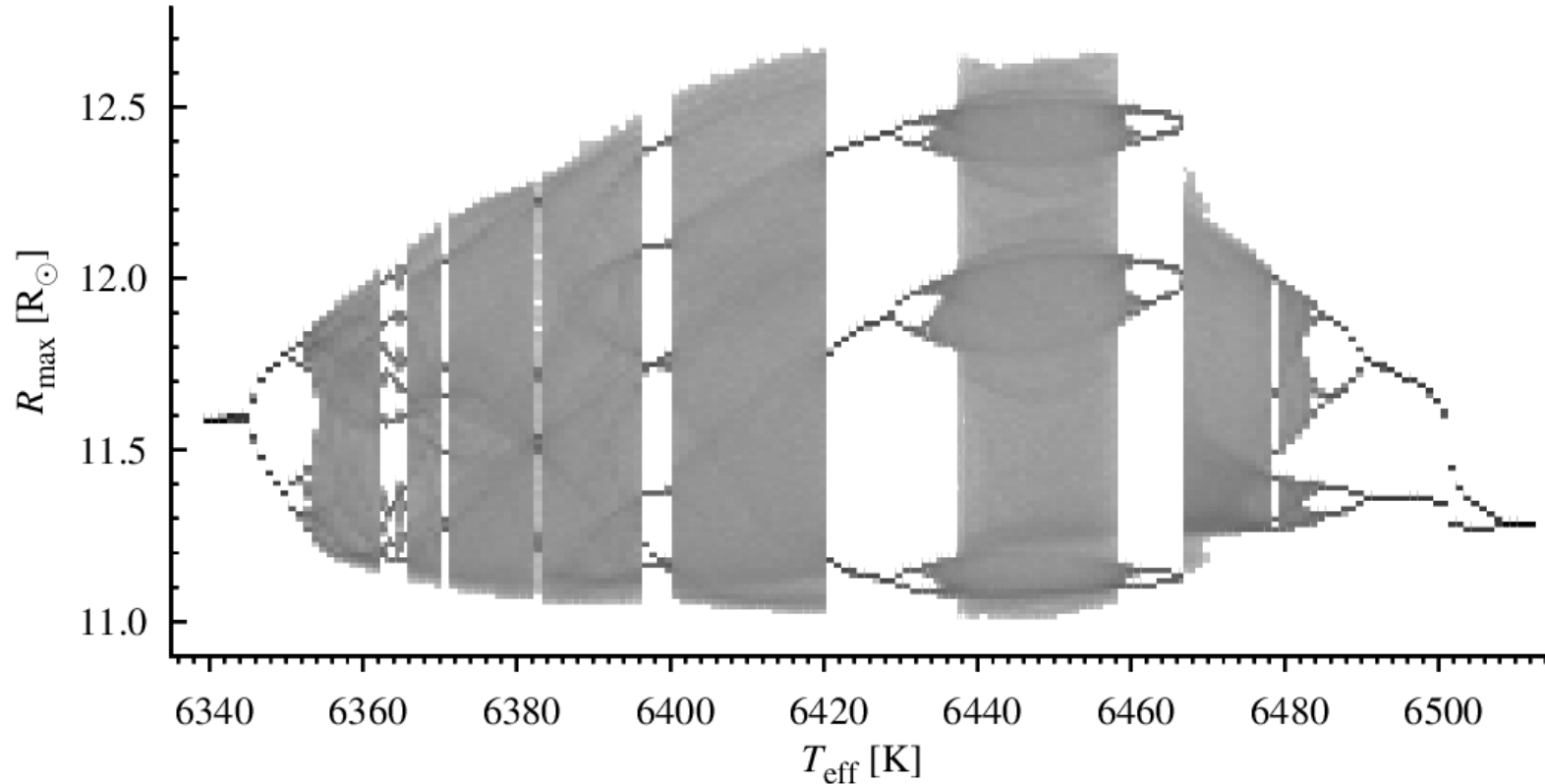


Chaos in hydrodynamic models of pulsating BL Her-type stars

Radosław Smolec

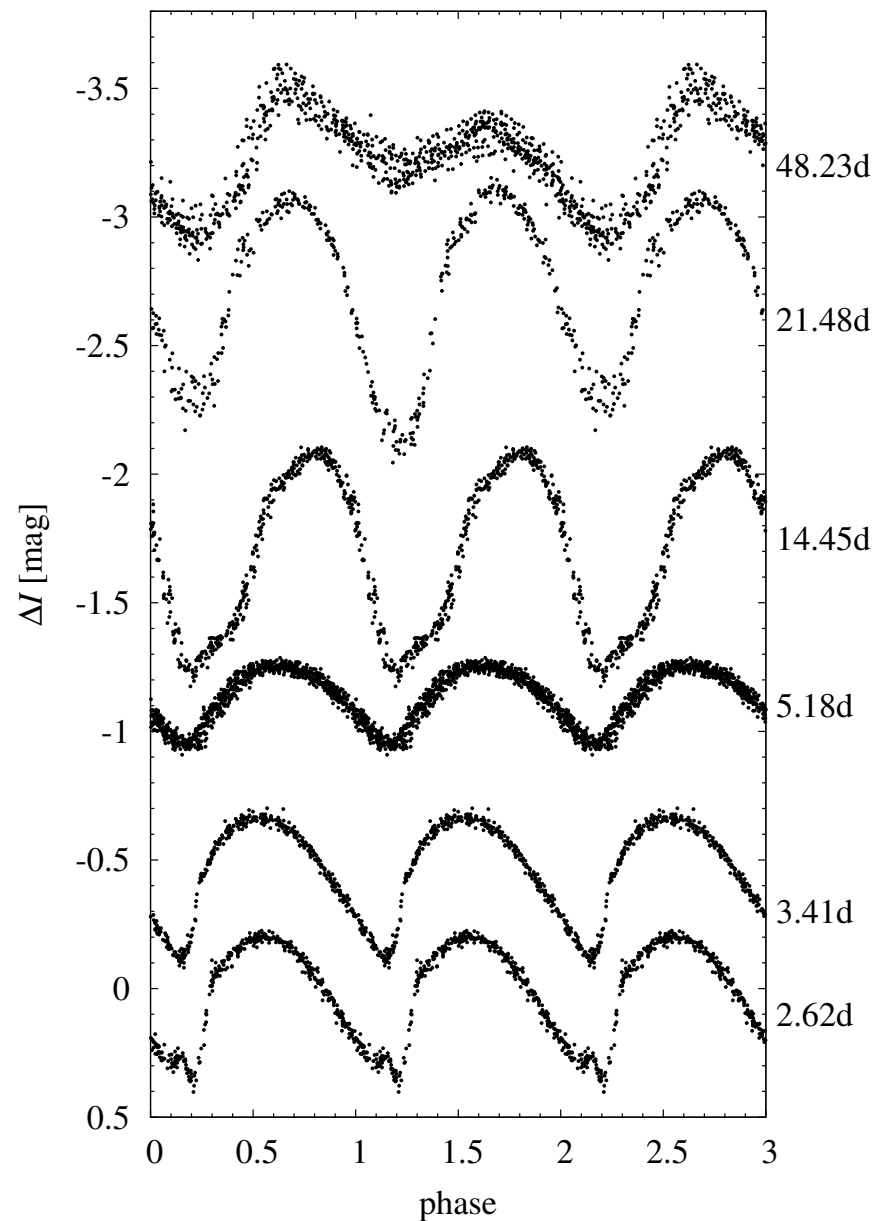


- *Chaos in hydrodynamic BL Herculis models*
Smolec & Moskalik, *MNRAS*, **441**, 101 (2014)

Outline

- ▶ BL Her stars – observational status
- ▶ Hydrodynamical modelling
- ▶ Chaos in BL Her models

