Mode selection and amplitude limitation in pulsating stars

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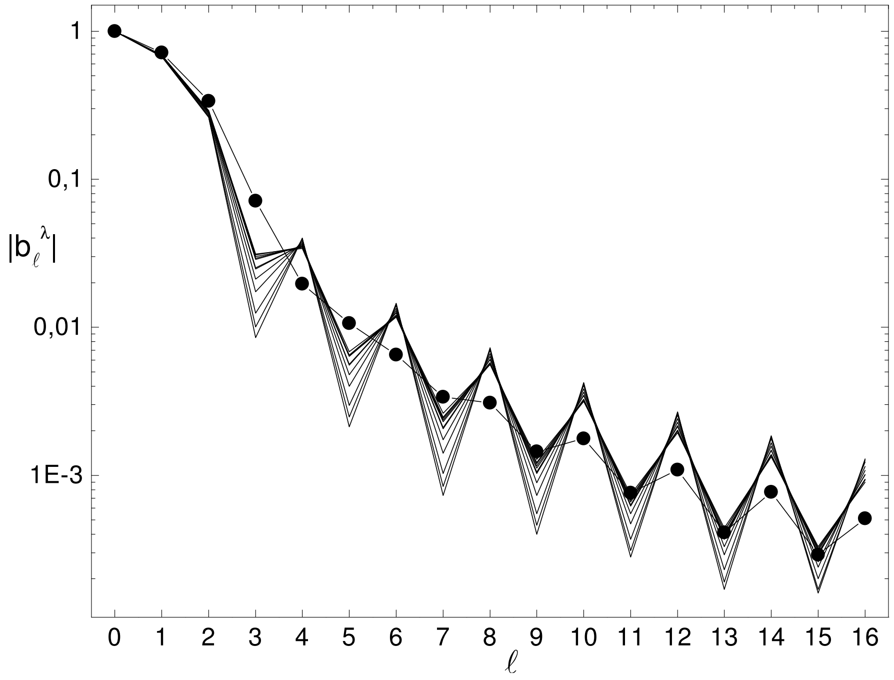
Mode selection problem

- ⋆ in many stars simultaneous instability of many pulsation modes is predicted, however lower number of modes is detected
  - ▸ which of the unstable pulsation modes are selected and why?
  - ▸ what is the amplitude limitation mechanism?
- ⋆ these are essentially non-linear problems!

Observational mode selection



disc averaging factor vs. ℓ for bolometric light and different pass-bands

Dziembowski (1977)

- geometrical cancellation (+limb darkening effect)

figure: Daszyńska-Daszkiewicz et al. (2002)

Tools for studying mode selection

- ⋆ linear non-adiabatic analysis
  - ▸ driving mechanism, instability ranges
  - ▸ linear growth rates, trapped modes
- ⋆ non-linear tools: non-linear pulsation codes (hydrodynamics)
  - ▸ so far only 1D for radial pulsators
- ⋆ non-linear tools: amplitude equations
  - ▸ both radial and non-radial pulsation
  - ▸ unknown saturation/coupling coefficients

Linear growth rate

$$\gamma = \frac{\int dW}{2\sigma I},$$

where

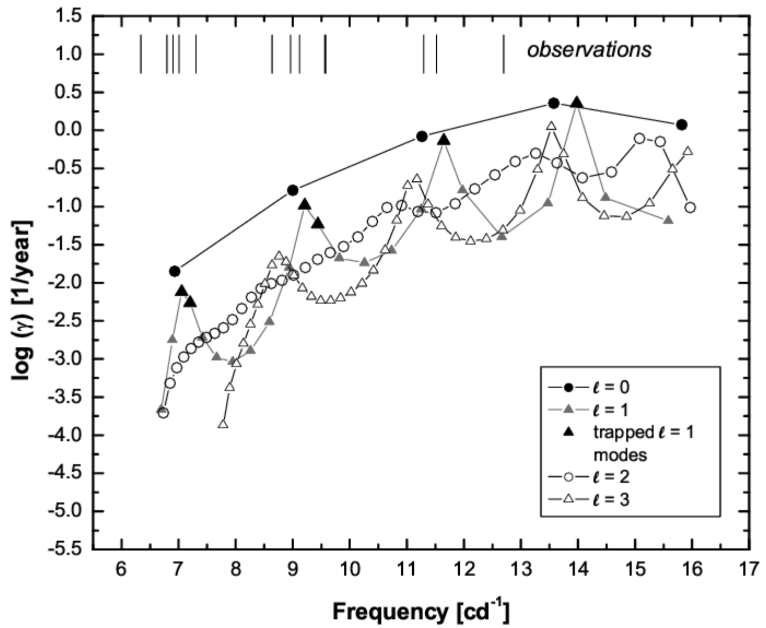
$$dW = \Im\left[\delta P \left(\frac{\delta\rho}{\rho}\right)^*\right], \quad I = \int_M |\boldsymbol{\xi}|^2 dm,$$

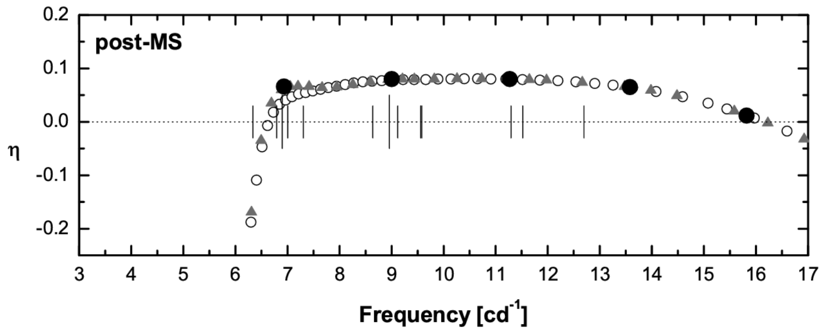
are local contribution to the work integral (dW) and mode inertia (I).

Stellingwerf (1978) growth rate:

$$\eta = \frac{\int dW}{\int |dW|},$$

with $\eta \in (-1:1)$



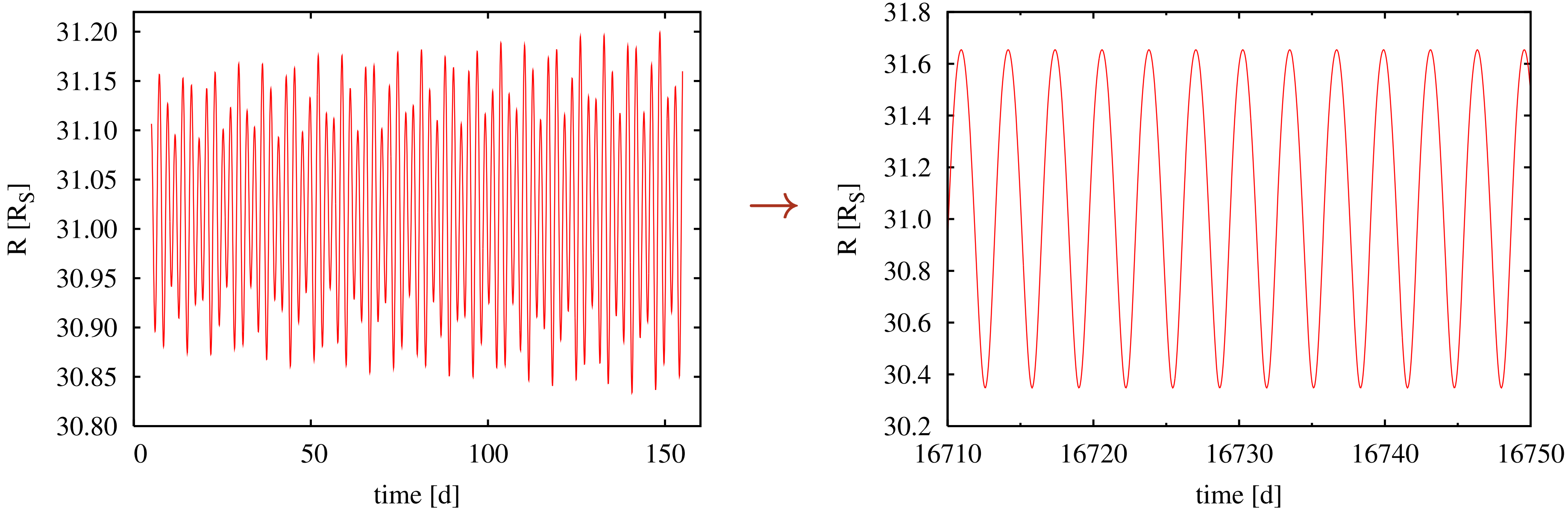


figures: Lenz et al. (2008)

Nonlinear pulsation codes

⋆ 1D, direct time integration codes with simple 1-equation model for turbulent convection (Stellingwerf 1982, Kuhfuß 1986)

⋆ several codes (e.g., Italian, Florida-Budapest, Warsaw, Vienna)



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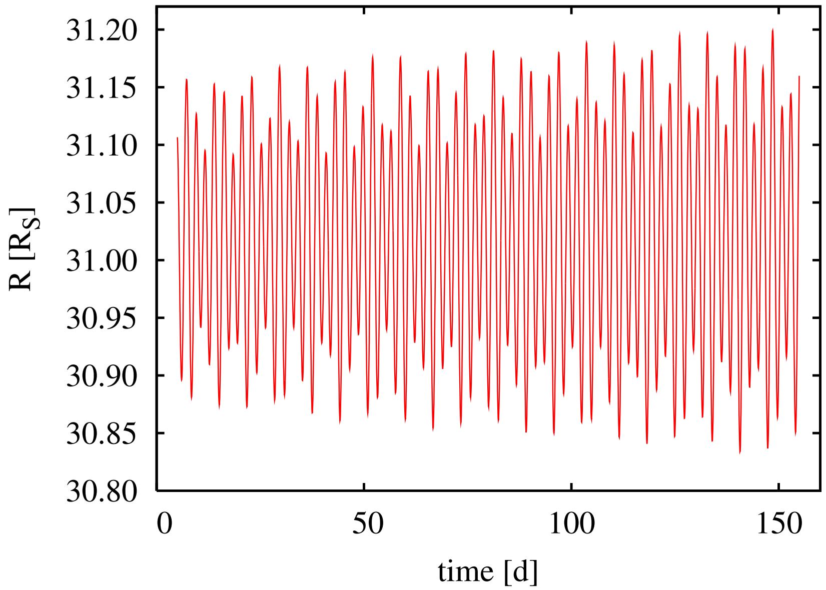
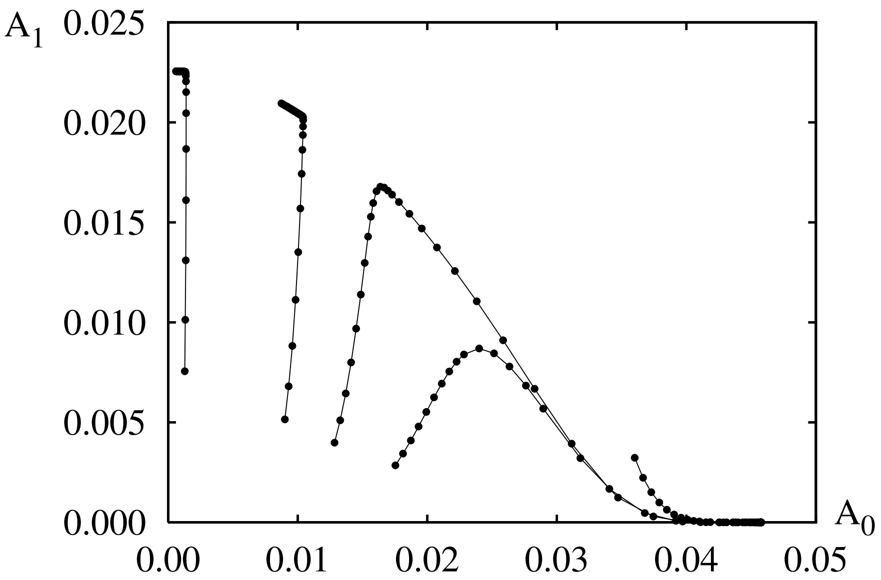


→



Amplitude equations

- assumption: $\gamma/\sigma \ll 1$ (e.g., Buchler & Goupil 1984, Dziembowski 1982)

⋆ non-resonant mode coupling:

$$\frac{dA_i}{dt} = \tilde{\gamma}_i A_i\,, \quad \tilde{\gamma}_i = \gamma_i\Big(1 + \sum_j \alpha_{ij} A_j^2\Big)$$

⋆ resonant mode coupling $\sigma_a = \sigma_b + \sigma_c + \Delta\sigma$ (exact form depends on resonance considered)

$$\frac{dA_a}{d\tau} = \gamma_a A_a - i\frac{C}{2} A_b A_c \mathrm{e}^{-i\Delta\sigma\tau}$$
$$\frac{dA_{b,c}}{d\tau} = \gamma_{b,c} A_{b,c} - i\frac{C}{2} A_a A_{c,b}^{*} \mathrm{e}^{i\Delta\sigma\tau}$$

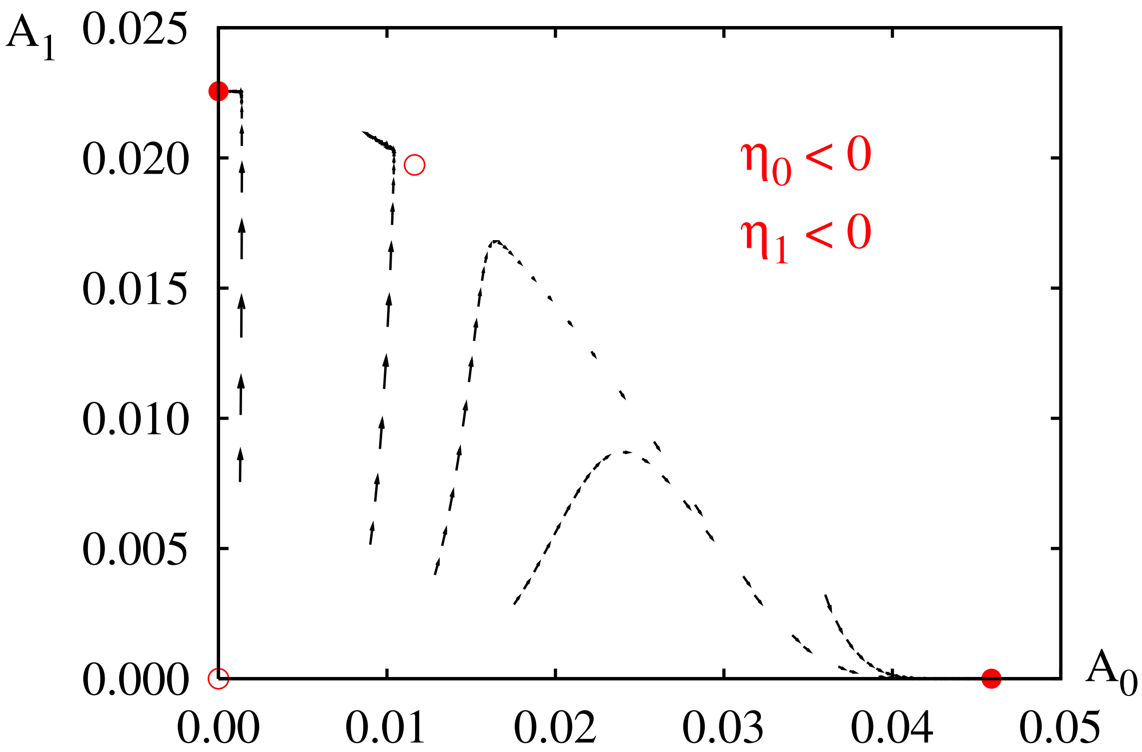
- weak point: saturation/coupling coefficients are hard to compute even for the simplified cases

Amplitude equations coupled with nonlinear pulsation codes

⋆ saturation/coupling coefficients obtained through fitting the hydrodynamic trajectories

- ▸ detailed mapping of mode selection in Cepheids/RR Lyrae stars (e.g., Szabó et al. 2004, Smolec & Moskalik 2008)
- ▸ mode selection problem in β Cephei variables (Smolec & Moskalik 2007)

Hydrodynamics + Amplitude Equations:



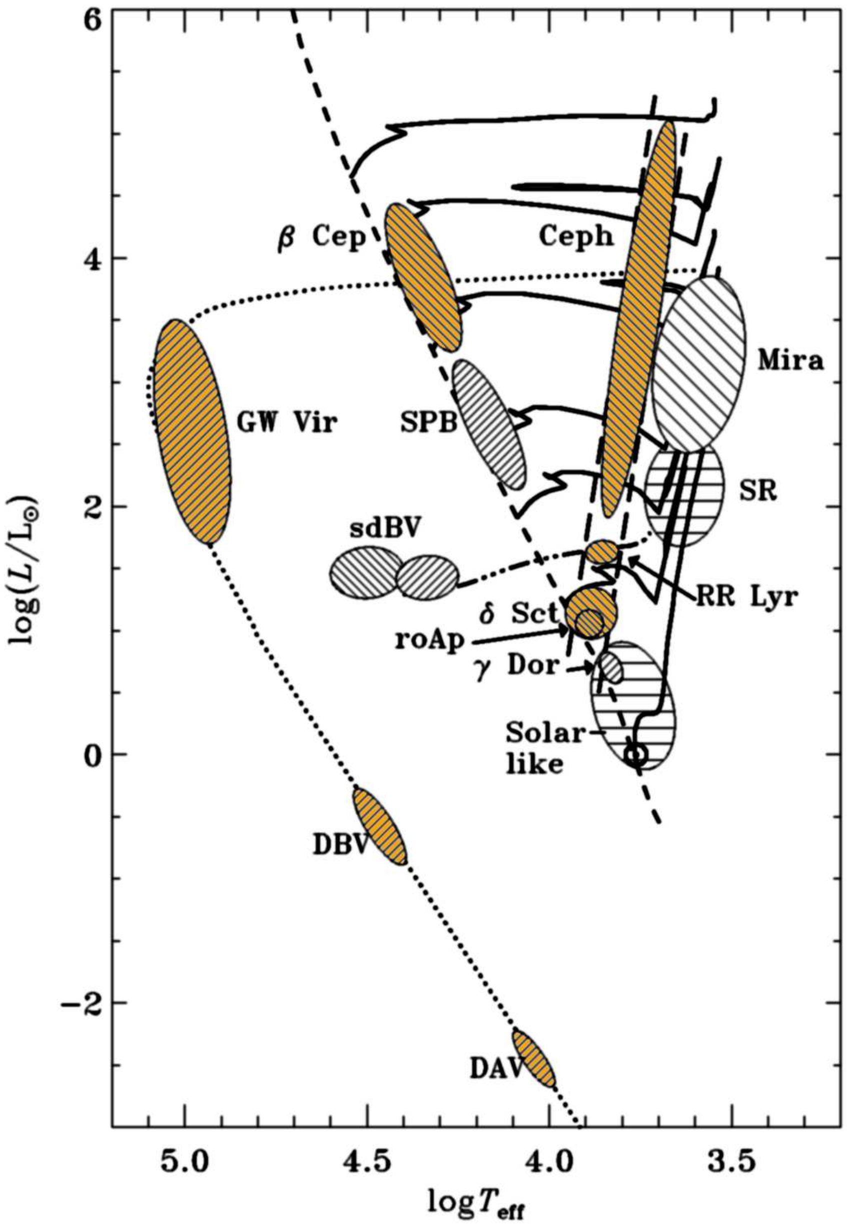
← Hydrodynamics

Amplitude Equations →

$$\dot{A}_0 = (\gamma_0 + q_{00}A_0^2 + q_{01}A_1^2)A_0$$

$$\dot{A}_1 = (\gamma_1 + q_{11}A_1^2 + q_{10}A_0^2)A_1$$

Mode selection in self-excited pulsators



Cepheids & RR Lyr →

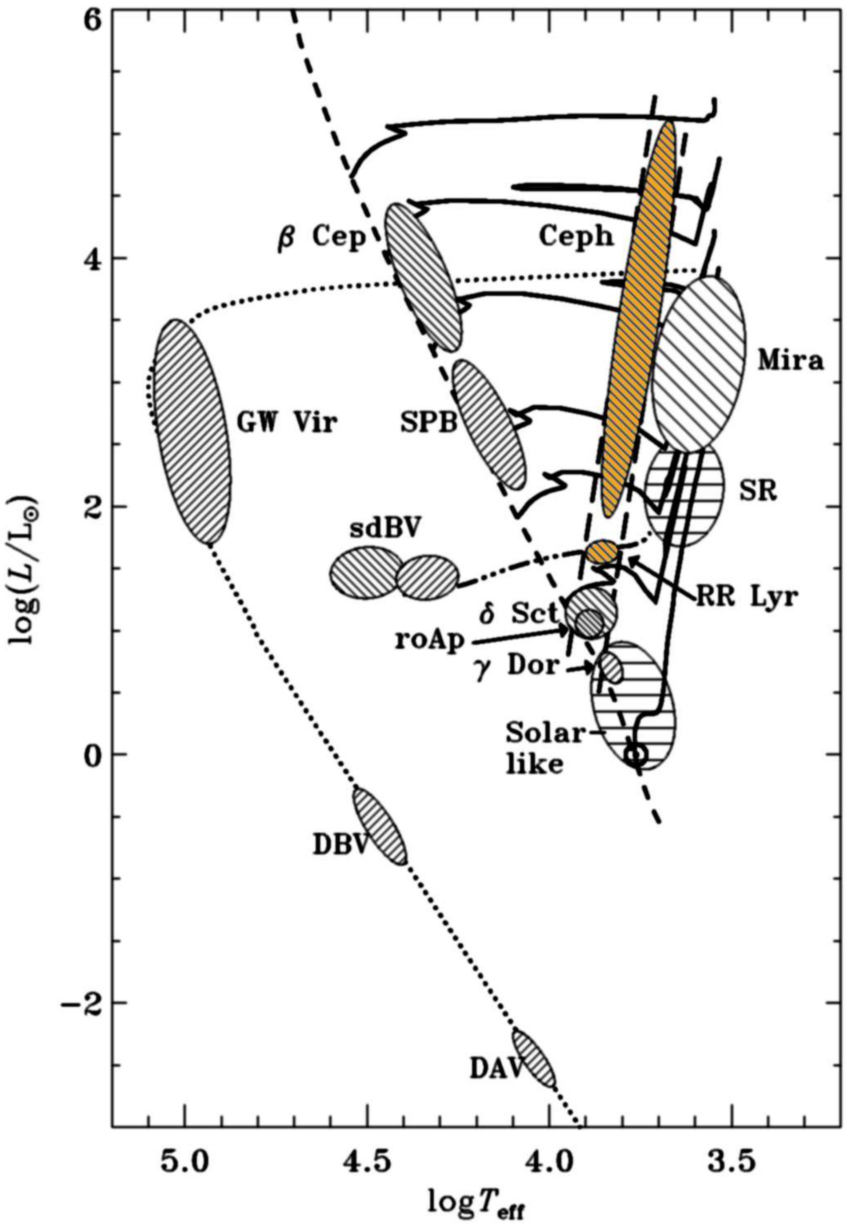
→ β Cephei →

→ WDs →

→ δ Scuti

figure: J. C.-D./Stellar Oscillations

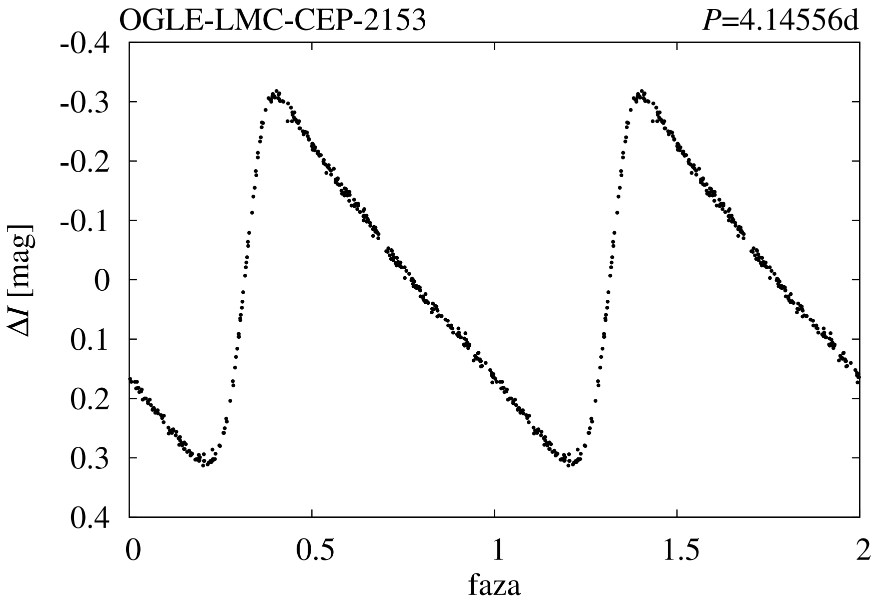
Cepheids and RR Lyrae stars

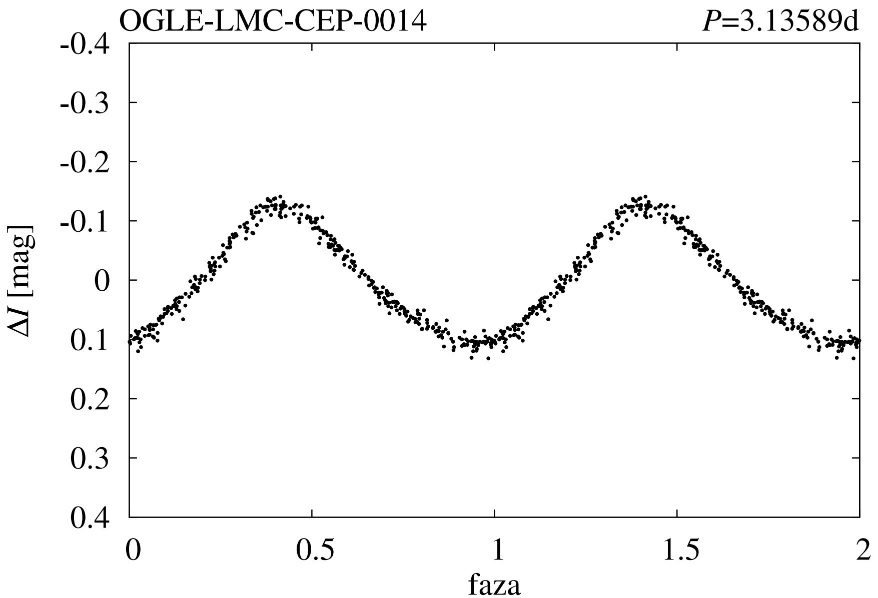


- large amplitude mostly single-periodic radial pulsators (F and 1O)
- multi-periodic pulsation is rare (DM, TM)
- non-radial pulsators are even more scarce

figure: J. C.-D./Stellar Oscillations

Single-periodic pulsators

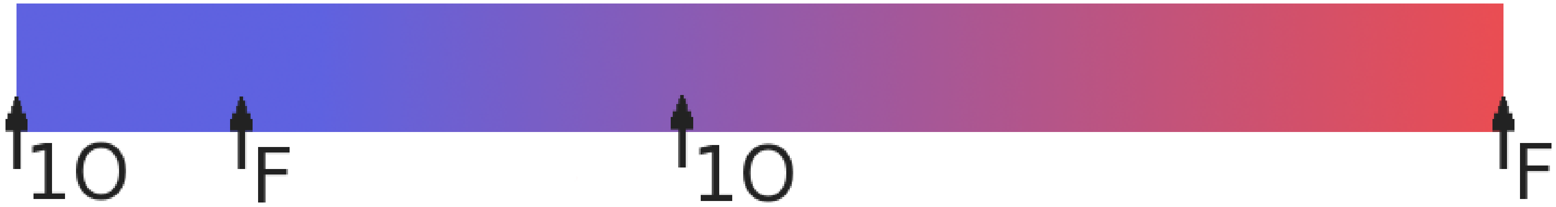




- instability is saturated (Christy 1964, Stellingwerf 1975, and others)

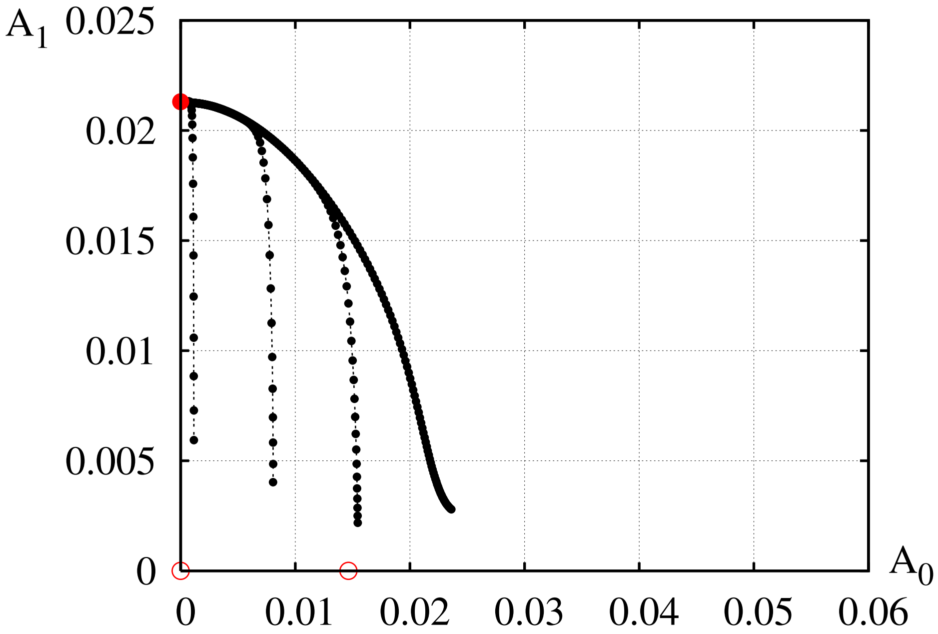
Double-period pulsators

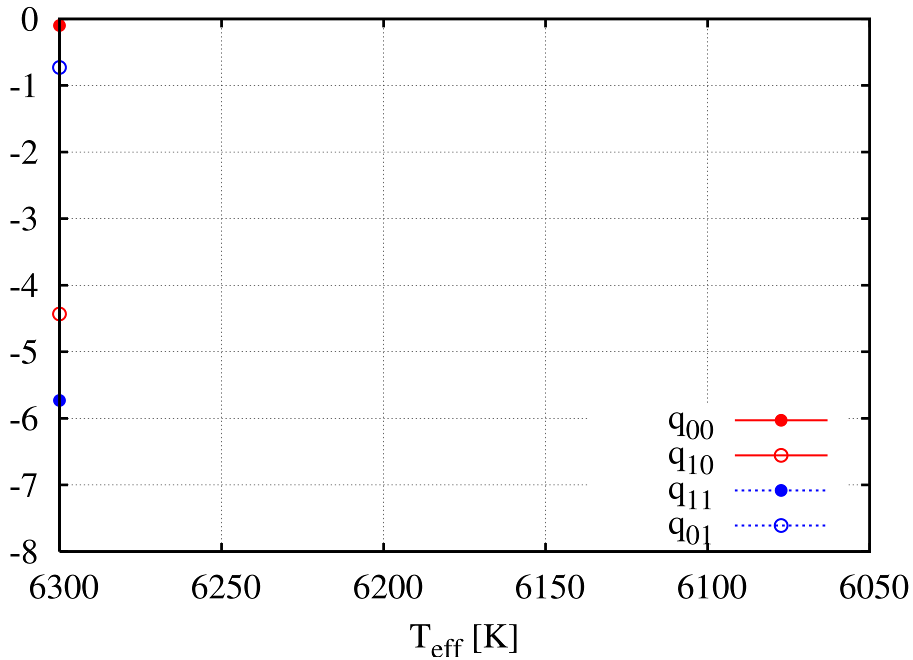
- what if $\gamma_{\rm F} > 0$ and $\gamma_{\rm 1O} > 0$?



Mode selection along sequence of Cepheid models

- $M,\ L\ =\ \text{const},\ T_{\text{eff}} = 6300\,\mathbf{K}$

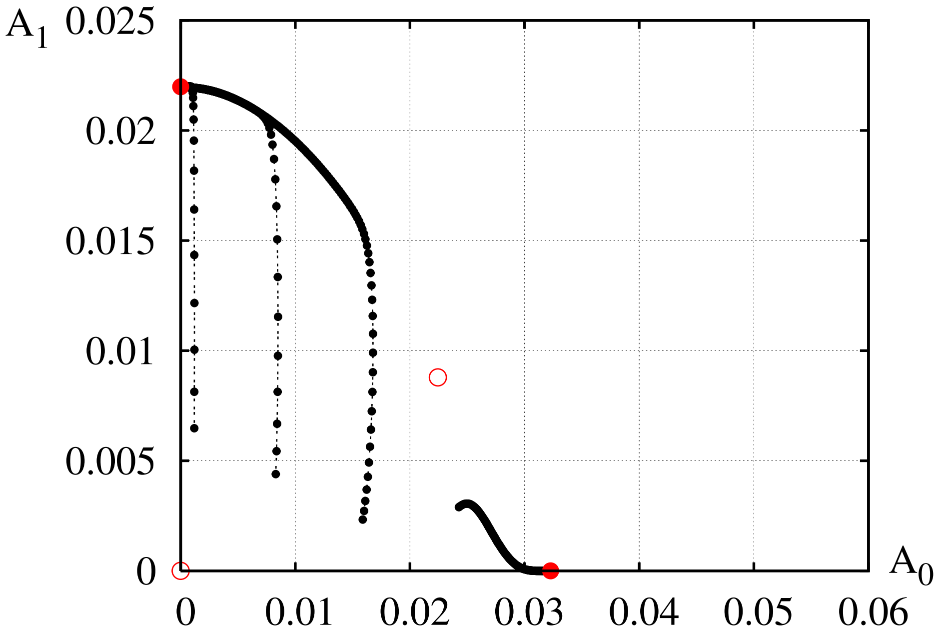


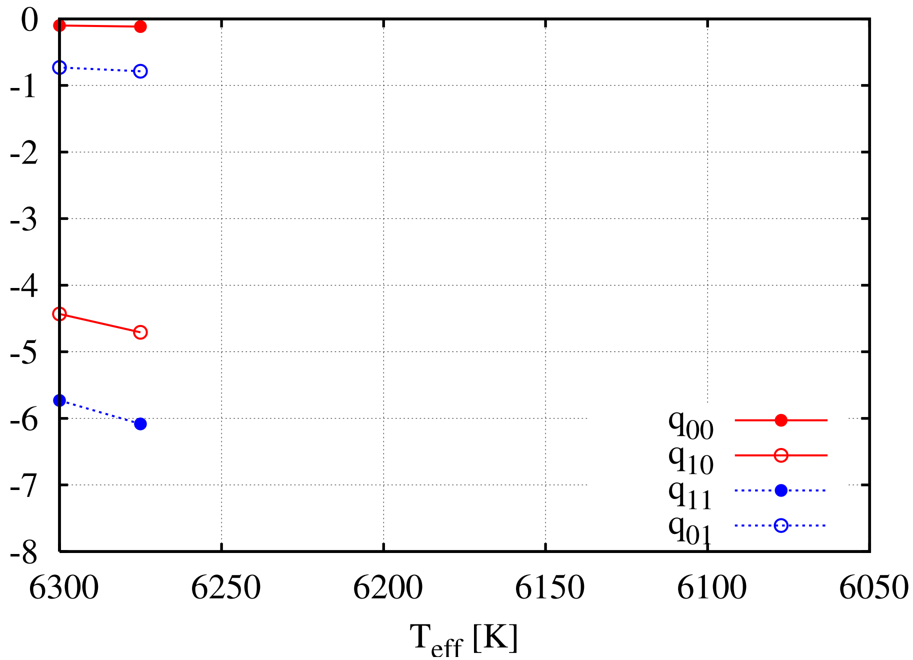


Smolec & Moskalik (2008)

Mode selection along sequence of Cepheid models

- $M,\ L\ =\ \text{const},\ T_{\text{eff}} = 6275\,\mathbf{K}$

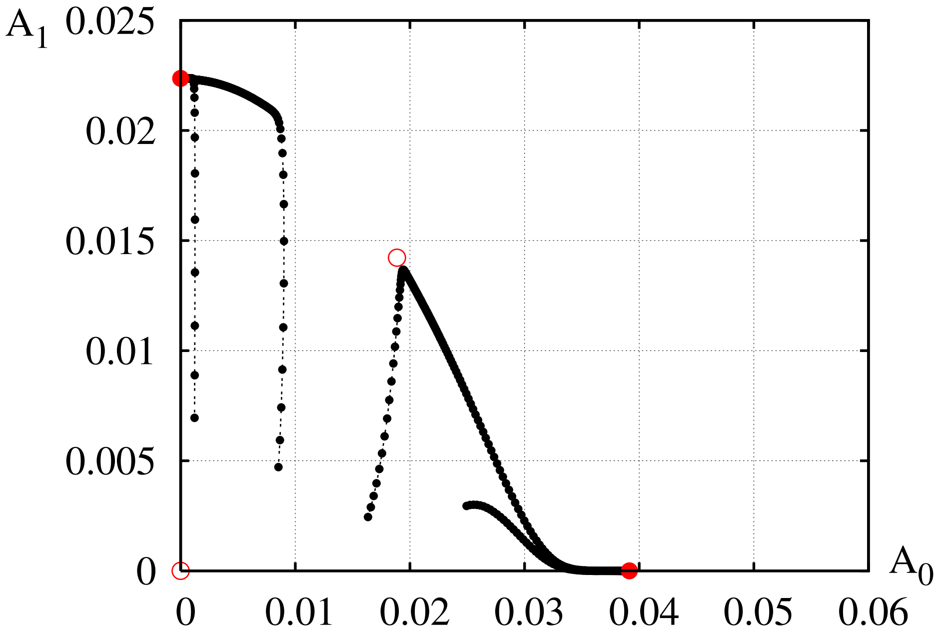


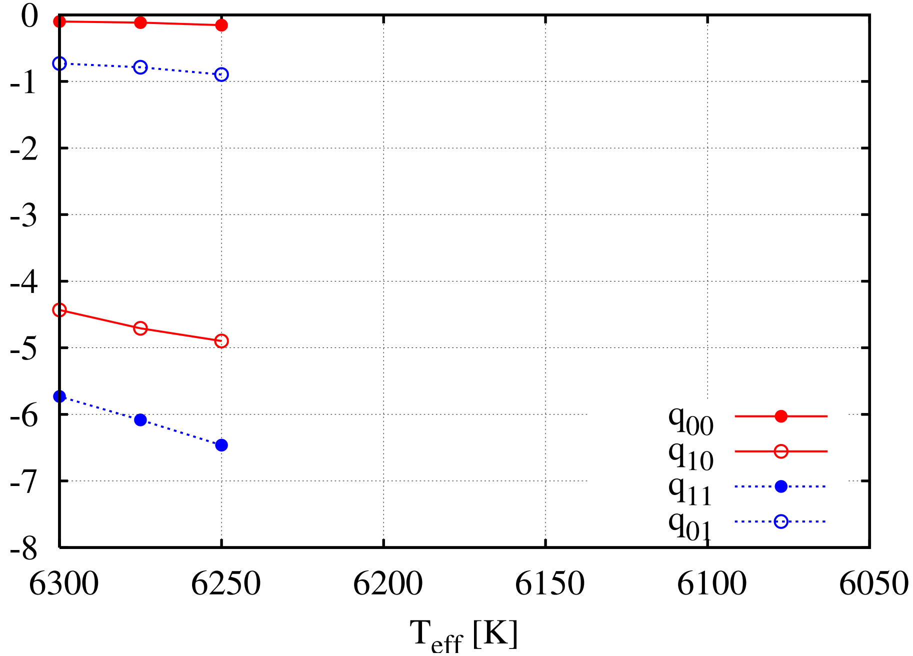


Smolec & Moskalik (2008)

Mode selection along sequence of Cepheid models

- $M,\ L\ =\ \text{const},\ T_{\text{eff}} = 6250\,\mathbf{K}$

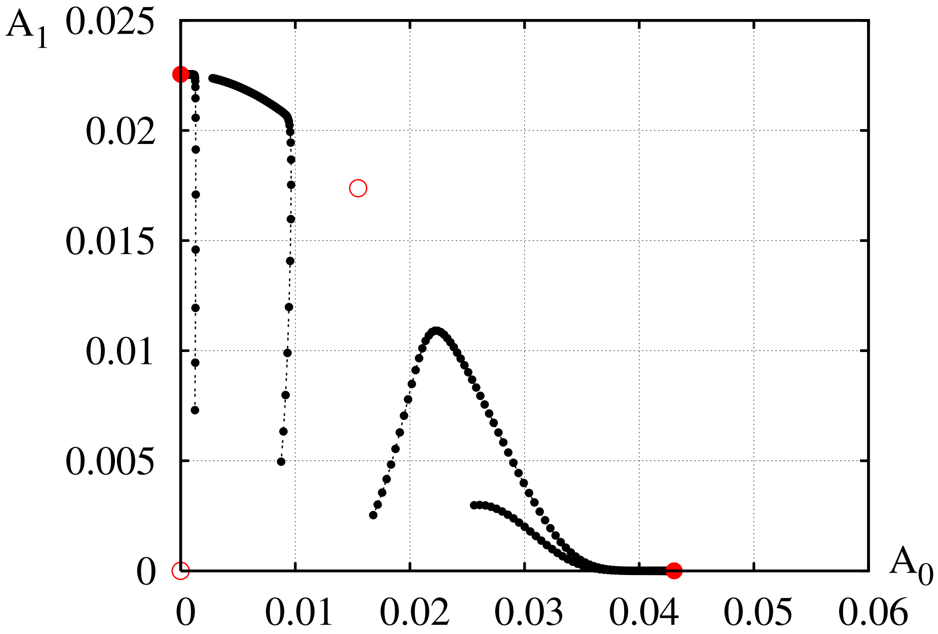


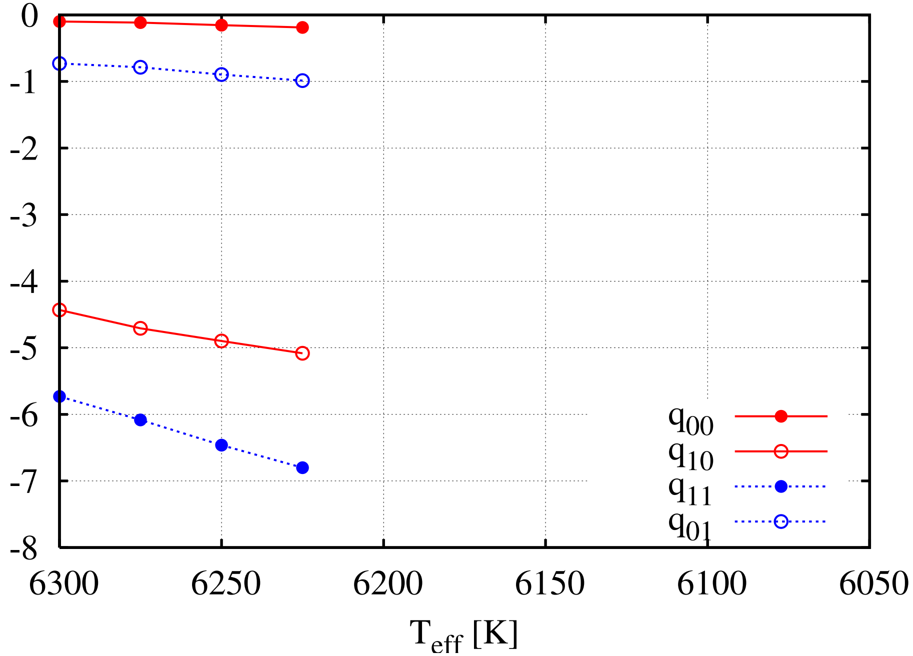


Smolec & Moskalik (2008)

Mode selection along sequence of Cepheid models

- $M,\ L\ =\ \mathrm{const},\ T_{\mathrm{eff}} = 6225\,\mathbf{K}$

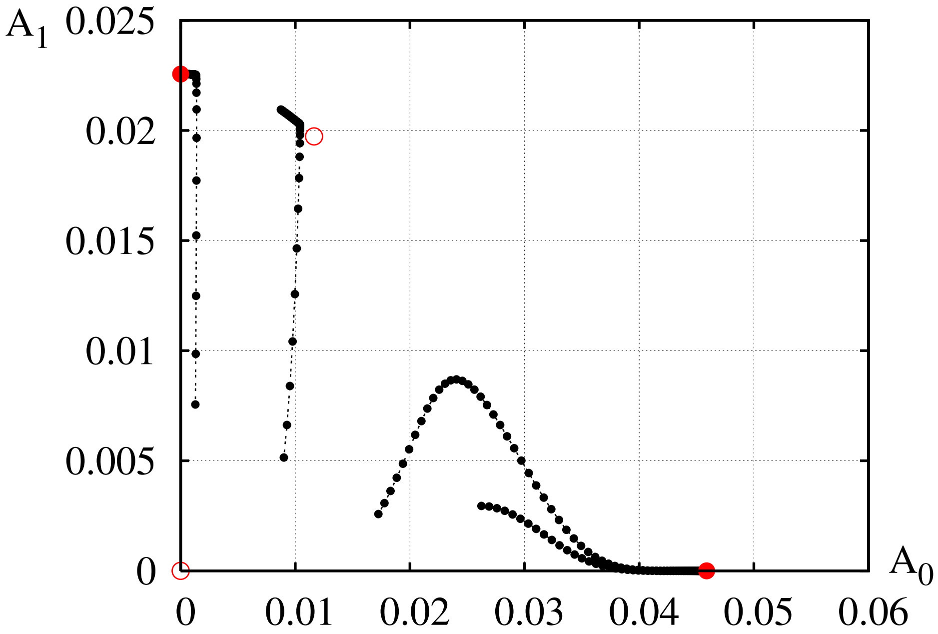


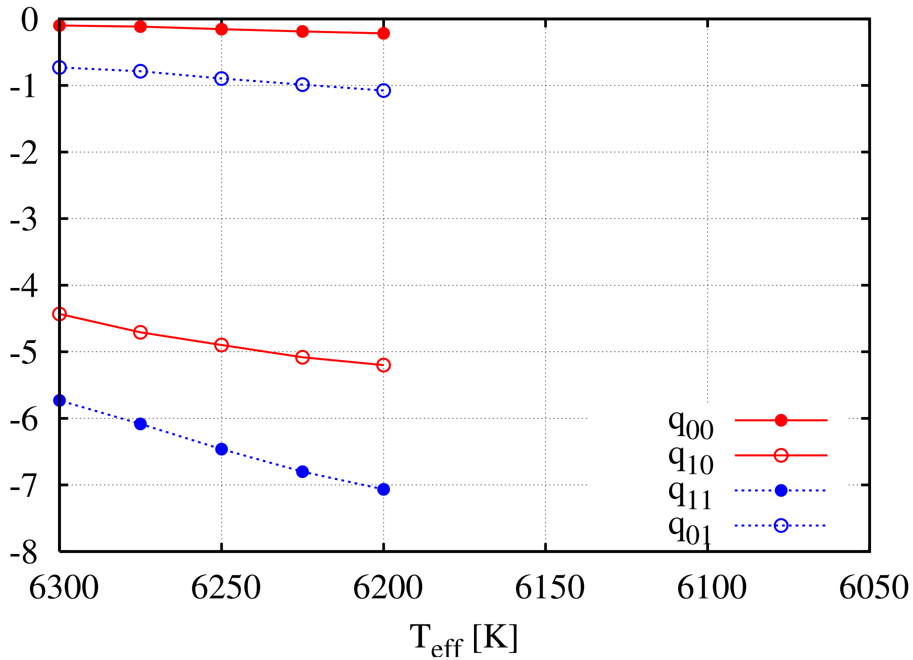


Smolec & Moskalik (2008)

