1962.

1969

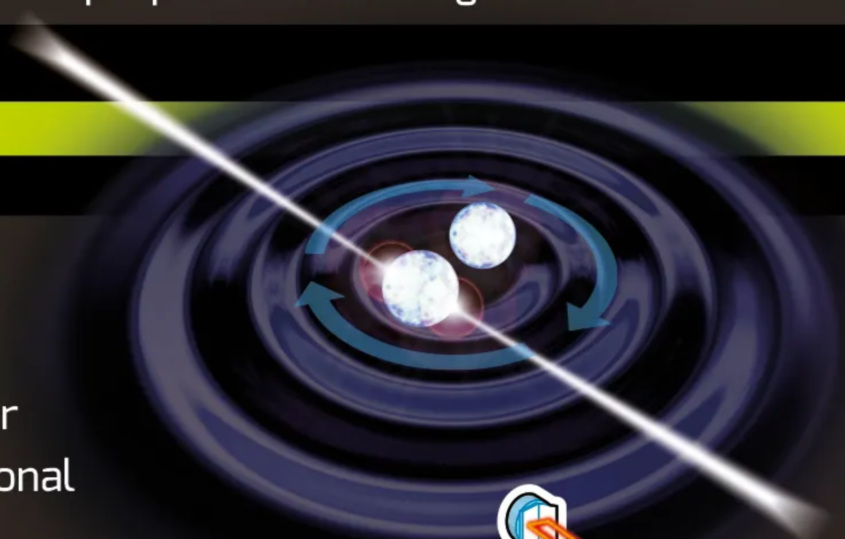


Joseph Weber claims to have detected gravitational waves using a device called a resonant bar detector, but no one can replicate his results.

1974

Rainer Weiss meets physicist Kip Thorne and convinces Thorne that a laser-based instrument would give them the best chance of finding gravitational waves. They start working on the project that would become LIGO.

1978



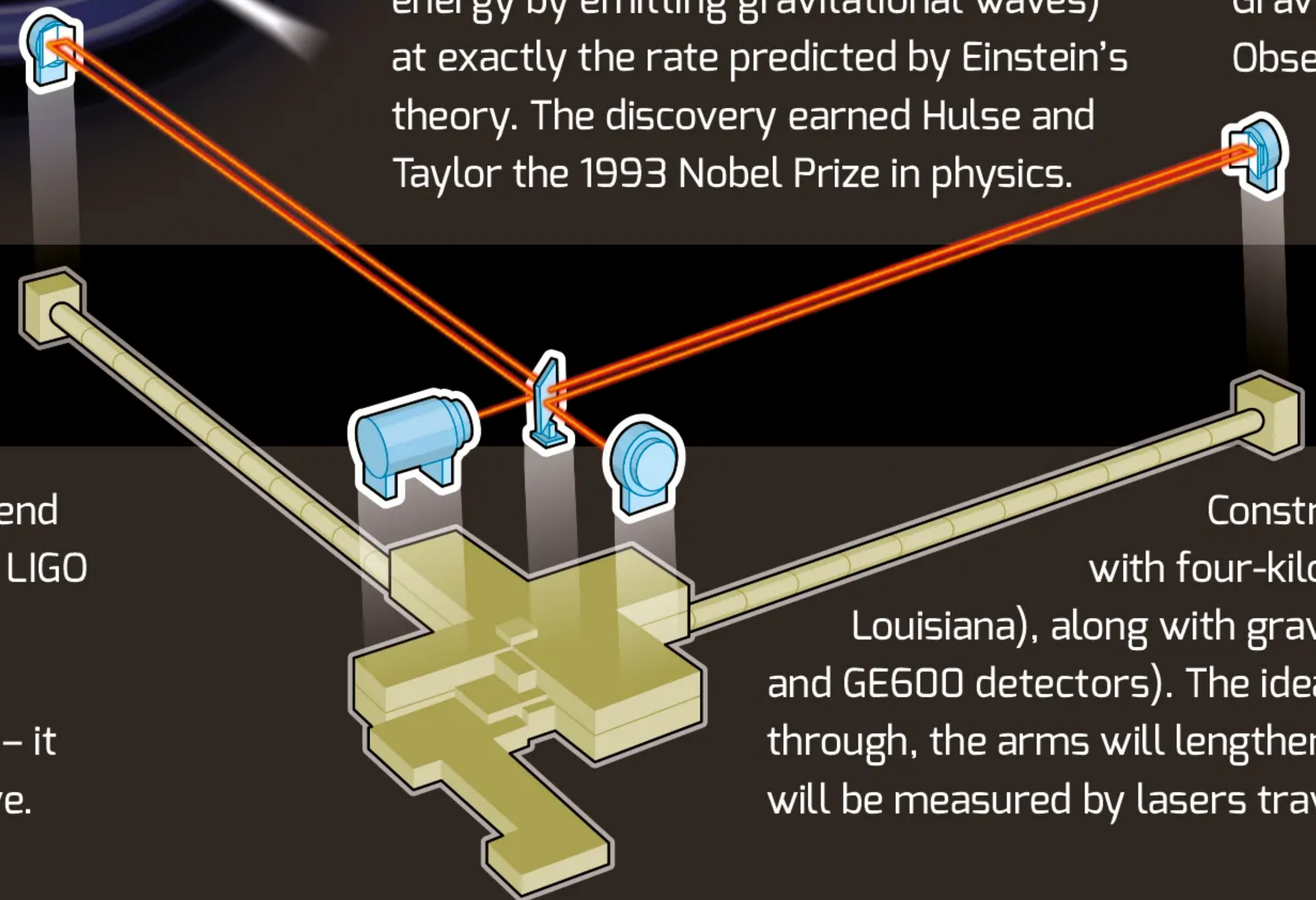
Russell Hulse and Joseph Taylor provide the first experimental evidence for the existence of gravitational waves by observing two neutron stars orbiting each other (a binary system).

They noticed that, rather than remaining in a stable orbit, they were moving closer together (because they were losing energy by emitting gravitational waves) at exactly the rate predicted by Einstein's theory. The discovery earned Hulse and Taylor the 1993 Nobel Prize in physics.

1984

Kip Thorne, Ronald Drever, and Rainer Weiss found the LIGO (Laser Interferometer Gravitational-wave Observatory) Project.

1990s



Construction begins on LIGO – two L-shaped detectors with four-kilometre-long arms (one in Washington and one in Louisiana), along with gravitational wave detectors in Europe (the VIRGO and GE600 detectors). The idea being that, when a gravitational wave passes through, the arms will lengthen and shorten by a fraction – the precise shift will be measured by lasers travelling along the arms.

2001

LIGO begins operations. At the end of its run in 2010, as expected, LIGO had found no evidence of gravitational waves. LIGO had proved the technology worked – it just needed to be more sensitive.

2007

The VIRGO laser interferometer based in Italy, designed to detect gravitational waves begins operation.

NEXT PAGE

Infographics: Ben Gilliland/STFC

2010

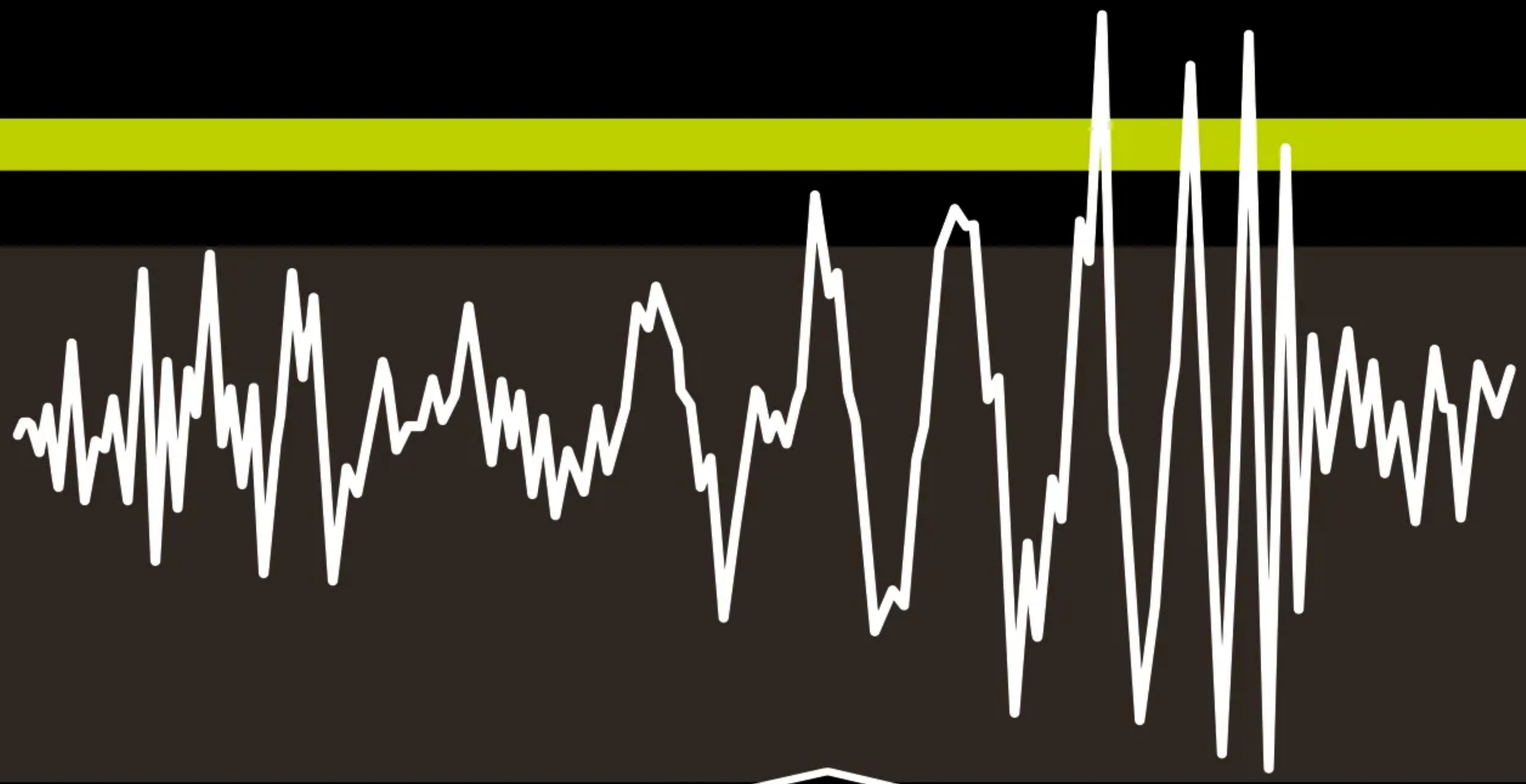
LIGO begins upgrades to become Advanced LIGO. This new facility will be ten times more sensitive than the old one, and includes technology from the UK-German GEO600 detector and from Australia.