Type II Cepheids, BL Her stars



RV Tau stars

- ▶ characteristic feature: period doubling
- ▶ chaos

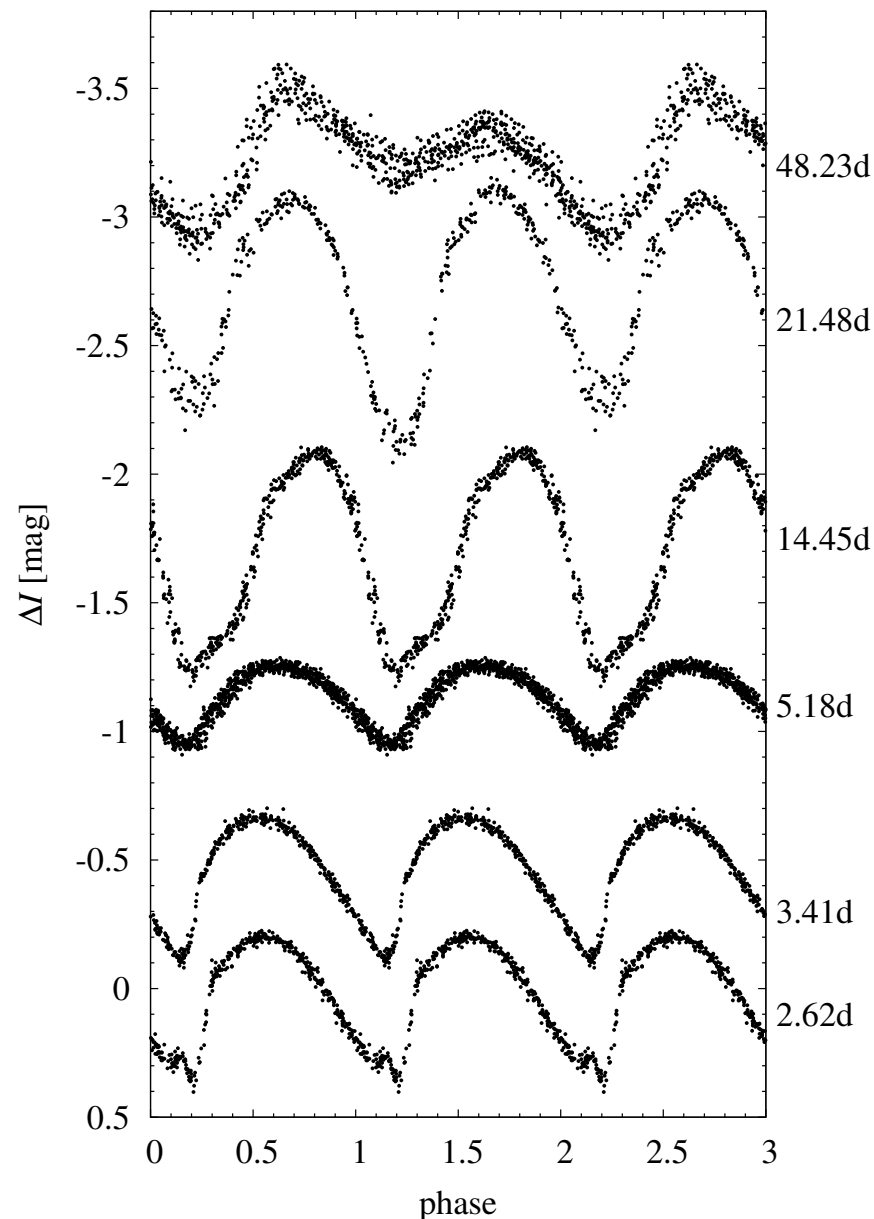
W Vir stars

- ▶ irregular variability observed

BL Her stars

- ▶ very regular variation

Type II Cepheids, BL Her stars



RV Tau stars

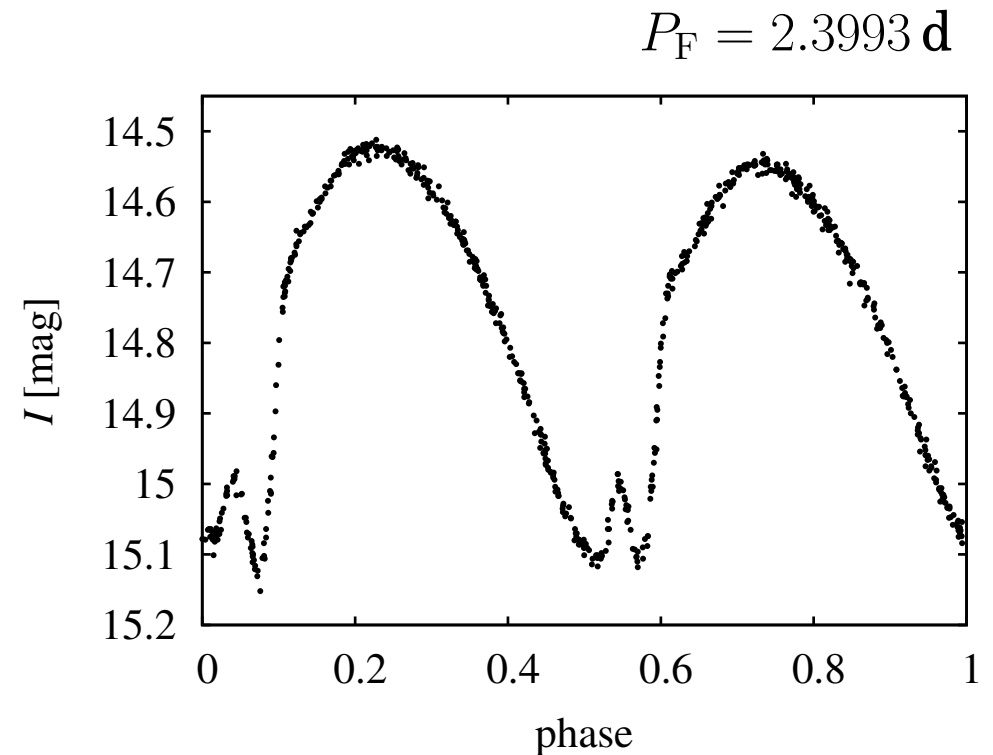
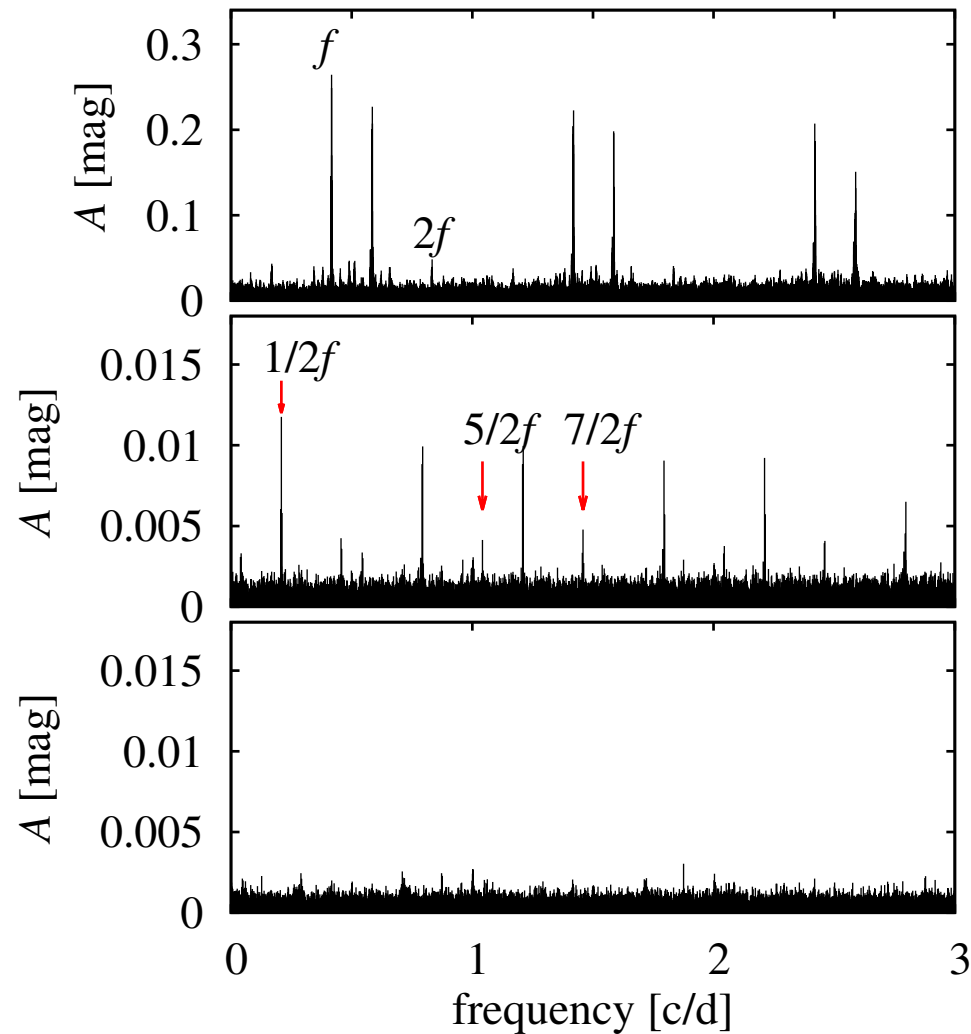
- ▶ characteristic feature: period doubling
- ▶ chaos

W Vir stars

- ▶ irregular variability observed

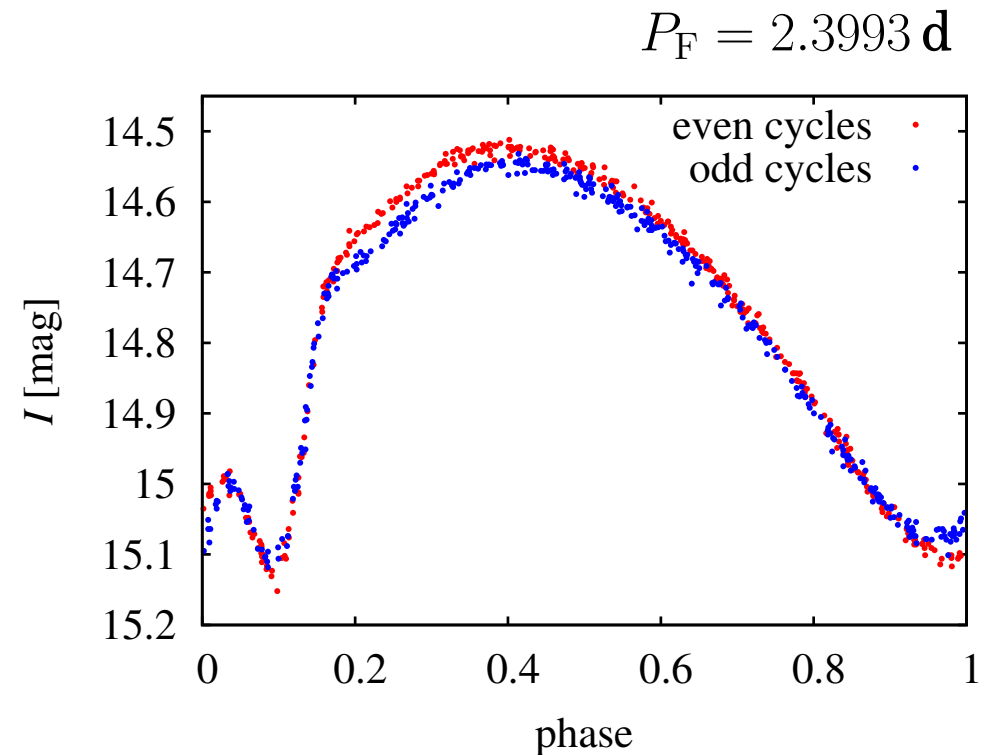
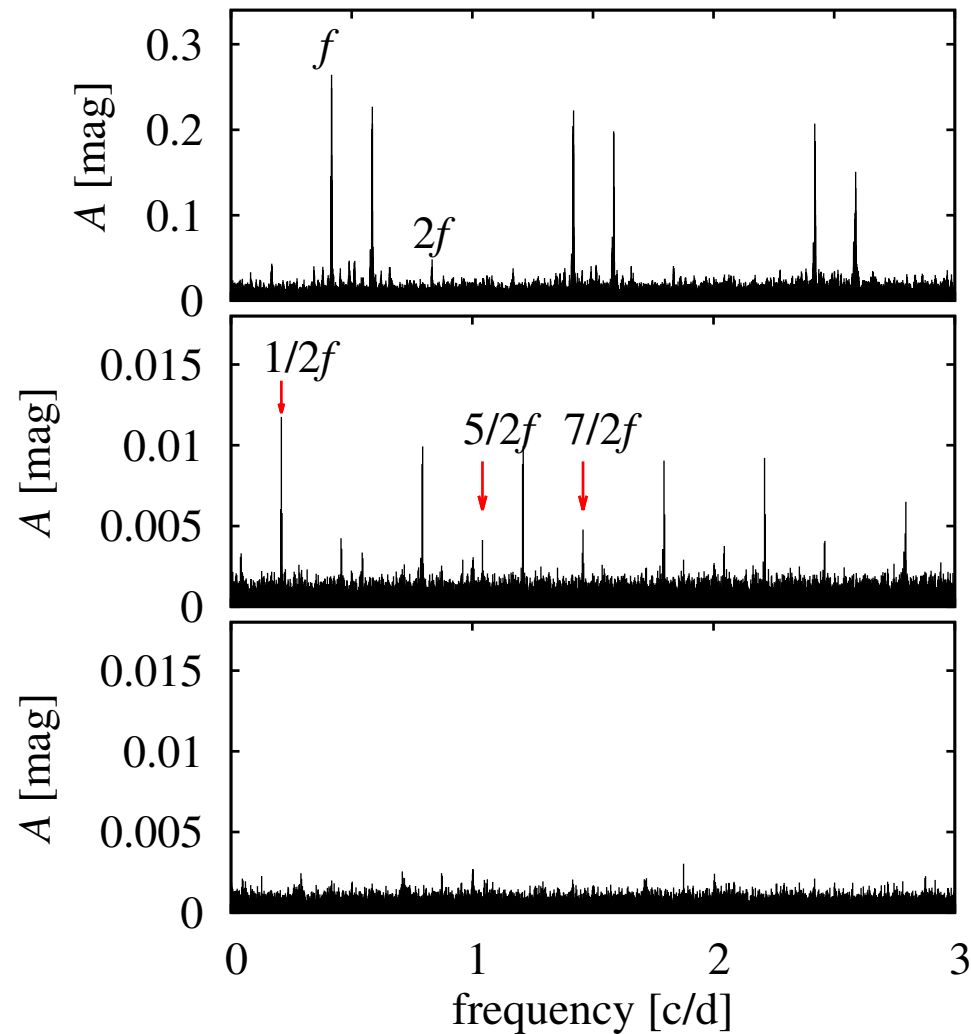
BL Her stars