Mode selection along sequence of Cepheid models

- $M,\ L\ =\ \text{const},\ T_{\text{eff}} = 6200\,\mathbf{K}$

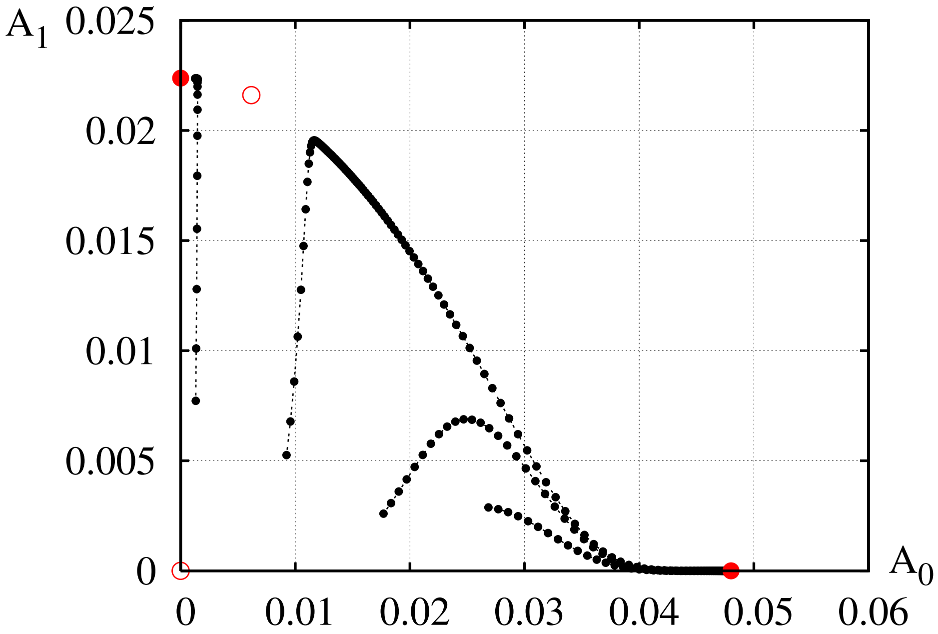


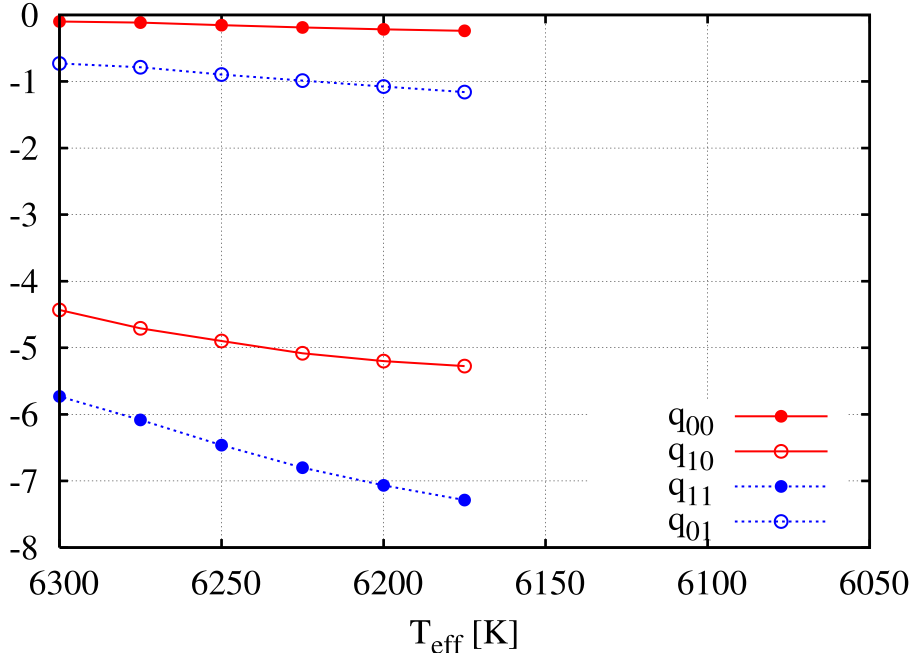


Smolec & Moskalik (2008)

Mode selection along sequence of Cepheid models

- $M,\ L\ =\ \text{const},\ T_{\text{eff}} = 6175\,\mathbf{K}$

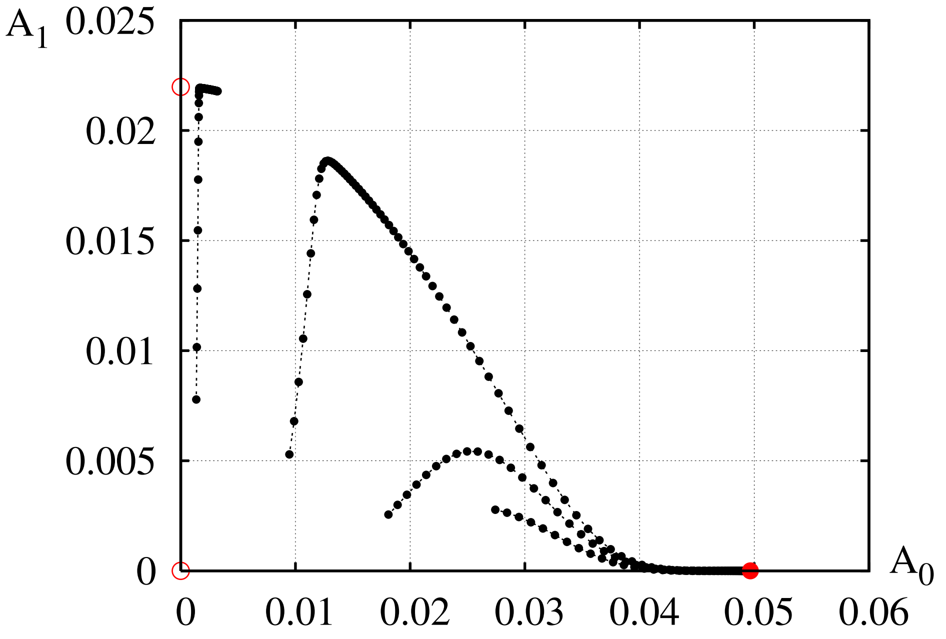


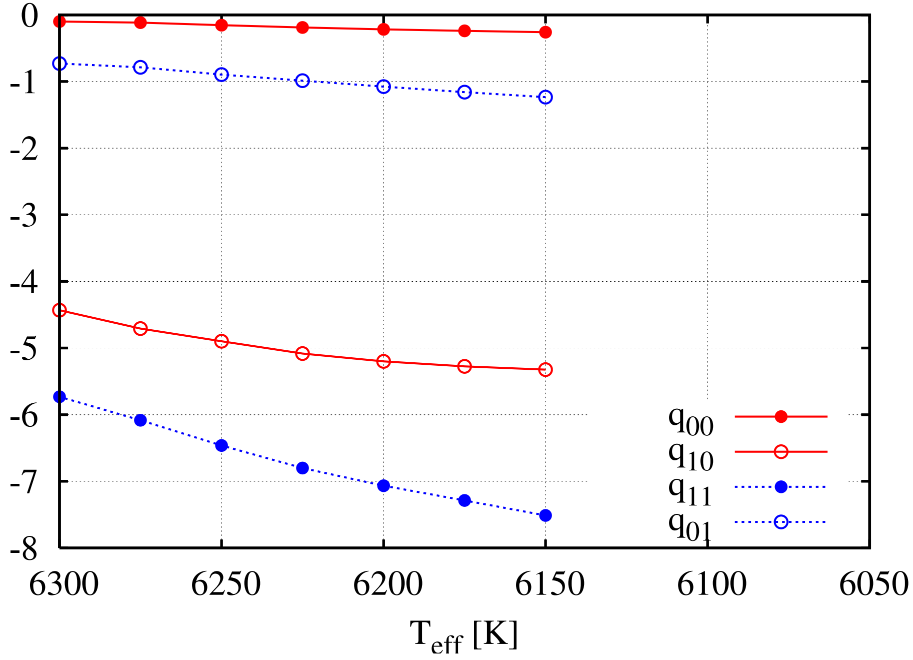


Smolec & Moskalik (2008)

Mode selection along sequence of Cepheid models

- $M,\ L\ =\ \text{const},\ T_{\text{eff}} = 6150\,\mathbf{K}$

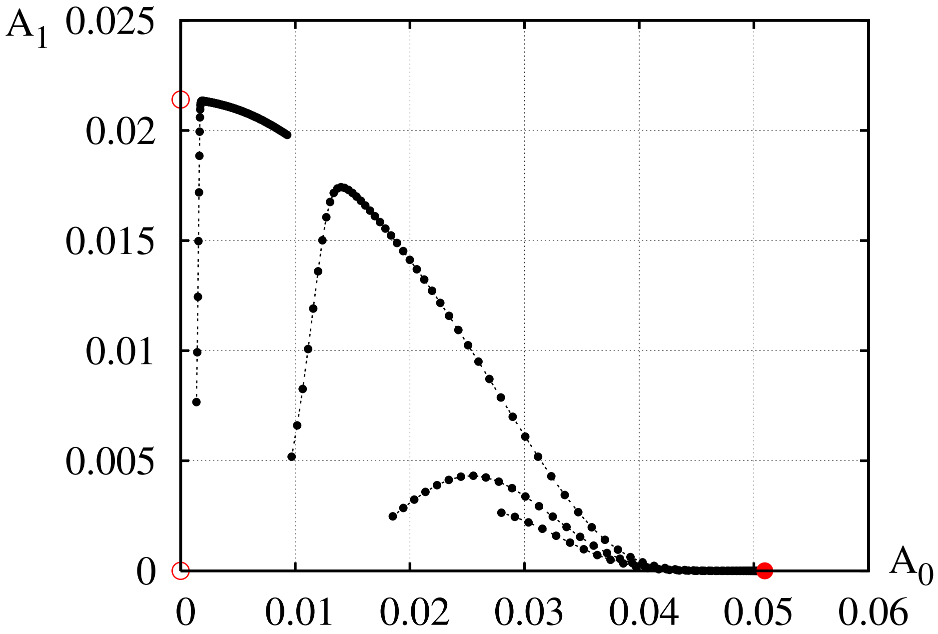


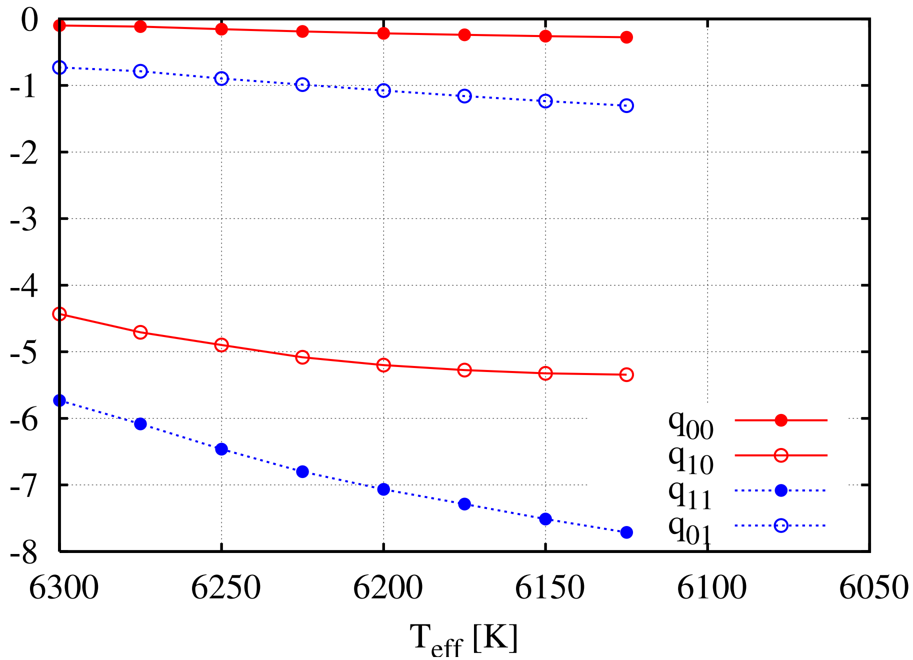


Smolec & Moskalik (2008)

Mode selection along sequence of Cepheid models

- $M, L = \text{const}, T_{\text{eff}} = 6125\,\mathbf{K}$

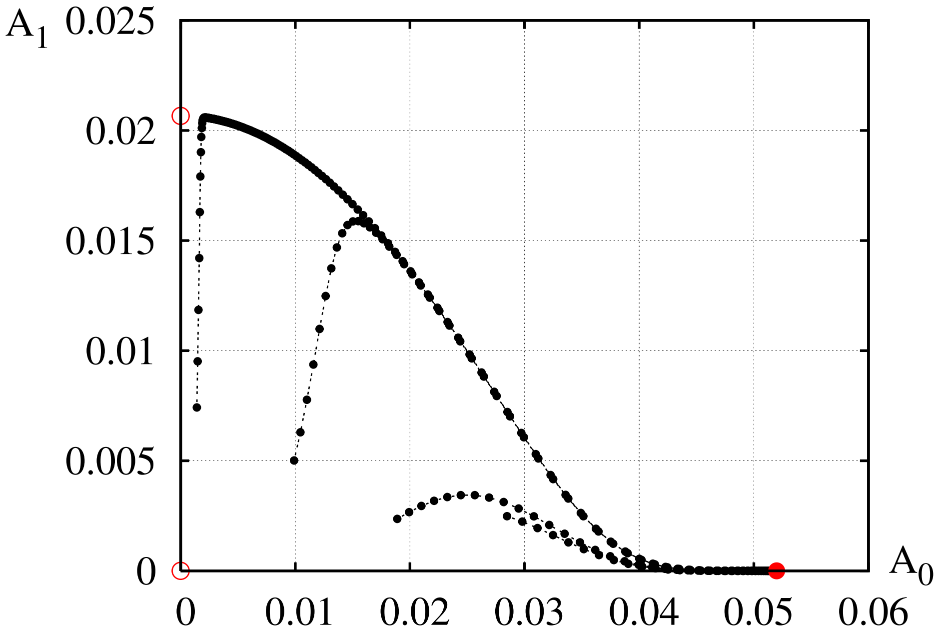


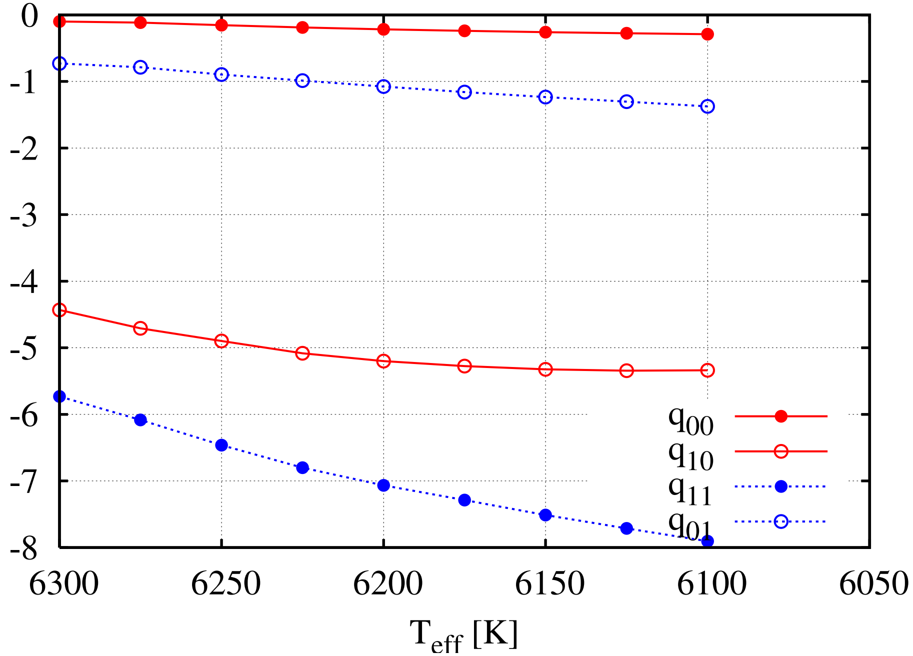


Smolec & Moskalik (2008)

Mode selection along sequence of Cepheid models

- $M,\ L\ =\ \mathrm{const},\ T_{\mathrm{eff}} = 6100\,\mathbf{K}$

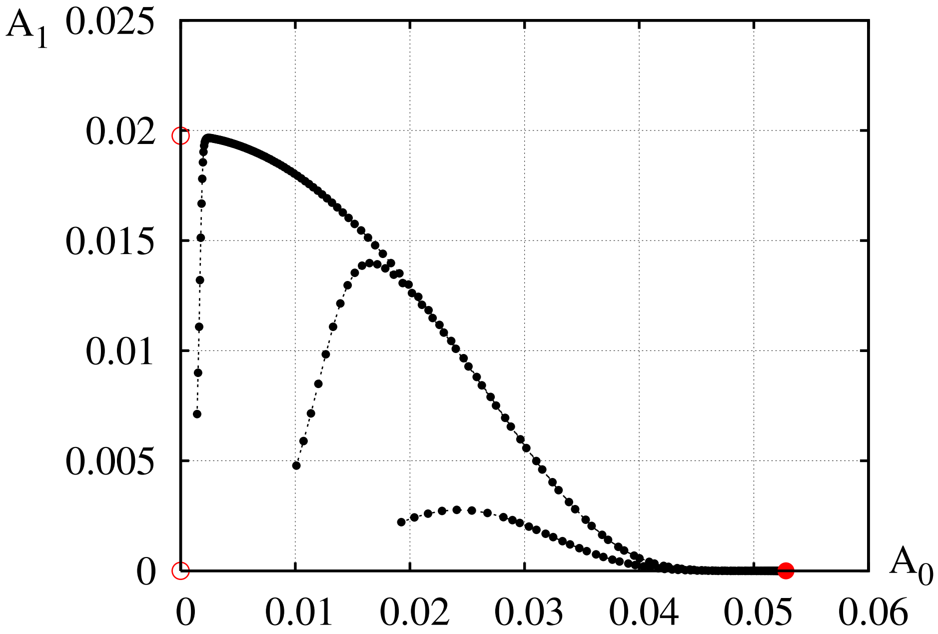


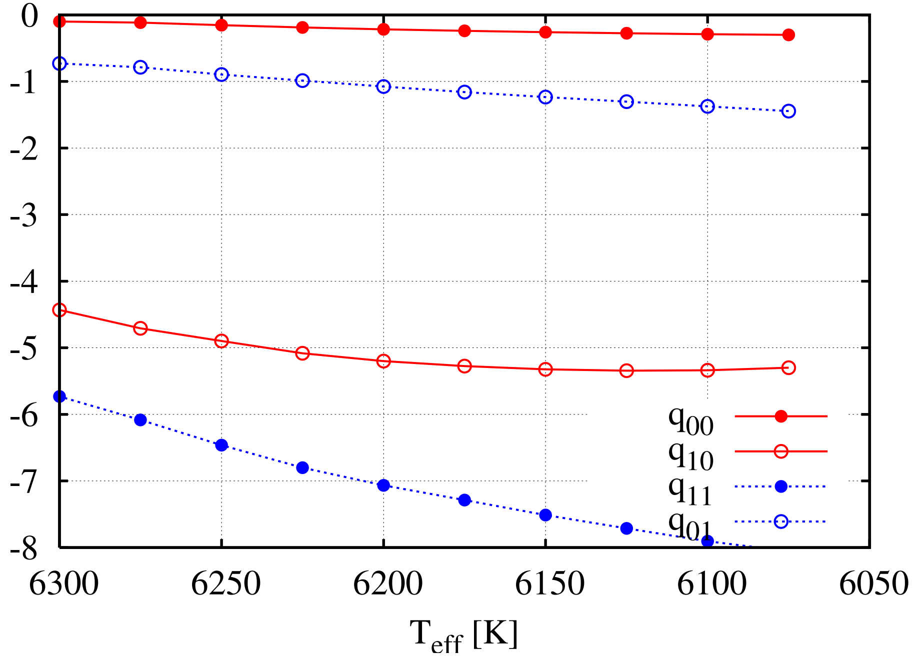


Smolec & Moskalik (2008)