2011

VIRGO upgrade commences that will eventually improve the sensitivity by a factor of ten.

2015

In September, Advanced LIGO begins its first engineering and test run. Although only operating at less than half its final sensitivity, it detects its first gravitational wave event on September 14.



2015

LISA Pathfinder is launched – a test bed mission for the first space-based gravitational wave detector. LISA Pathfinder will test technology for the planned LISA (Laser Interferometer Space Antenna) mission.

2015

On December 26, Advanced LIGO makes a second observation of gravitational waves. This time from two black holes, 14 and 8 times the mass of the Sun, merging into a more massive spinning single black hole 21 times the mass of the Sun.

In the 'Boxing Day' binary at least one of the black holes was spinning

2016

In February, the LIGO Scientific Collaboration announce that they had indeed detected gravitational waves on September 14, 2015. The waves had been created by two black holes, spiralling in toward each other and merging into a single black hole.

2016

On June 15, the Boxing Day event is announced. This new observation indicates that there is a rich population of binary black holes in the Universe, whose properties are gradually starting to emerge. Gravitational-wave astronomy is no longer a field of single detections, but of regular observations. This discovery transforms the LIGO detector into a true astronomical observatory.

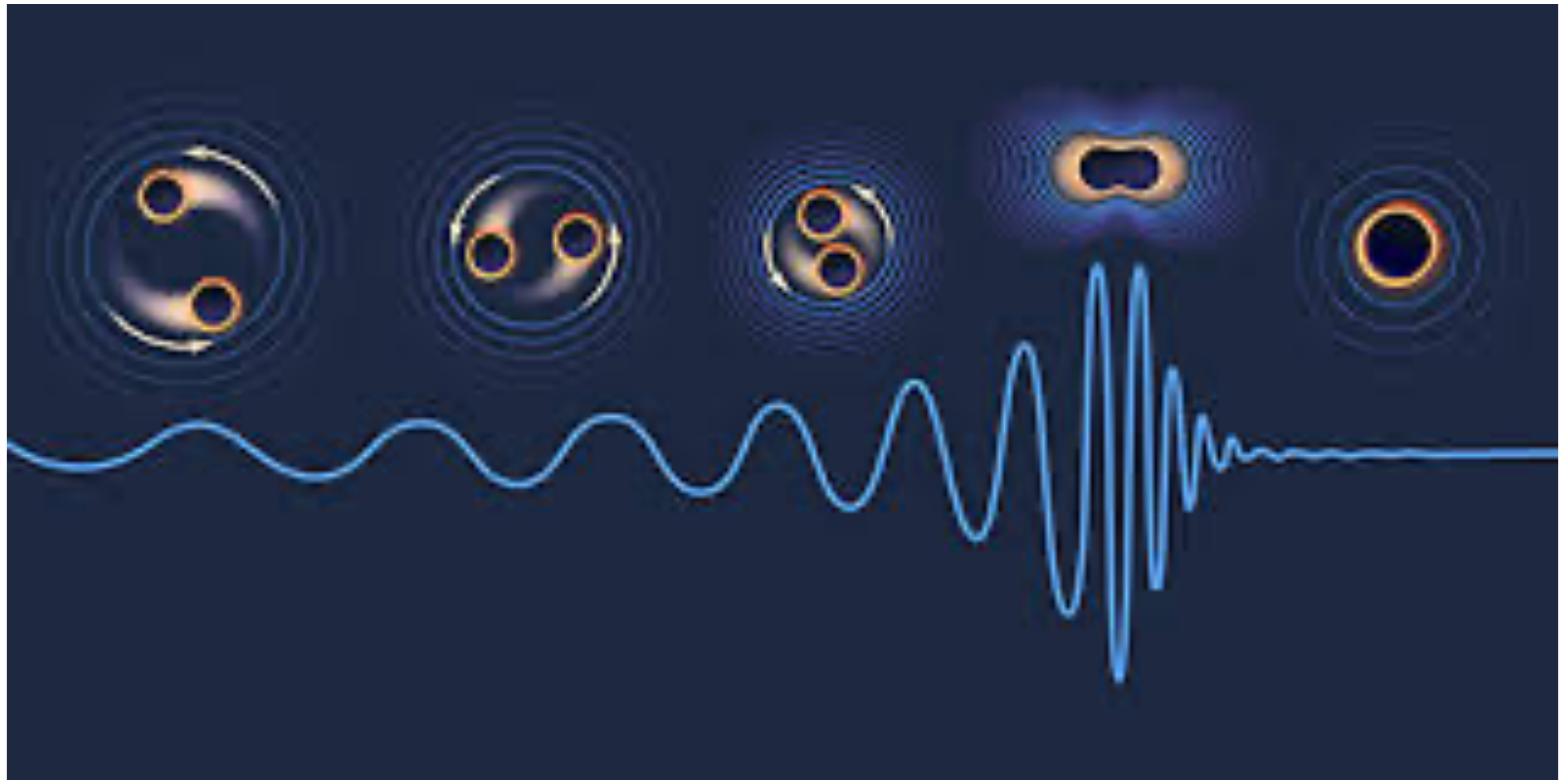
THE FUTURE

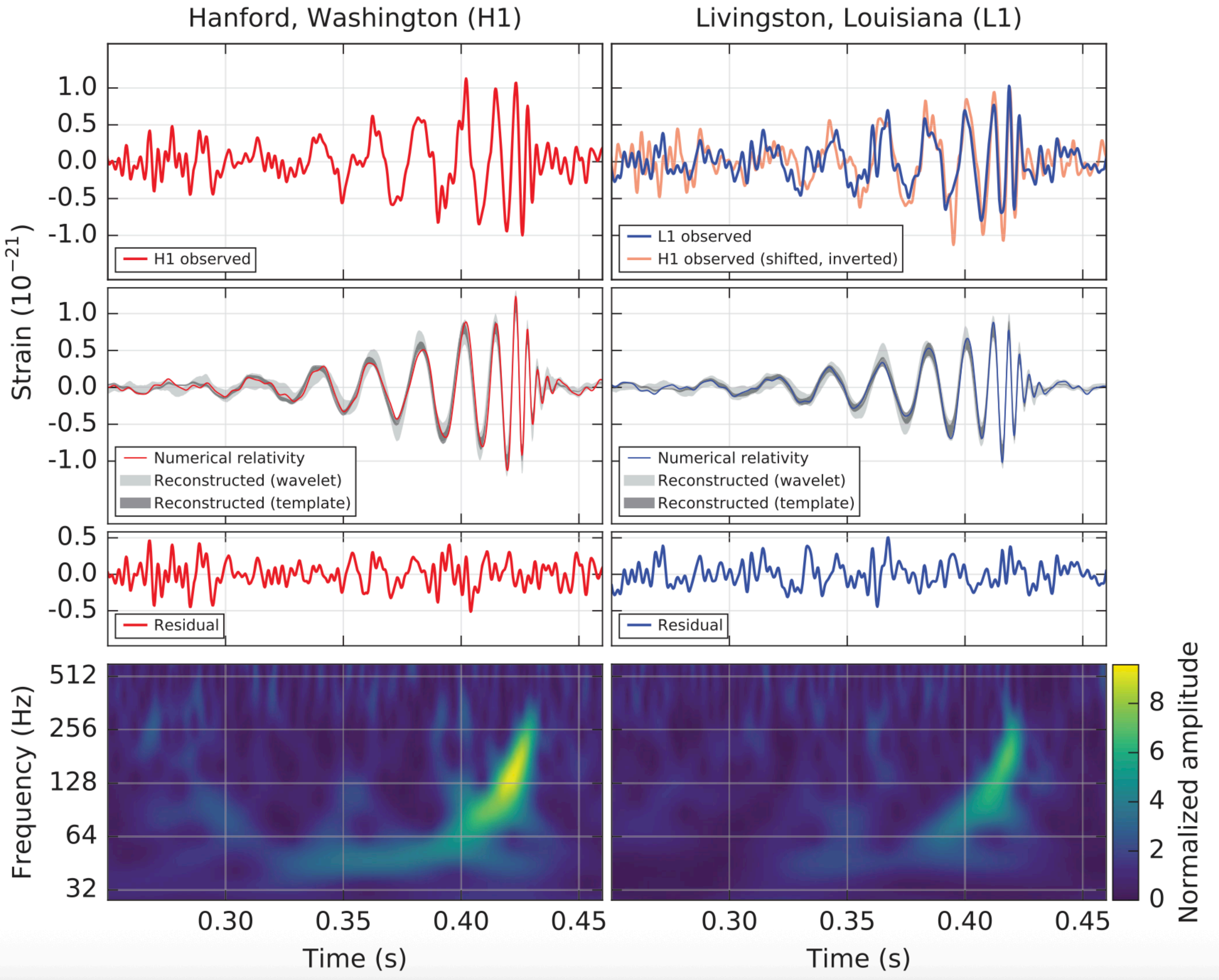
Following Advanced LIGO going on-line in 2017, a third LIGO detector is due for completion in India in 2024. In the 2030s more sensitive ground-based detectors are foreseen, and LISA will be launched. LISA will extend our capabilities to 'listen' to new kinds of dark phenomena in the Universe.