- ▶ very regular variation
- ▶ one period doubled star, predicted with models

OGLE-BLG-T2CEP-279: period-doubled BL Her star

Soszyński et al. (2011)

Smolec et al. (2012)

OGLE-BLG-T2CEP-279: period-doubled BL Her star

Soszyński et al. (2011)

Smolec et al. (2012)

Pulsation codes

(Smolec & Moskalik 2008)

- ★ static model builder, LNA code, nonlinear code
- ★ lagrangean codes based on old radiative [Stellingwerf's \(1975\)](#) codes
 - + convection model ([Kuhfuß 1986](#))
 - + updated physics, new model builder and LNA scheme

Pulsation codes

(Smolec & Moskalik 2008)

$$\frac{dR}{dt} = U$$

$$\frac{dU}{dt} = -\frac{1}{\rho} \nabla (P + P_t + P_\nu) - \nabla \phi$$

$$\frac{de}{dt} + P \frac{dV}{dt} = -\frac{1}{\rho} \nabla (F_r + F_c) - (S - D)$$

$$\frac{de_t}{dt} + (P_t + P_\nu) \frac{dV}{dt} = -\frac{1}{\rho} \nabla F_t + (S - D)$$

e_t turbulent energy

S source function $\alpha \alpha_s T P Q e_t^{1/2} Y / H_P$

D turbulent dissipation $(\alpha_d / \alpha) (e_t^{3/2} / H_P)$

P_t turbulent pressure $\alpha_p \rho e_t$

P_ν eddy-viscous effects $-(4/3) \alpha \alpha_\nu H_P e_t^{1/2} R \frac{\partial(U/R)}{\partial R}$

F_c convective heat flux $\alpha \alpha_c \rho T c_P e_t^{1/2} Y$

F_t turbulent flux $-\alpha_t \alpha \rho H_P e_t^{1/2} \frac{\partial e_t}{\partial R}$

8 free parameters

$$F_c \sim Y, S \sim Y, Y = \nabla - \nabla_a$$

Pulsation codes

(Smolec & Moskalik 2008)

$$\begin{aligned}\frac{dR}{dt} &= U \\ \frac{dU}{dt} &= -\frac{1}{\rho} \nabla (P + P_t + P_\nu) - \nabla \phi \\ \frac{de}{dt} + P \frac{dV}{dt} &= -\frac{1}{\rho} \nabla (F_r + F_c) - (S - D) \\ \frac{de_t}{dt} + (P_t + P_\nu) \frac{dV}{dt} &= -\frac{1}{\rho} \nabla F_t + (S - D)\end{aligned}$$

Static model builder & LNA code:

- ★ construction of the static model (typically 150–200 mass shells)
- ★ LNA stability analysis

Pulsation codes

(Smolec & Moskalik 2008)

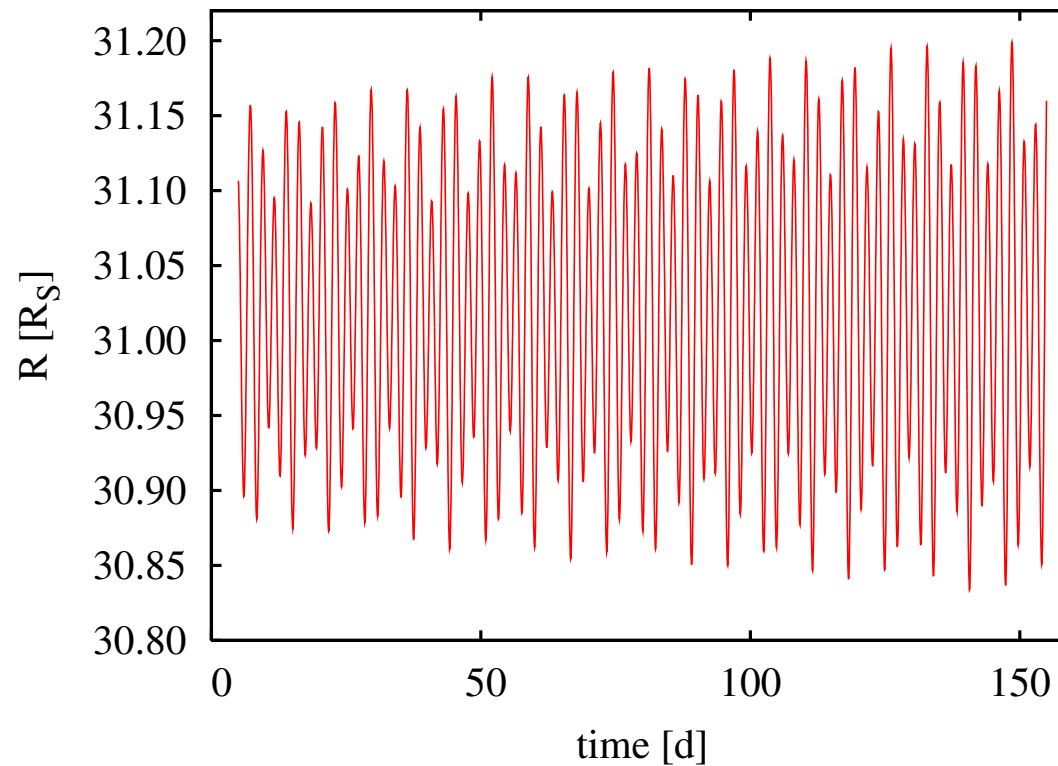
$$\begin{aligned}\frac{dR}{dt} &= U \\ \frac{dU}{dt} &= -\frac{1}{\rho} \nabla (P + P_t + P_\nu) - \nabla \phi \\ \frac{de}{dt} + P \frac{dV}{dt} &= -\frac{1}{\rho} \nabla (F_r + F_c) - (S - D) \\ \frac{de_t}{dt} + (P_t + P_\nu) \frac{dV}{dt} &= -\frac{1}{\rho} \nabla F_t + (S - D)\end{aligned}$$

Nonlinear code:

- ★ initial model: static
- ★ initial perturbation (kick): scaled velocity eigenvectors
- ★ forward time-integration till attractor is reached