Mode selection along sequence of Cepheid models

- $M,\ L\ =\ \mathrm{const},\ T_{\mathrm{eff}} = 6075\,\mathbf{K}$

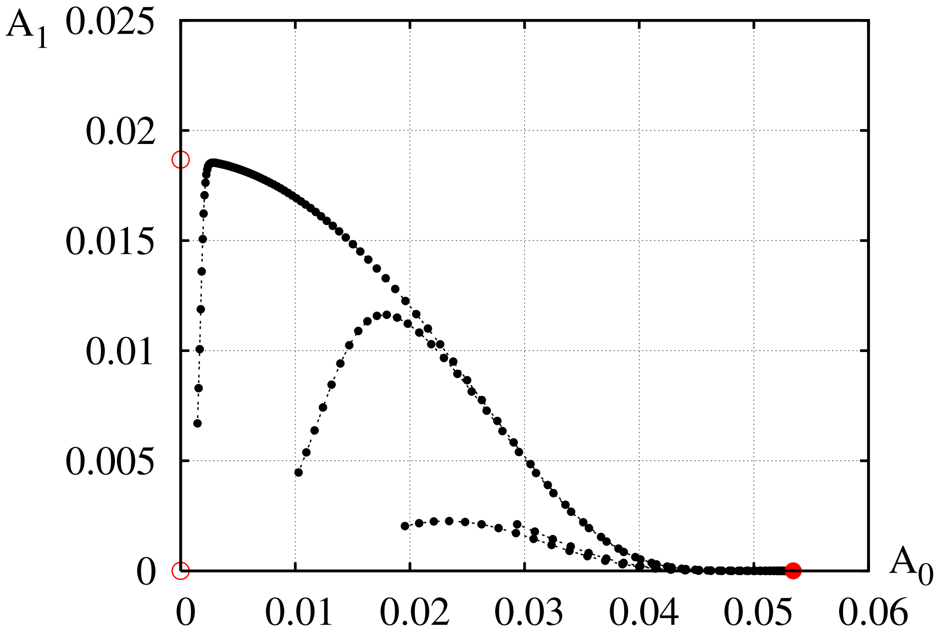


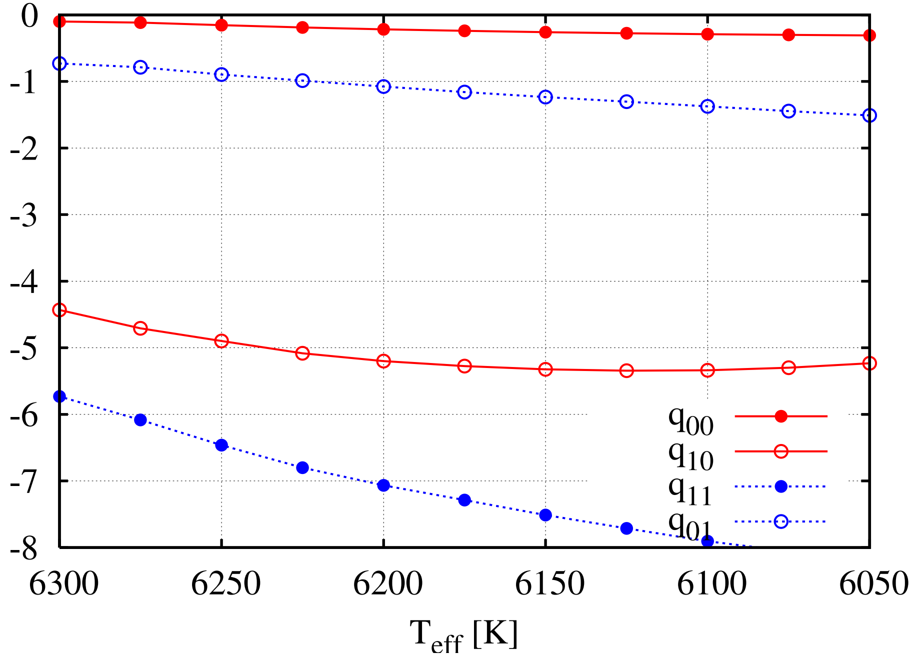


Smolec & Moskalik (2008)

Mode selection along sequence of Cepheid models

- $M,\ L\ =\ \text{const},\ T_{\rm eff} = 6050\,\mathbf{K}$

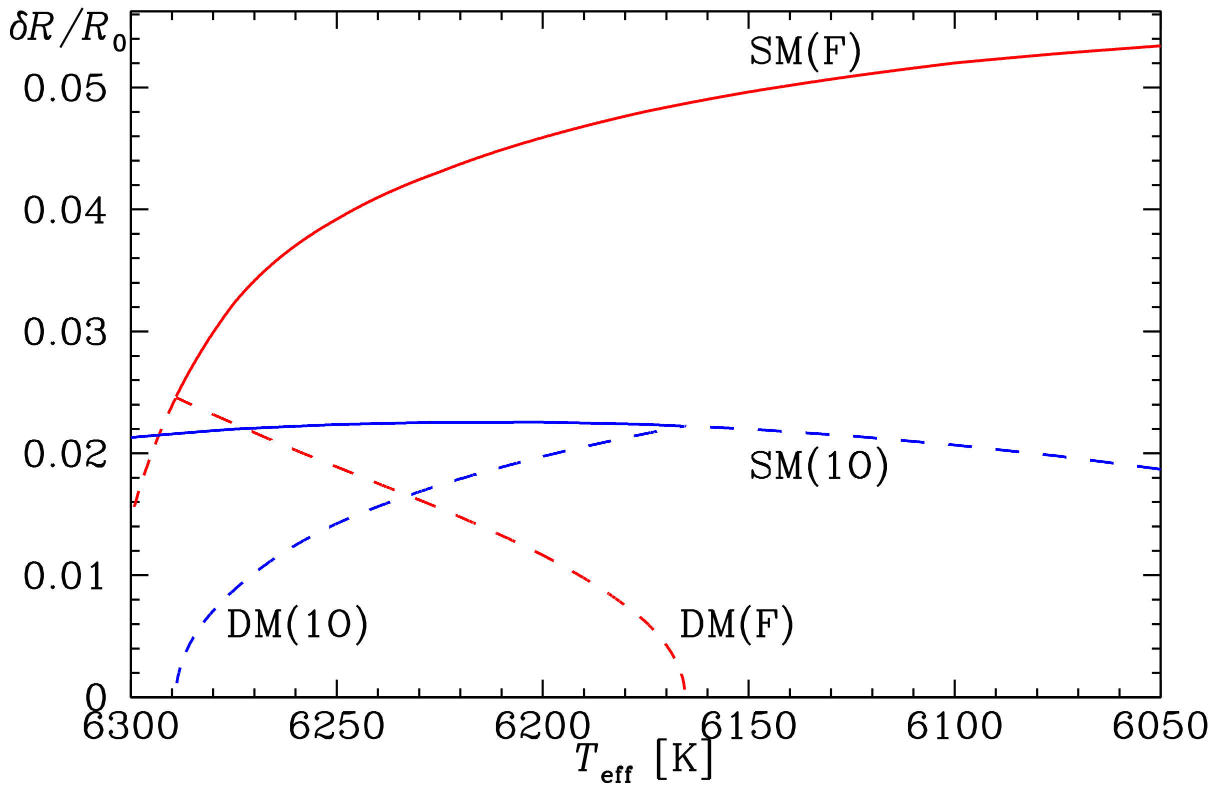




Smolec & Moskalik (2008)

Mode selection along sequence of Cepheid models

Amplitudes and stability of the fixed points:



⋆ only single-periodic solutions

- F only
- 1O only
- E/O either-or

Mode selection along sequence of Cepheid models

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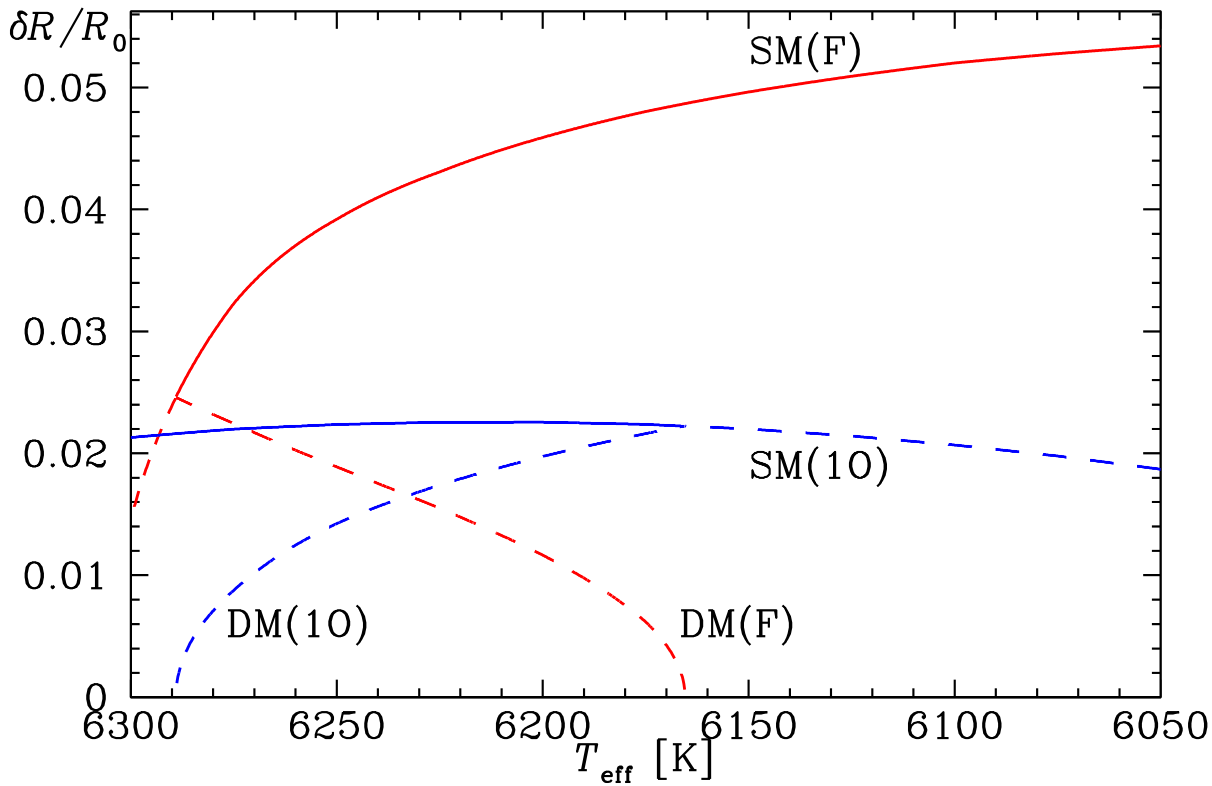
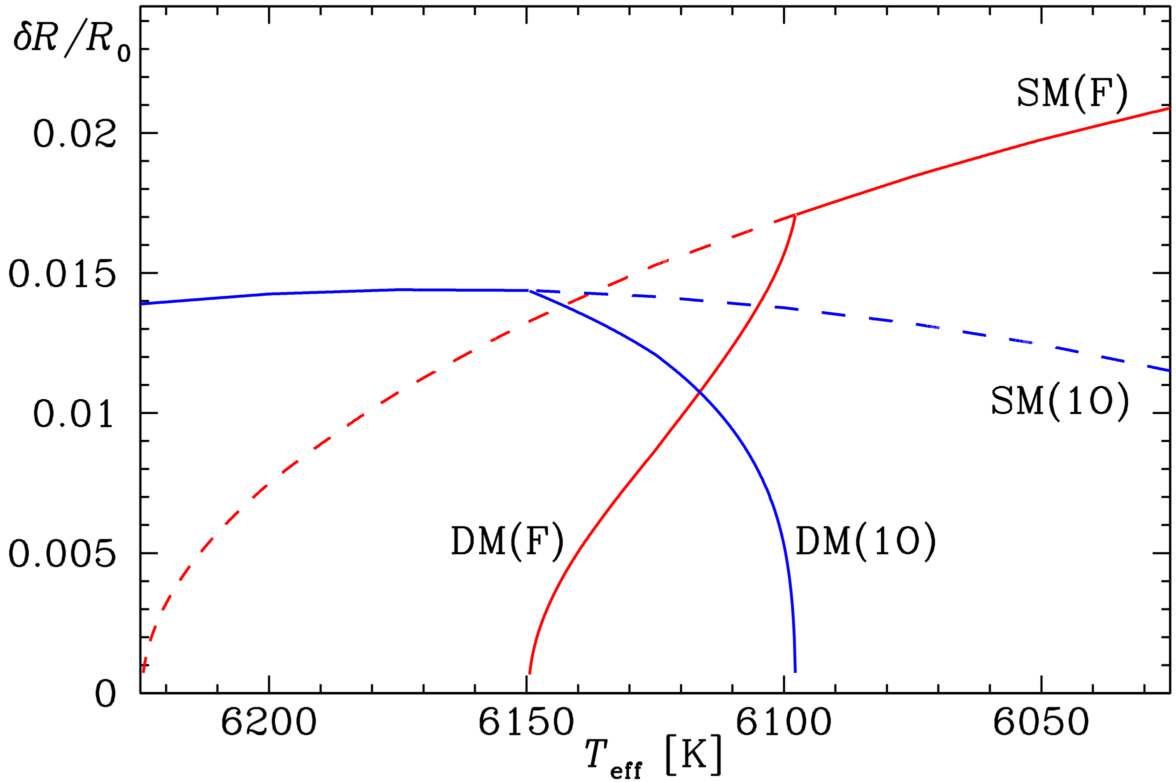


- ⋆ only single-periodic solutions
  - F only
  - 1O only
  - E/O either-or
- how to get DM?
  - neglect buoyancy in convectively stable regions

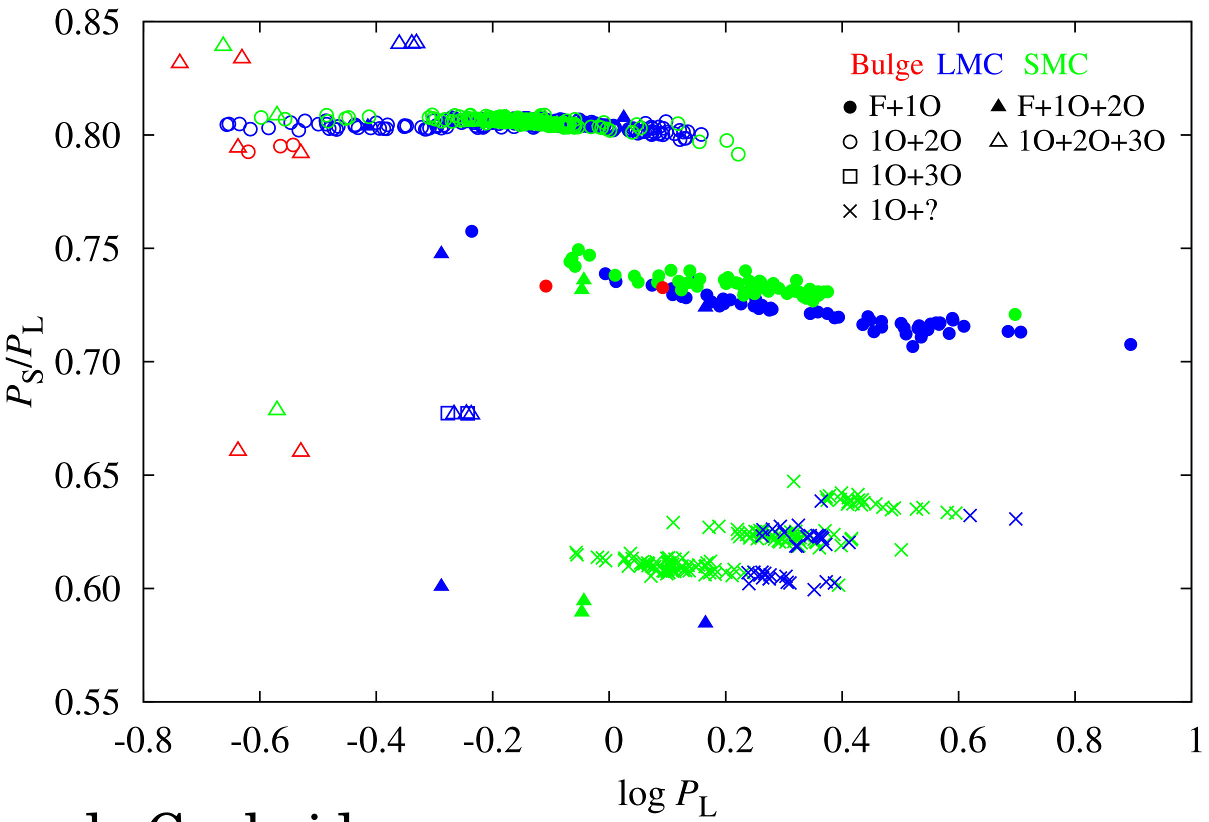
Mode selection along sequence of Cepheid models

Amplitudes and stability of the fixed points:



- ⋆ buoyancy neglected (Florida-Budapest approach; e.g. Kolláth et al. 1998, 2002)
  - ▸ DM is present, but
- ⋆ results from unphysical assumption!

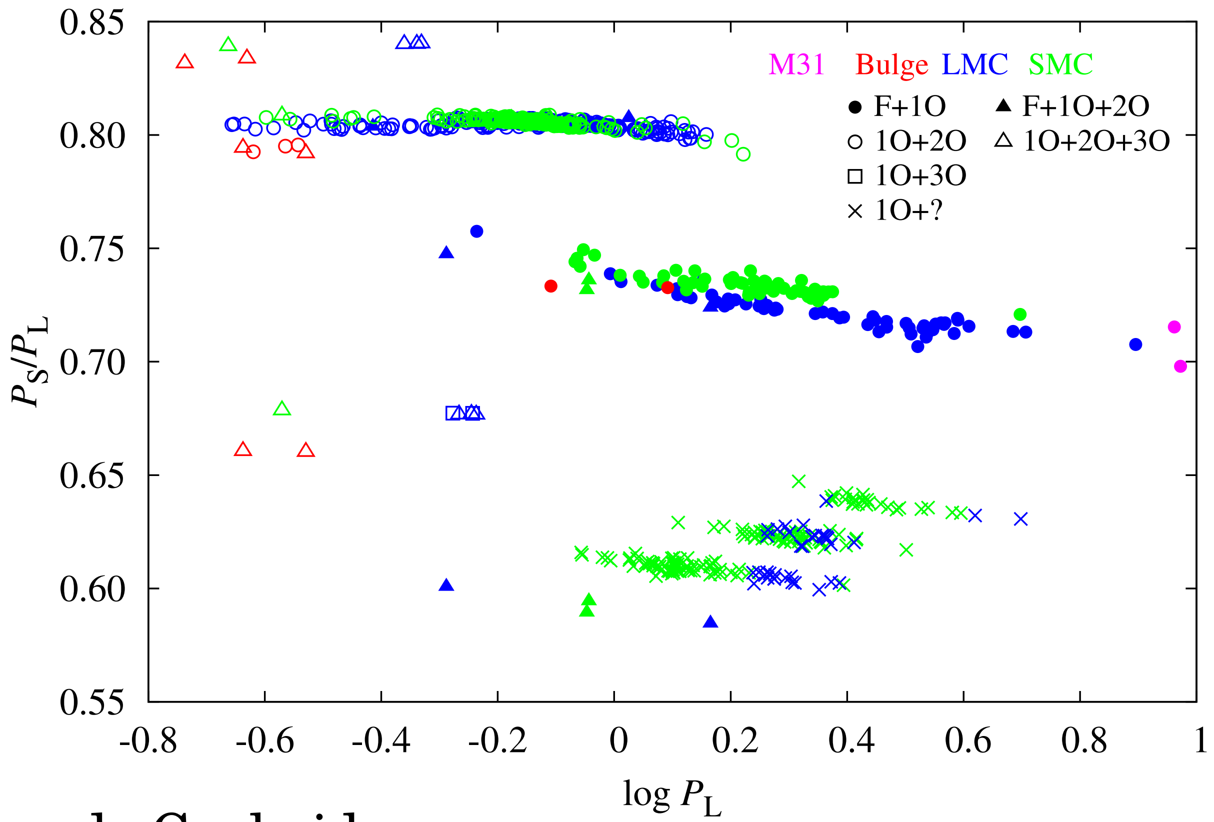
Cepheids and RR Lyrae stars – more puzzles



- triple mode Cepheids
- mysterious 1O/X stars

data: OGLEIII-CVS

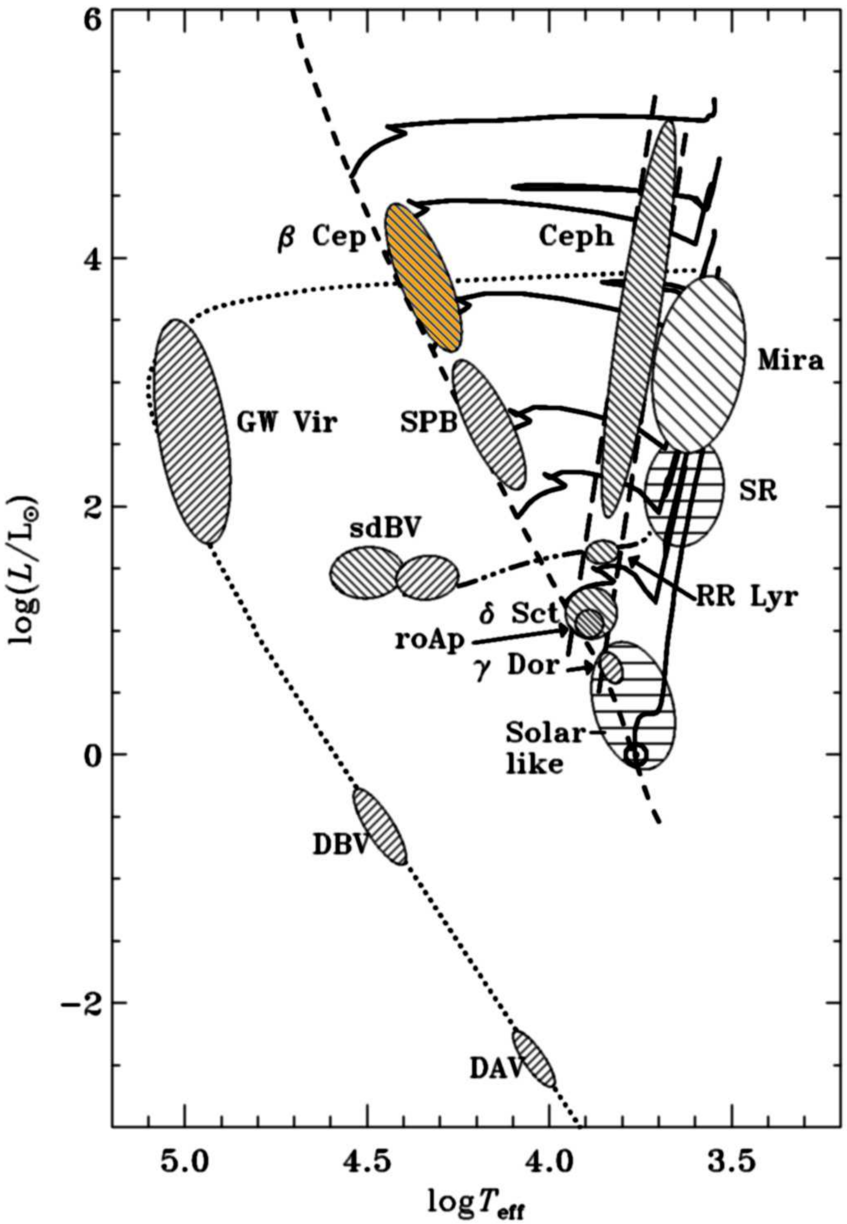
Cepheids and RR Lyrae stars – more puzzles



