On Resonant and Non-Resonant Origin of Double-Mode Cepheid Pulsation

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Origin of double-mode pulsation:

There are two mechanisms that can be conductive in bringing up stable double-mode behaviour:

- non-resonant – pulsation modes compete with each other to saturate the pulsation instability
- resonant – amplitude of one of the two linearly unstable modes is limited through the resonant coupling to the damped, parasite mode, allowing the simultaneous growth of the other linearly unstable mode
- linear analysis indicate, that in most of the double-mode Cepheids non-resonant mechanism should be operational

Modeling of double-mode Cepheid pulsation:

⋆ unsuccessful with radiative hydrocodes

⋆ inclusion of turbulent convection into the models (Kolláth et al. 1998, Feuchtinger 1998) led to success

▸ one equation model for generation of turbulent energy, e_t (Kuhfuß 1986)

- ▸ turbulent motions are driven by the buoyancy (S) if $\mathcal{Y} = \nabla - \nabla_\mathrm{a} > 0$,
- ▸ e_t are always generated through the eddy-viscous forces, E_q,
- ▸ e_t are always damped by turbulent dissipation (cascade), D,
- ▸ turbulent motions are braked by the buoyancy (S) if $\mathcal{Y} = \nabla - \nabla_\mathrm{a} < 0$

$\mathcal{Y} > 0$

$$\frac{de_t}{dt} = \boldsymbol{S} - \boldsymbol{D} + \boldsymbol{E_q}$$

$\mathcal{Y} < 0$

$$\frac{de_t}{dt} = \cancel{\boldsymbol{S}} - \boldsymbol{D} + \boldsymbol{E_q}$$

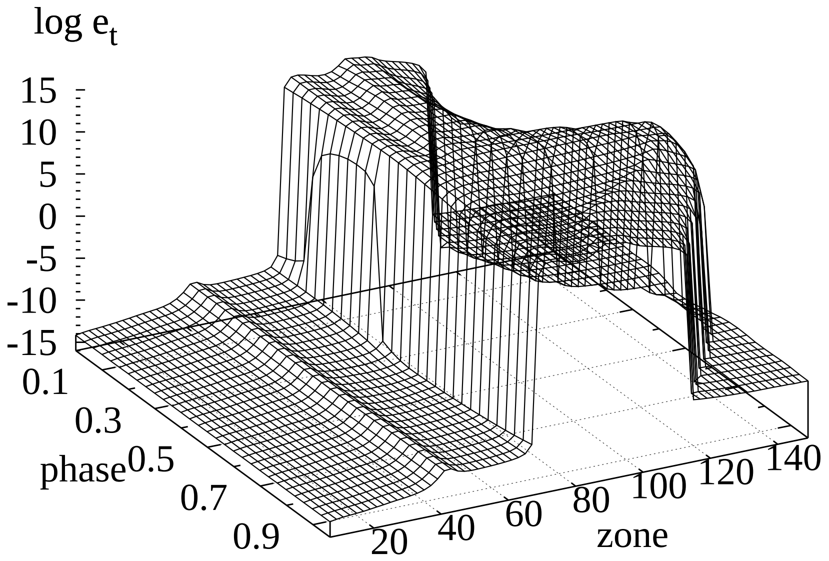
Florida-Budapest codes

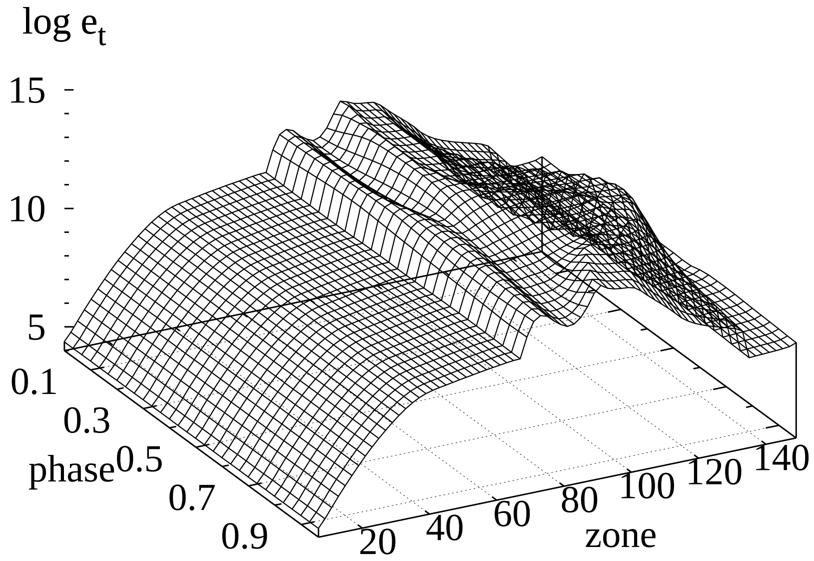
Treatment of negative buoyancy: *single-mode models*

Negative buoyancy included: Negative buoyancy neglected:

$$\mathcal{Y} < 0$$

$$\frac{de_t}{dt} = \boldsymbol{S} - \boldsymbol{D} + \boldsymbol{E_q} \qquad\qquad \frac{de_t}{dt} = - \boldsymbol{D} + \boldsymbol{E_q}$$





Treatment of negative buoyancy: *single-mode models*

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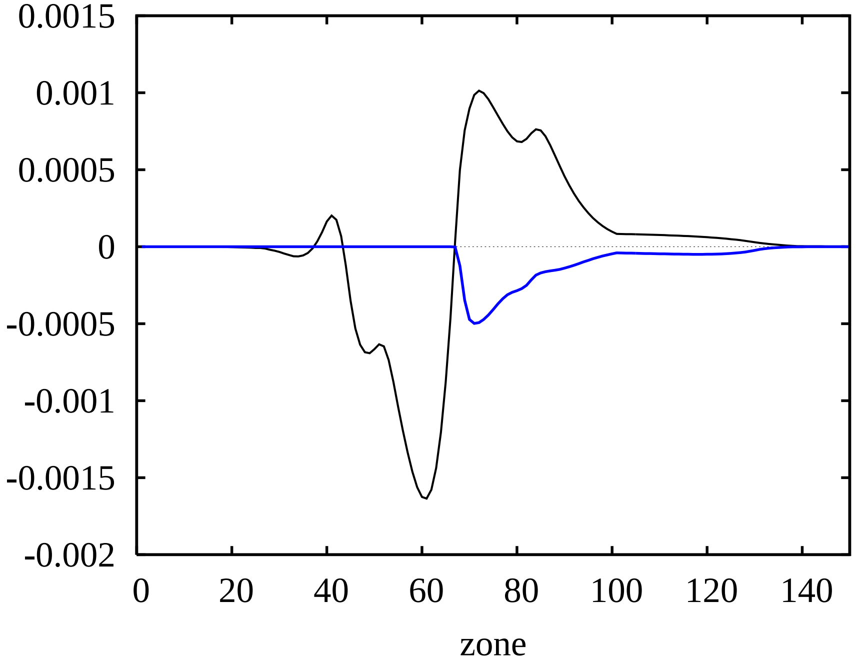