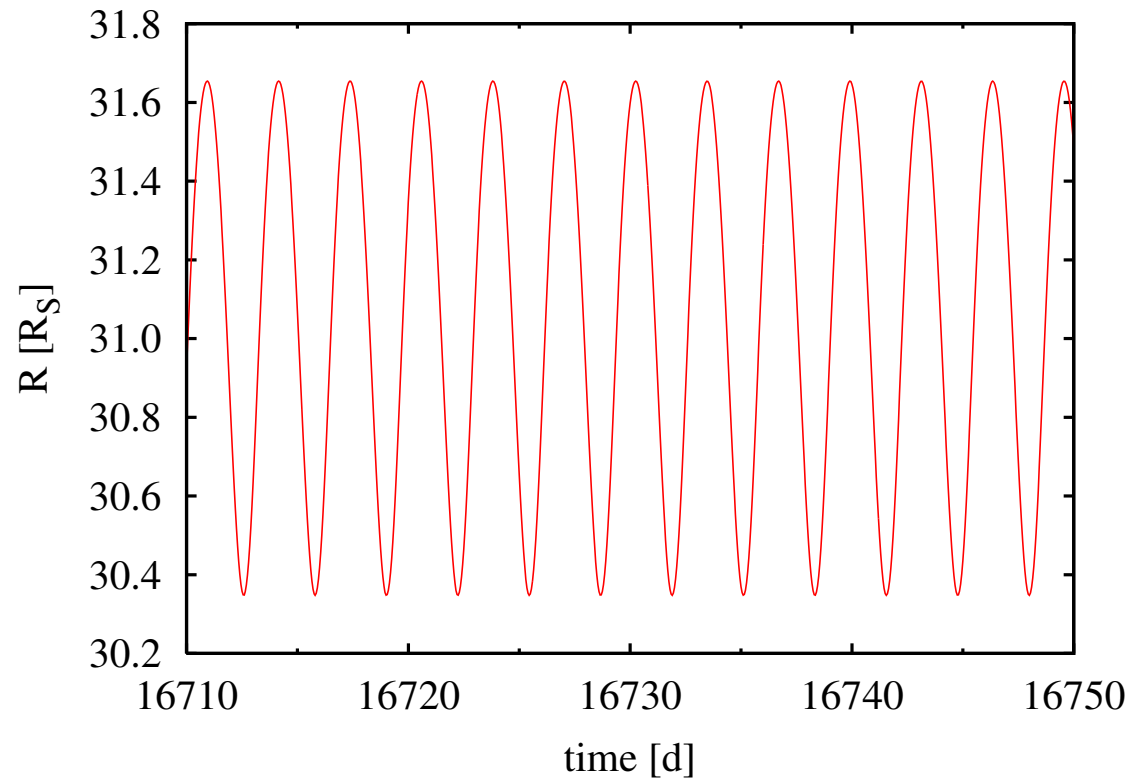
Nonlinear code

Initial state: growth of F and 10 modes

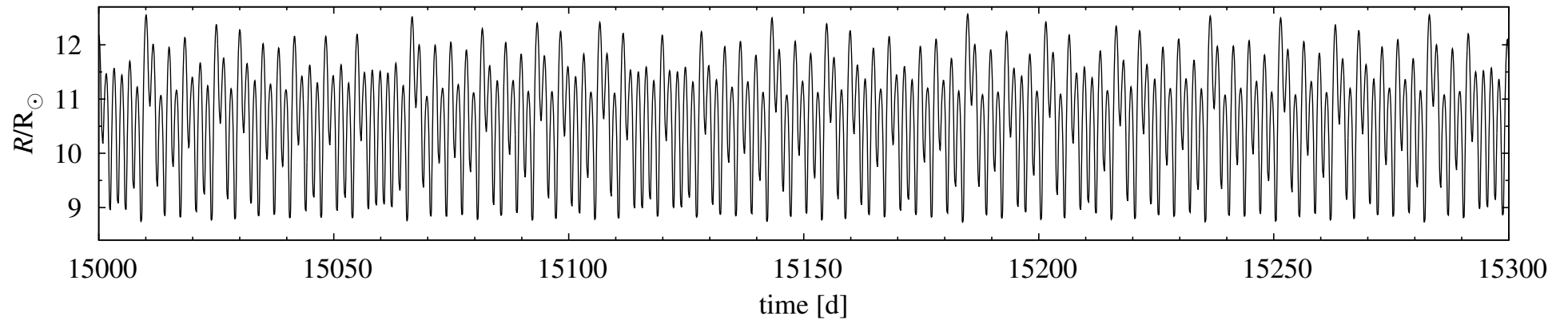


Nonlinear code

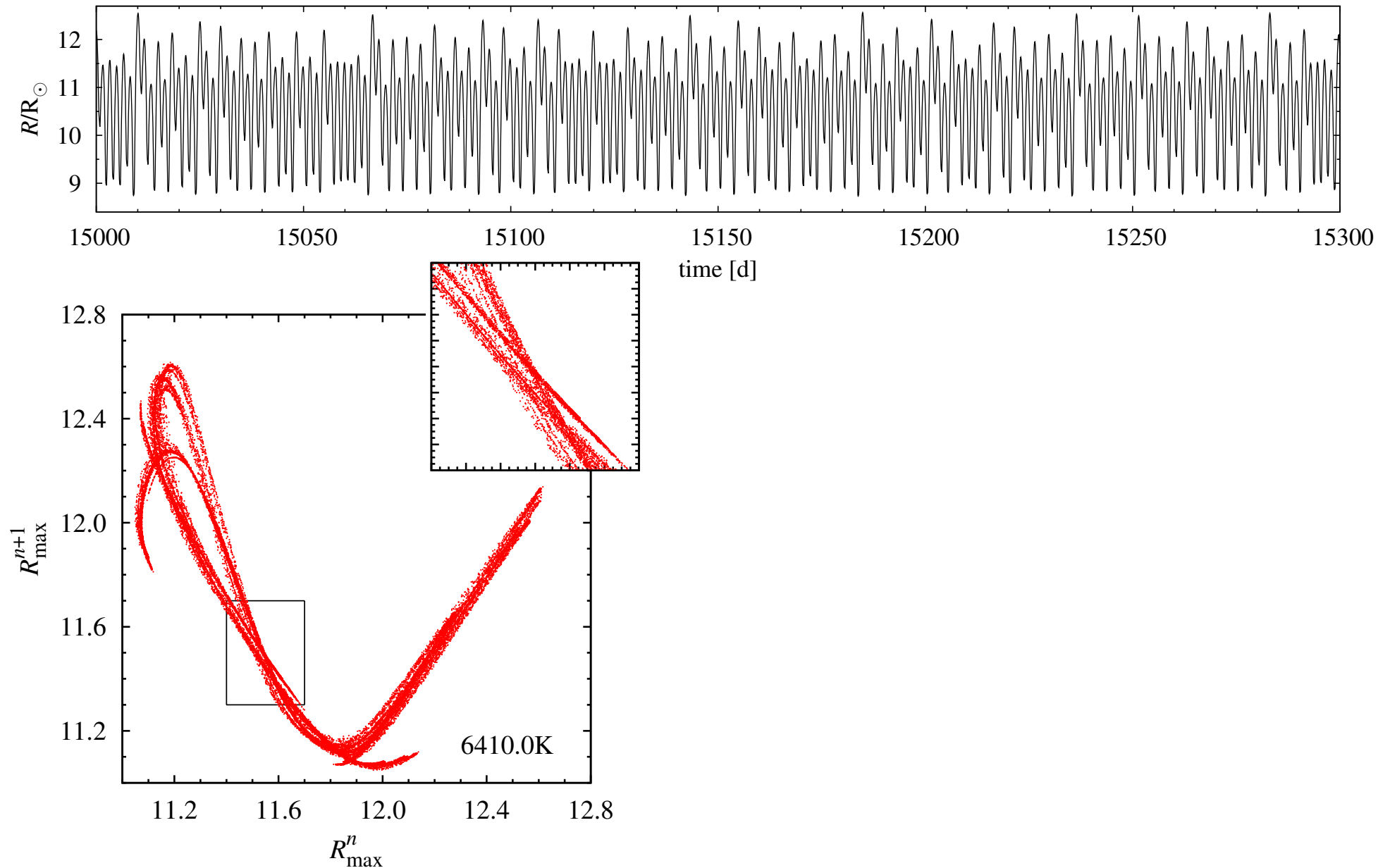
Possible final state: single-mode pulsation (limit cycle)