- triple mode Cepheids
- mysterious 1O/X stars
- F+1O at ~ 10 days (Poleski 2013) $\rightarrow$ *resonant* DM (Dziembowski & Kovács 1984); *modelling in progress*

data: OGLEIII-CVS

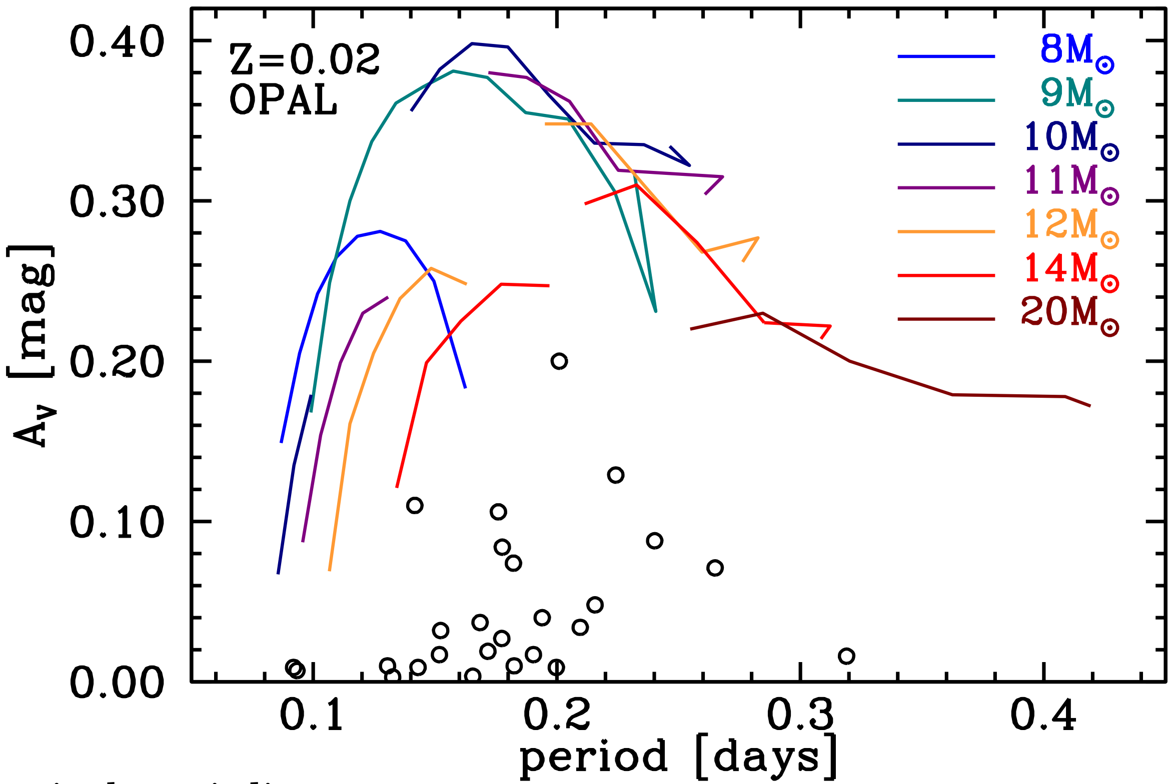
β Cephei stars



- low amplitude, mostly multi-periodic radial and non-radial pulsators
- a large group of single-periodic variables is also observed

figure: J. C.-D./Stellar Oscillations

Single mode saturation amplitudes:



○ single-periodic stars

Smolec & Moskalik (2007)

Collective saturation of the instability mechanism

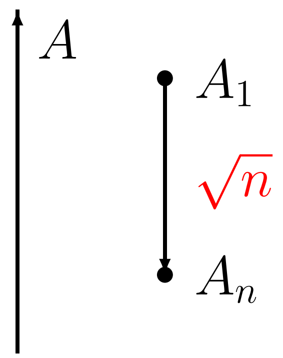
Nonresonant amplitude equations:

$$\frac{dA_i}{dt} = \gamma_i\left(1 - \alpha_{i1}A_1^2 + \ldots - \alpha_{in}A_n^2\right)A_i$$

Simplifying assumption: $\boldsymbol{\alpha_{i,j} \equiv \alpha}$

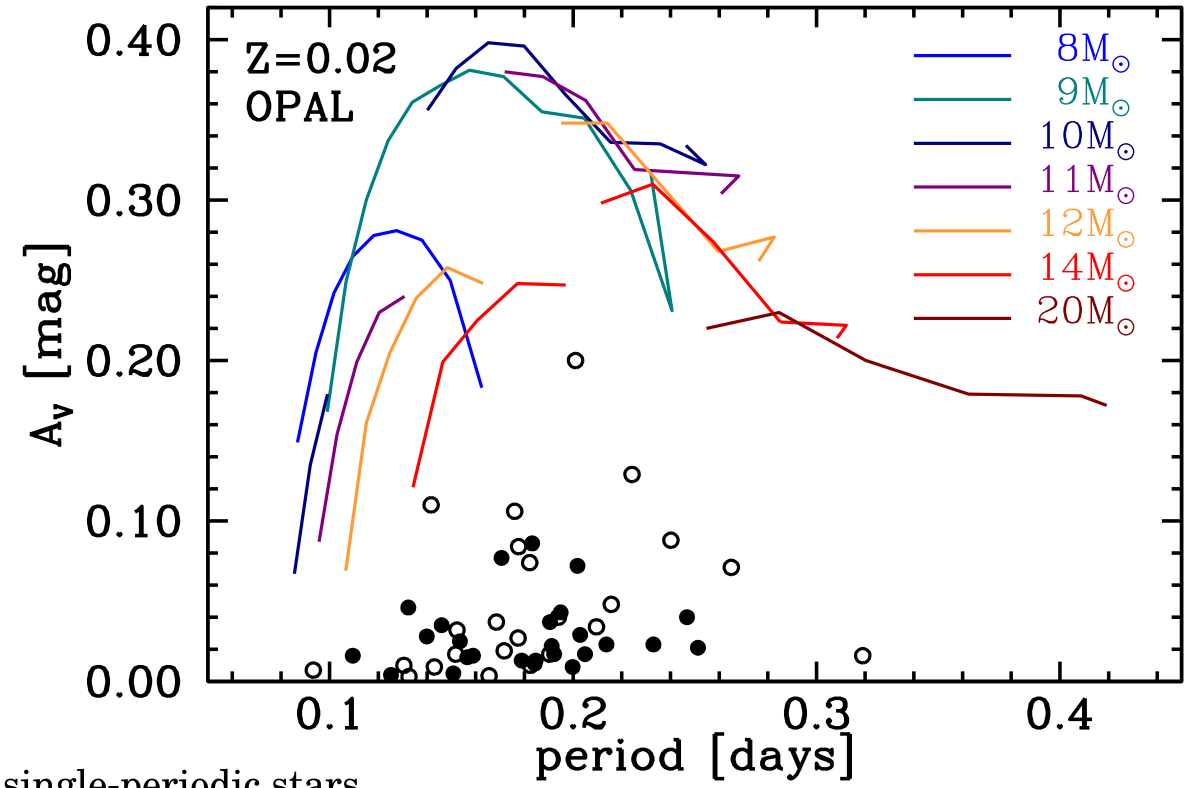
Amplitude saturation: $dA_i/dt = 0 \quad \Rightarrow \quad \mathbf{1 - n\alpha A_i^2 = 0}$

- single mode of amplitude $A_1 = 1/\sqrt{\alpha}$
- n modes of amplitude $A_n = 1/(\sqrt{n}\sqrt{\alpha})$



Collective saturation of the instability mechanism

Reduction of amplitudes:



○ single-periodic stars

● multi-periodic stars

Smolec & Moskalik (2007)

Collective saturation of the instability mechanism

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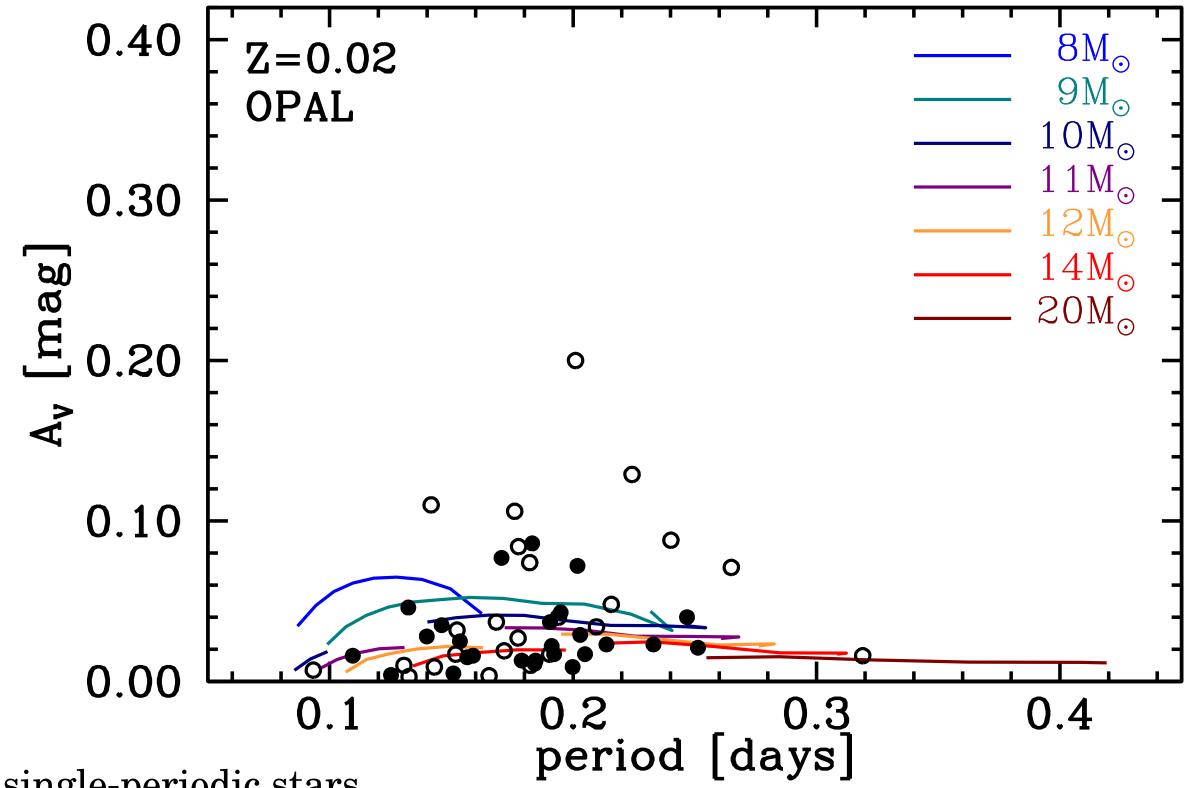

○ single-periodic stars

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Smolec & Moskalik (2007)

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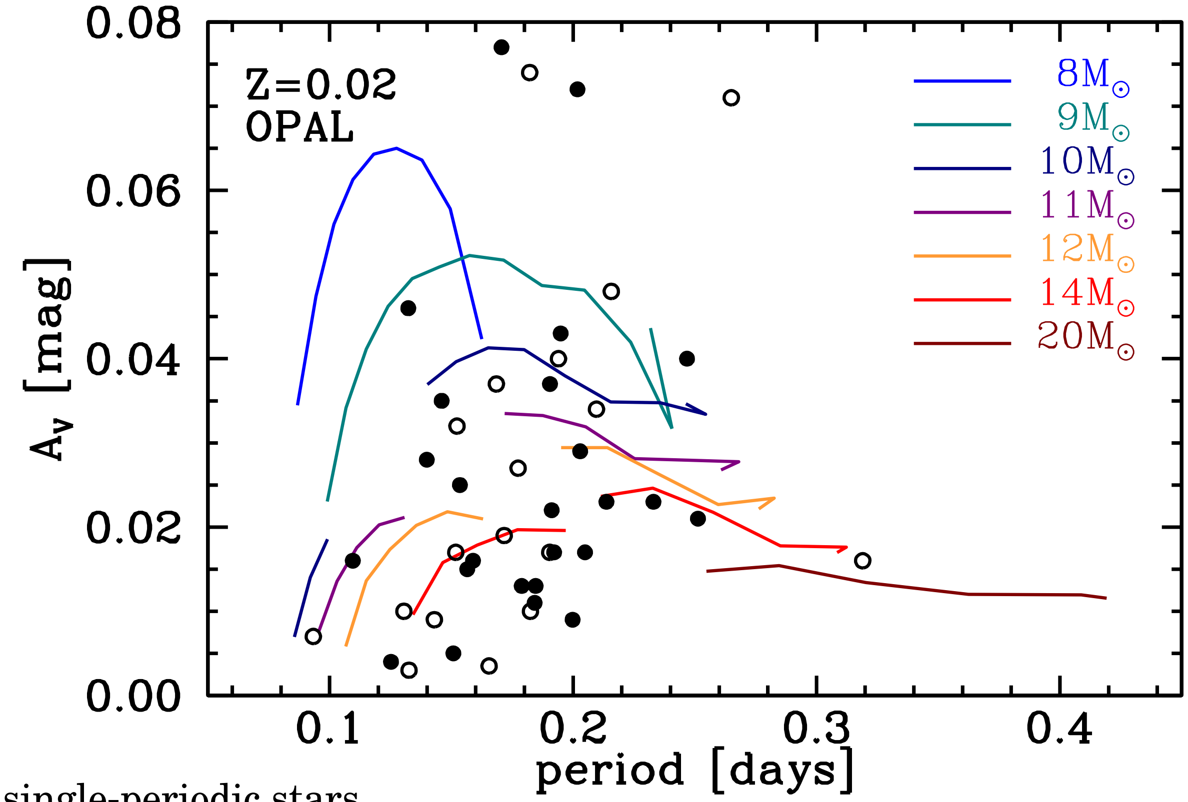

○ single-periodic stars

● multi-periodic stars

Smolec & Moskalik (2007)

Collective saturation of the instability mechanism

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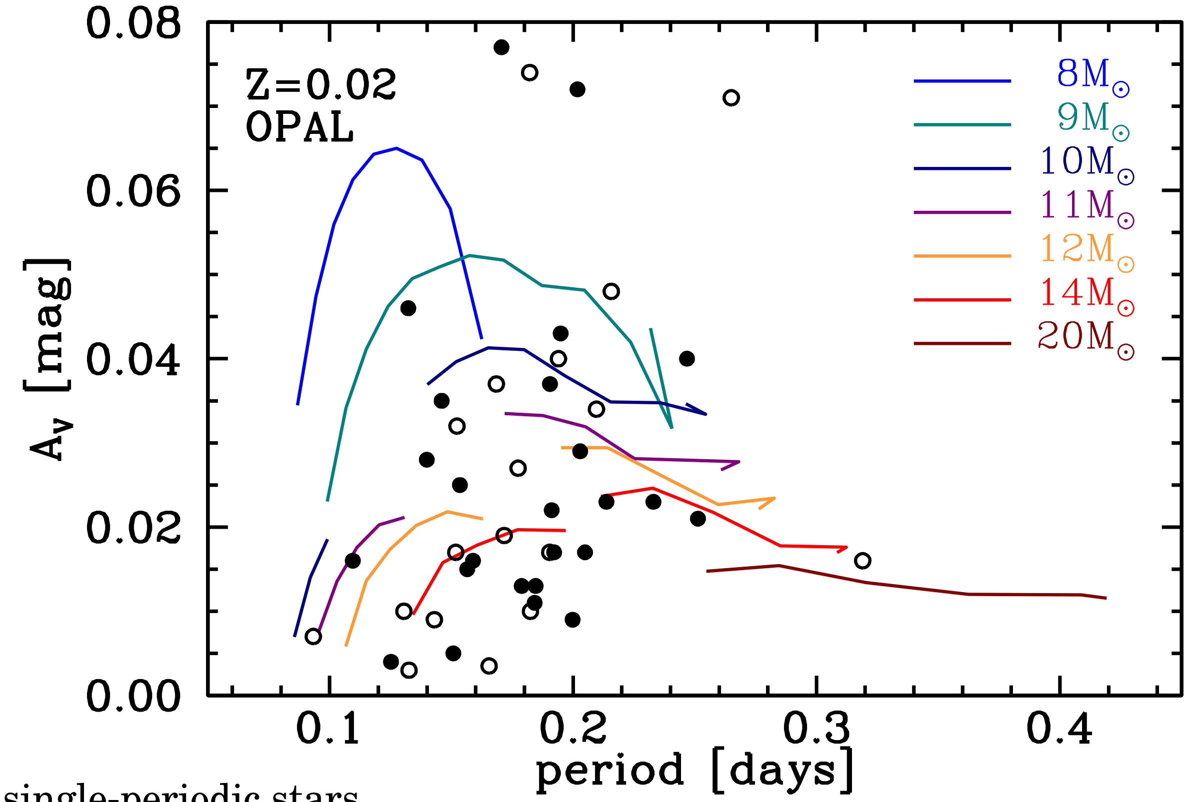


∘ single-periodic stars
• multi-periodic stars

only a fraction (~ 0.3) of the linearly unstable acoustic modes suffices

Smolec & Moskalik (2007)

Collective saturation of the instability mechanism

- collective saturation is sufficient to explain amplitude limitation

Line-broadening problem:

⋆ contributions from hundreds of modes, with different periods add up to form a line profile (*macro-turbulence*)

⋆ rms macro-turbulence velocity is roughly $V/\sqrt{n}$

⋆ this is of order of 100 km/s in our models – for many β Cephei stars observed line broadening is smaller

⋆ inclusion of g-modes into saturation process may solve the problem (photospheric variations of the g-modes should be smaller than those of p-modes (calculations by WD)

White dwarfs

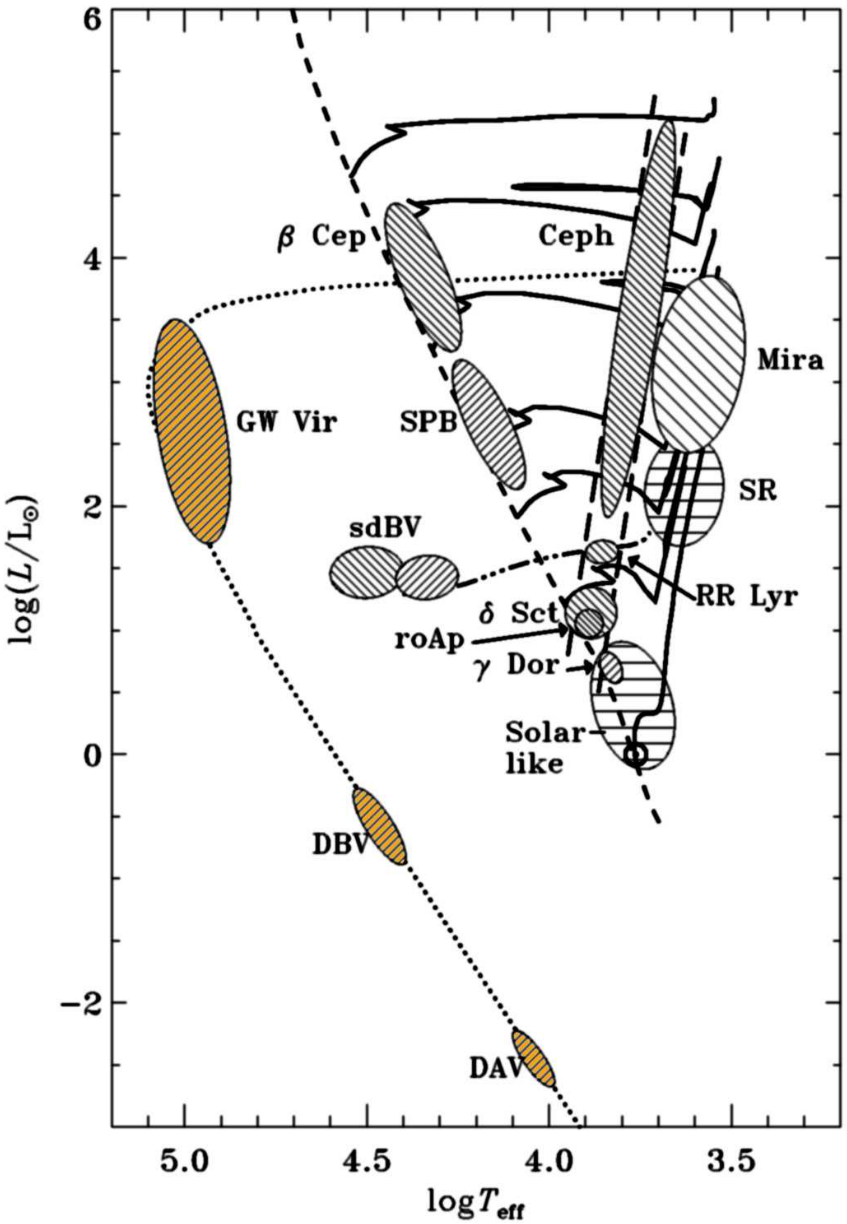
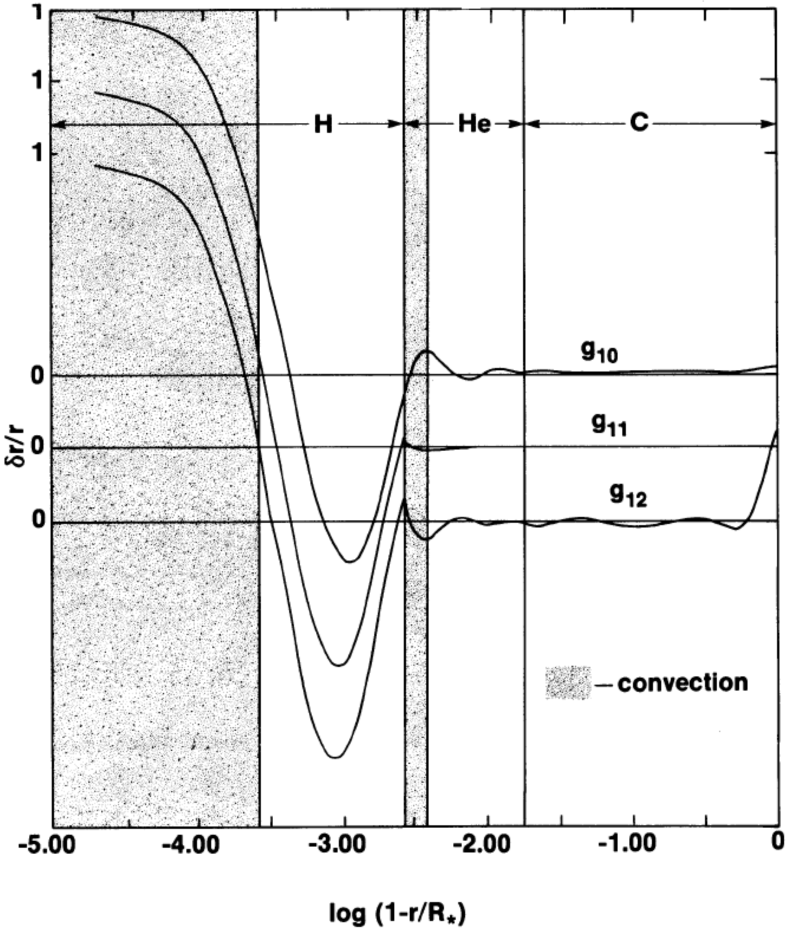
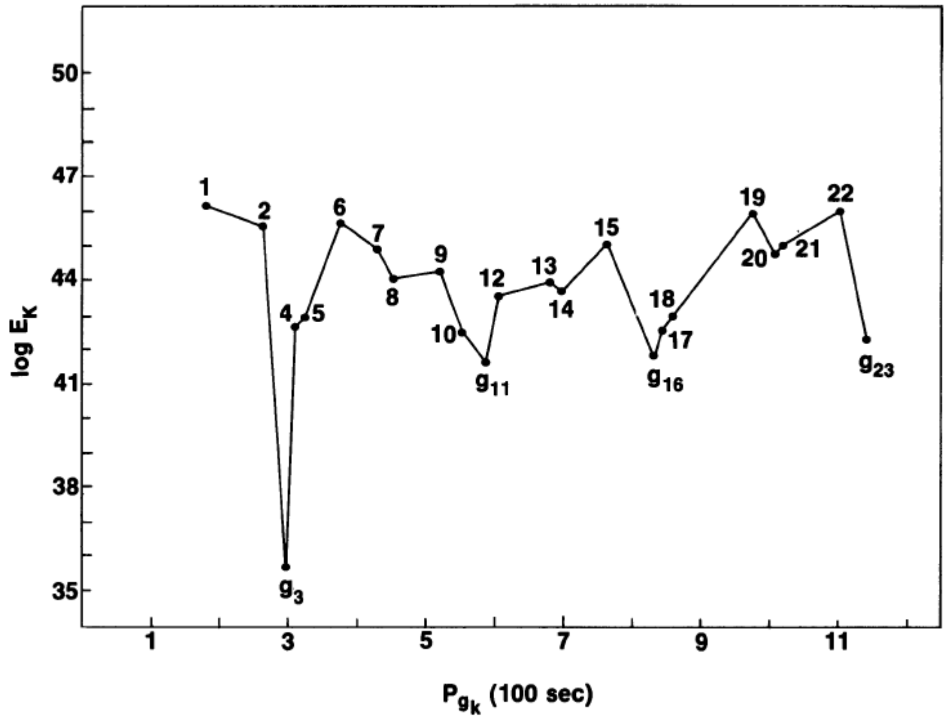