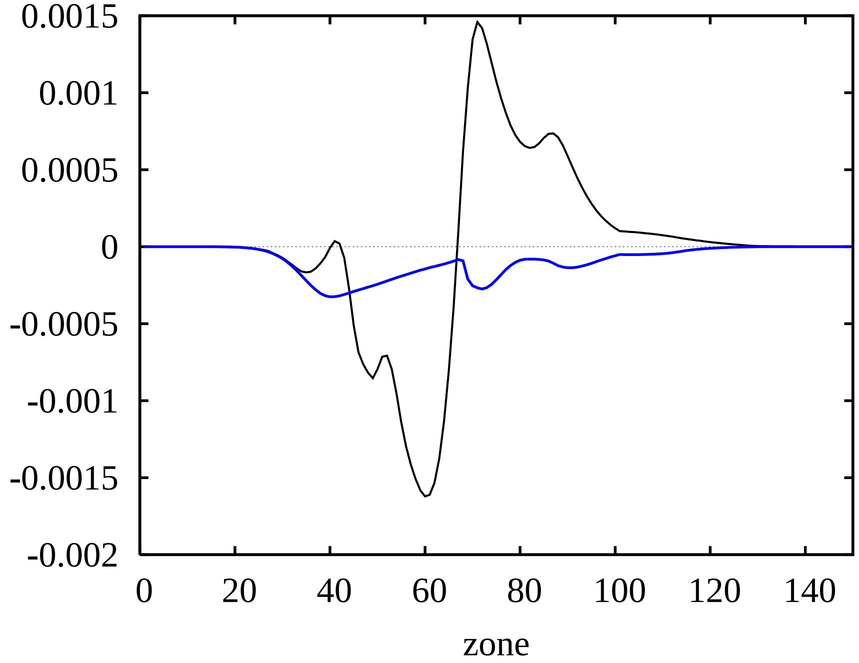

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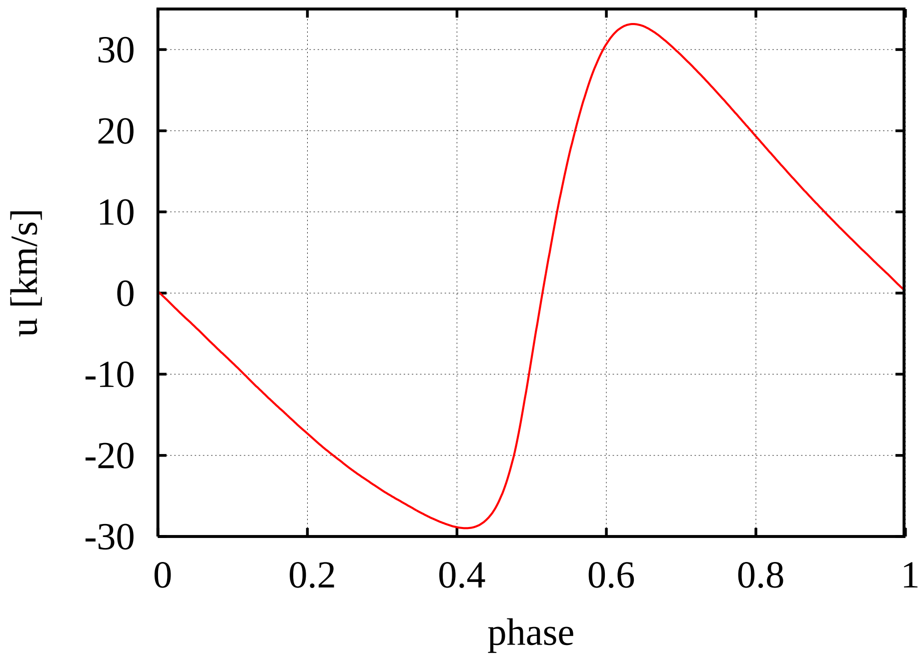
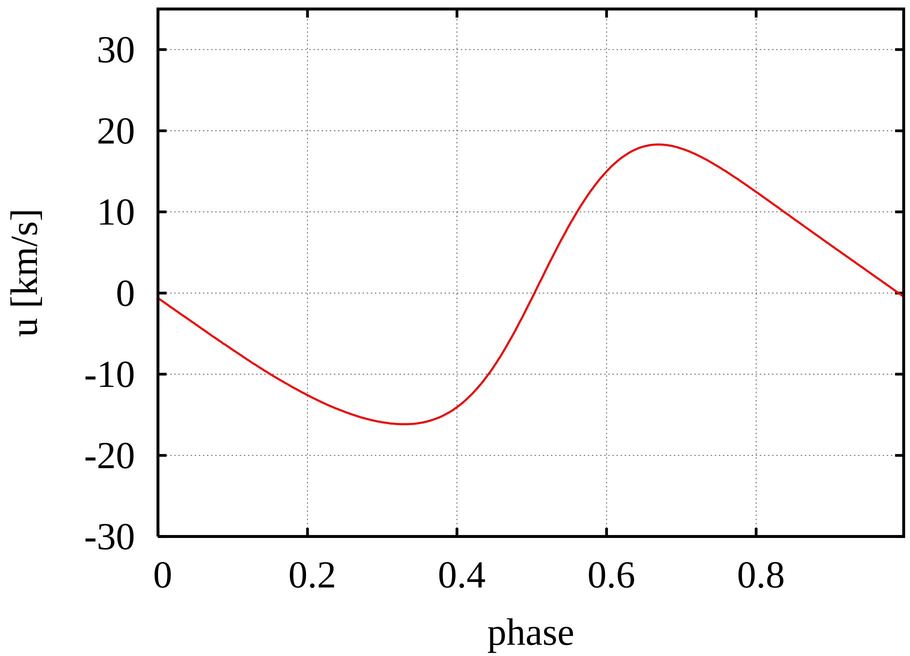
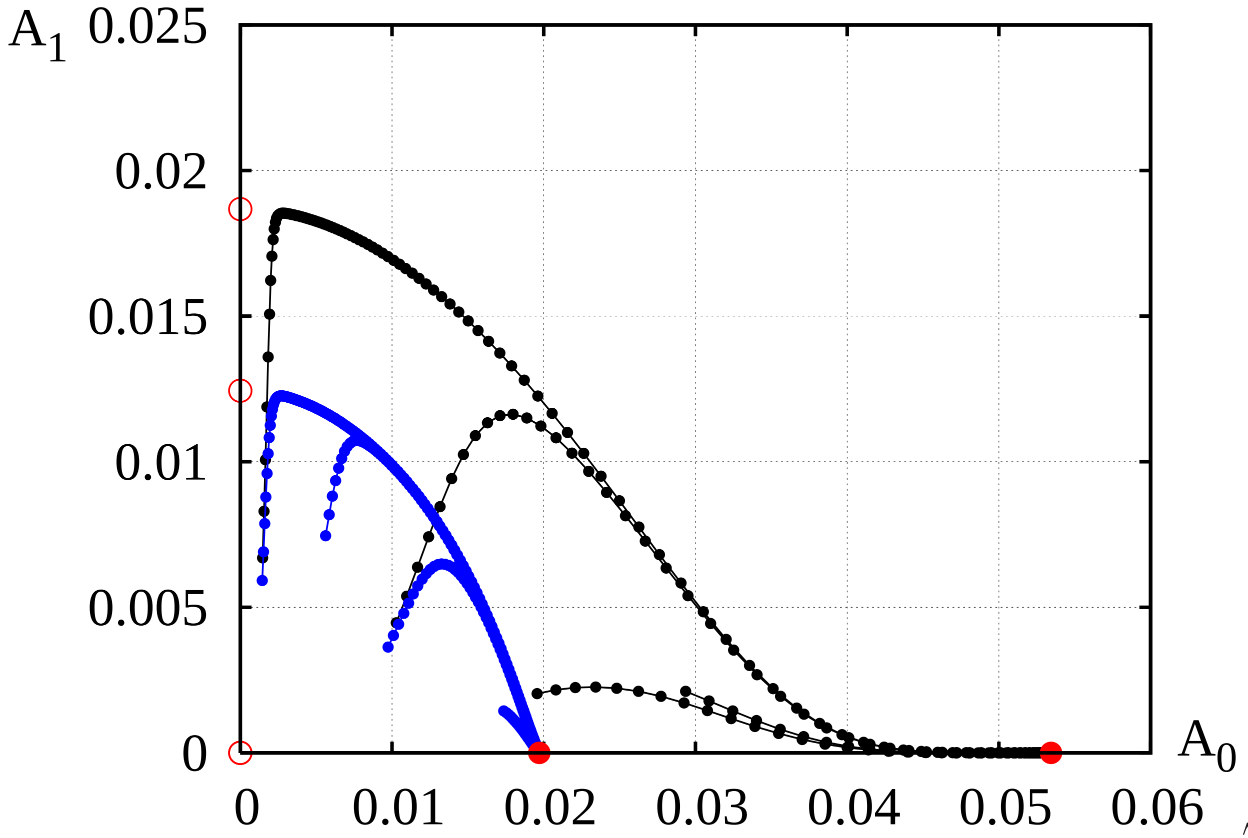




Treatment of negative buoyancy: *modal selection*

⋆ Sequence of models of the same physical, convective and numerical parameters, **including** and **neglecting** negative buoyancy



$T_{\mathrm{eff}} = 6025\,\mathbf{K}$

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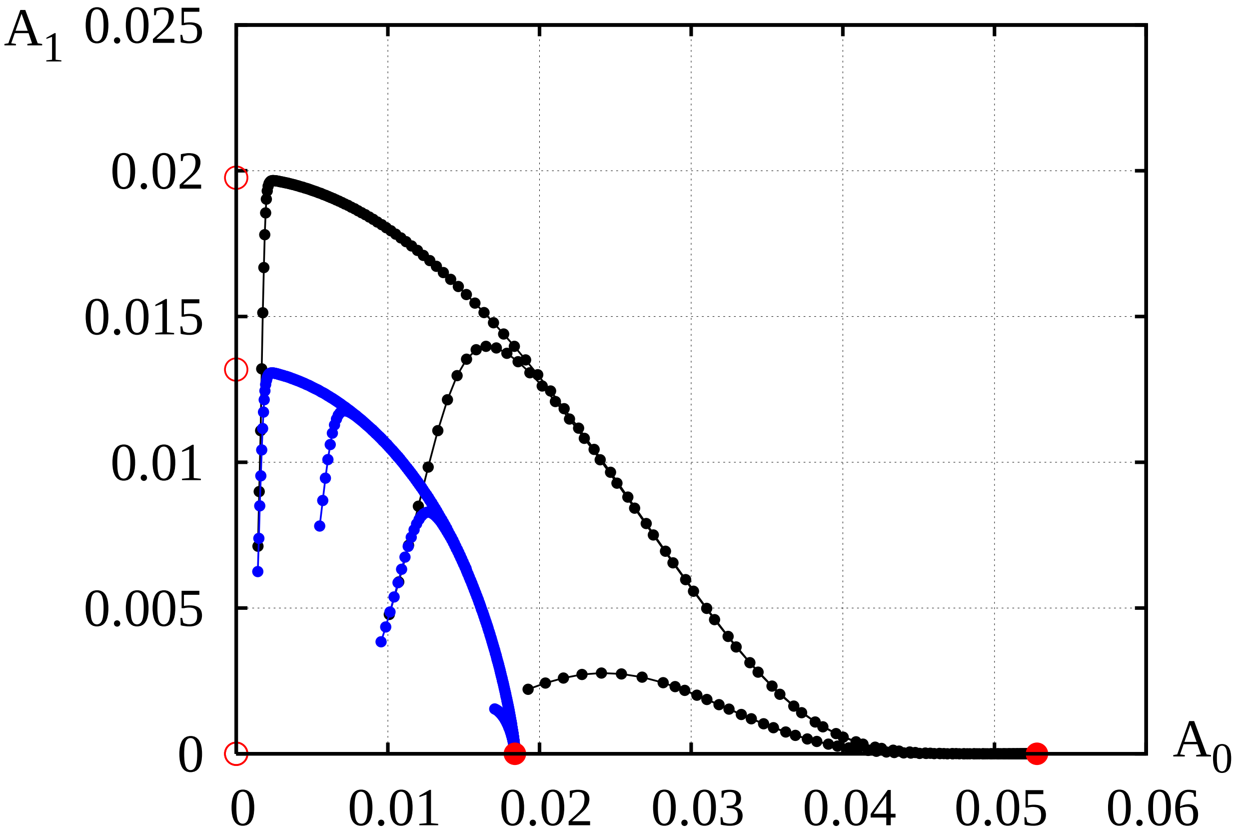

$T_{\rm eff} = 6075\,\mathbf{K}$

Treatment of negative buoyancy: *modal selection*

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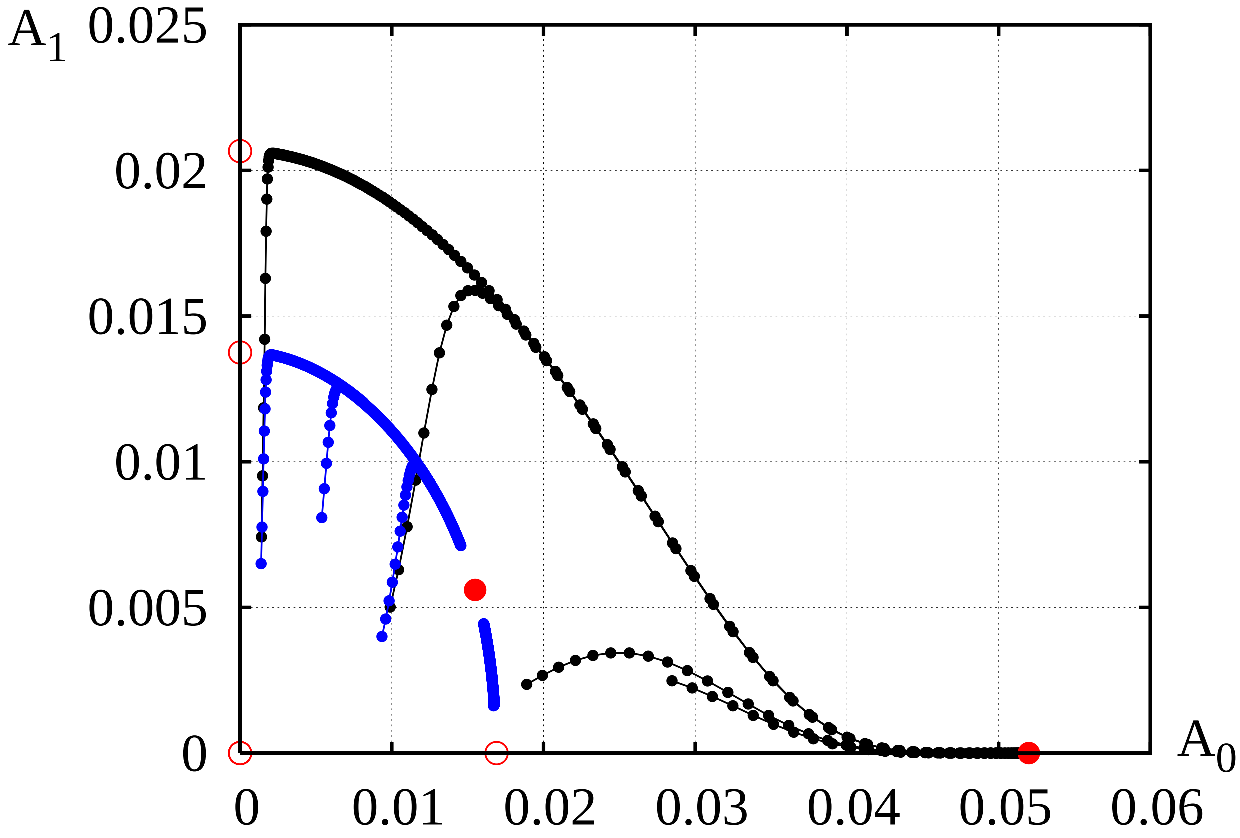
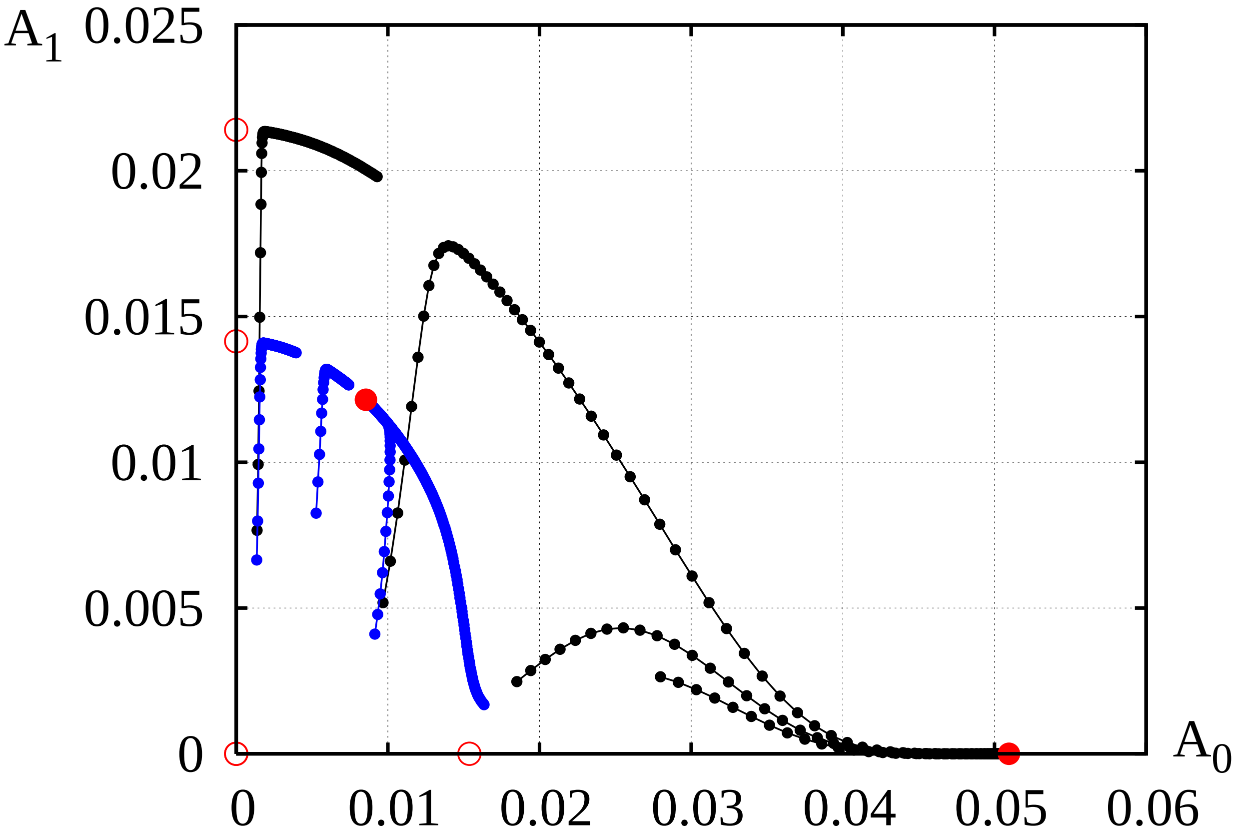