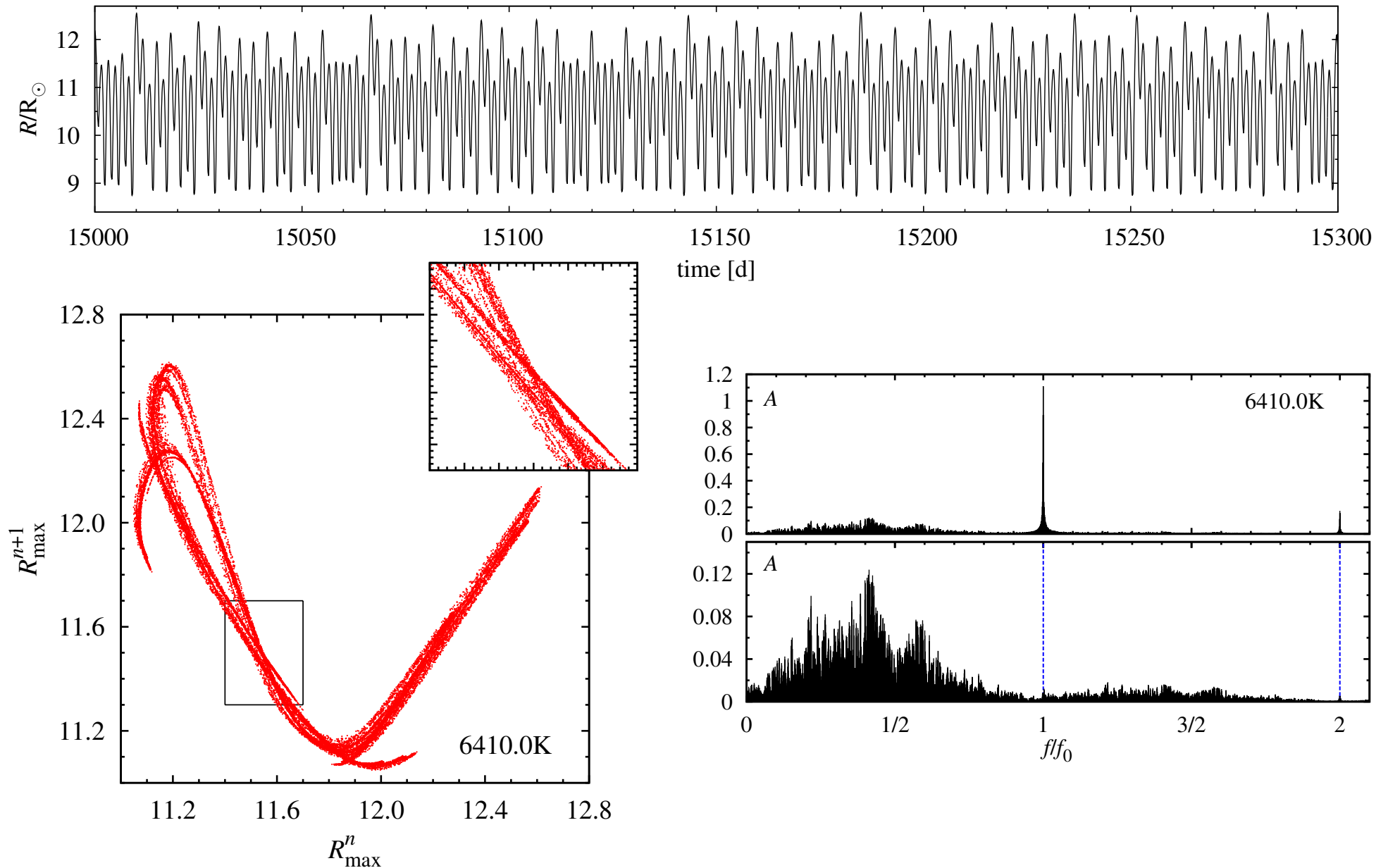
Typical chaotic model, 6410K



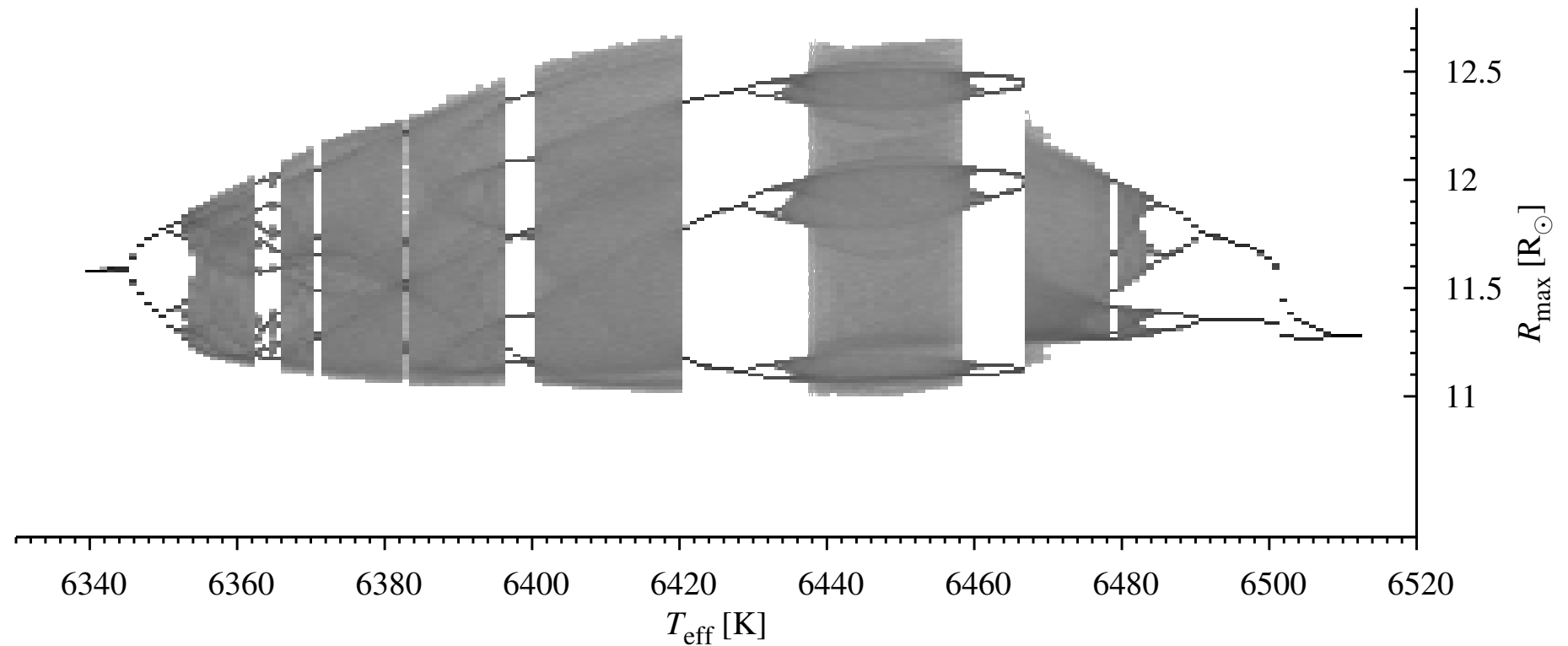
Typical chaotic model, 6410K



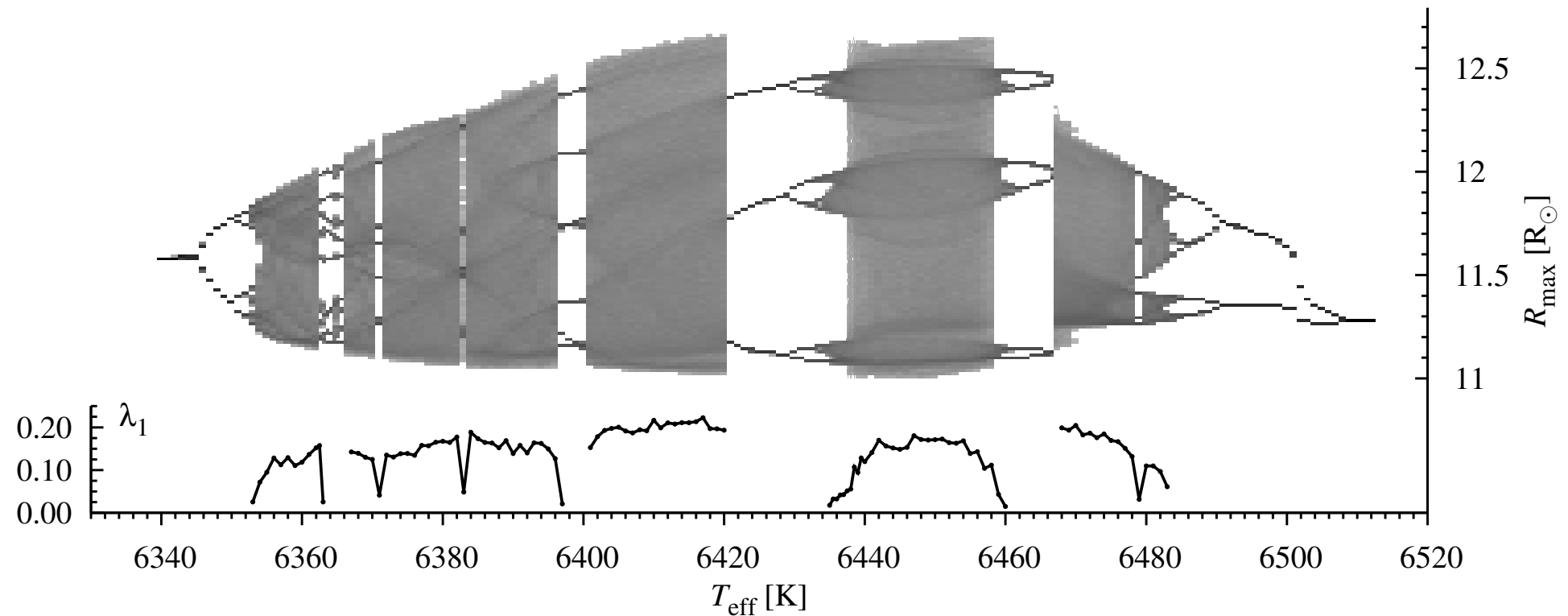
Typical chaotic model, 6410K



Bifurcation diagram and largest Lyapunov exponents

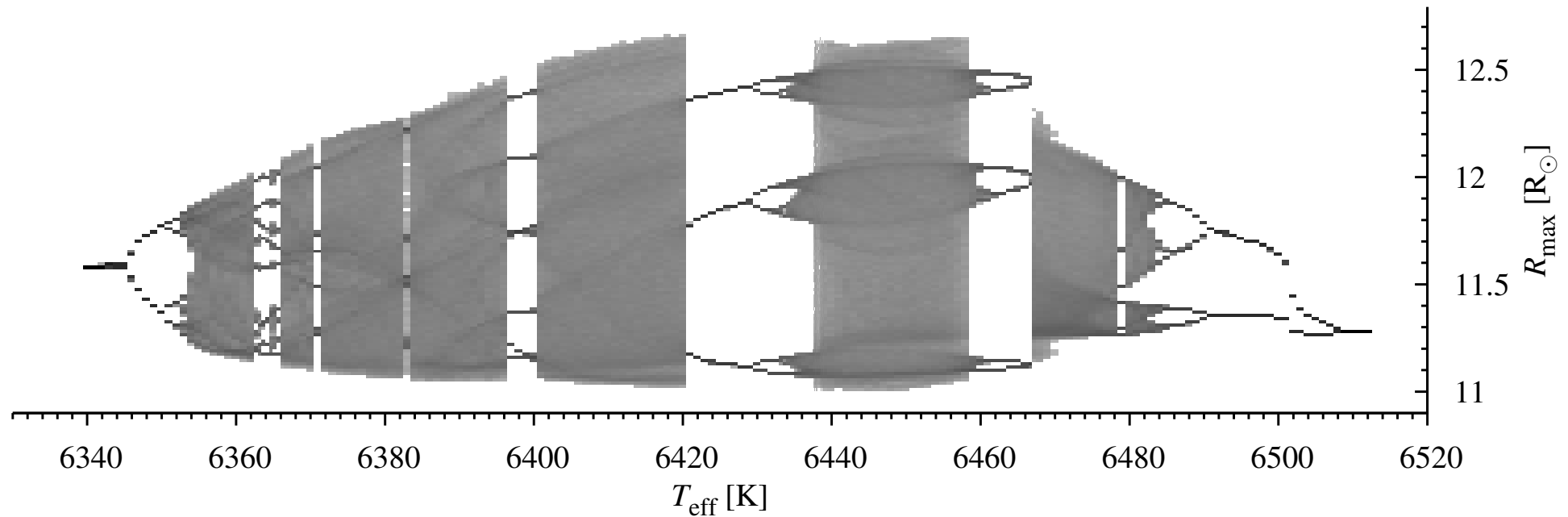


Bifurcation diagram and largest Lyapunov exponents



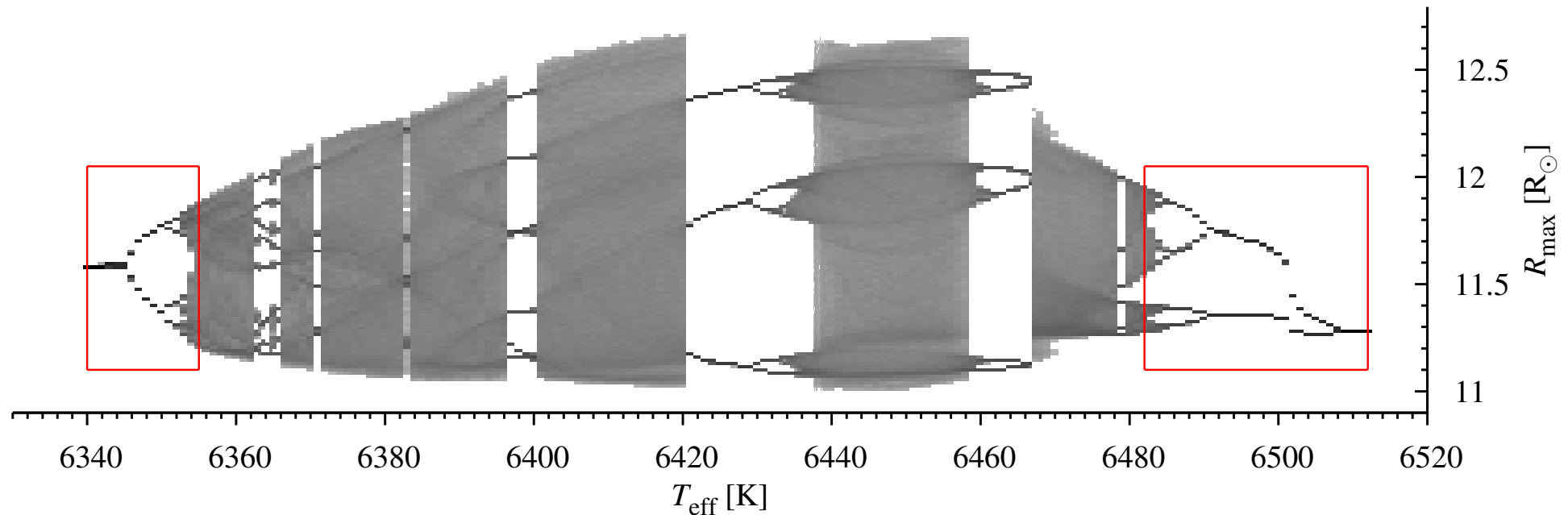
- λ_1 determined with algorithm of [Rosenstein, Collins & DeLuca \(1993\)](#)

Diversity of dynamic phenomena



- ★ period doubling cascades
- ★ chaotic domains and periodic windows
- ★ type-I intermittency
- ★ crises bifurcation
- ★ type-III intermittency