figure: J. C.-D./Stellar Oscillations

White dwarfs – mode trapping



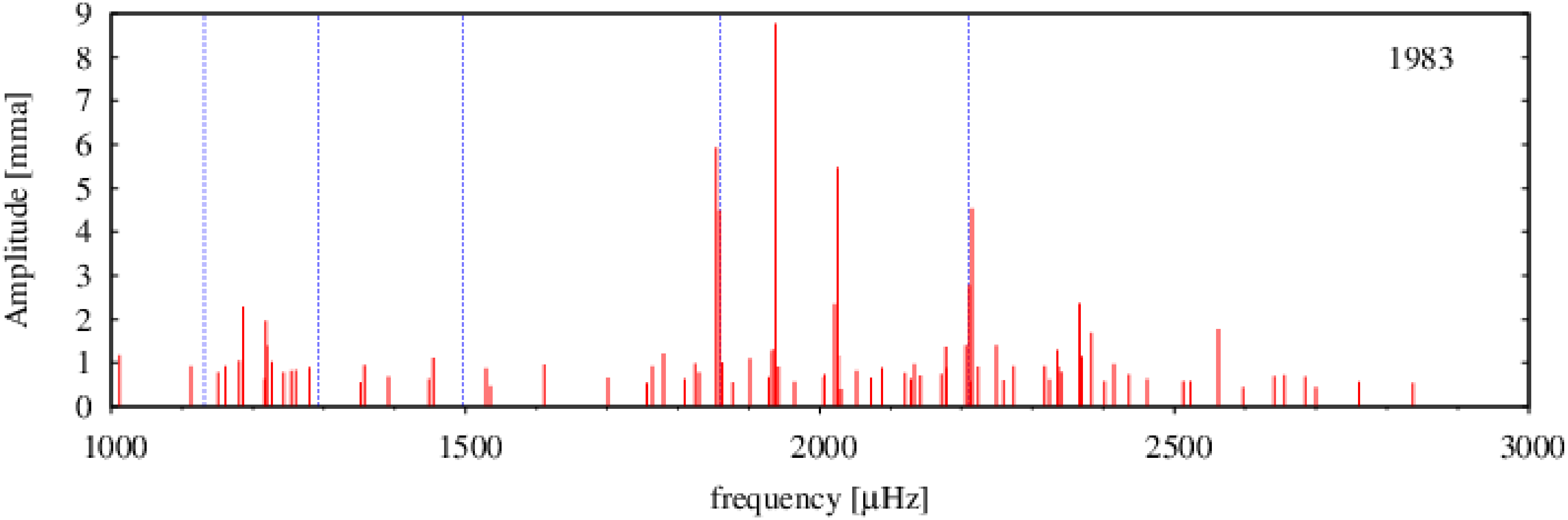


- cause: resonance between the wavelength of the oscillation and the thickness of the composition layers
- minima in the kinetic energy of the mode, $E_{\rm kin} = (\omega^2/2)I$

Winget, van Horn & Hansen (1981)

White dwarfs – mode trapping

PG 1159-035 – data from Costa et al. 2008



White dwarfs – mode trapping

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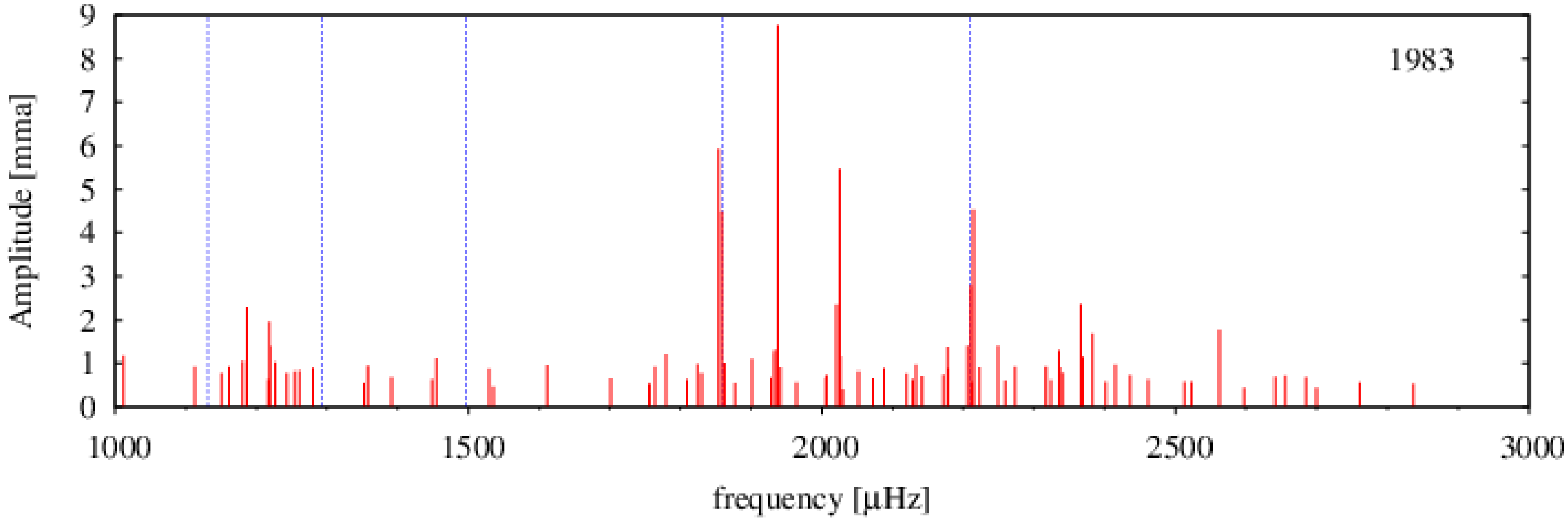
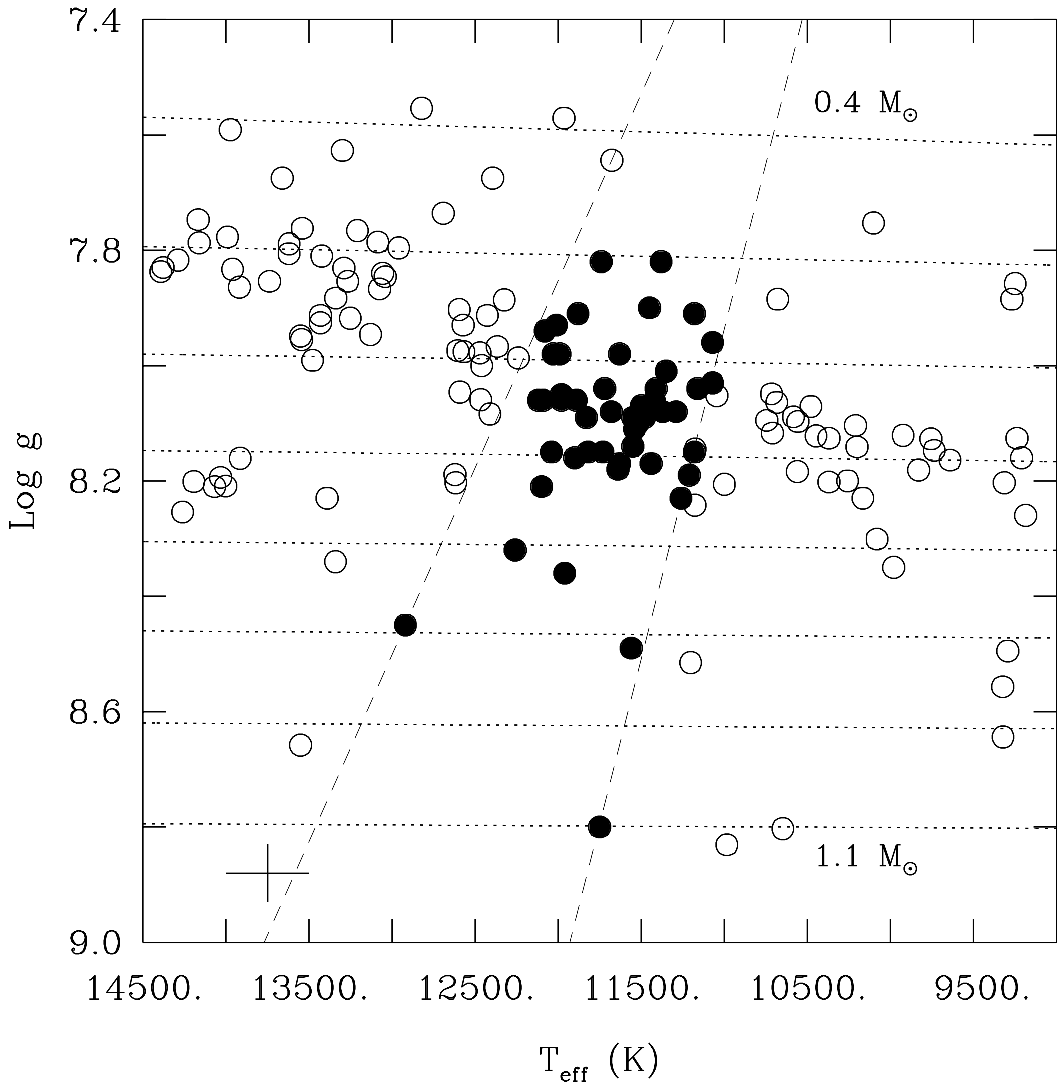


- the trapped modes are not the highest amplitude ones
- out of five $\ell = 1$ trapped modes only 2 present in all data sets
- amplitudes of pulsation modes vary
- models for ZZ Ceti stars with diffusive stratification (e.g. Corsico et al. 2002, Althaus et al. 2010) show much less pronounced minima in $E_{\rm kin}$

White dwarfs – *pure* ZZ Ceti instability strip



- result of systematic study aimed at defining the empirical IS for ZZ Ceti stars (Gianninas, Bergeron, Fontaine 2007)
- *also a talk by Barbara Castanheira today*

figure: Van Grootel et al. (2012) **adapted from** Fontaine & Brassard (2008)

δ Scuti stars

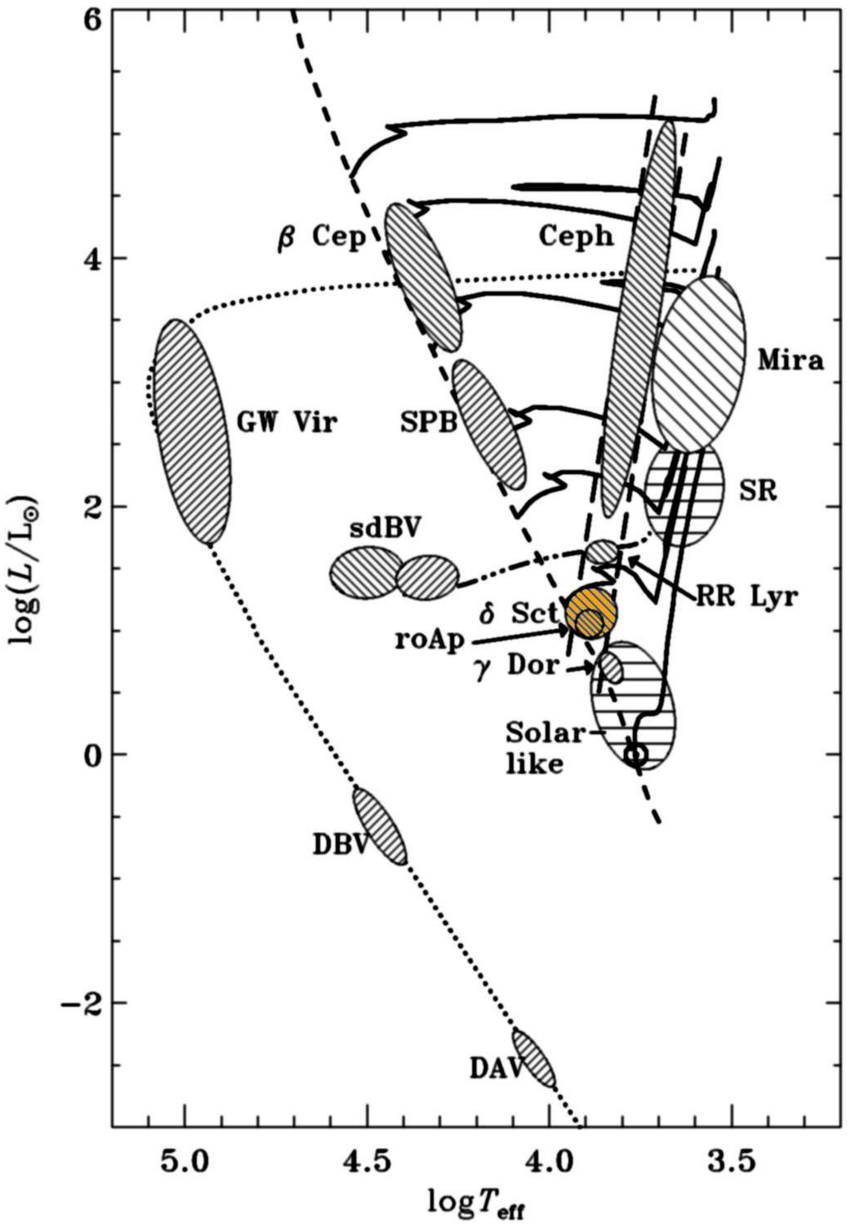


figure: J. C.-D./Stellar Oscillations

δ Scuti stars

- ⋆ low amplitude multi-mode pulsators
- ⋆ high amplitude δ Sct stars (HADS), one-two modes
- ▸ only 30-50 percent of stars in the IS pulsate (Breger 1979, 2000)
- ▸ saturation of the instability excluded – Stellingwerf's (1980) catastrophe ($\Delta M_{\mathrm{bol}} = 2.7\,$mag)

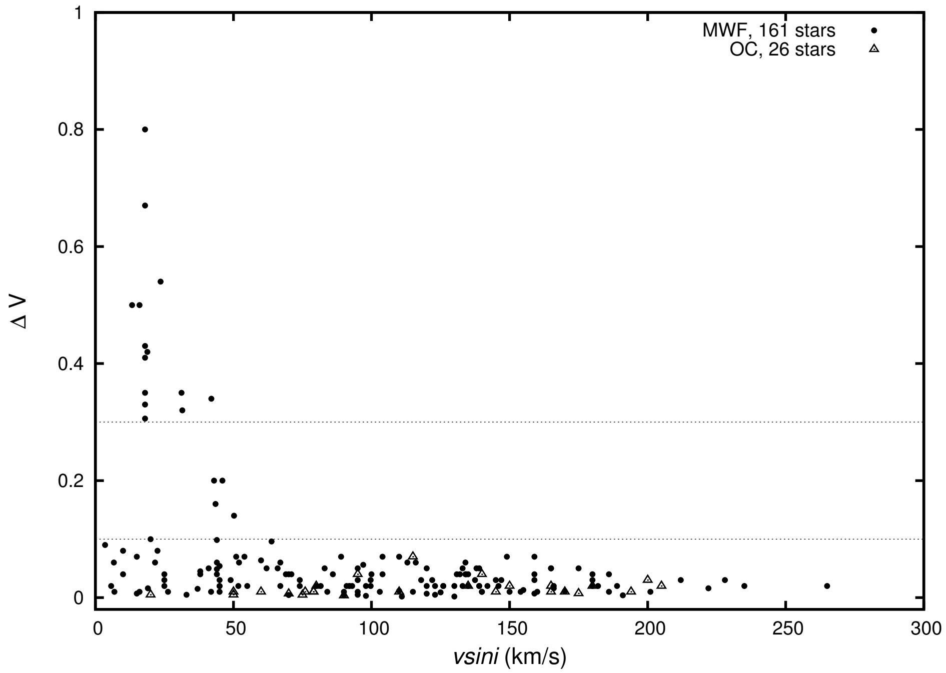


figure: Chang et al. (2013)

δ Scuti stars: parametric resonance

⋆ Dziembowski (1982): resonant mode interaction may be an effective amplitude limiting mechanism

- coupling of the unstable p-mode ($\gamma_a > 0$) to a pair of high degree, linearly damped ($\gamma_b, \gamma_c < 0$) g-modes; $\boldsymbol{\sigma_a = \sigma_b + \sigma_c + \Delta\sigma}$
- the parametric excitation starts when critical amplitude of the p-mode is exceeded (ν – quadratic coupling coefficient):

$$Q_a > Q_{a,c} = \sqrt{\frac{4\gamma_b\gamma_c}{\nu^2}\left[1 + \left(\frac{\Delta\sigma}{\gamma_b + \gamma_c}\right)^2\right]}.$$

δ Scuti stars: parametric resonance

Steady state solution is possible:

$$Q_a = \sqrt{\frac{4\gamma_b\gamma_c}{\nu^2}(1+q^2)}\,, \quad Q_{b,c} = Q_a\sqrt{-\frac{I_a\gamma_a\sigma_a}{I_{b,c}\gamma_{b,c}\sigma_{b,c}}}\,,$$

and is stable if

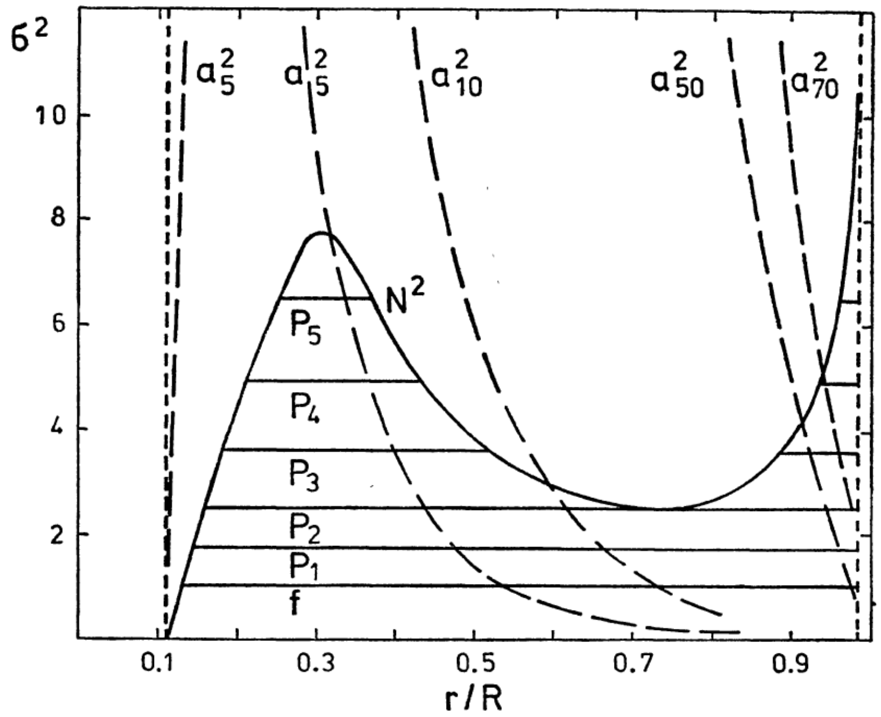
$$\gamma = \gamma_a + \gamma_b + \gamma_c < 0\,,$$

$$4\gamma^3(1+q^2) - (1+3q^2)\left[4\gamma_a\gamma_b\gamma_c(1+3q^2) + 2\gamma^3(1+q^2) - 8q^2\gamma(\gamma_a\gamma_b + \gamma_a\gamma_c + \gamma_b\gamma_c)\right] > 0\,,$$

where $q = \Delta\sigma/\gamma$.