Listening to the black holes for the first time - GW150914

at 11:50:45 am CET on 14 September 2015

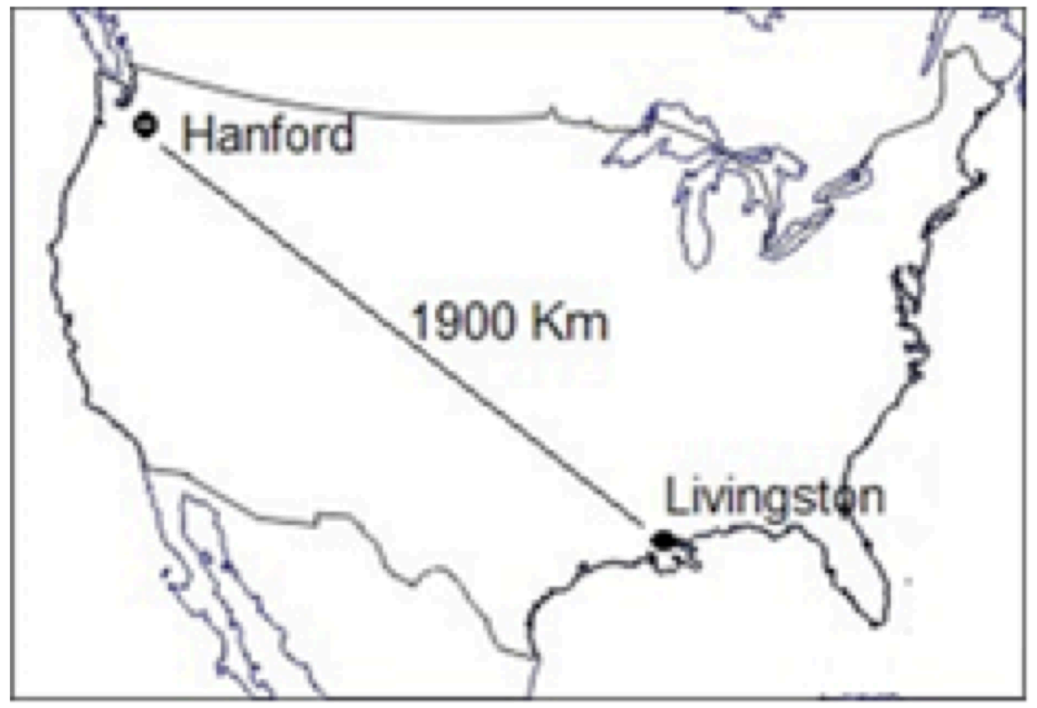
Phys. Rev. Lett. 116, 061102





This marked the birth of gravitational wave astronomy

Most important discovery in this century



GW150914

YY MM DD

60 years of experimental efforts and 100 years since its prediction we detected it.

GW170817

Binary neutron star merger

A LIGO / Virgo gravitational wave detection with associated electromagnetic events observed by over 70 observatories.

H L V

Distance
130 million light years

Discovered
17 August 2017

Type
Neutron star merger

12:41:04 UTC
A gravitational wave from a binary neutron star merger is detected.

gravitational wave signal
Two neutron stars, each the size of a city but with at least the mass of the sun, collided with each other.

gamma ray burst
A short gamma ray burst is an intense beam of gamma ray radiation which is produced just after the merger.

+ 2 seconds
A gamma ray burst is detected.

kilonova
Decaying neutron-rich material creates a glowing kilonova, producing heavy metals like gold and platinum.

+10 hours 52 minutes
A new bright source of optical light is detected in a galaxy called NGC 4993, in the constellation of Hydra.

+11 hours 36 minutes
Infrared emission observed.

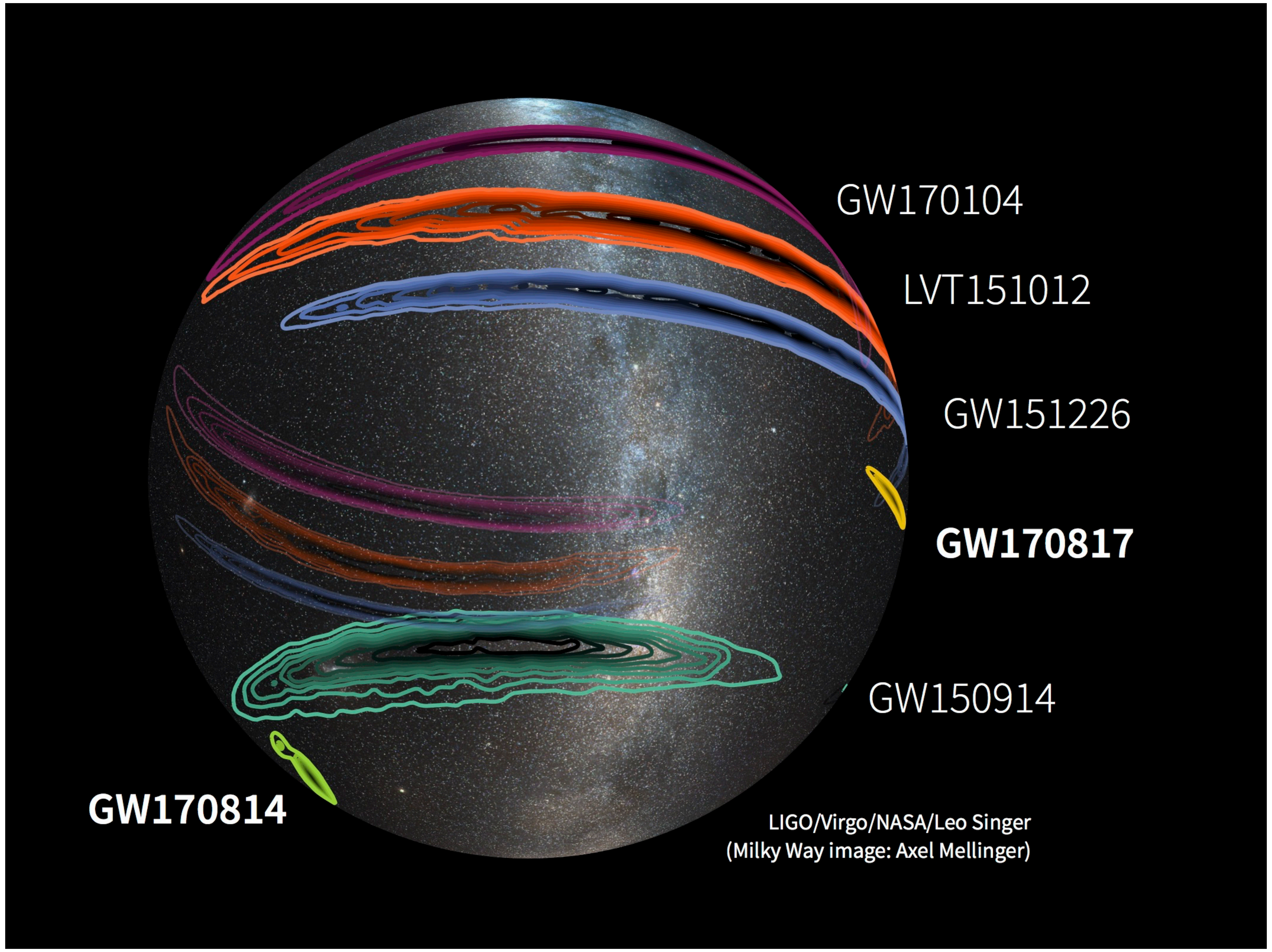
+15 hours
Bright ultraviolet emission detected.