$T_{\rm eff} = 6100\,\mathbf{K}$

Treatment of negative buoyancy: *modal selection*

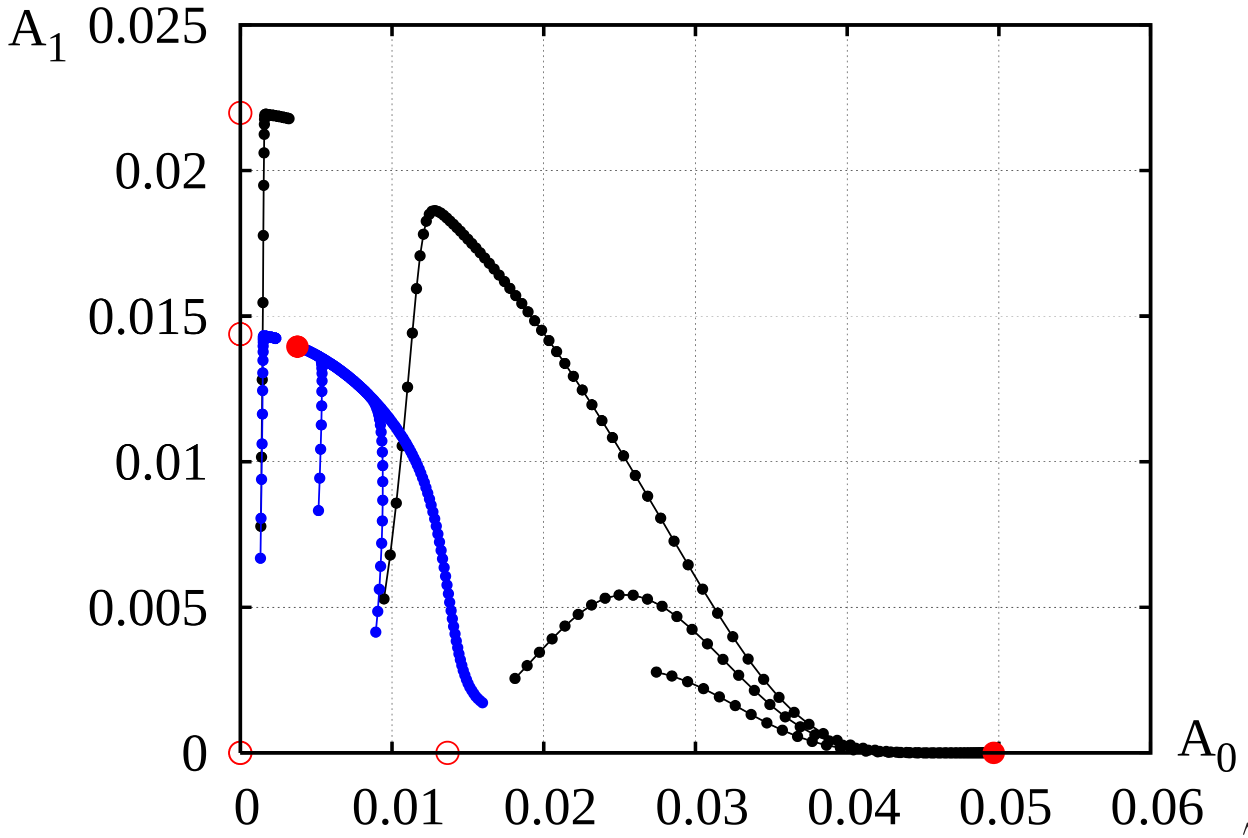
⋆ Sequence of models of the same physical, convective and numerical parameters, **including** and **neglecting** negative buoyancy



$T_{\rm eff} = 6125\,\mathbf{K}$

Treatment of negative buoyancy: *modal selection*

⋆ Sequence of models of the same physical, convective and numerical parameters, **including** and **neglecting** negative buoyancy



$T_{\rm eff} = 6150\,\mathbf{K}$

Treatment of negative buoyancy: *modal selection*

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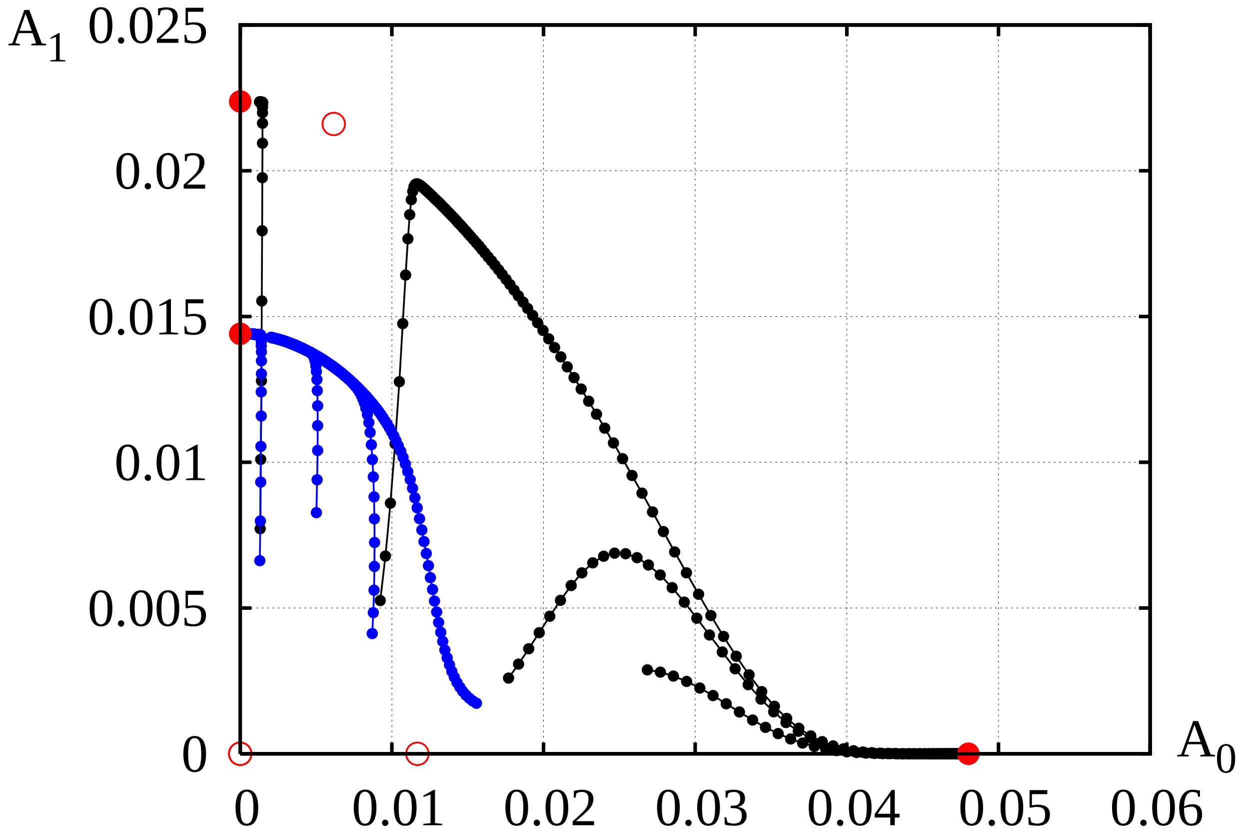
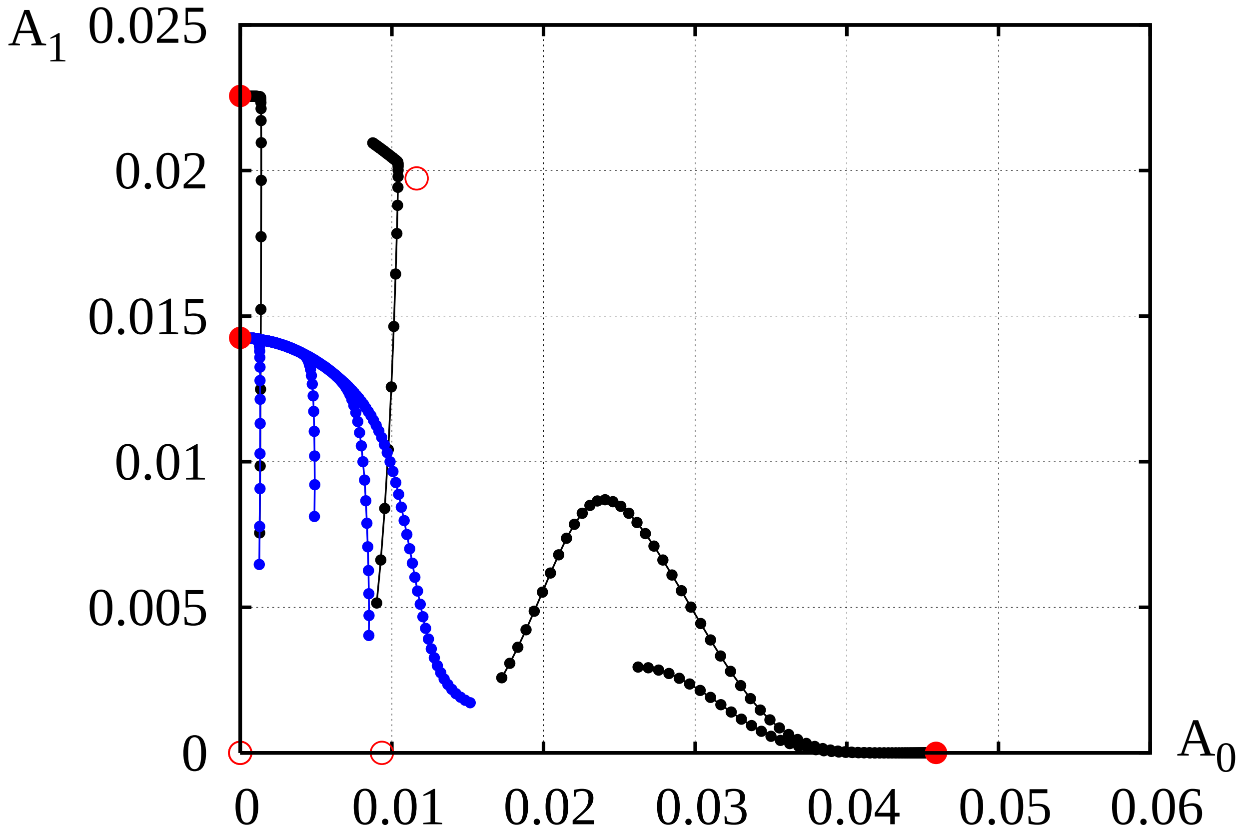