δ Scuti stars: parametric resonance

⋆ Dziembowski & Królikowska (1985) – application to δ Sct stars



⋆ low order p-modes can couple to

▸ *global* g-modes ($\gamma_a \ll -\gamma_{b,c}$)

⋆ high order p-modes couple to

▸ *inner* g-modes (large n and ℓ)

$\gamma_a > -(\gamma_b + \gamma_c)$

▸ *outer* g-modes ($n = 1, 2$)

$\gamma_a \ll -\gamma_{b,c}$, $\nu \gg 1$

▸ strong coupling arises only if the radial orders of the gravity modes are close $\Rightarrow \sigma_b \approx \sigma_c \approx \sigma_a/2$

▸ many g-mode pairs may be excited through the parametric resonance $\rightarrow$ probability distribution for p-mode amplitude at onset of the instability

δ Scuti stars: parametric resonance

- the mean value of critical amplitude exceeds 0.01 mag only for the lowest frequency modes (and is always below 0.02 mag)
- for lower frequency p-modes
  - there is a large probability that equilibrium is stable with $Q_a \approx Q_c$ and much smaller amplitudes of the g-modes

δ Scuti stars: parametric resonance

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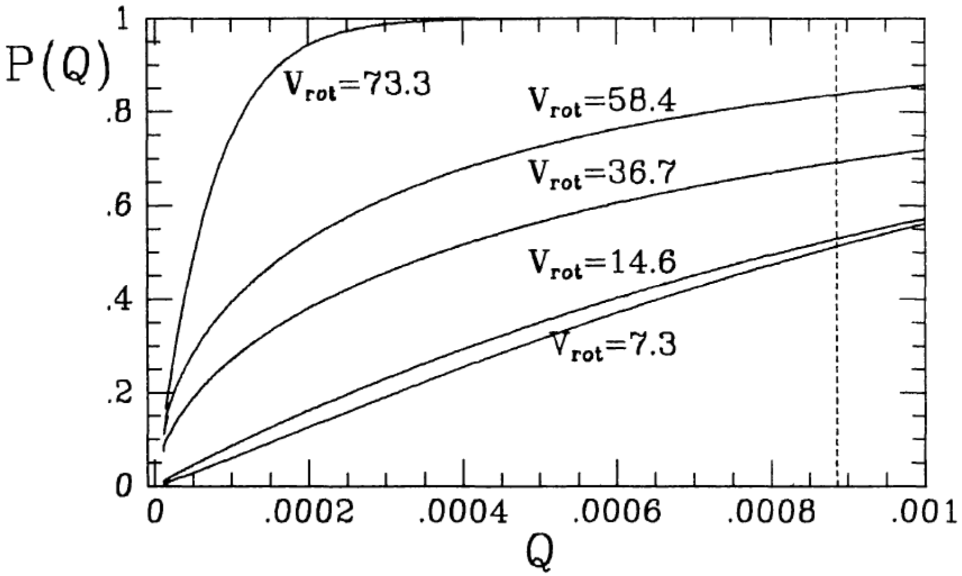
⋆ Dziembowski, Królikowska & Kosovitchev (1988) studied the effects of rotation, which are

  - frequency degeneracy in m is lifted ($m_a = m_b + m_c$) and frequency spectrum of g-modes is denser – allows for precise tuning of the resonance
  - changes to the growth rates (negligible), coupling coefficient and probability distribution for the coupling coefficient

δ Scuti stars: parametric resonance



probability that the parametric instability of the F mode occurs at the amplitude $\Delta R/R$ less than Q

- effects of rotation significant starting from $20\,$km/s
- at $60\,$km/s probability that the instability occurs before F mode reaches $0.01\,$mag reduces from 0.5 to 0.1
- explains the difference between HADS and normal δ Sct stars

figures: Dziembowski, Królikowska & Kosovitchev (1988)

δ Scuti stars: parametric resonance

- for higher frequency p-modes
  - coupled to the *inner* g-modes: equilibrium is unstable (unbounded amplitude growth
  - coupled to the *outer* g-modes: equilibrium is most likely unstable

→ excitation of new g-mode pairs is inevitable

δ Scuti stars: parametric resonance – dynamical solutions

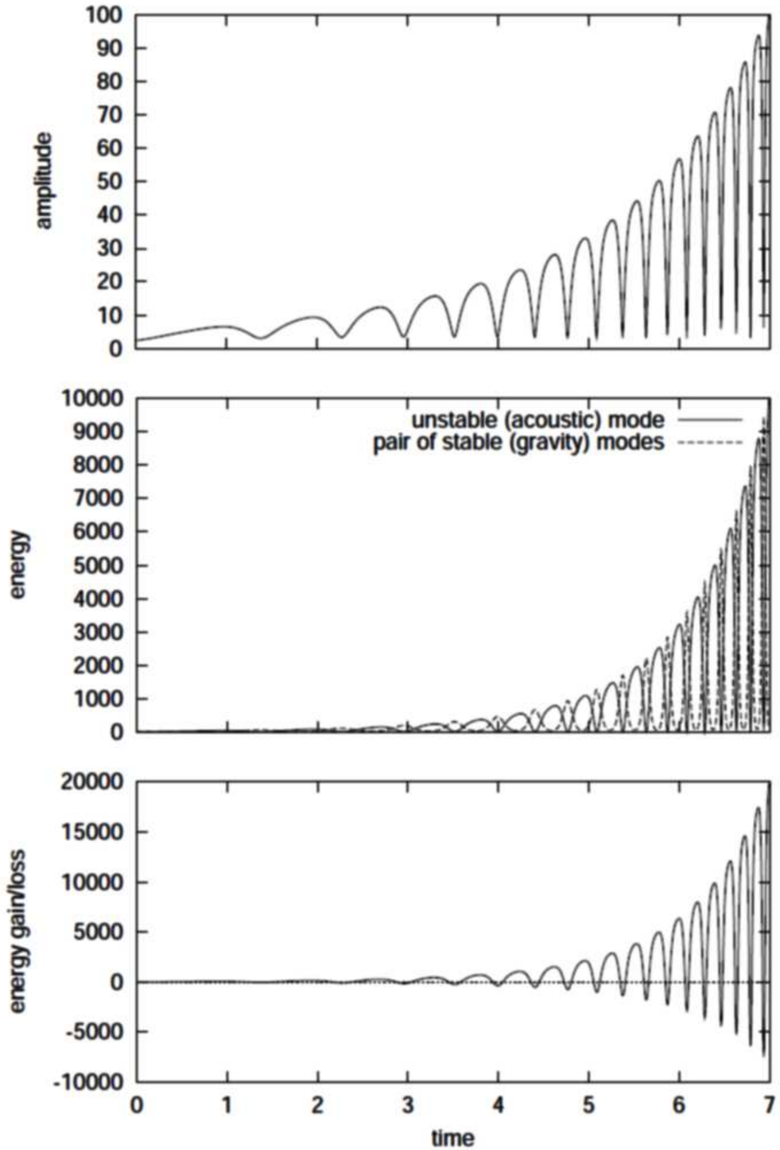


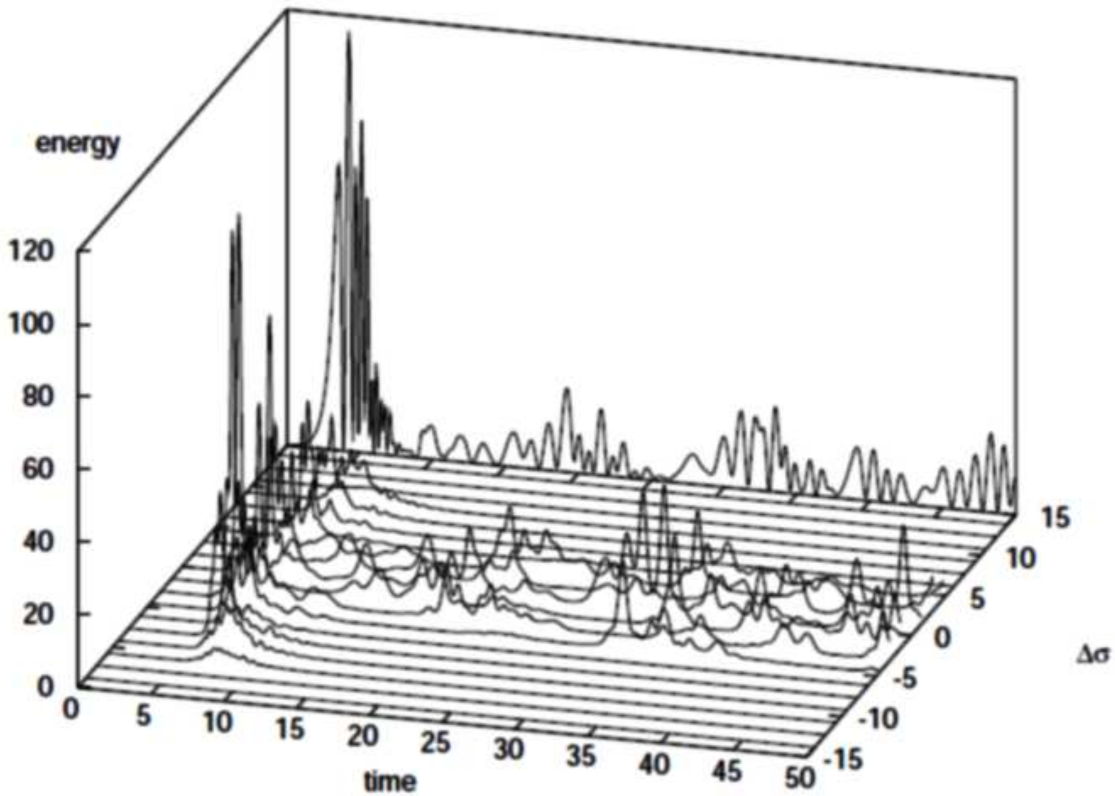
figure: Nowakowski (2005)

Nowakowski (2005)

⋆ interaction with single g-mode pair ($-\gamma_{b,c} < \gamma_a$; *inner* g-modes)

- case of unbounded amplitude growth (no stable equilibrium and no stable limit cycle)
- g-mode damping is low and the pair does not manage to loose enough energy to balance the unstable mode driving

δ Scuti stars: parametric resonance – dynamical solutions



Nowakowski (2005)

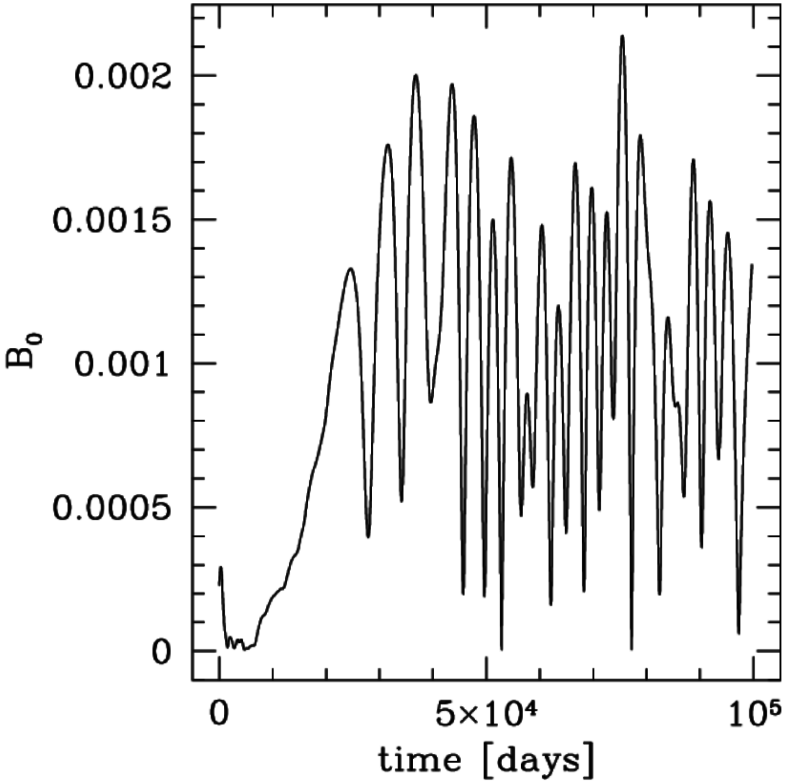
⋆ interaction with ensemble of g-mode pairs

- static multi-mode solution does not exist
- irregular variability of the p-mode amplitude on the timescale given by γ_a^{-1}

figure: Nowakowski (2005)

δ Scuti stars: parametric resonance – dynamical solutions

application to XX Pyx model



acoustic $\ell = 2$ mode together with an ensemble of several hundred inner g-mode pairs

Results

- mode amplitudes are too high as compared with observation even taking into account the rotation
- resonant mode coupling cannot be a dominant amplitude limiting effect in evolved δ Sct stars.
- saturation of the driving mechanism must play a role

figure: Nowakowski (2005)

δ Scuti stars: mode trapping (Dziembowski & Królikowska 1990)

- mode trapping significant only for $\ell = 1$ frequencies of which are close to the radial mode frequencies
- trapping is significant for evolved models
- for evolved models the parametrically excited g modes are deep interior modes and hence strong interaction with modes trapped in the envelope is less likely

⋆ this selection rule (mode trapping) relies only on the linear theory → non-linear theory needed

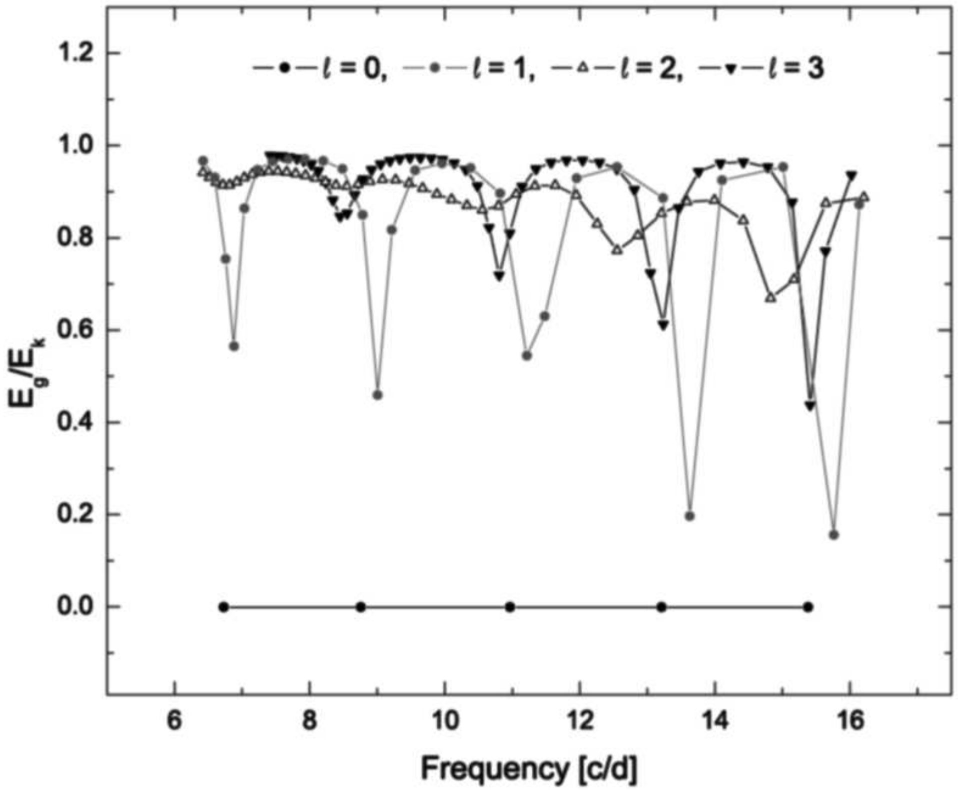
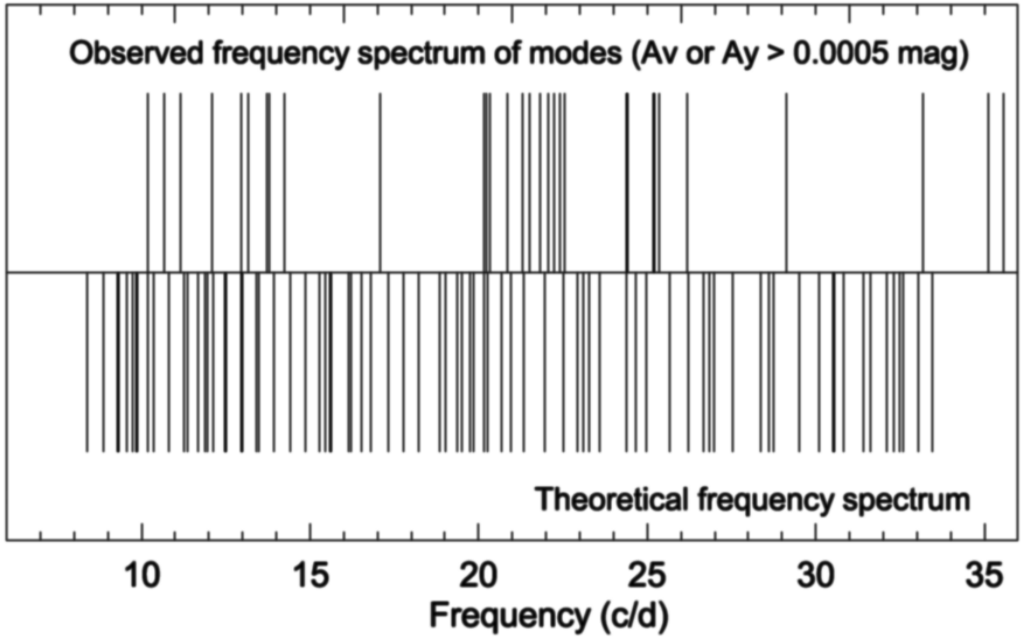


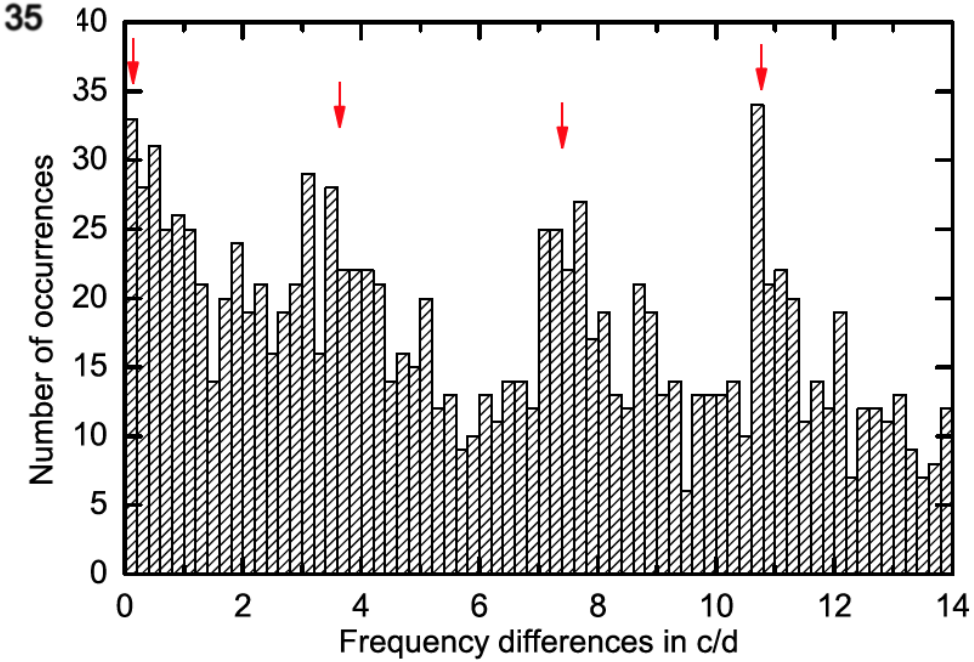
figure: Breger, Lenz & Pamyatnykh (2009)

δ Scuti stars from ground – FG Vir



- 68 independent frequencies
- predicted spectrum is denser than observed

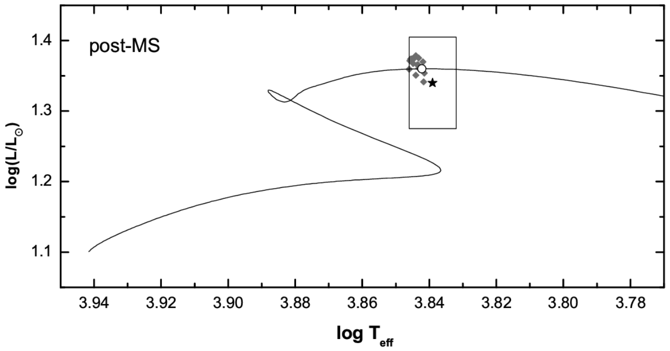
- preferred spacing between the non-radial modes, which corresponds to the spacing between radial modes
- mode trapping?
- Caution!

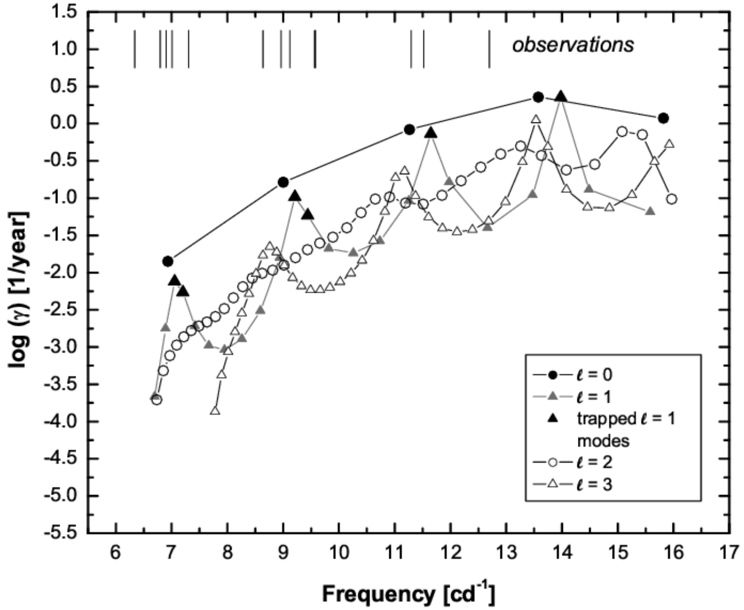


Breger, Lenz & Pamyatnykh (2009)

δ Scuti stars from ground – 44 Tau

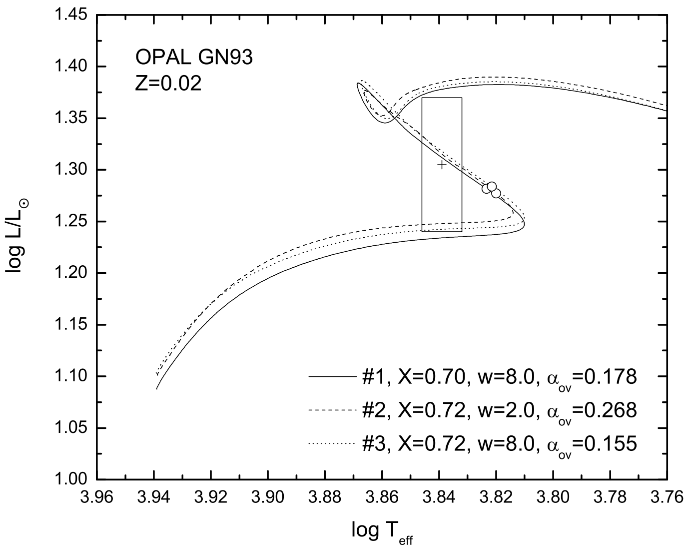
Lenz et al. (2008): post-MS

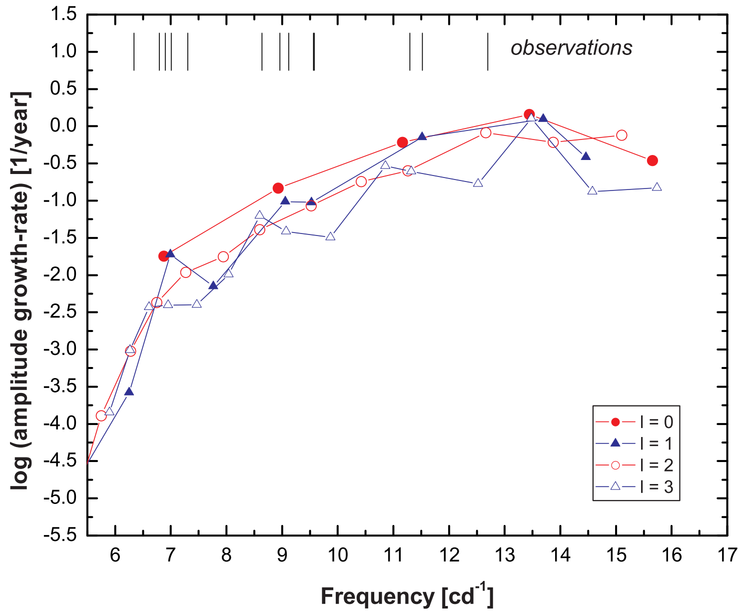




trapping?