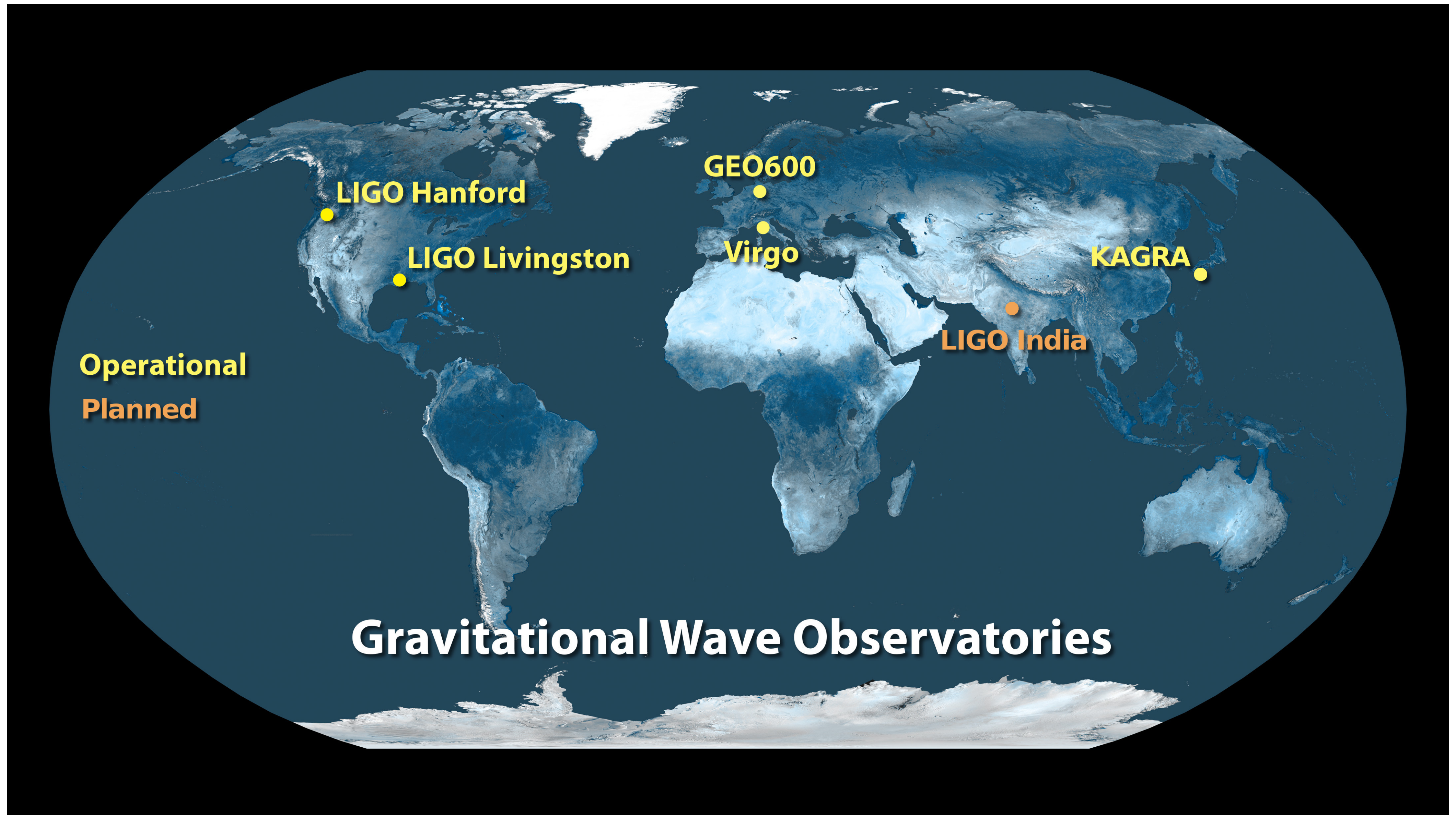
+9 days
X-ray emission detected.

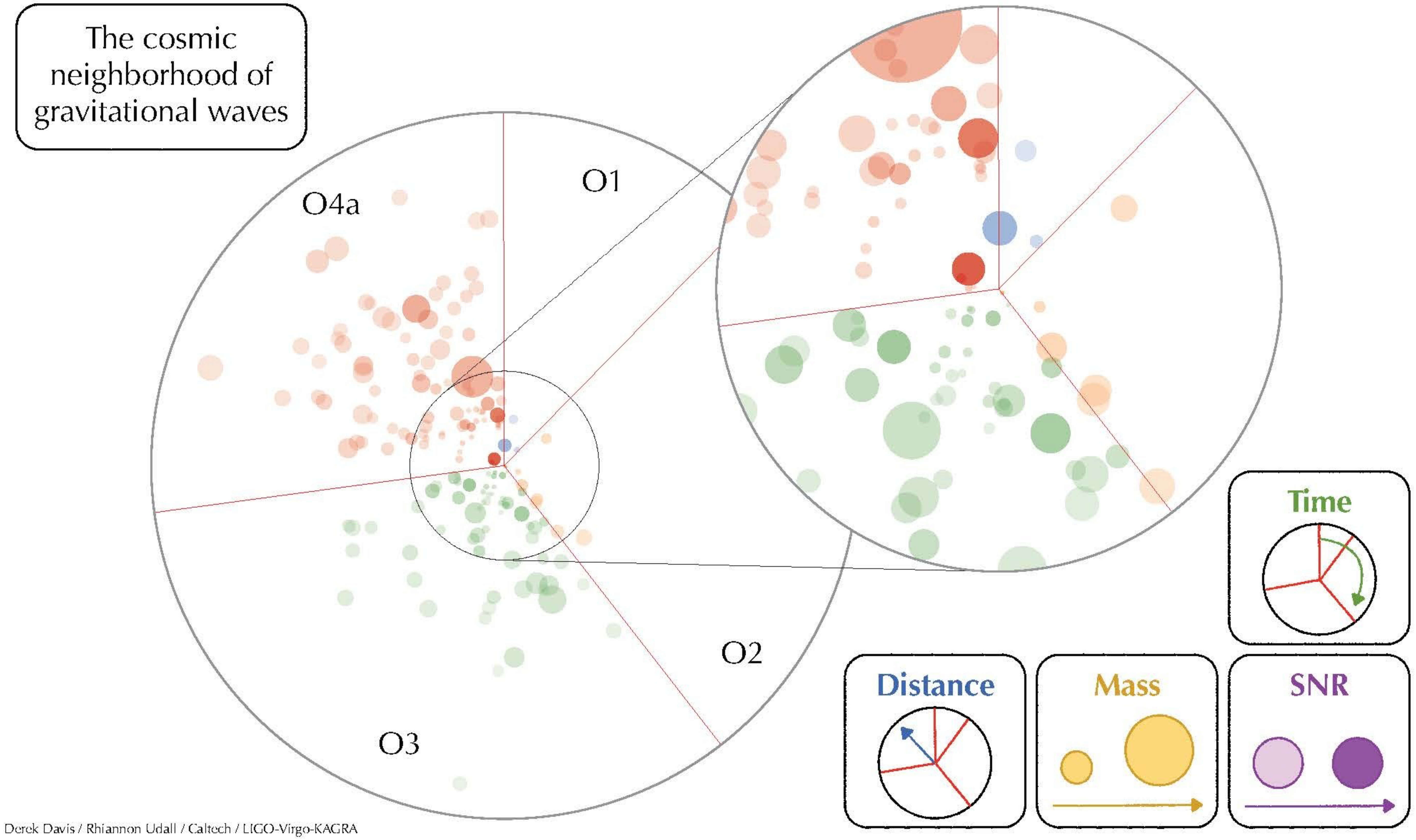
radio remnant
As material moves away from the merger it produces a shockwave in the interstellar medium - the tenuous material between stars. This produces emission which can last for years.

+16 days
Radio emission detected.

The electromagnetic counter helped in better sky localization





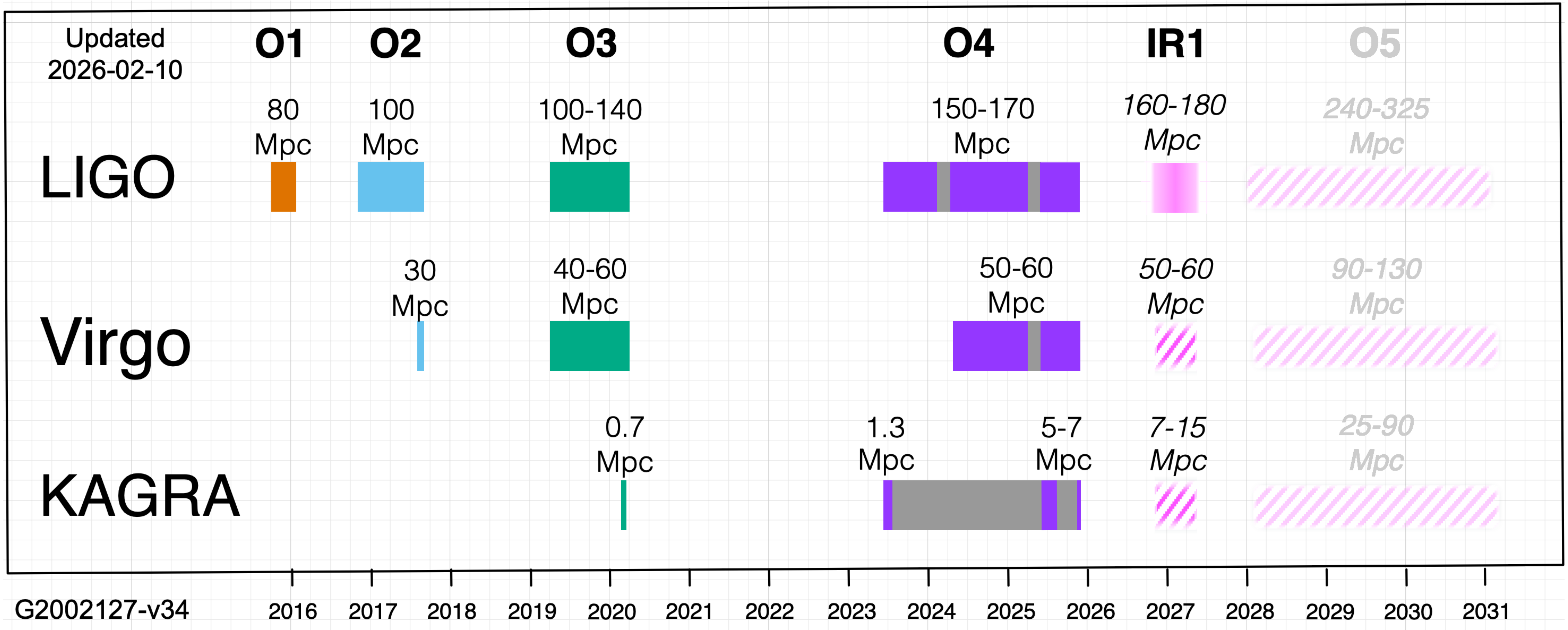


- Four functional detectors, LIGO India under construction
- More than 300 GW detections.
- These include **BBH, NSBH, BNS**
- One multi-messenger event GW170817
- Evidence of Stochastic GW background (PTA)