$T_{\rm eff} = 6175\,\mathbf{K}$

Treatment of negative buoyancy: *modal selection*

⋆ Sequence of models of the same physical, convective and numerical parameters, **including** and **neglecting** negative buoyancy



$T_{\rm eff} = 6200\,\mathbf{K}$

Treatment of negative buoyancy: *modal selection*

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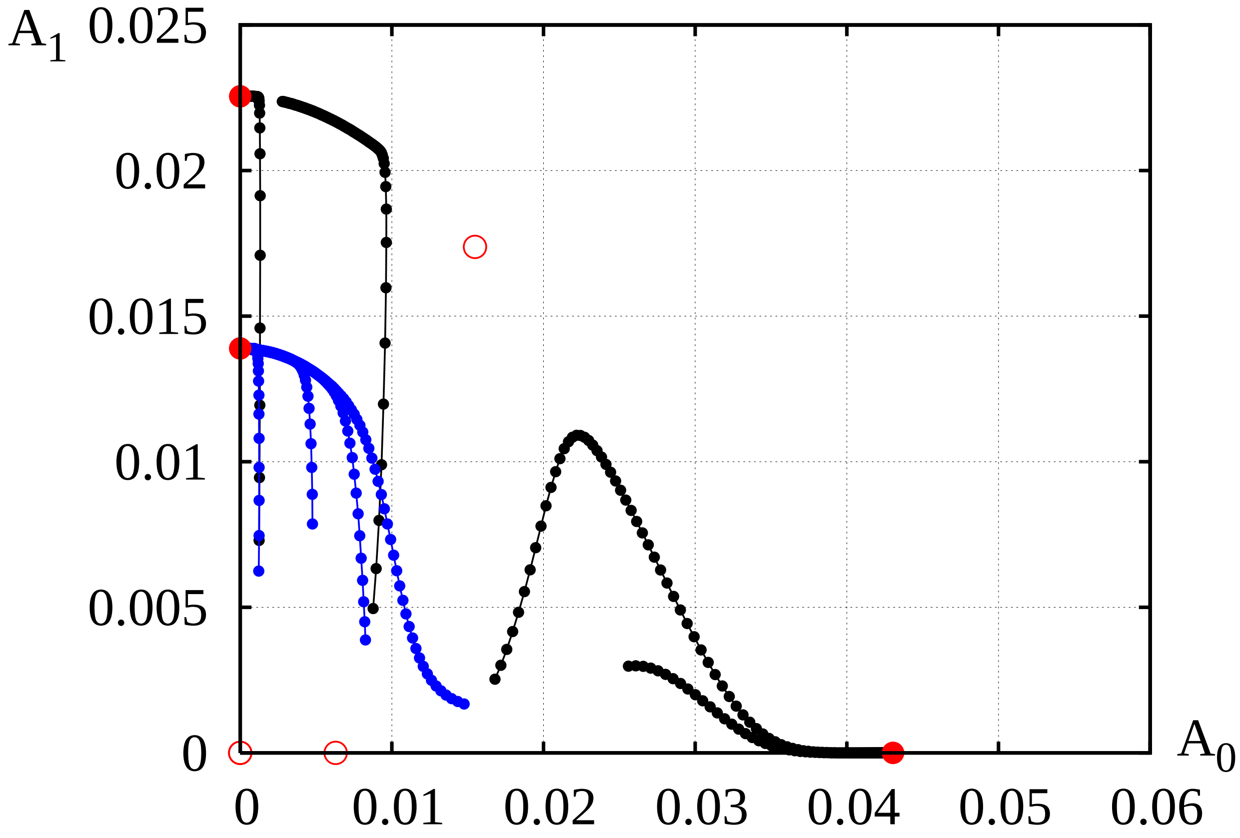

$T_{\rm eff} = 6225\,\mathbf{K}$

Treatment of negative buoyancy: *modal selection*

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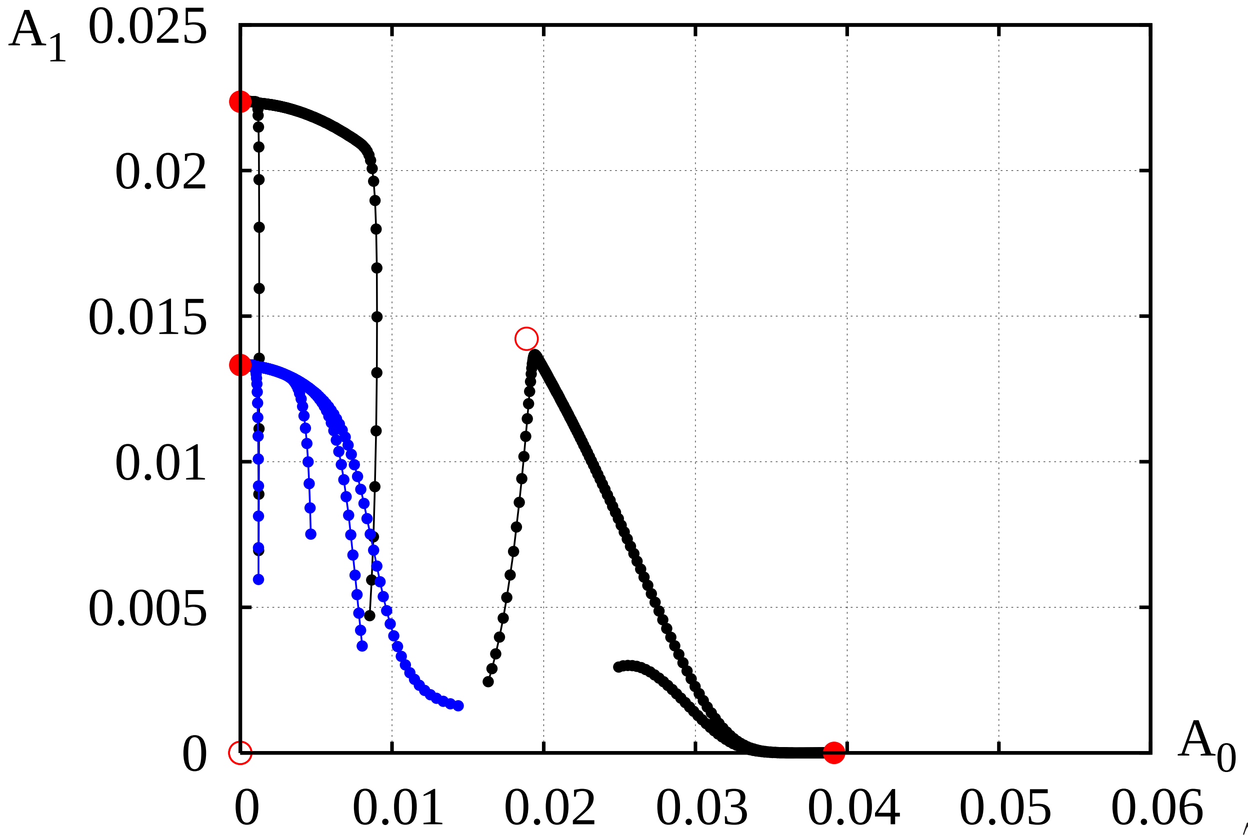


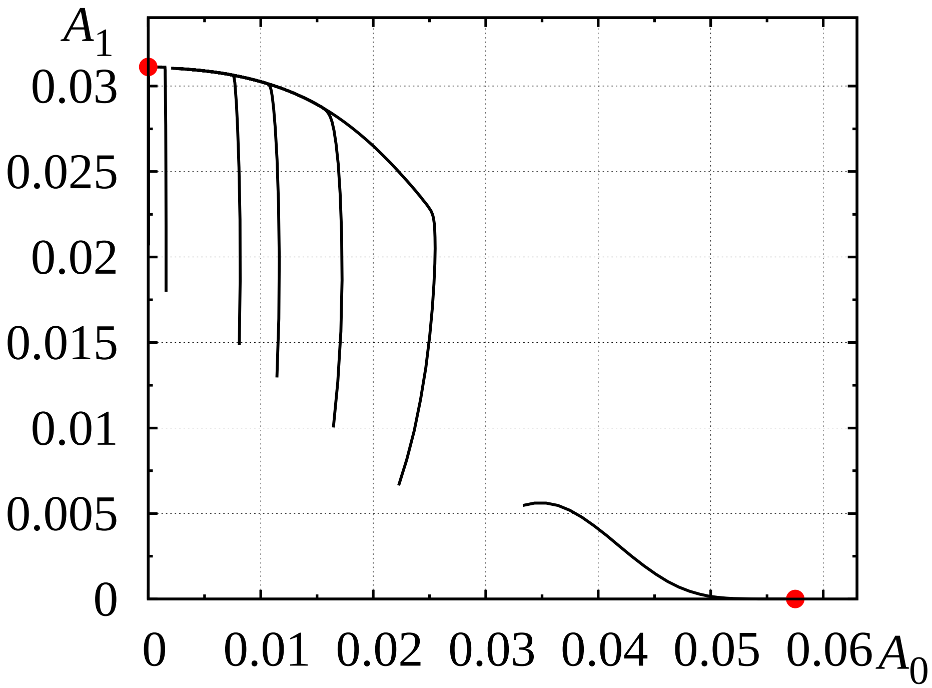
$T_{\mathrm{eff}} = 6250\,\mathbf{K}$

Double-mode nonresonant F/1O Cepheid models:

- ▸ Nonresonant F/1O double-mode models were not found if negative buoyancy was included in the models
- ▸ Once negative buoyancy is neglected, double-mode behaviour appears quite naturally, not depending on exact values of convective parameters
- ⋆ Mechanism at operation is analogous to the resonant one
- ⋆ Double-mode behaviour is caused by the neglect of negative buoyancy, which significantly reduces amplitude of the fundamental mode
- ⋆ Consequently, it is no longer able to saturate the pulsation instability alone, allowing the growth of the first overtone
- ▸ Some resonant multi-mode models were found with hydrocode including negative buoyancy

Multi-mode resonant F/1O Cepheids:

Sequence of models crossing the $2\omega_1 = \omega_0 + \omega_2$ resonance center