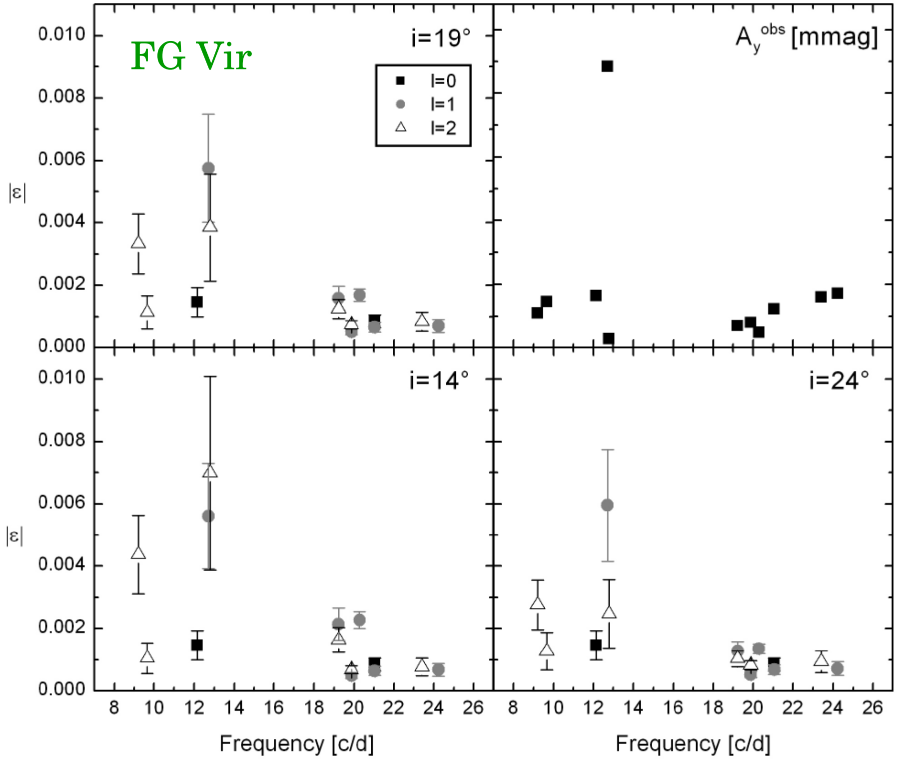
Lenz et al. (2010): post-MS contraction

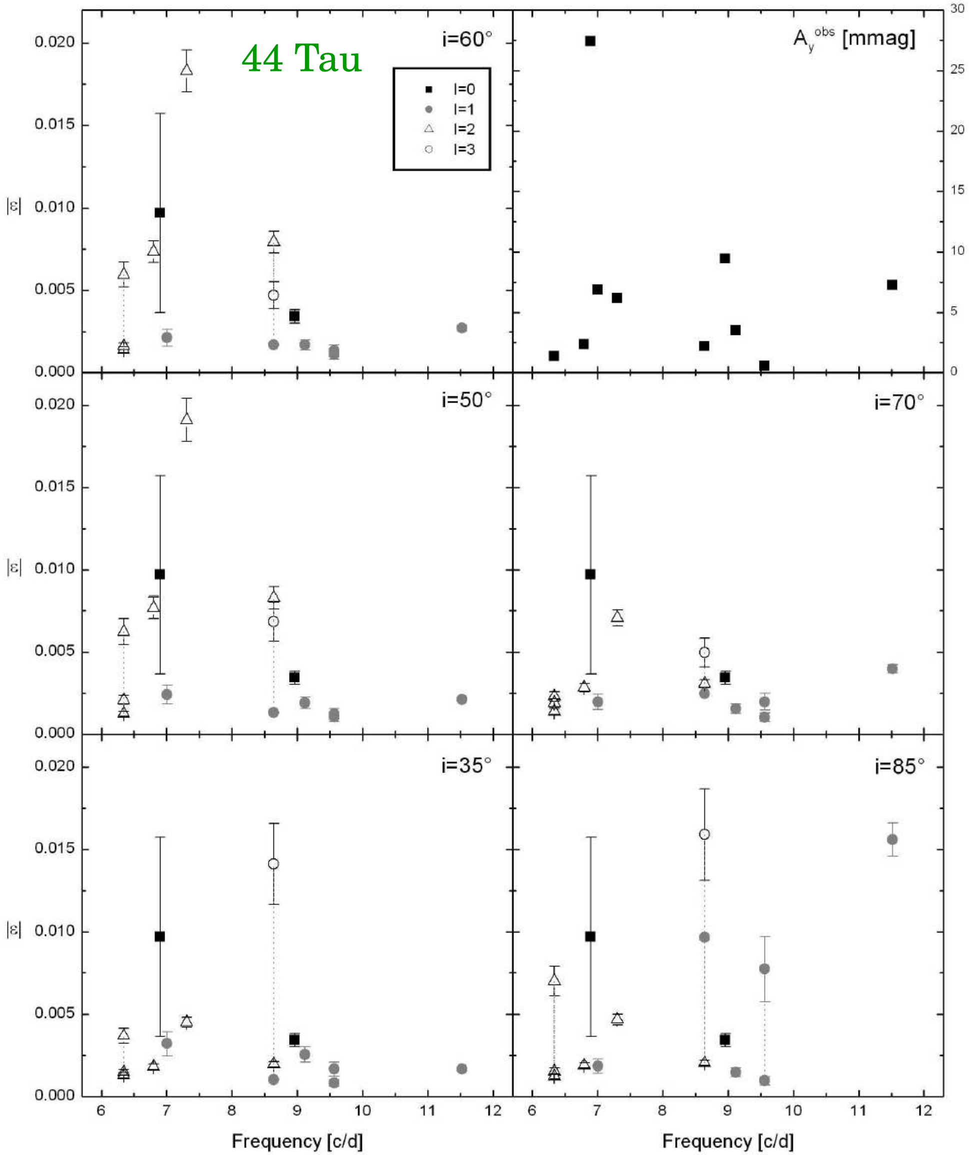




no trapping!

δ Scuti stars from ground – intrinsic amplitudes





- in both stars some $\ell = 2$ modes have very high intrinsic amplitudes

figures: Lenz et al. (2008b)

δ Scuti stars from space

Excitation and visibility of high-degree modes in stars

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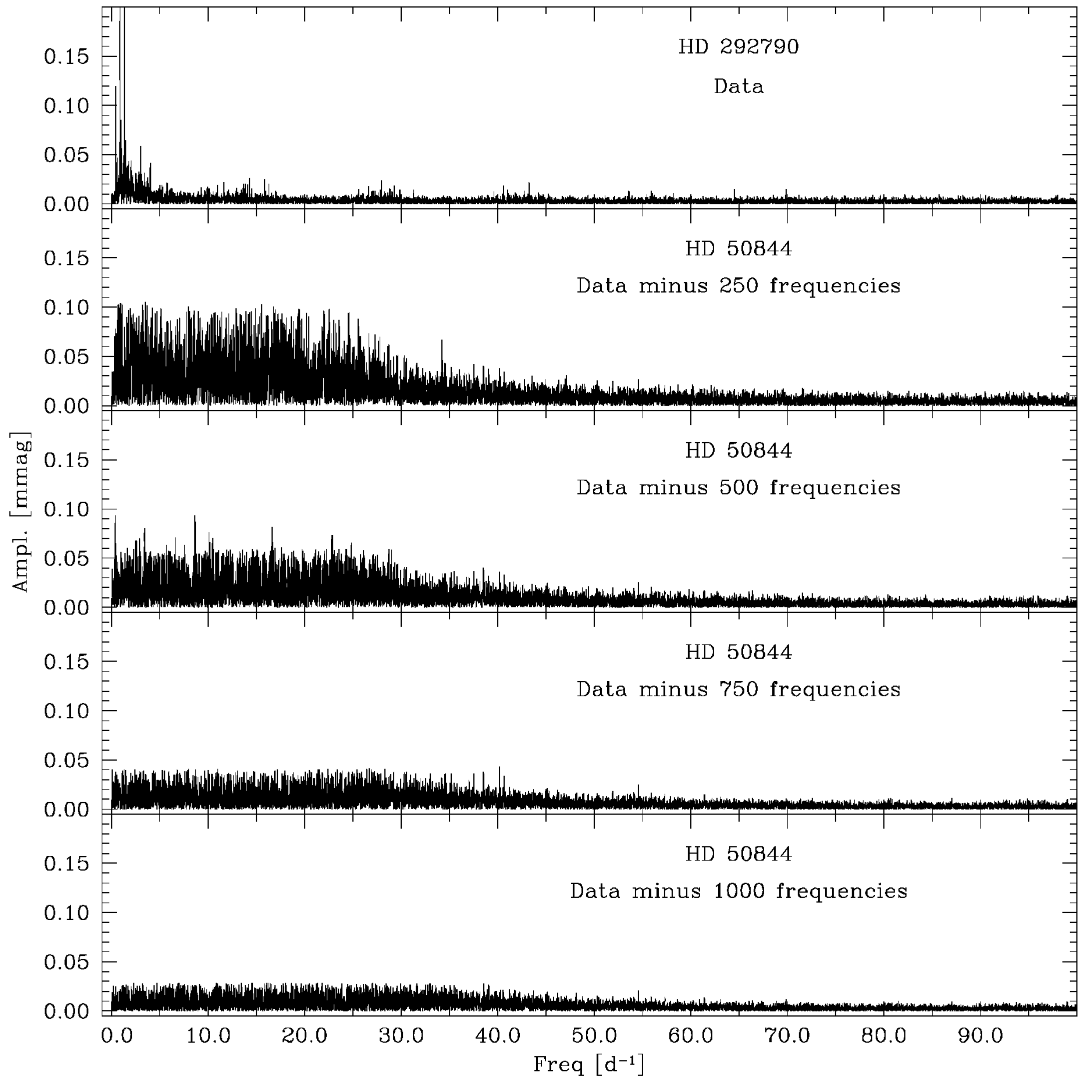
ABSTRACT
Observational evidence for excitation of non-radial modes in stars is confronted with the results of linear stability surveys for stellar models. We consider various types of pulsators on the upper main sequence as well as stars in the Cepheid strip. Our stability survey covers the whole range of spherical harmonic degrees, ℓ, where instability is found. There is fair agreement between the theoretical instability strip and the location of ζ Oph stars, but the observed and calculated periods do not agree in some stars. We suggest that either pulsation is not responsible for the ζ Oph phenomenon or else there are serious errors in mode identification in these cases. We do not find instability at long periods for early B-type stars, supporting the idea that pulsation is not responsible for the periodic variations in Be stars. The agreement between the observed and calculated periods of high-degree modes in δ Sct stars is not very satisfactory. This is attributed to problems in mode identification. We discuss unstable modes of high degree in Cepheid models as a possible mechanism for the low-amplitude radial velocities seen in some stars within the instability strip. We find, however, that the observed periods are at least a factor of 2 longer than the calculated periods. Finally, we discuss the possibility of observing modes of high degree photometrically. We suggest that a large number of high-degree modes may become detectable by future space-borne photometric missions. The confusion arising from these modes may greatly reduce the value of such observations for asteroseismology. However, they will be very important in studying the mechanism of mode selection.

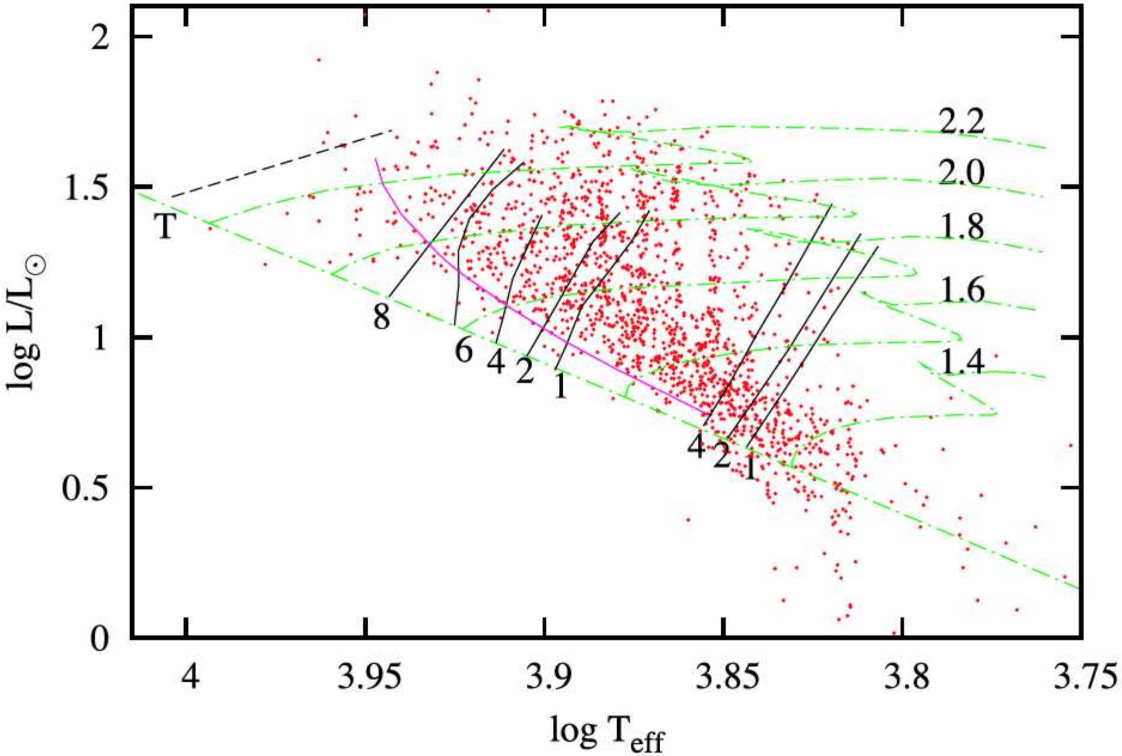
δ Scuti stars from space – HD 50844 (by CoRoT)



- ⋆ 1000+ frequencies identified
- ⋆ degrees up to $\ell = 14$ (spectroscopy)
- ▸ the star is *unique* – in no other *CoRoT* and *Kepler* stars such large number of frequencies are reported
  - ▸ non-white granulation background noise, (< 100 of the frequencies are p-modes) (Kallinger & Matthews 2010)?

Poretti et al. (2009)

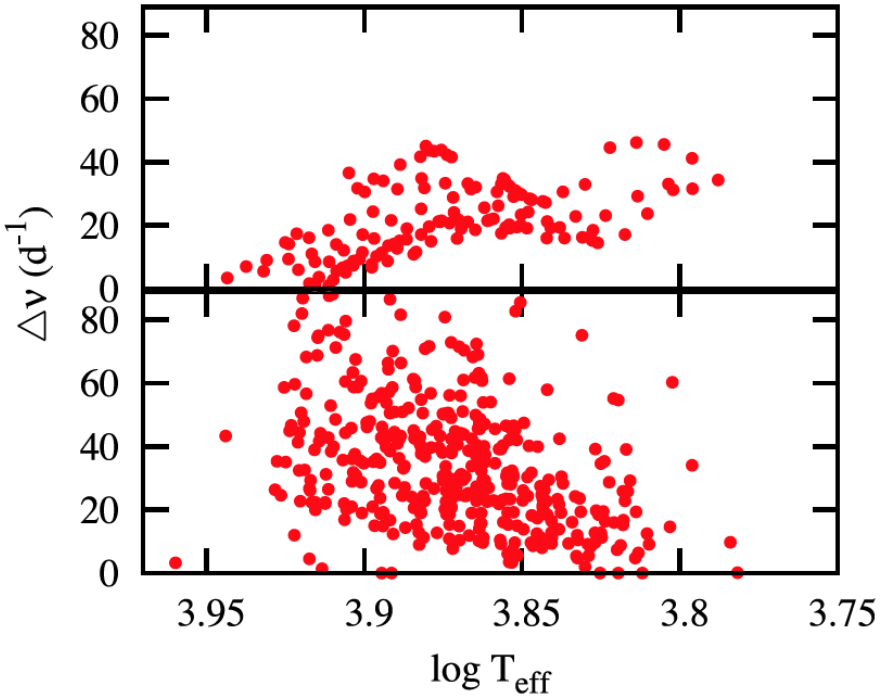
δ Scuti stars – Kepler observations (Balona & Dziembowski 2011)



- ⋆ 1568 δ Sct stars
- ⋆ high radial orders
- ⋆ high-ℓ trapped modes
  - ▸ $\ell > 12$: strongly unstable modes trapped in the envelope
  - ▸ practically no δ Sct stars at the expected region

figure: Balona & Dziembowski (2011)

δ Scuti stars – Kepler observations vs. linear theory



freq. range of unstable modes ($0 \leq \ell \leq 2$) (models)
and freq. range as a function of T_{eff}

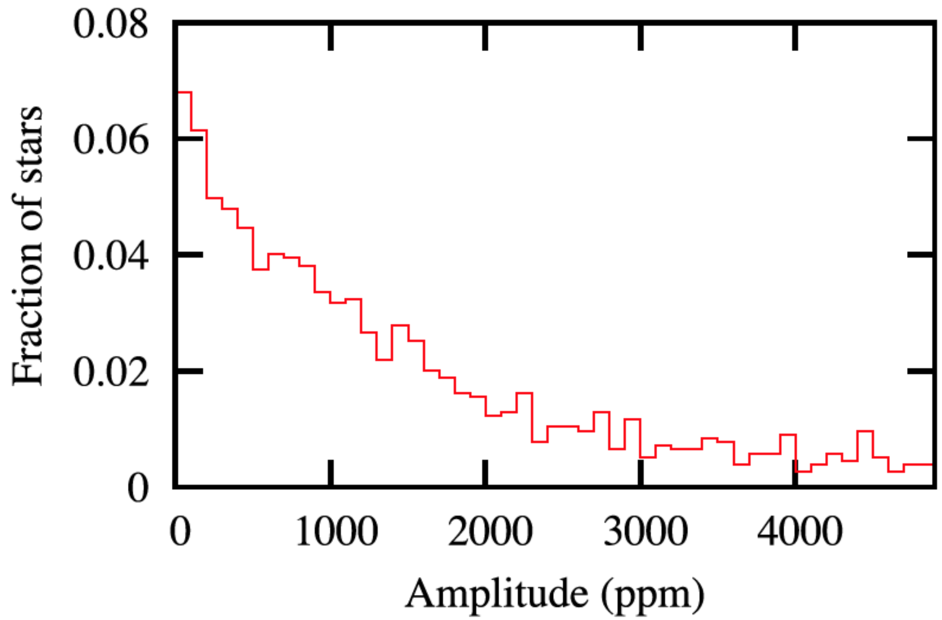
- observed freq. range is significantly higher than models predict

  $\rightarrow$ *rotational splitting, mode identification needed*

figures: Balona & Dziembowski (2011)

δ Scuti stars from space: instability strip

- more than 50% of stars in IS do not pulsate (*also J. Guzik talk on Monday*)
- this is not an effect of too high detection limit!



distribution of maximum amplitudes

⋆ 6.8% of stars have amplitudes of $0 < A < 100$ ppm. If constant stars are included (assuming they pulsate) this increases to 57.6% (cf. to 2.7% in the next bin)

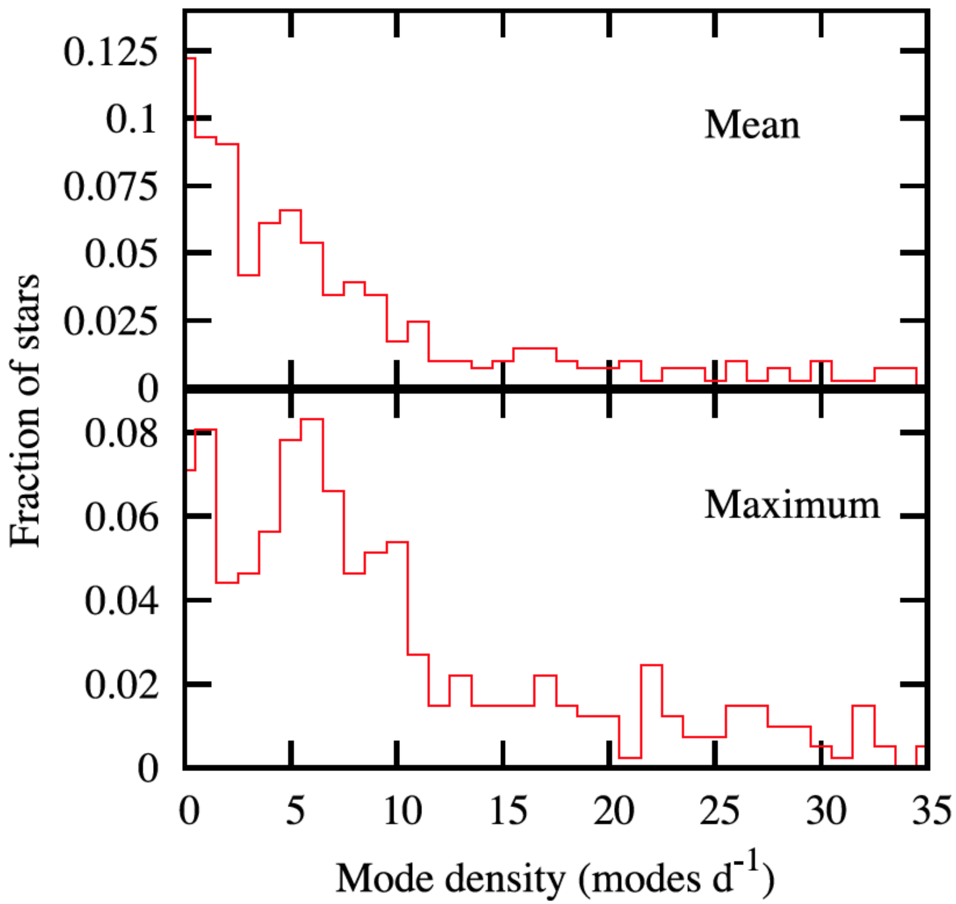
- caution: KIC parameters are not accurate

→ *spectroscopic program akin to WD program needed*

figure: Balona & Dziembowski (2011)

δ Scuti stars from space: mode content

- HD 50844 seems unique
- high mode densities only in limited frequency range
- typical max mode density 5/d^{-1}
  - $\ell \leq 4$ enough to explain such mode densities



distribution of mean mode density ($5 < f < 30$d^{-1}) and max mode density (modes per cycle d^{-1})

figure: Balona & Dziembowski (2011)

Conclusions

- we do not understand mode selection in any group of the self-excited pulsators
- linear growth rates are not predictors of mode amplitude

⋆ more theoretical work is needed

  - development of nonlinear non-radial pulsation codes (talk by F. Kupka, poster by C. Geroux)
  - studies of AEs for reliable coupling coefficients?

Conclusions

⋆ spectroscopic observations are needed

- ▸ determination of basic stellar parameters ($T_{\rm eff}$, L) (*also for non-pulsating stars*)
- ▸ mode identification

⋆ *Kepler* data needs to be analysed statistically with scrutiny

- ▸ artifacts, combination frequencies
- ▸ studies of amplitude variation are needed (mode coupling)
- ▸ distribution of mode amplitudes – can we infer the information about intrinsic amplitudes?