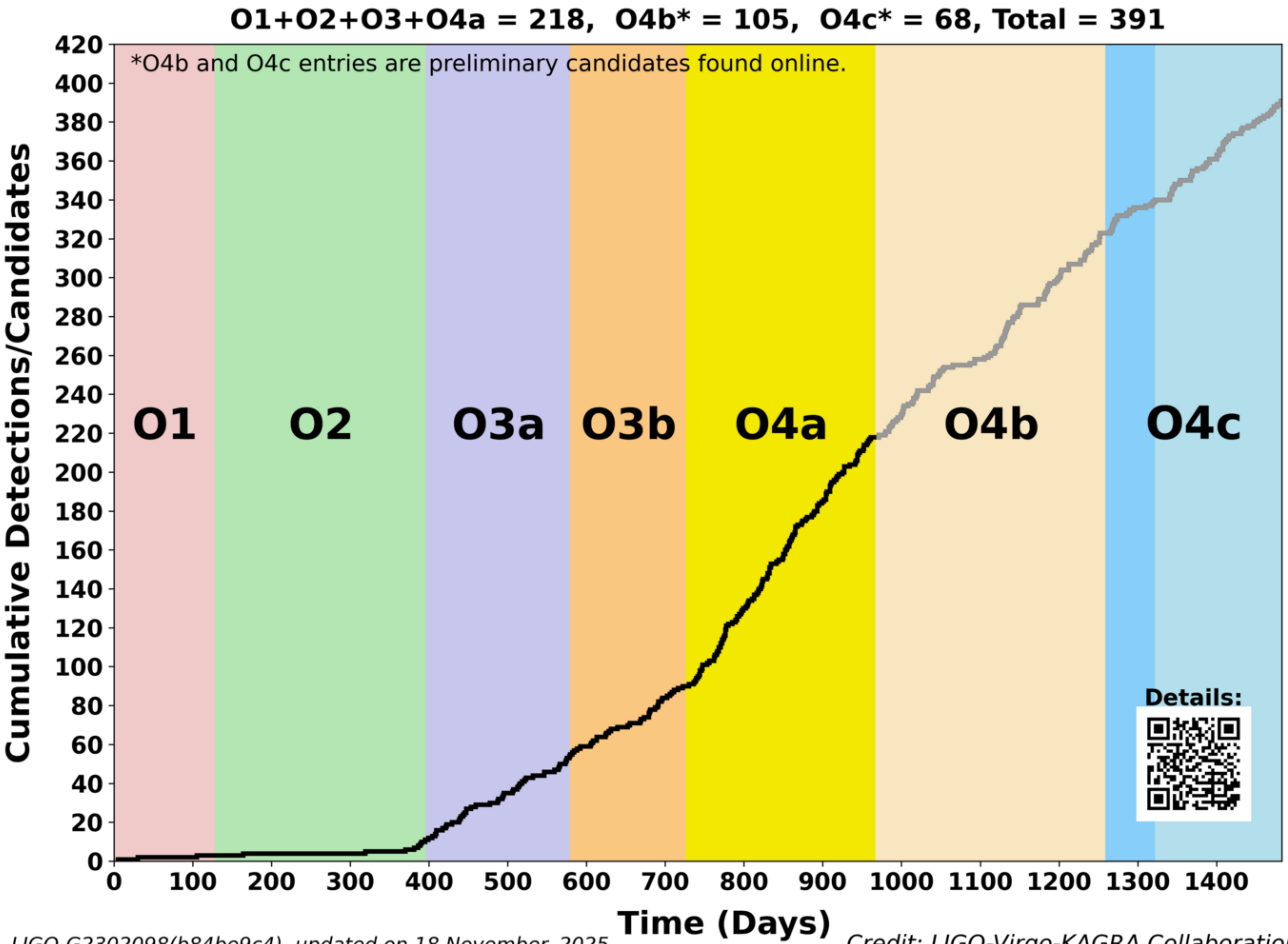
More about GW sources and detectors will be discussed in future lectures