Diversity of dynamic phenomena



★ **period doubling cascades**

★ chaotic domains and periodic windows

★ type-I intermittency

★ crises bifurcation

★ type-III intermittency

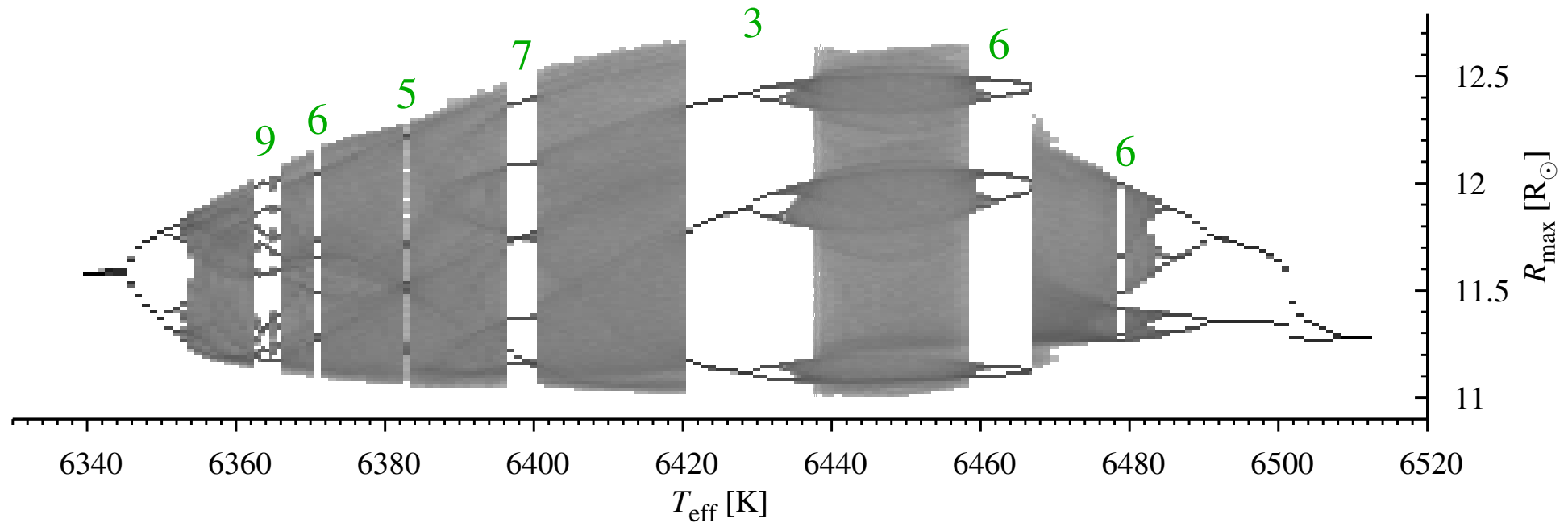
► *Feigenbaum constant*

$$\triangleright d_2/d_4 = 3.6 \pm 0.4,$$

$$d_4/d_8 = 5 \pm 2.5 \text{ (hot side)}$$

$$\triangleright d_2/d_4 = 3.5 \pm 0.9 \text{ (cool side)}$$

Diversity of dynamic phenomena



★ period doubling cascades

★ **chaotic domains and periodic windows**

★ type-I intermittency

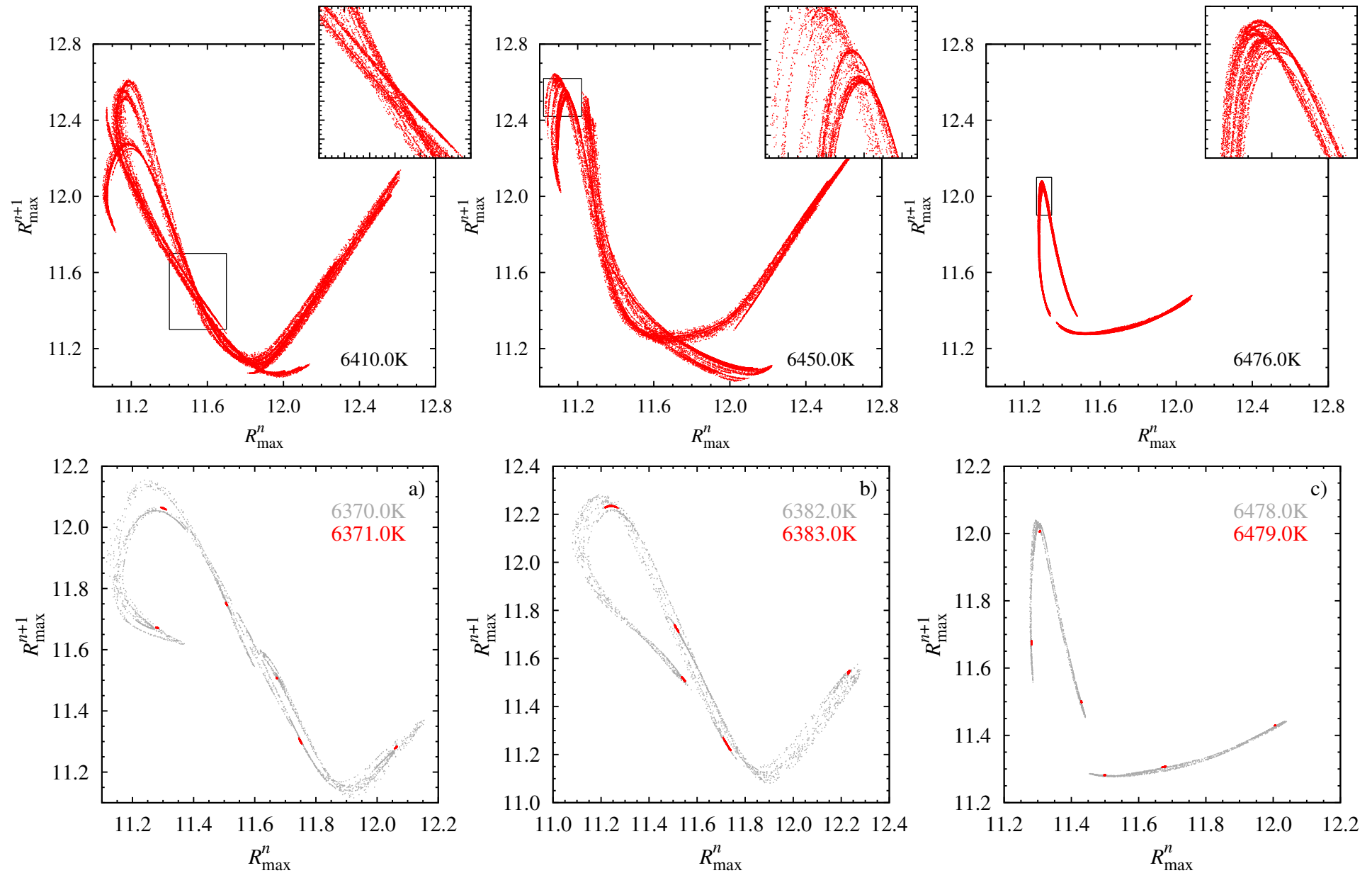
► 7 windows of order

★ crises bifurcation

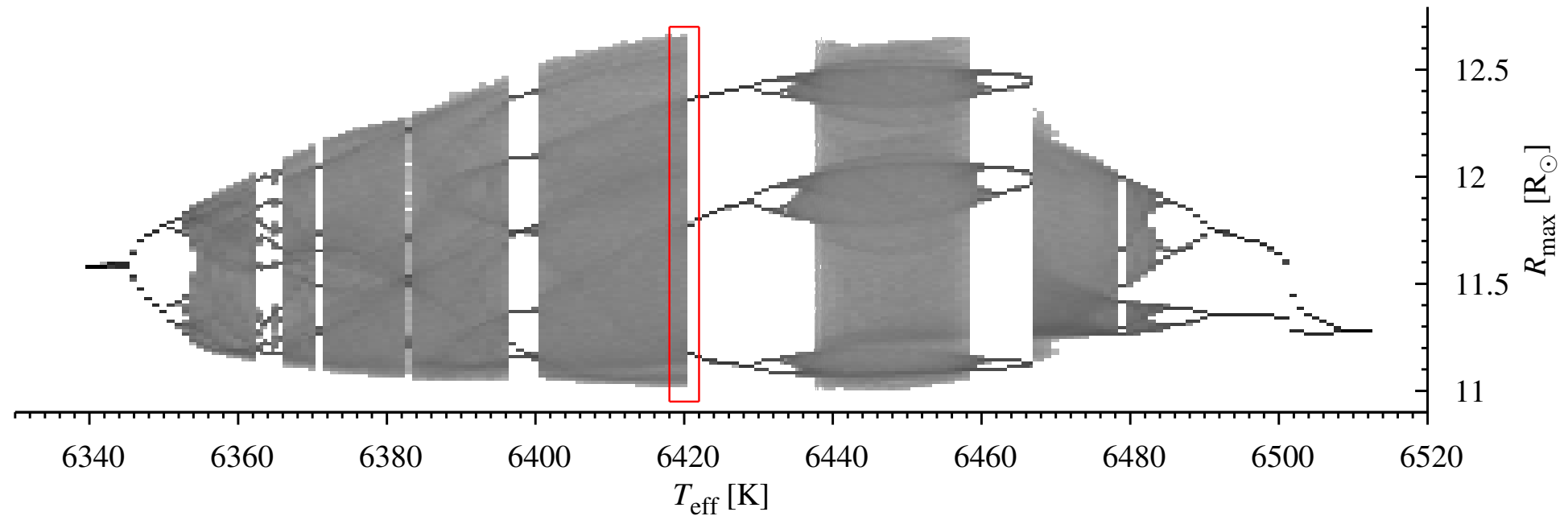
★ type-III intermittency

First return maps

Chaotic domains and periodic windows

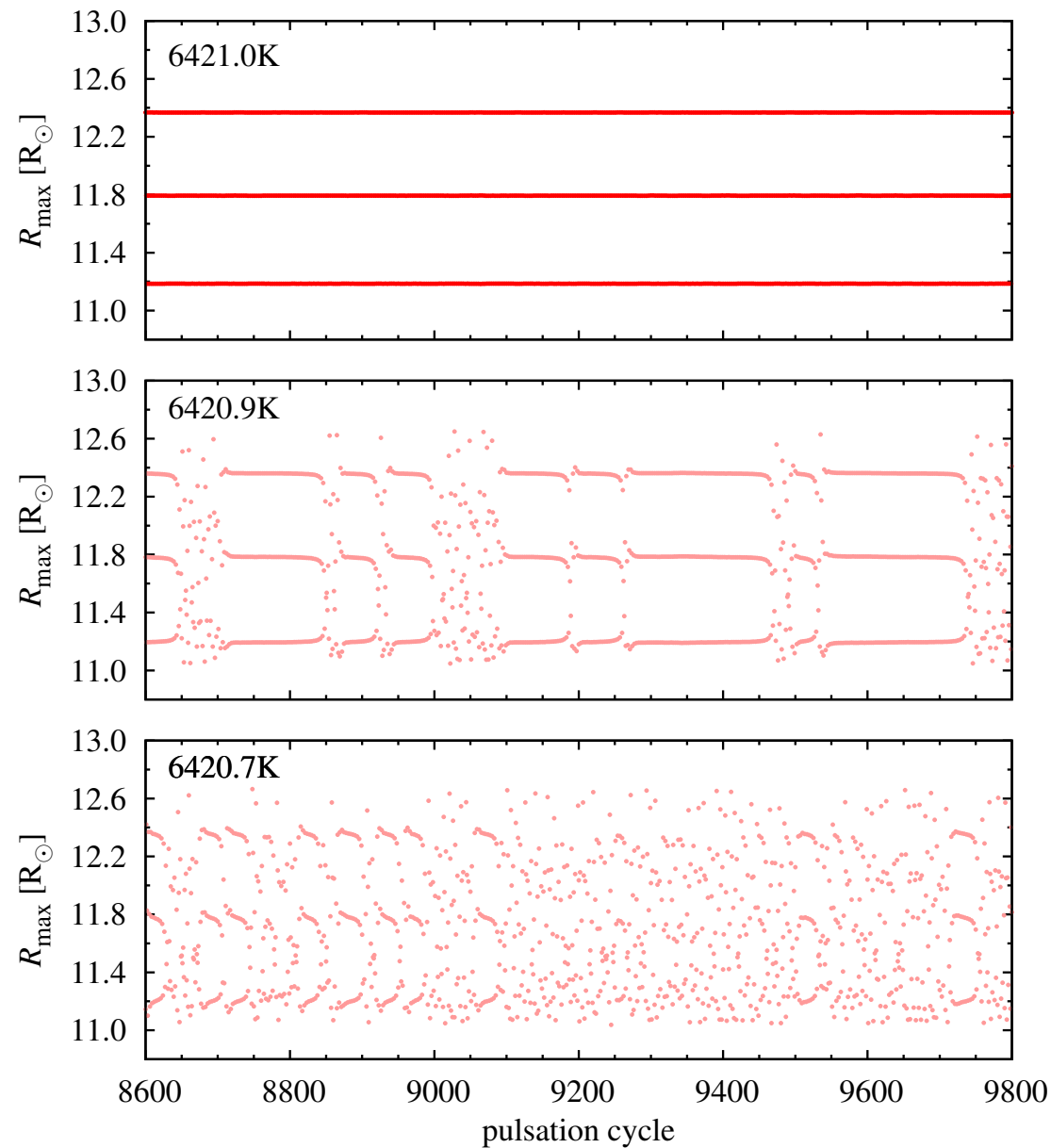


Diversity of dynamic phenomena

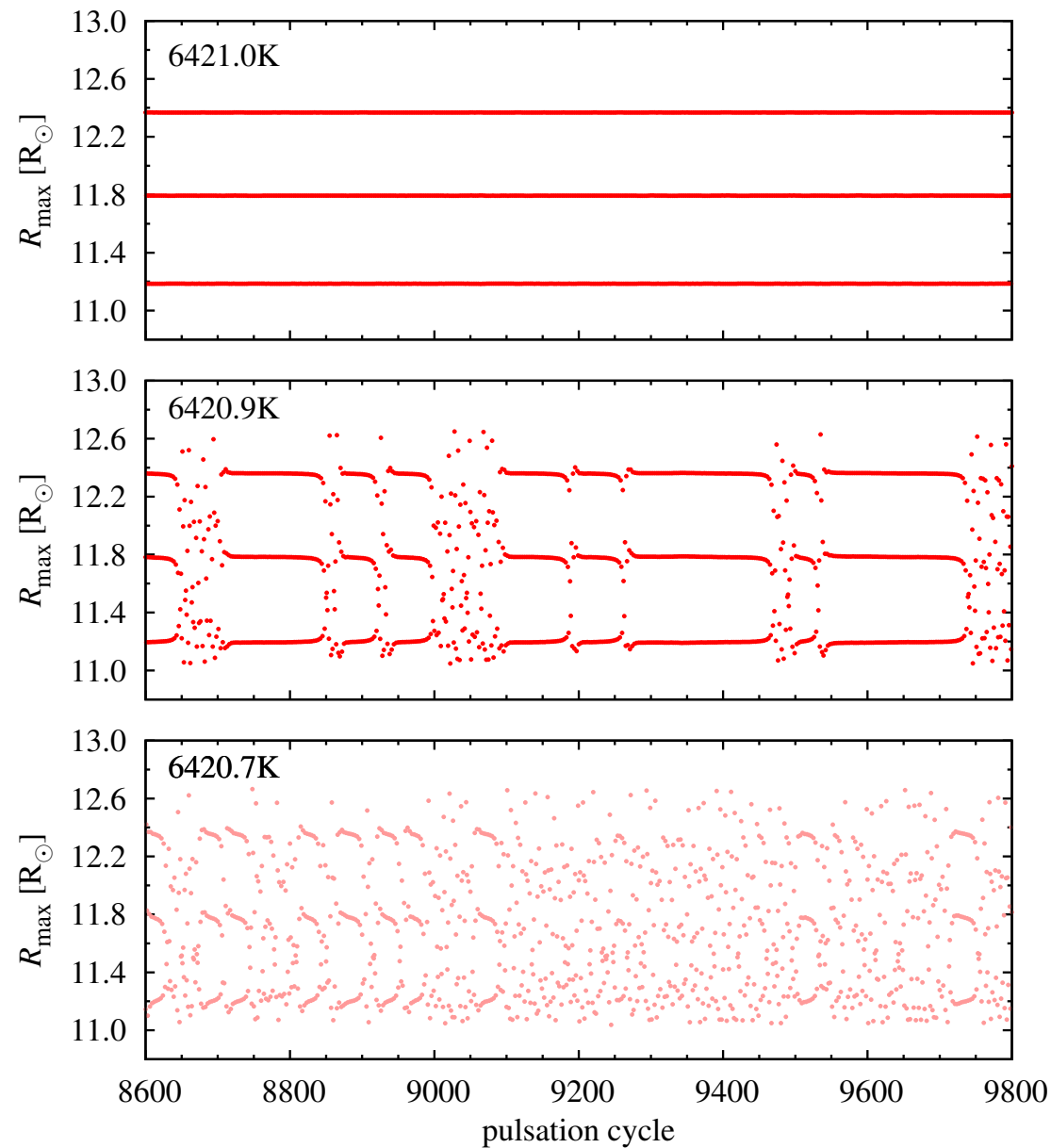


- ★ period doubling cascades
- ★ chaotic domains and periodic windows
- ★ **type-I intermittency**
- ★ crises bifurcation
- ★ type-III intermittency