1O / F

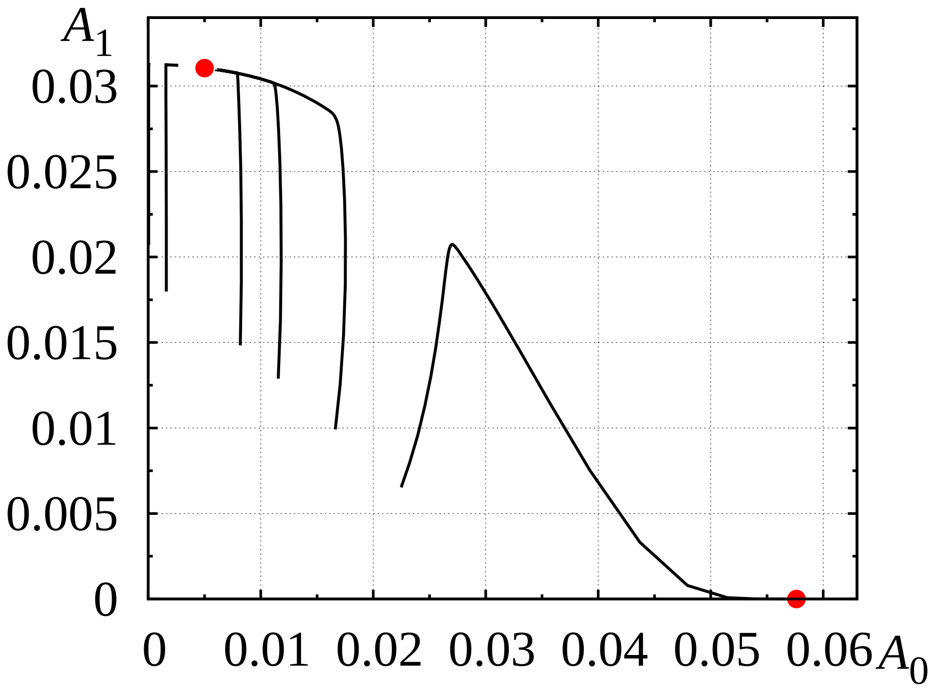
$T_{\rm eff} = 5850\,\mathrm{K}$

$\Delta = 1.00041$

$\Delta = \frac{2\omega_1}{\omega_0+\omega_2}$

Multi-mode resonant F/1O Cepheids:

Sequence of models crossing the $2\omega_1 = \omega_0 + \omega_2$ resonance center



DM / F

$T_{\rm eff} = 5840\,{\rm K}$

$\Delta = 0.99996$

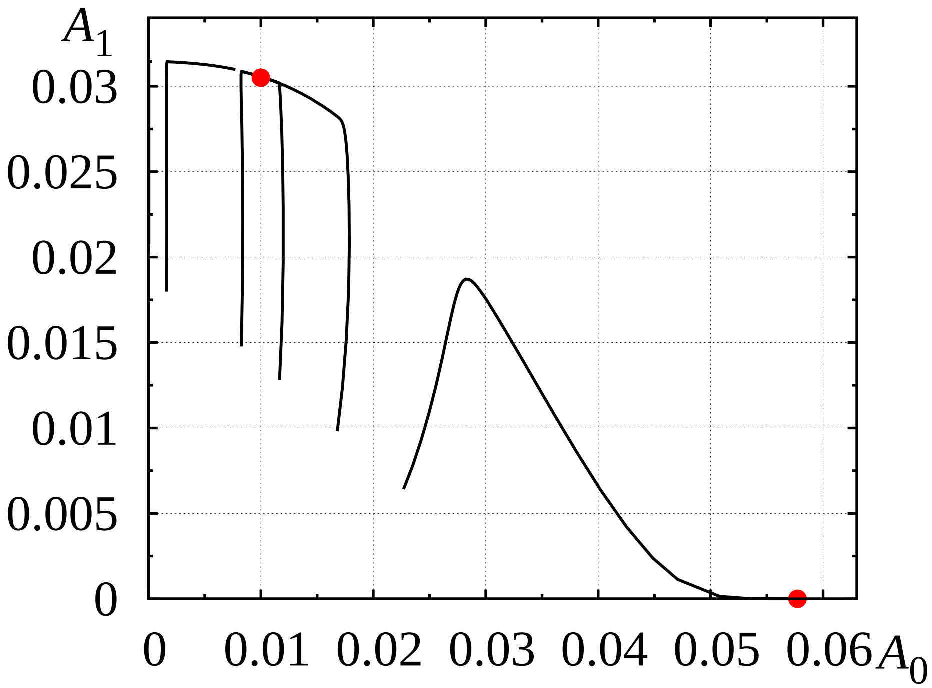
$P_0 = 4.54\,{\rm d}$

$P_1/P_0 = 0.6946$

$\Delta = \frac{2\omega_1}{\omega_0+\omega_2}$

Multi-mode resonant F/1O Cepheids:

Sequence of models crossing the $2\omega_1 = \omega_0 + \omega_2$ resonance center



DM / **F**

$T_{\rm eff} = 5830\,{\rm K}$

$\Delta = 0.99952$

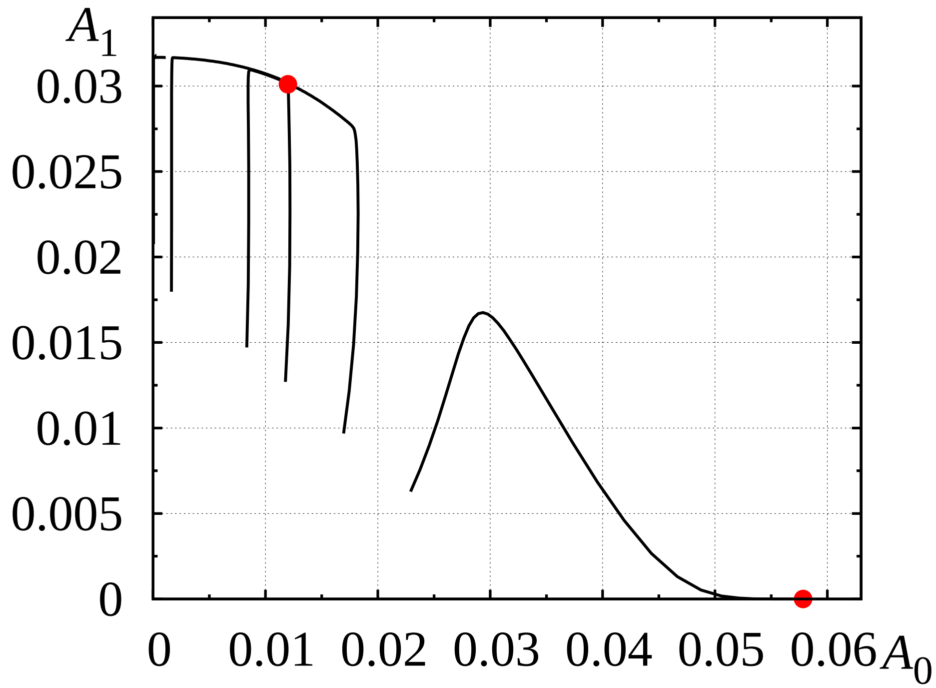
$P_0 = 4.57\,{\rm d}$

$P_1/P_0 = 0.6947$

$\Delta = \frac{2\omega_1}{\omega_0+\omega_2}$

Multi-mode resonant F/1O Cepheids:

Sequence of models crossing the $2\omega_1 = \omega_0 + \omega_2$ resonance center



DM / F

$T_{\mathrm{eff}} = 5820\,\mathrm{K}$

$\Delta = 0.99907$

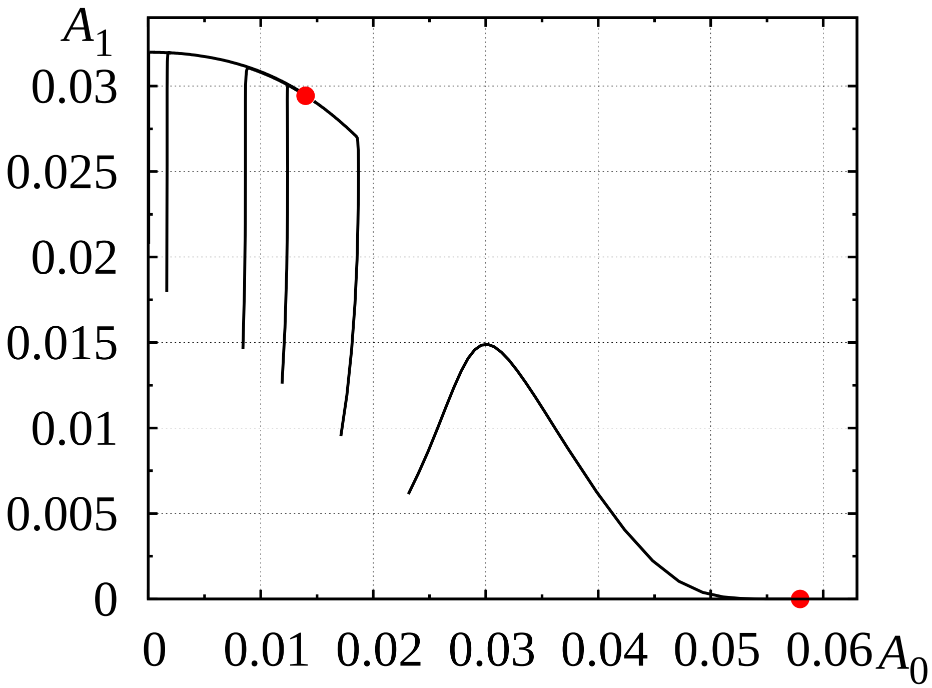
$P_0 = 4.60\,\mathrm{d}$

$P_1/P_0 = 0.6948$

$\Delta = \frac{2\omega_1}{\omega_0+\omega_2}$

Multi-mode resonant F/1O Cepheids:

Sequence of models crossing the $2\omega_1 = \omega_0 + \omega_2$ resonance center



DM / F

$T_{\rm eff} = 5810\,{\rm K}$

$\Delta = 0.99862$

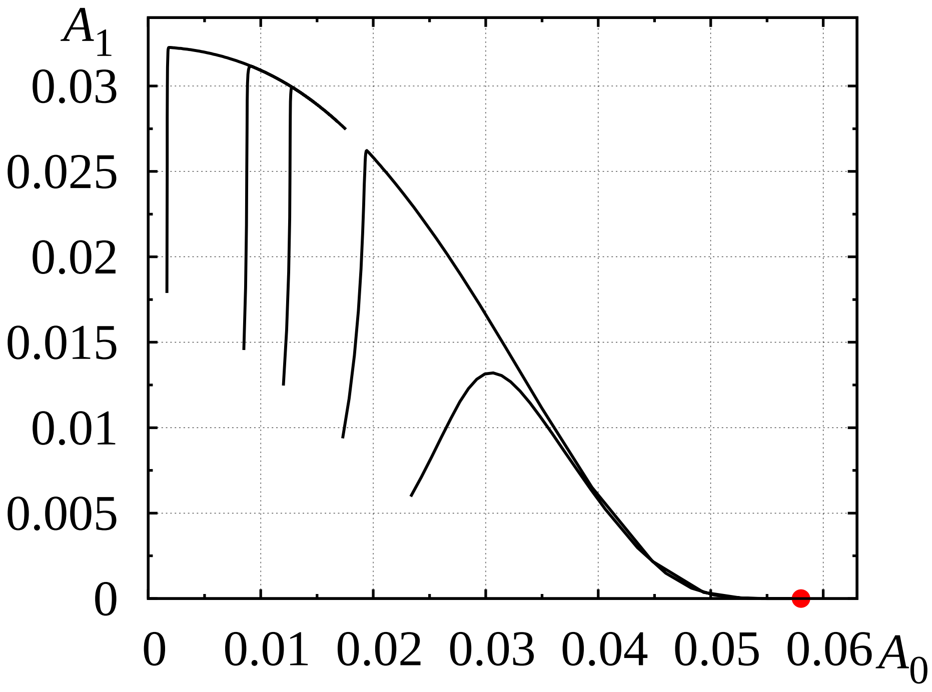
$P_0 = 4.63\,{\rm d}$

$P_1/P_0 = 0.6948$

$\Delta = \frac{2\omega_1}{\omega_0+\omega_2}$

Multi-mode resonant F/1O Cepheids:

Sequence of models crossing the $2\omega_1 = \omega_0 + \omega_2$ resonance center



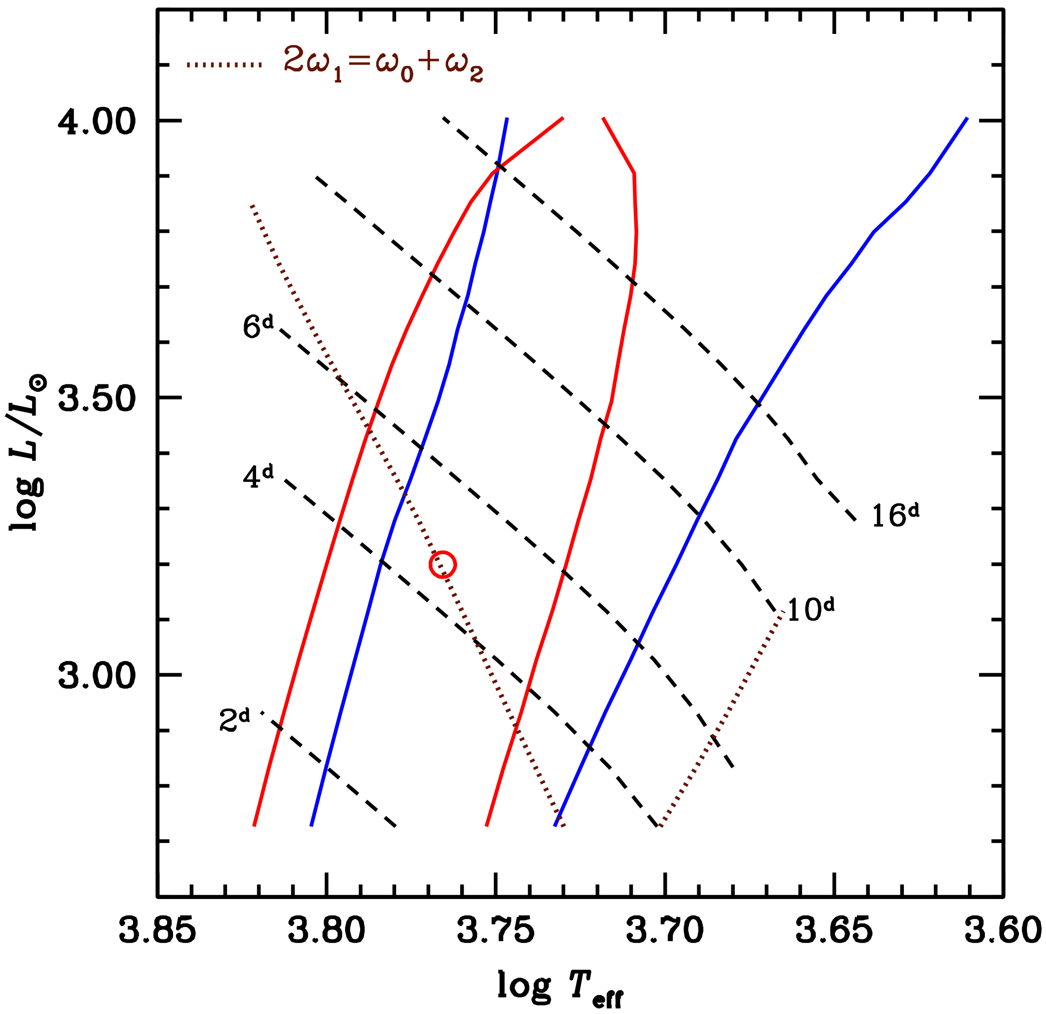
F

$T_{\rm eff} = 5800\,{\rm K}$

$\Delta = 0.99817$

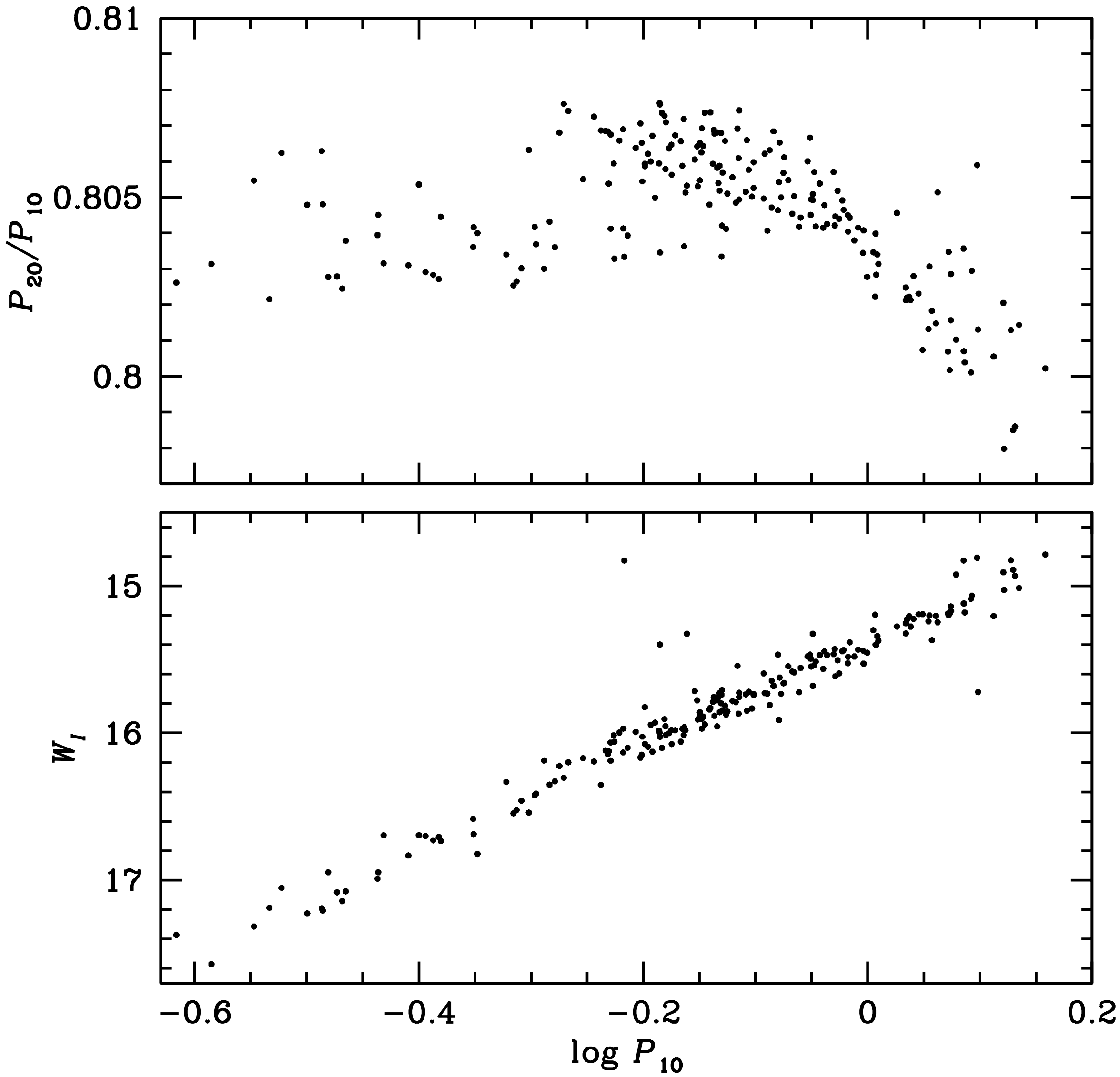
$\Delta = \frac{2\omega_1}{\omega_0+\omega_2}$

Multi-mode resonant F/1O Cepheids:



- ⋆ Multi-mode models were found in several sequences with different convective parameters
- ⋆ they are always restricted to a narrow temperature range
- ⋆ they are always located:
  - ▸ close to the resonance center
  - ▸ in between the 1O/F and F pulsation domains
- ⋆ amplitudes of the models are slightly too high

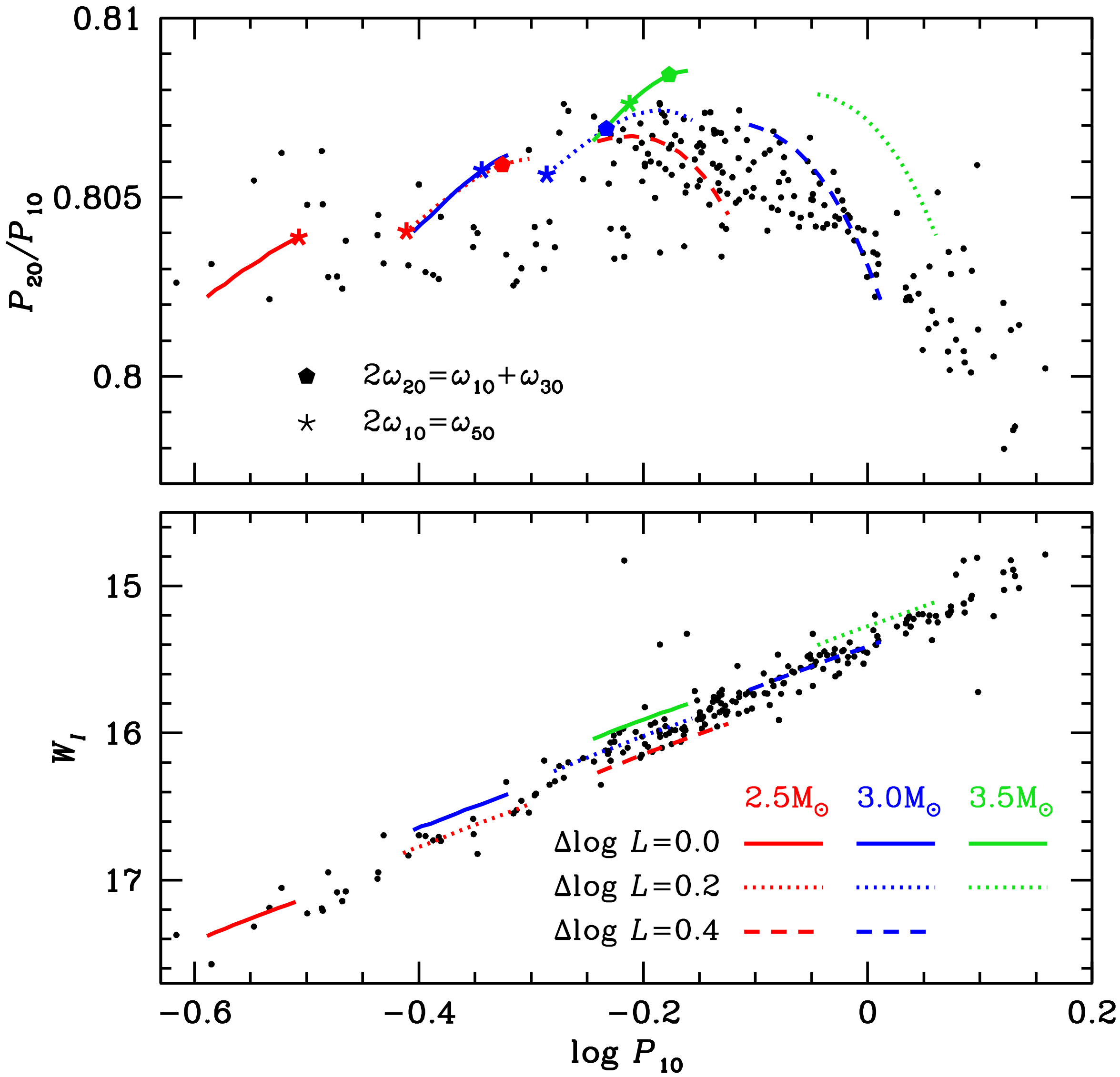
Double-overtone Cepheids in the LMC:



OGLE-III data (Soszyński et al. 2008)

- 203 1O/2O objects
- most common form of Cepheid pulsation in a period range $0.5\,\mathrm{d} < P_{1\mathrm{O}} < 0.9\,\mathrm{d}$

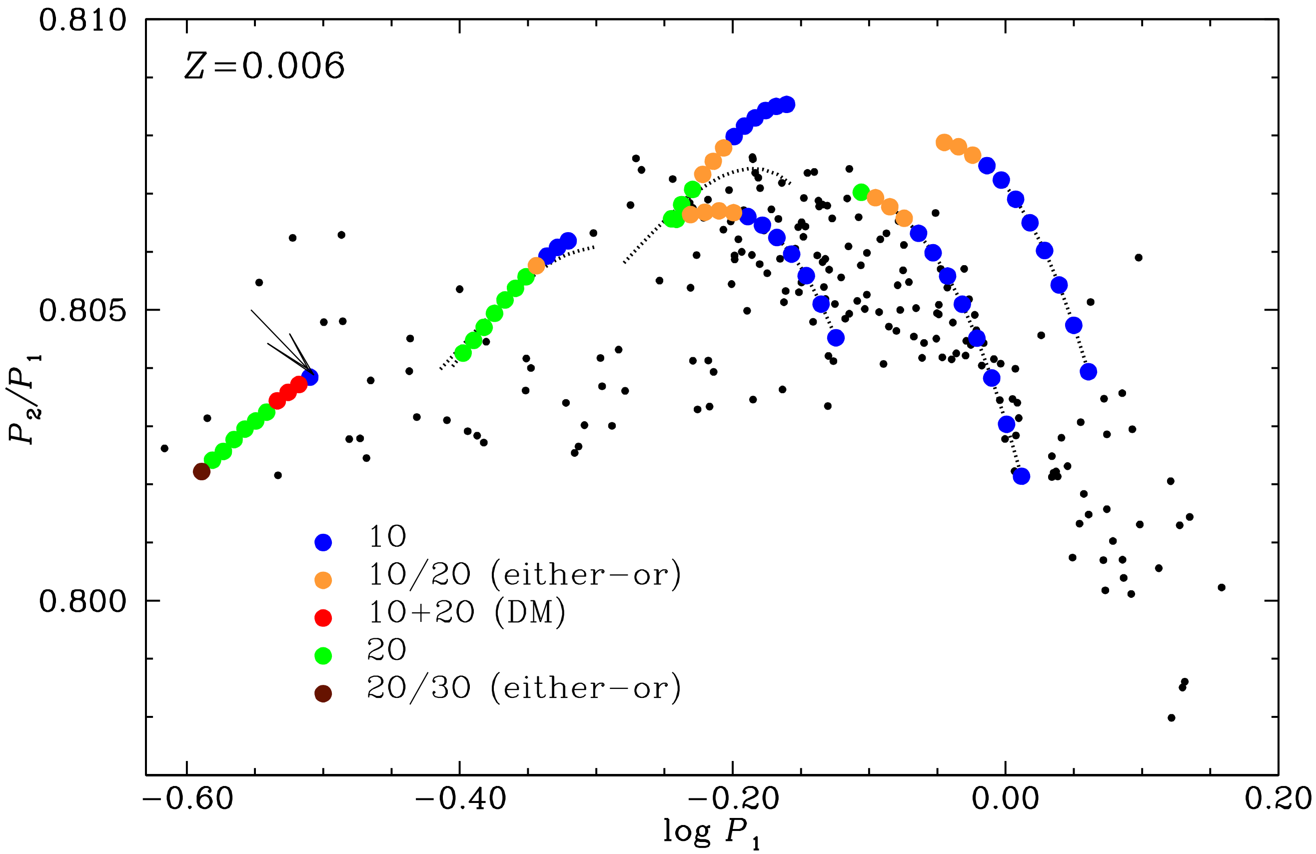
Double-overtone Cepheids in the LMC:



Linear modeling:

- simultaneous instability in 1O and 2O
- models along evolutionary track (I crossing)...
  ...and with luminosity increase
- challenge for stellar evolution theory – poster by Dziembowski & Smolec
- resonances between pulsation modes

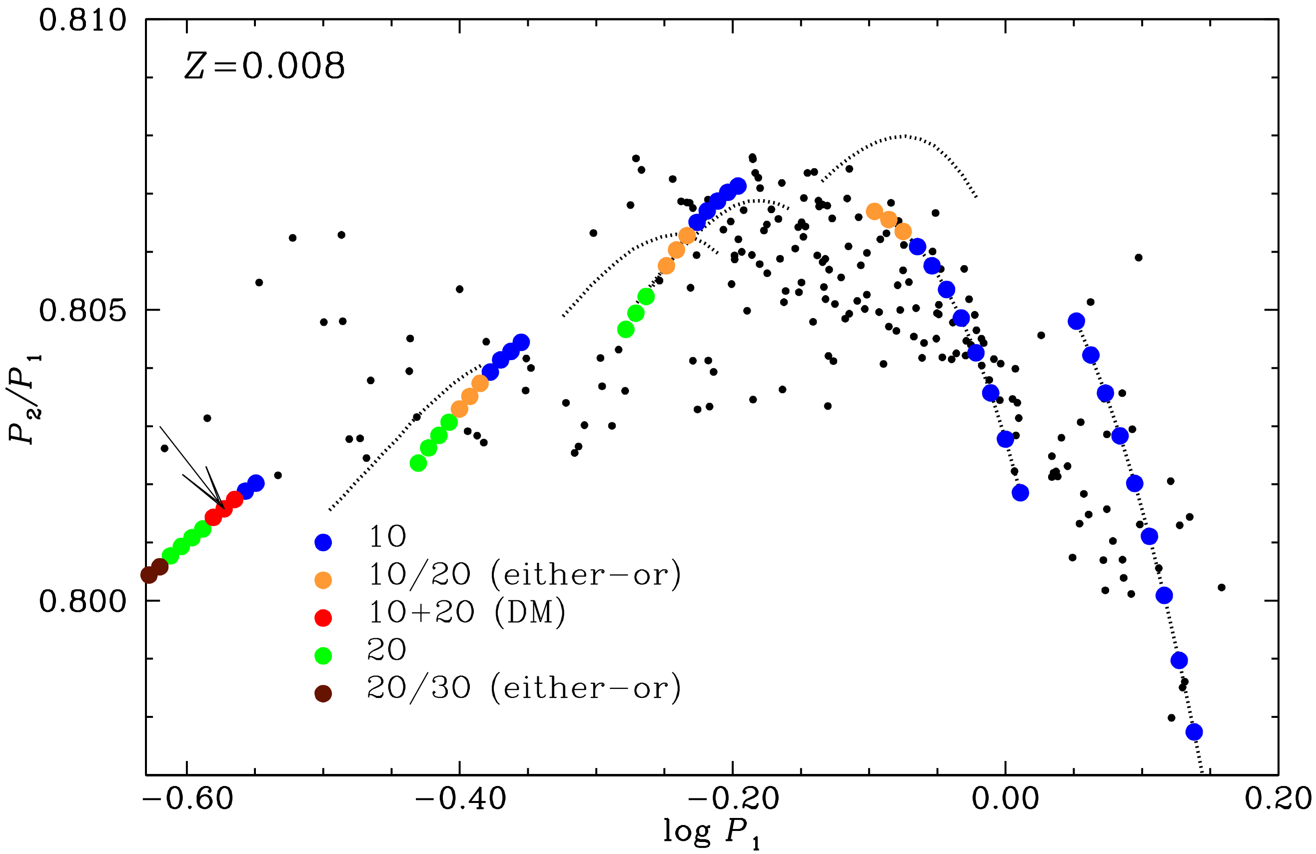
Double-overtone Cepheids in the LMC:



Nonlinear modeling:

- modal selection along selected tracks
- $Z = 0.006$
- DM only at shortest periods

Double-overtone Cepheids in the LMC:

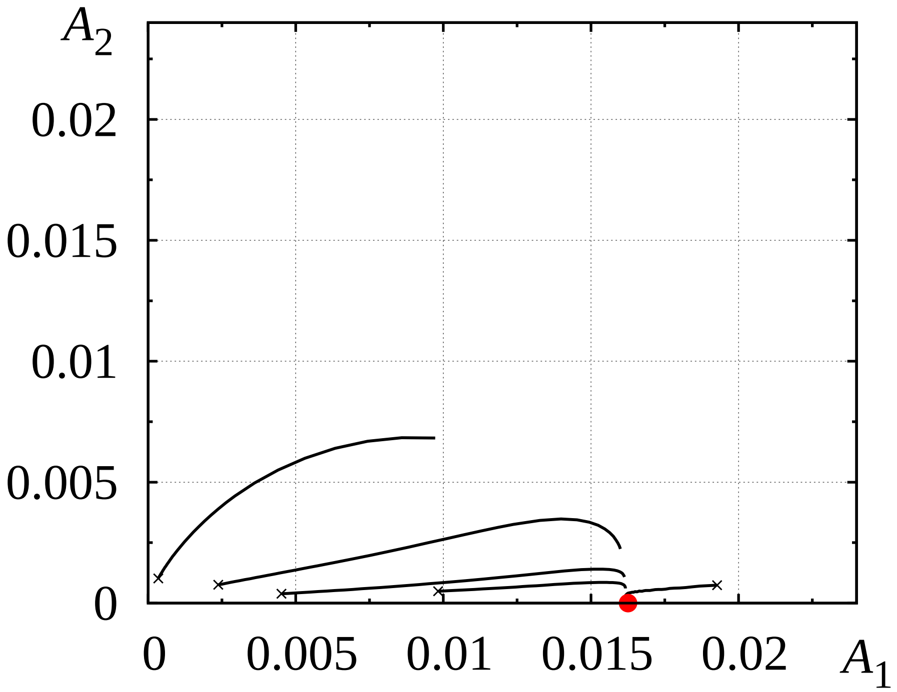


Nonlinear modeling:

- modal selection along selected tracks
- $Z = 0.008$
- DM only at shortest periods
- qualitatively the same scenario as for $Z = 0.006$

Double-overtone Cepheids in the LMC:

Sequence of models close to the $2\omega_1 = \omega_5$ resonance center



1O

$\log T_{\rm eff} = 3.8251$

$\Delta = 0.5002$

$\Delta = \frac{\omega_1}{\omega_5}$

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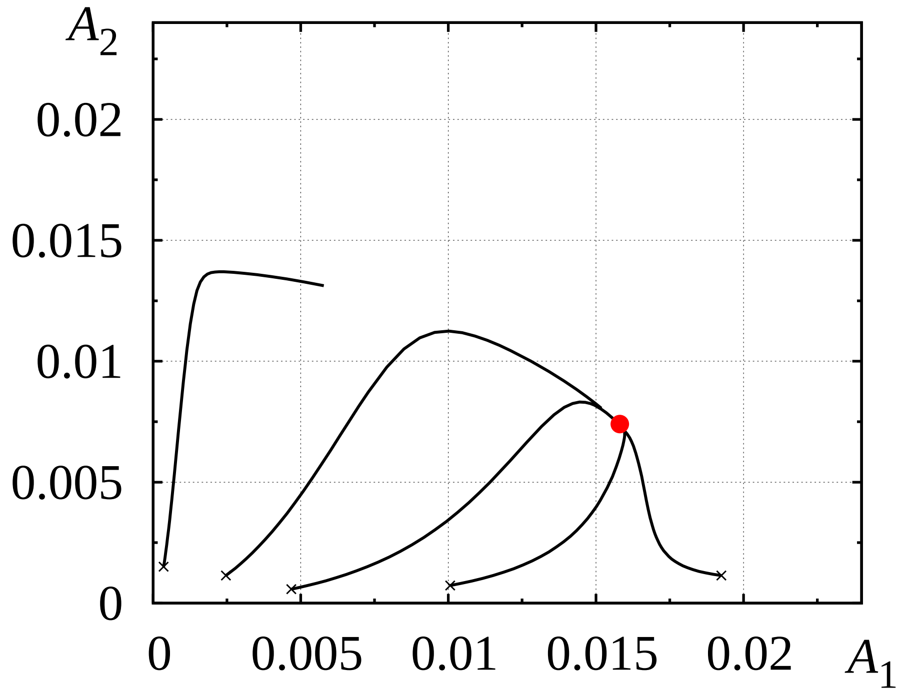