LIGO Virgo KAGRA (LVK) Observing runs

The sensitivity of different detectors during different observing runs are shown



LIGO Virgo KAGRA (LVK) Observing runs



LIGO-G2302098(b84be9c4), updated on 18 November, 2025

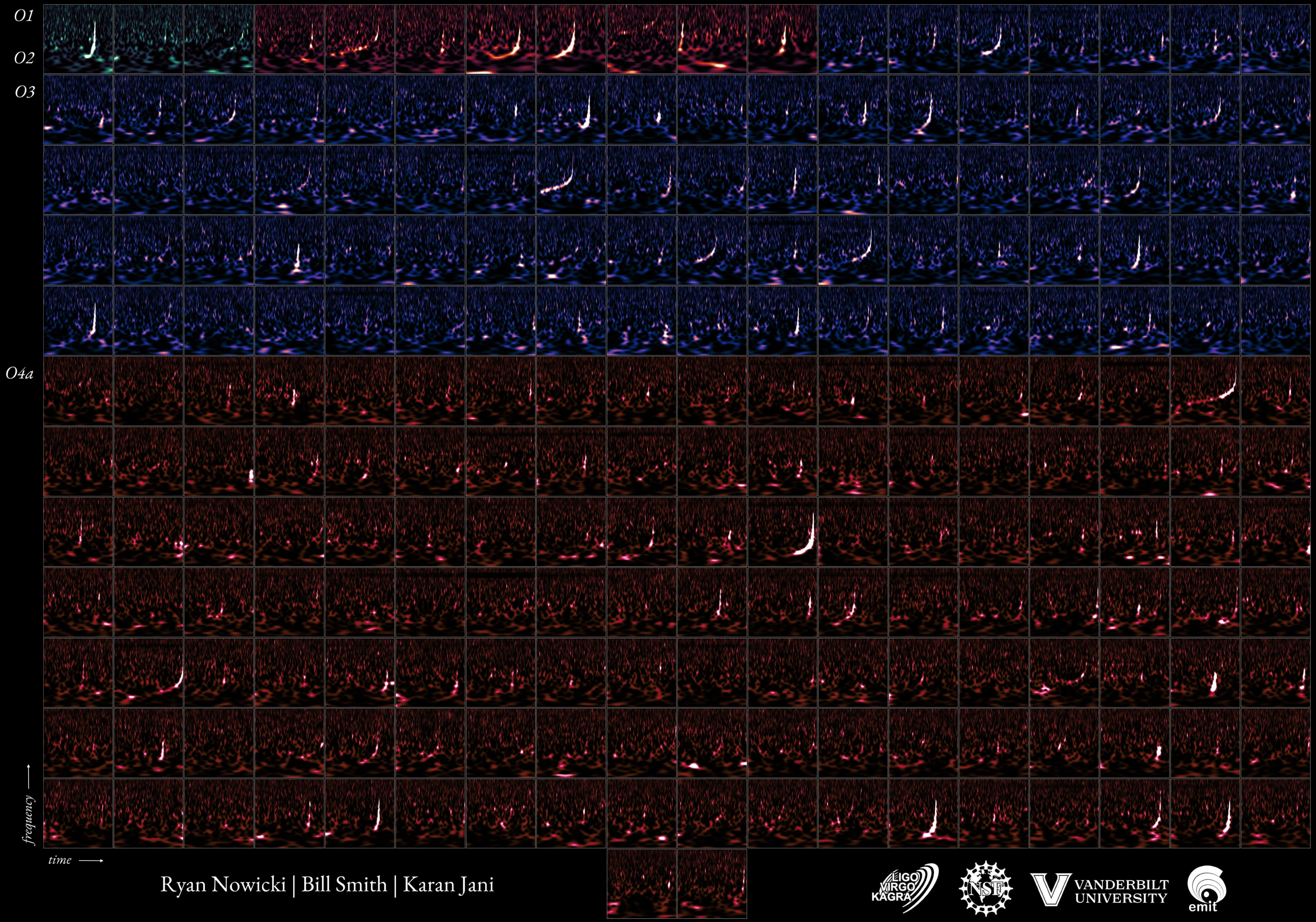
Credit: LIGO-Virgo-KAGRA Collaboration

Masses in the Stellar Graveyard

LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

Gravitational-Wave Transient Catalog

10 Years of Detections (2015-2024) of Compact Binary Coalescences with Black Holes and Neutron Stars

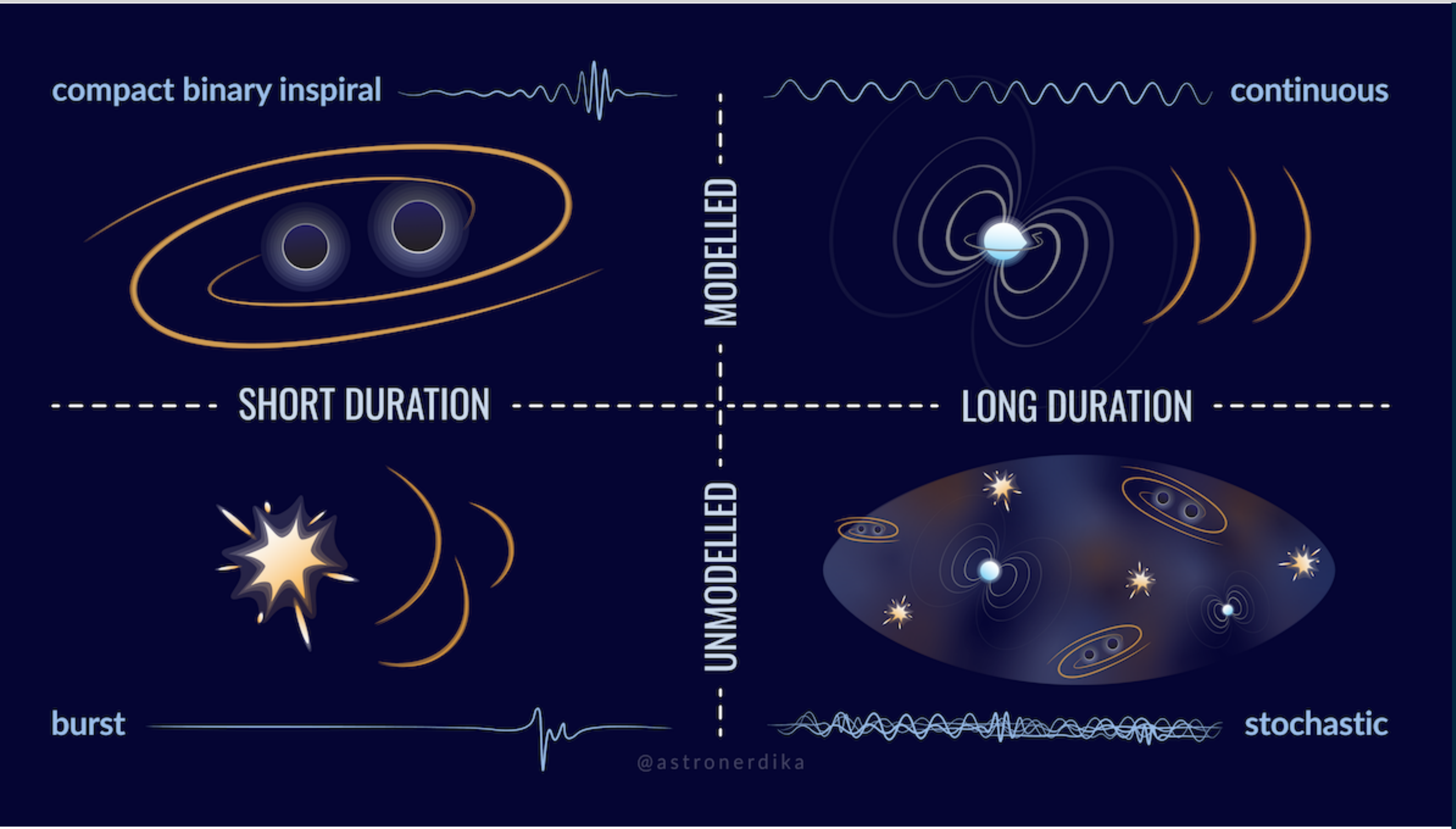


Ryan Nowicki | Bill Smith | Karan Jani

LIGO VIRGO KAGRA · NSF · VANDERBILT UNIVERSITY · emit

Sources of Gravitational Waves

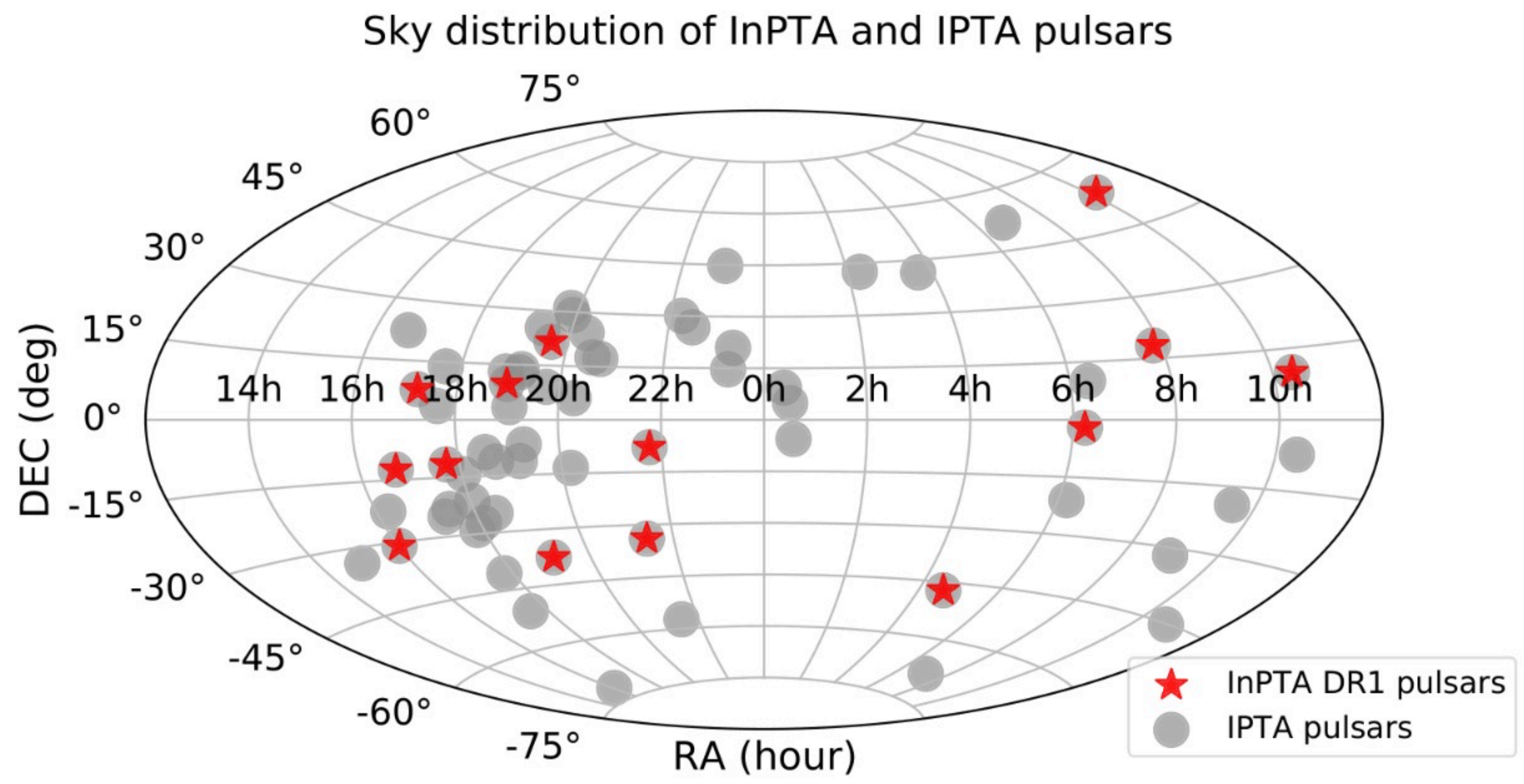
Typical sources of GWs

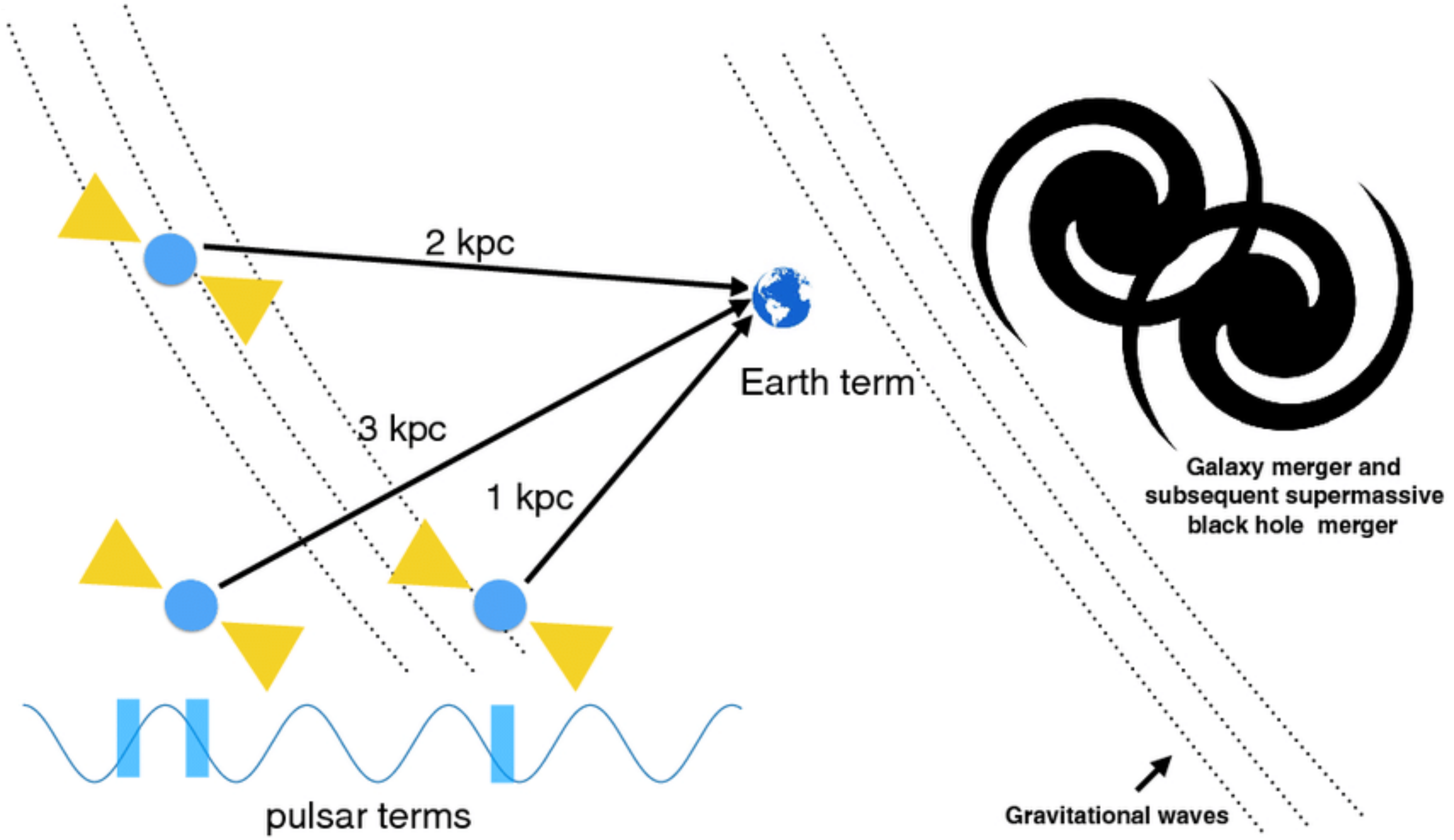


- Binary Black Holes
- Binary Neutron Stars
- Galaxy mergers
- SMBBH mergers
- EMRIs
- cosmic strings
- Supernovae
- primordial dark matter decay
- Primordial GW background

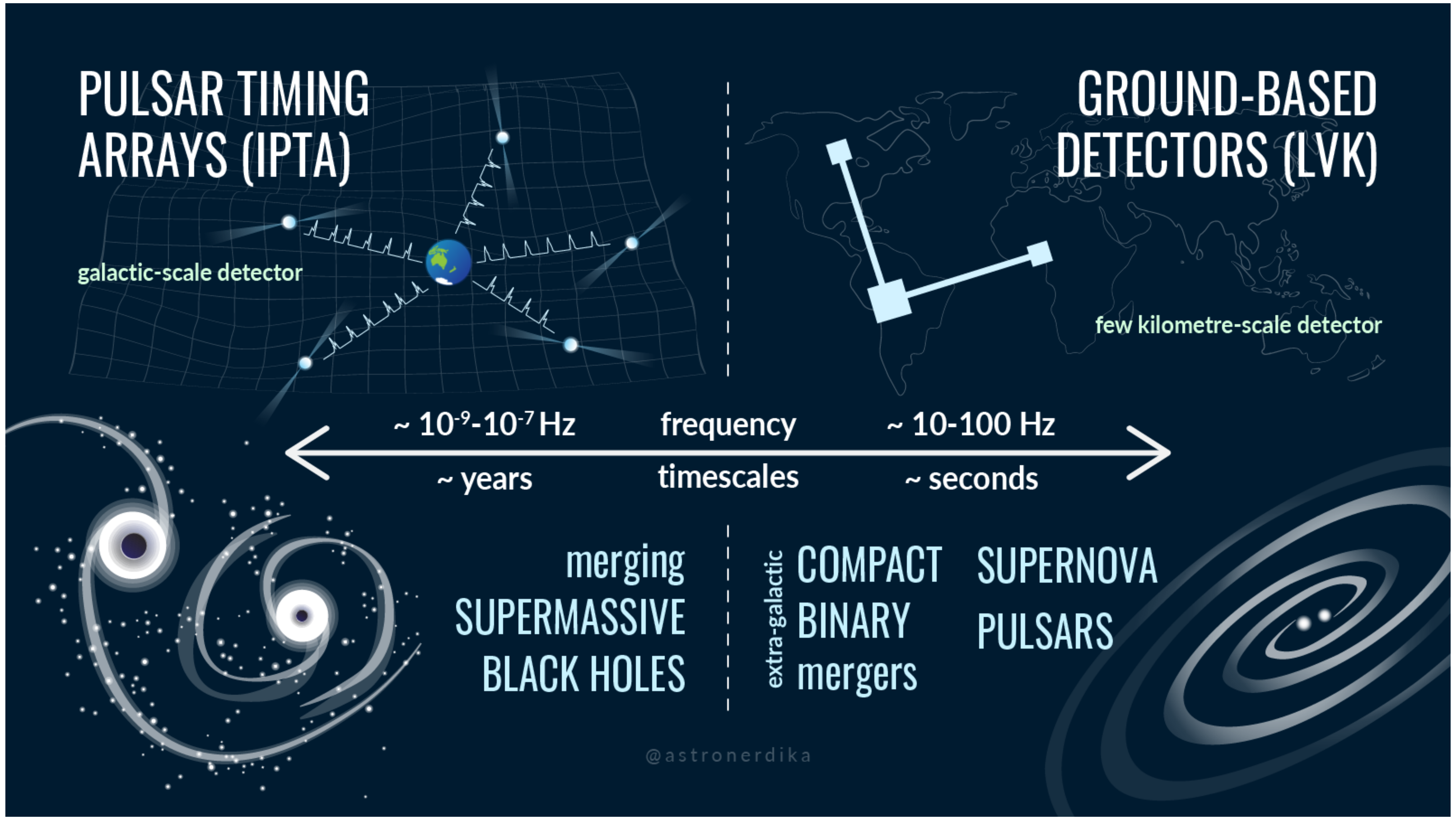
Any mass that undergoes an accelerated motion can generate gravitational signals

Pulsar Timing Array