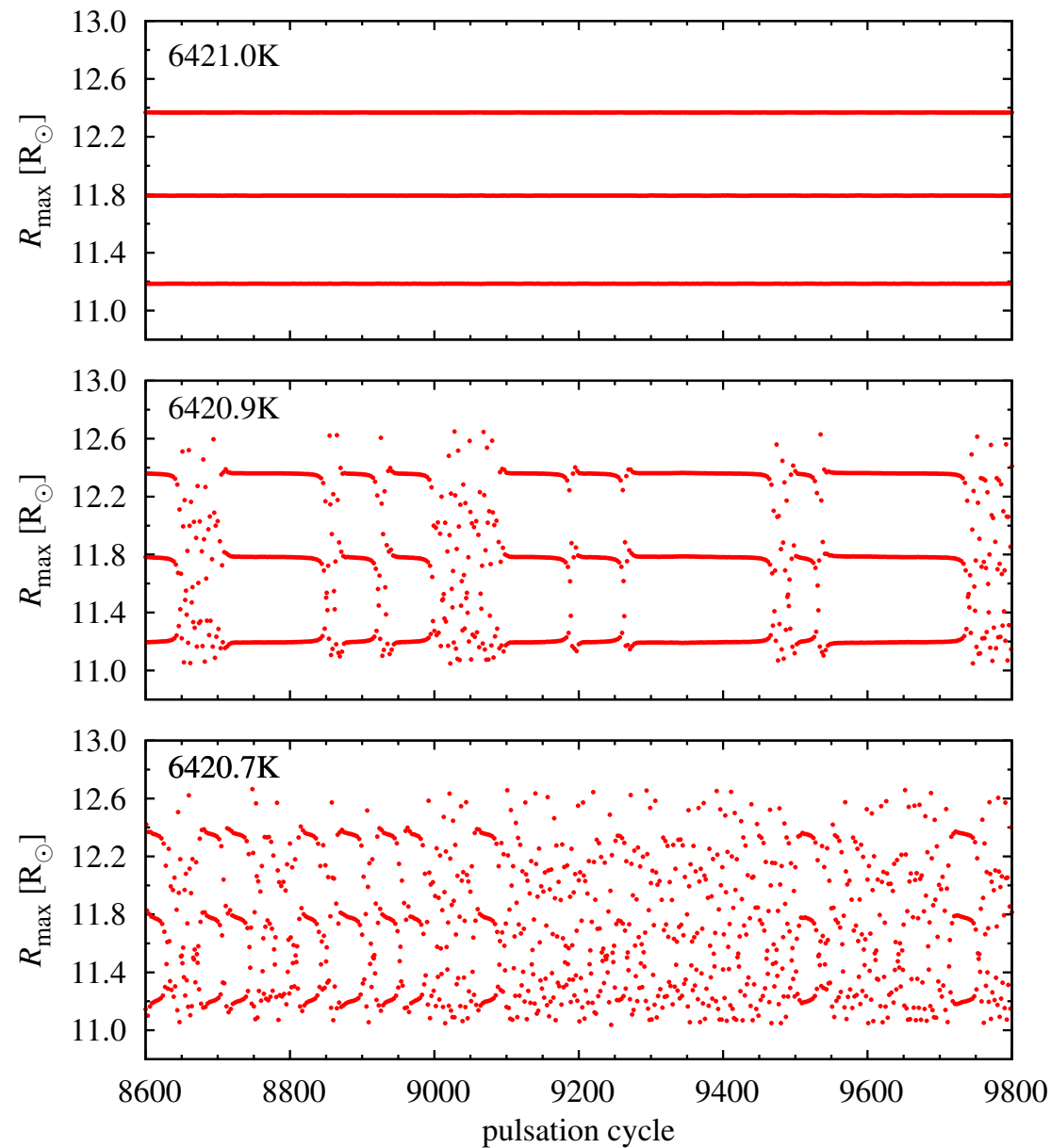
Type-I intermittency



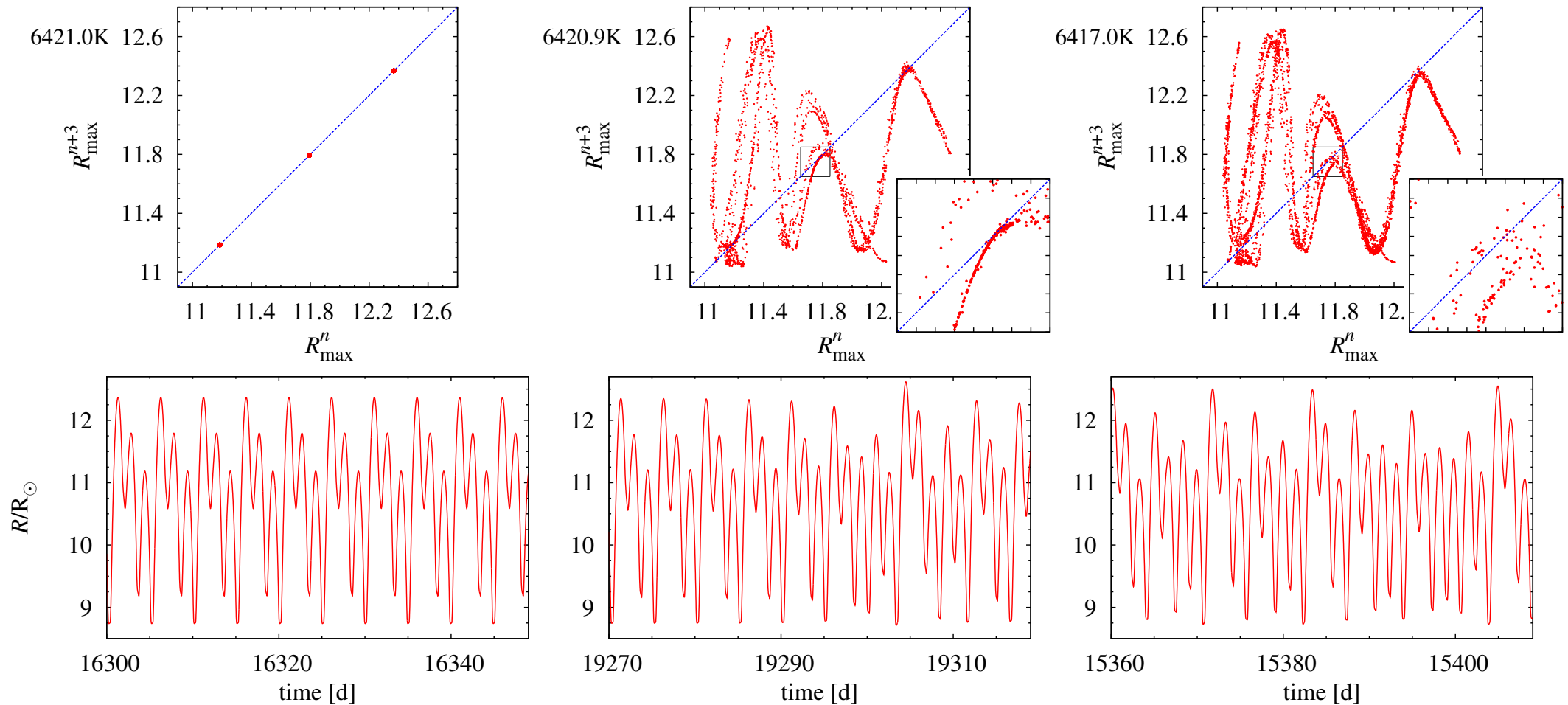
Type-I intermittency



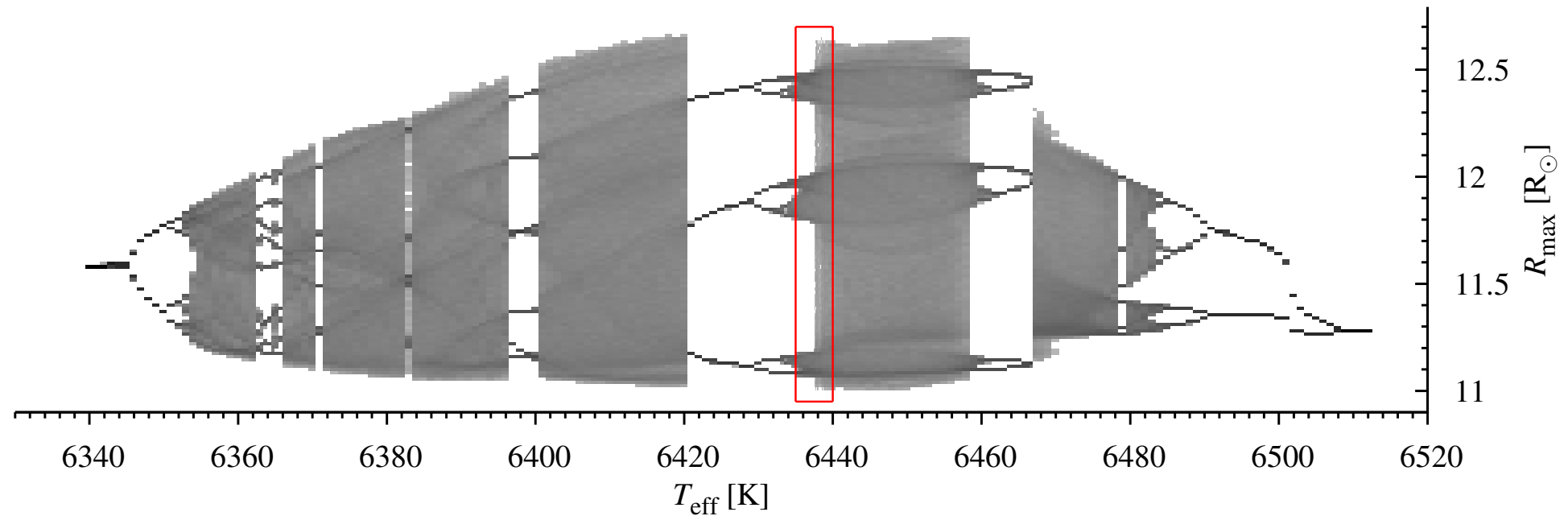
Type-I intermittency



Type-I intermittency

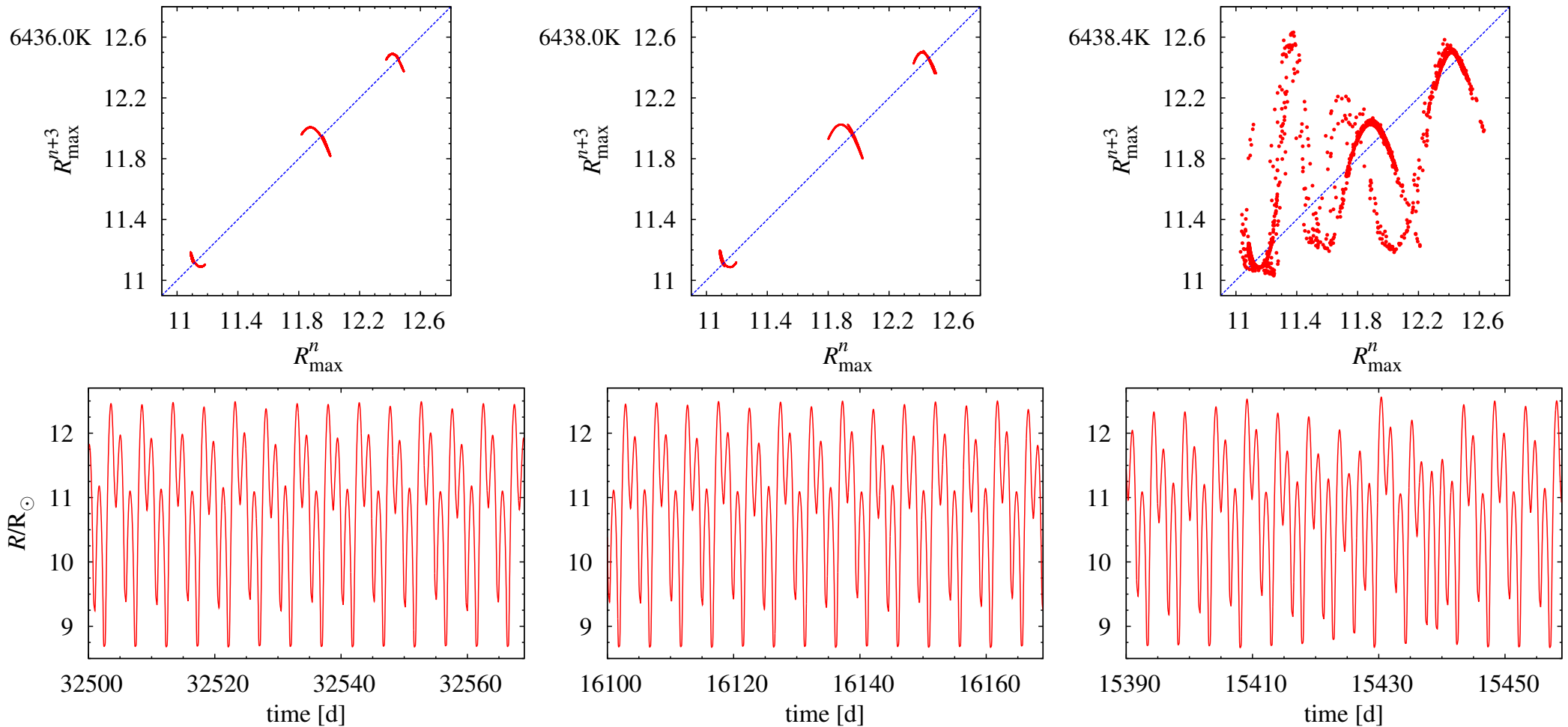