1O+2O

$\log T_{\rm eff} = 3.8284$

$\Delta = 0.5006$

$\Delta = \frac{\omega_1}{\omega_5}$

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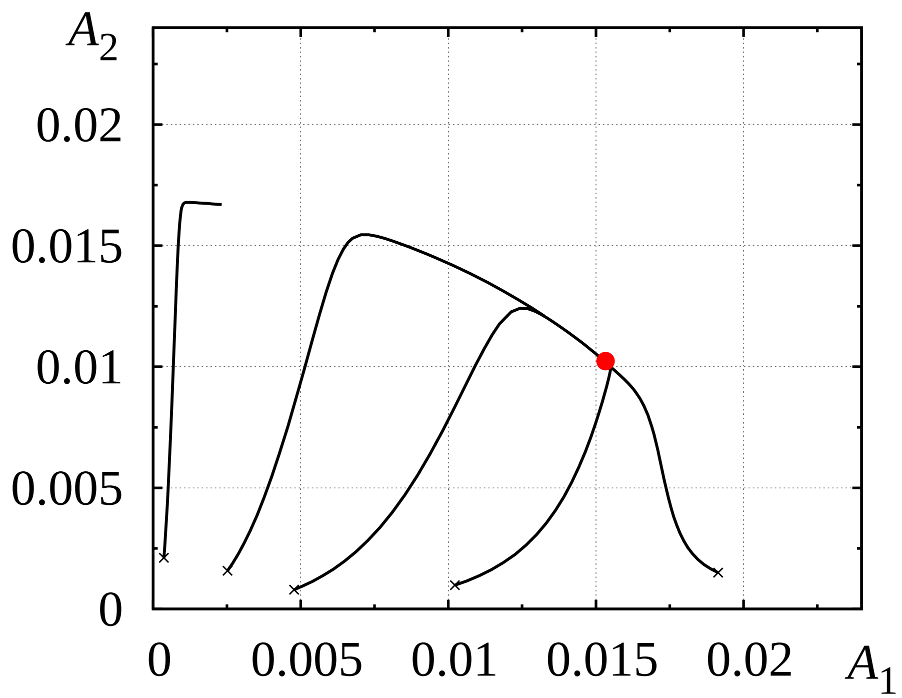


1O+2O

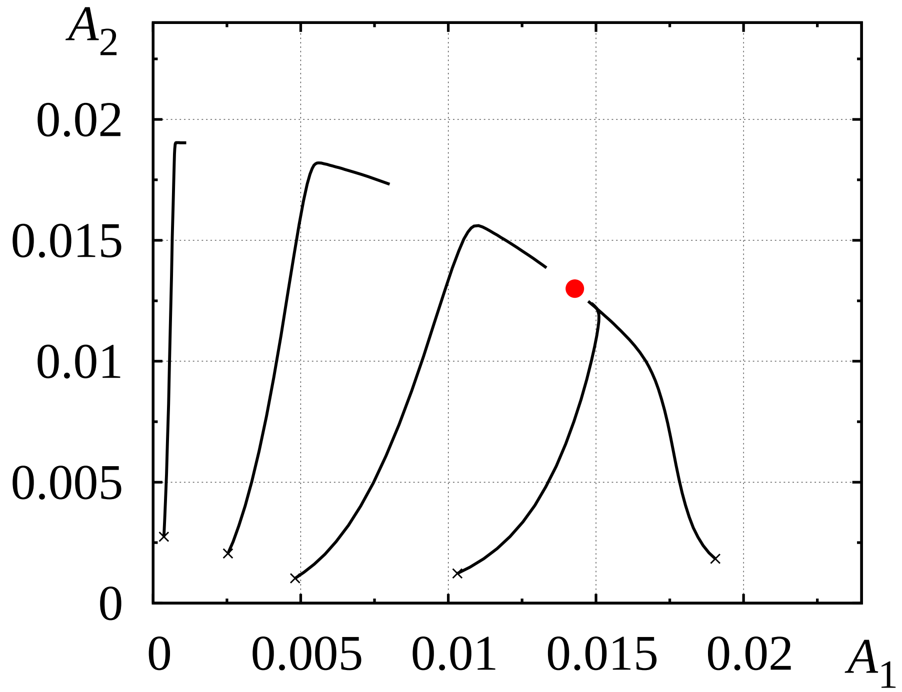
$\log T_{\rm eff} = 3.8316$

$\Delta = 0.5009$

$\Delta = \frac{\omega_1}{\omega_5}$

Double-overtone Cepheids in the LMC:

Sequence of models close to the $2\omega_1 = \omega_5$ resonance center



1O+2O

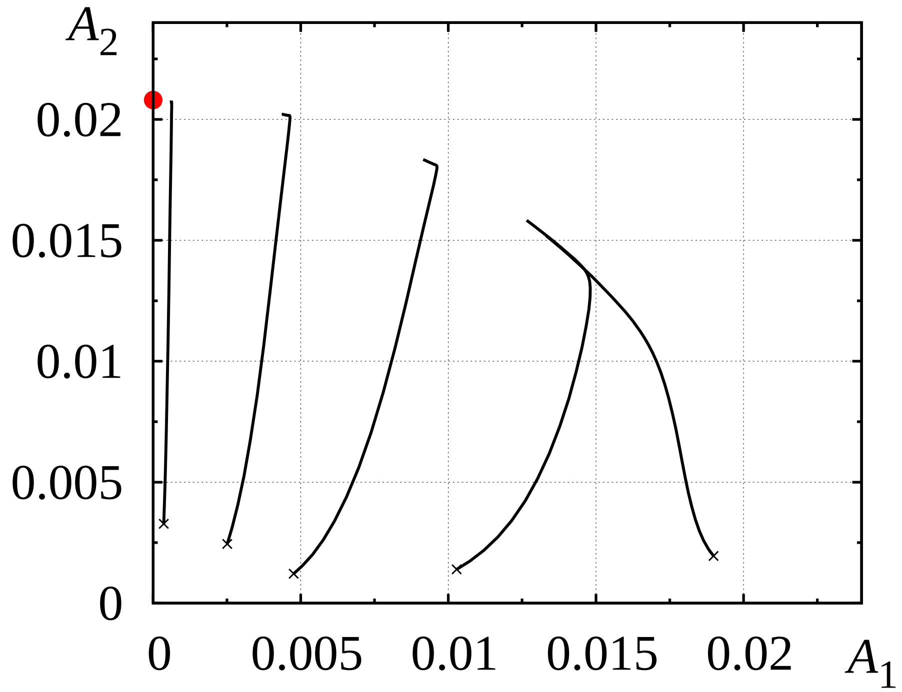
$\log T_{\rm eff} = 3.8349$

$\Delta = 0.5012$

$\Delta = \frac{\omega_1}{\omega_5}$

Double-overtone Cepheids in the LMC:

Sequence of models close to the $2\omega_1 = \omega_5$ resonance center



2O

$\log T_{\rm eff} = 3.8382$

$\Delta = 0.5015$

$\Delta = \frac{\omega_1}{\omega_5}$

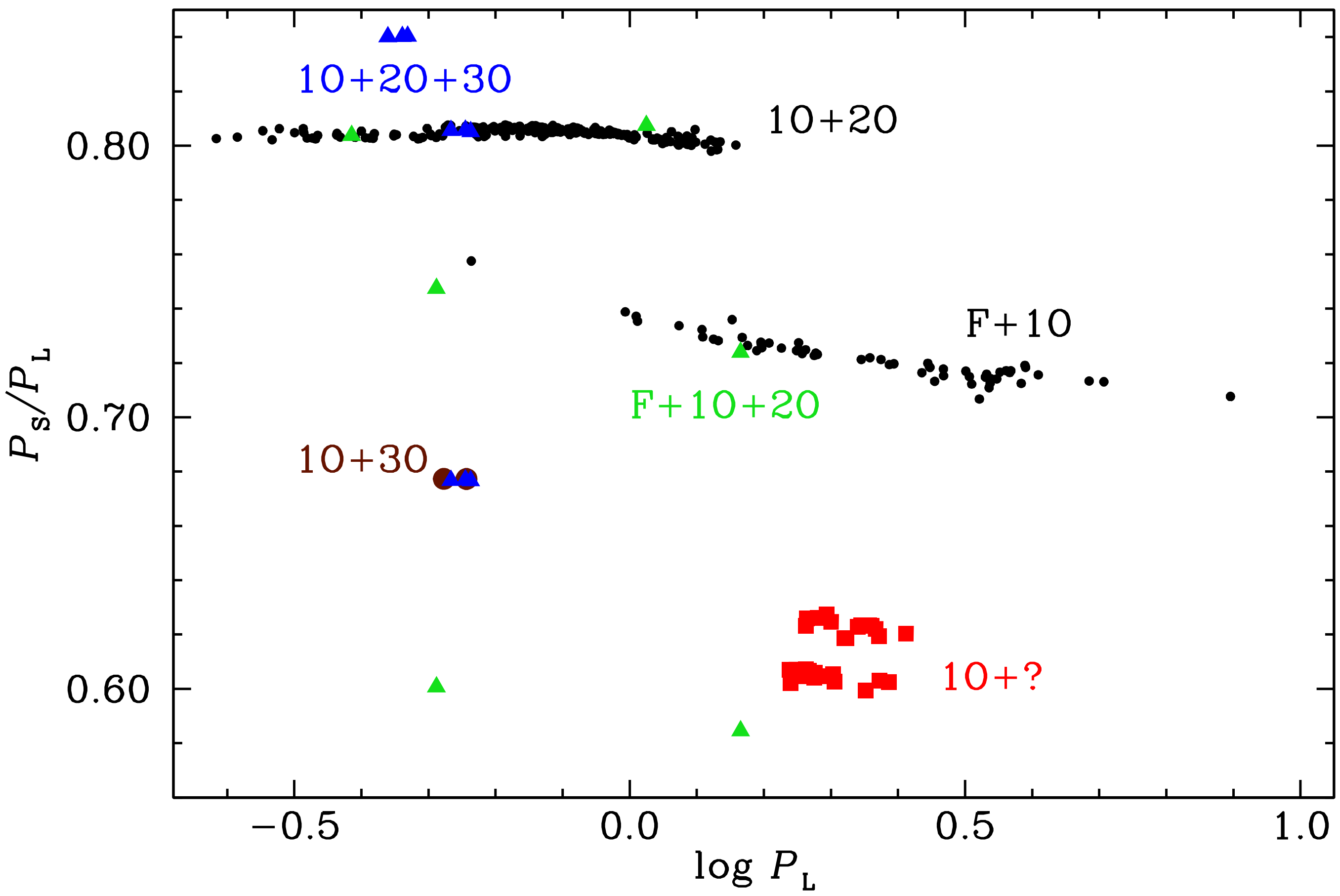
Double-overtone Cepheids in the LMC:

- ⋆ except for the shortest period Cepheids, double-mode 1O/2O behaviour is not found
- ⋆ many models exhibit 2O pulsation, contrary to observations

Conclusions:

- ▸ Modeling of the double-mode Cepheid pulsation is far from being satisfactory
- ▸ Better treatment of pulsation/convection coupling seems necessary
- ▸ OGLE-III data on multi-mode Cepheids represent new challenges to stellar pulsation and evolution theories and hydrodynamical modeling

Multiperiodic Cepheids in the LMC:



see poster by DZIEMBOWSKI & SMOLEC