Credit: Sarah Burke Spolaor et.al. 2019



Pulsar Timing Arrays such as NANOGrav detect gravitational waves by measuring correlated nanosecond-scale timing deviations in millisecond pulsars, providing evidence for a nanohertz stochastic background likely produced by inspiraling supermassive black hole binaries

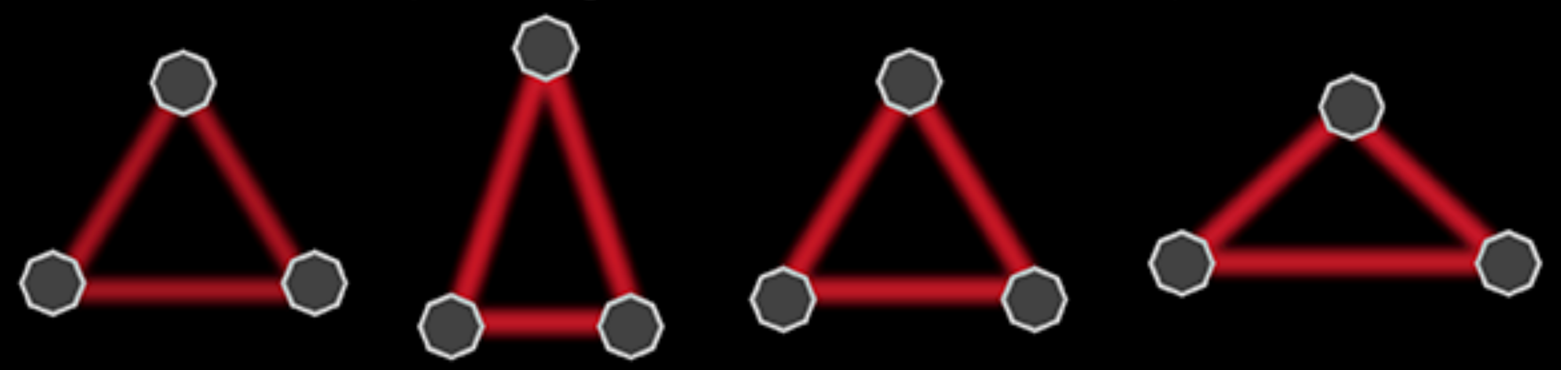

gamma ray
X-ray
ultraviolet
visible
infrared
microwave
radio
Credit: Observatory images from NASA, ESA (Herschel and Planck), Lavochkin Association (Specktr-R), HESS Collaboration (HESS), Salt Foundation (SALT), Rick Peterson/WMKO (Keck), Germini Observatory/AURA (Gemini), CARMA team (CARMA), and NRAO/AUI (Greenbank and VLA); background image from NASA

The Gravitational Wave Spectrum
Sources
Detectors
Big Bang
Supermassive Black Hole Binary Merger
Compact Binary Inspiral & Merger
Extreme Mass-Ratio Inspirals
Pulsars, Supernovae
age of the universe
Wave Period
years
hours
seconds
milliseconds
10^-16
10^-14
10^-12
10^-10
10^-8
10^-6
10^-4
10^-2
1
10^2
Wave Frequency
CMB Polarization
Radio Pulsar Timing Arrays
Space-based interferometers
Terrestrial interferometers

LISA - LASER INTERFEROMETER SPACE ANTENNA

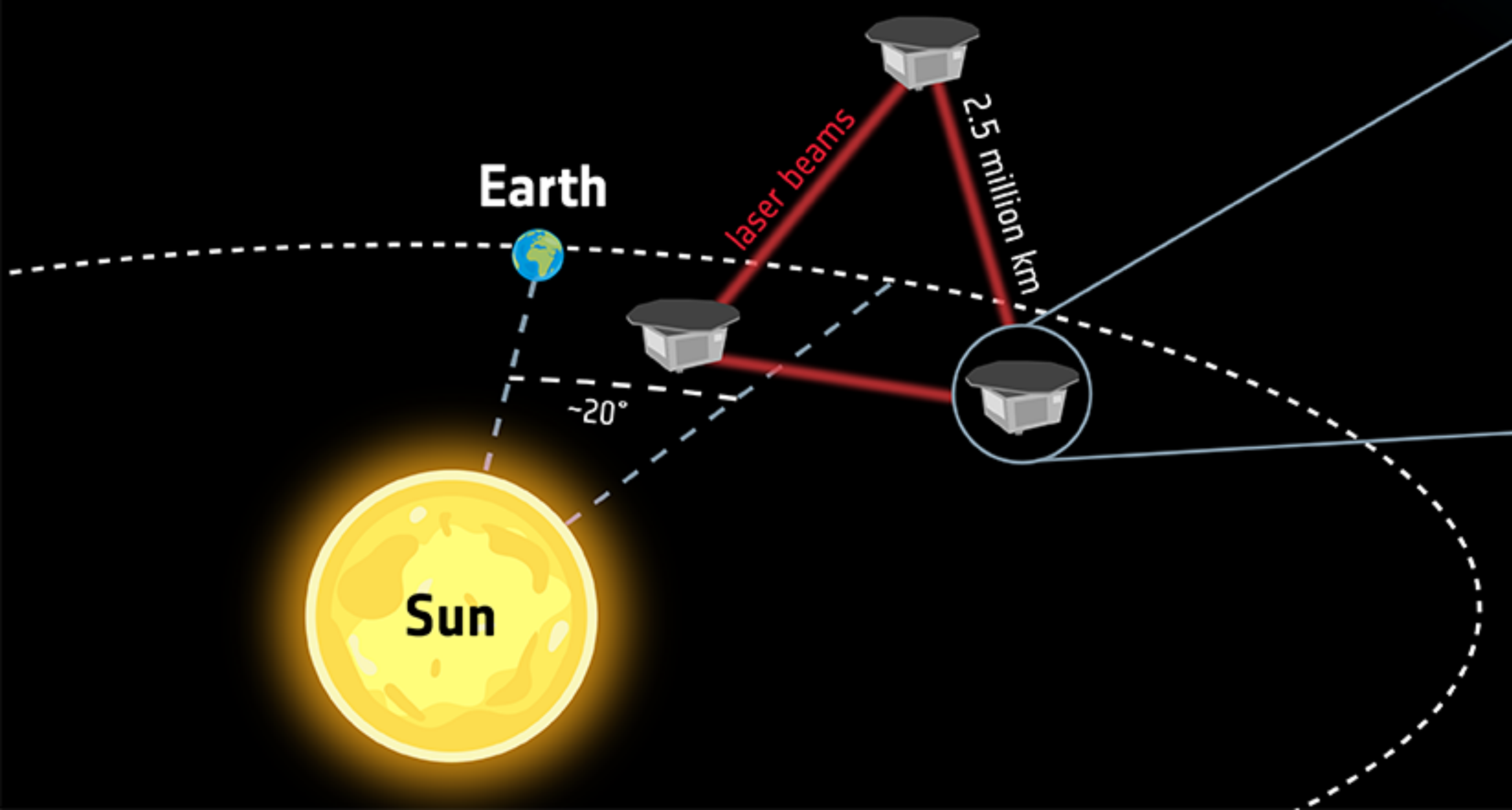
Gravitational waves are ripples in spacetime that alter the distances between objects. LISA will detect them by measuring subtle changes in the distances between **free-floating cubes** nestled within its three spacecraft.

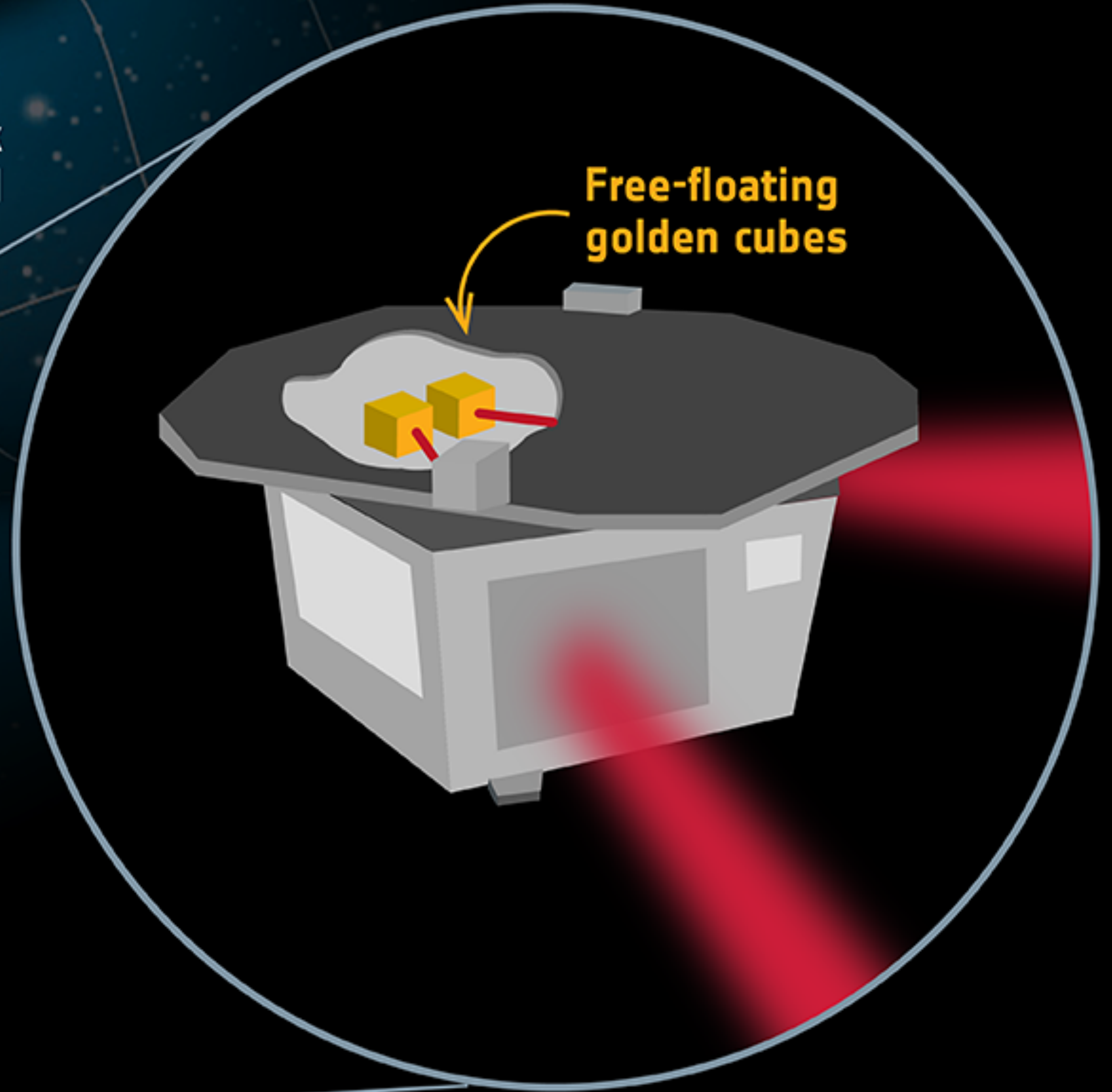
3 identical spacecraft exchange **laser beams**. Gravitational waves change the distance between the **free-floating cubes** in the different spacecraft. This tiny change will be measured by the laser beams.



* *Changes in distances travelled by the laser beams are not to scale and extremely exaggerated*

Powerful events such as **colliding black holes** shake the fabric of spacetime and cause gravitational waves






THE SPECTRUM OF GRAVITATIONAL WAVES
esa
Observatories & experiments
Ground-based experiment
Space-based observatory
Pulsar timing array
Cosmic microwave background polarisation
Timescales
milliseconds
seconds
hours
years
billions of years
Frequency (Hz)
100
1
10-2
10-4
10-6
10-8
10-16
Cosmic fluctuations in the early Universe
Cosmic sources
Supernova
Pulsar
Compact object falling onto a supermassive black hole
Merging supermassive black holes
Merging neutron stars in other galaxies
Merging stellar-mass black holes in other galaxies
Merging white dwarfs in our Galaxy
ESA | NASA
lisa
#lisa

- **First direct detection of gravitational waves (2015)** — GW150914 by LIGO Scientific Collaboration
- **Discovery of binary black hole mergers** as a common astrophysical population
- **First binary neutron star merger detection (2017)** — GW170817
- **Birth of multi-messenger astronomy** (GW + EM + neutrinos) via GW170817
- **Independent measurement of the Hubble constant** using “standard sirens”
- **Precision tests of General Relativity in the strong-field, dynamical regime**
- **Evidence for intermediate-mass black holes** (e.g., GW190521)
- **Detection of the gravitational-wave memory and higher-order modes (emerging evidence)**
- **Observation of a stochastic nanohertz GW background** by PTAs such as NANOGrav
- **Establishment of gravitational-wave astronomy as a precision cosmology tool**

References

1. A brief history of gravitational waves, https://arxiv.org/pdf/1609.09400