Diversity of dynamic phenomena

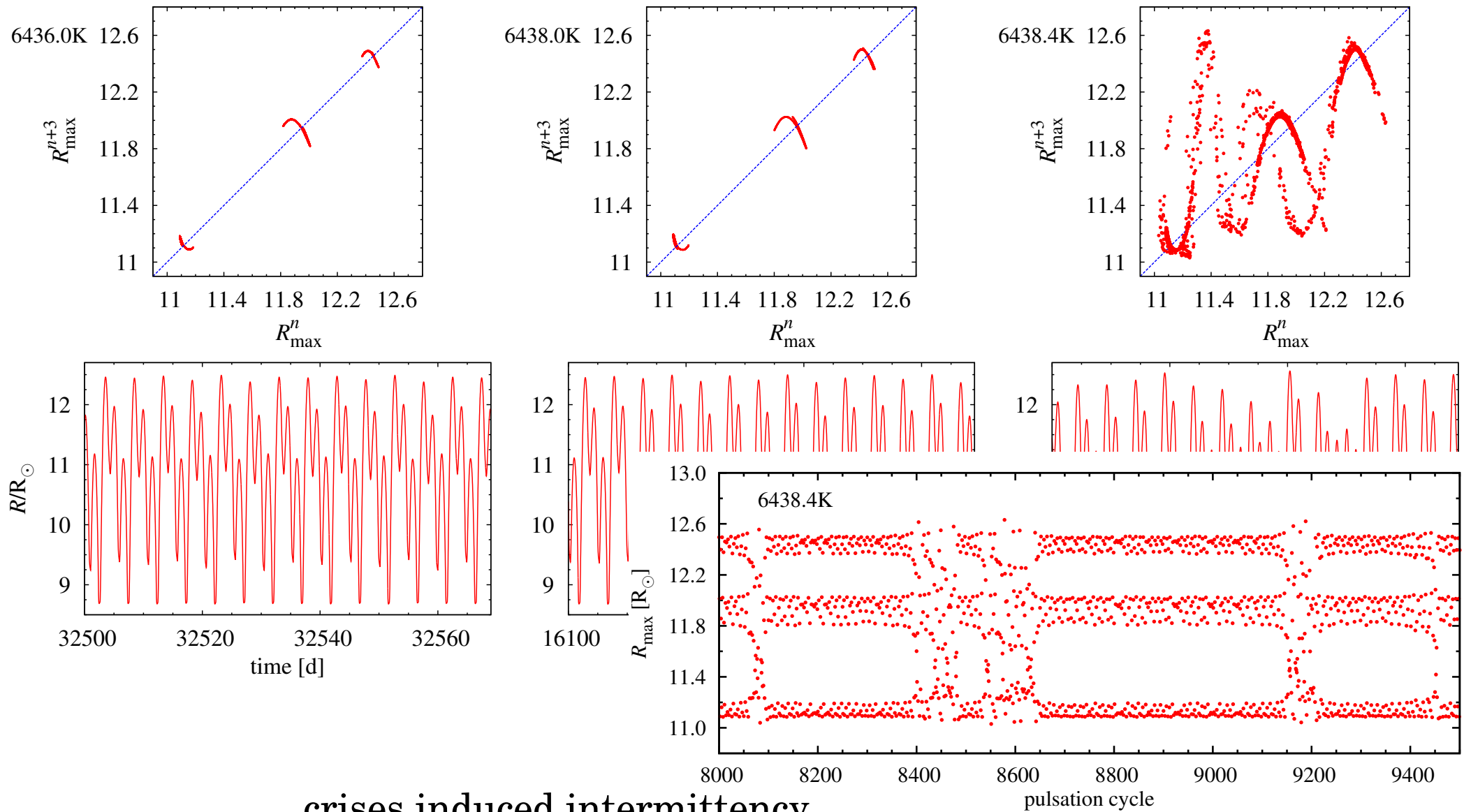


- ★ period doubling cascades
- ★ chaotic domains and periodic windows
- ★ type-I intermittency
- ★ **crises bifurcation**
- ★ type-III intermittency

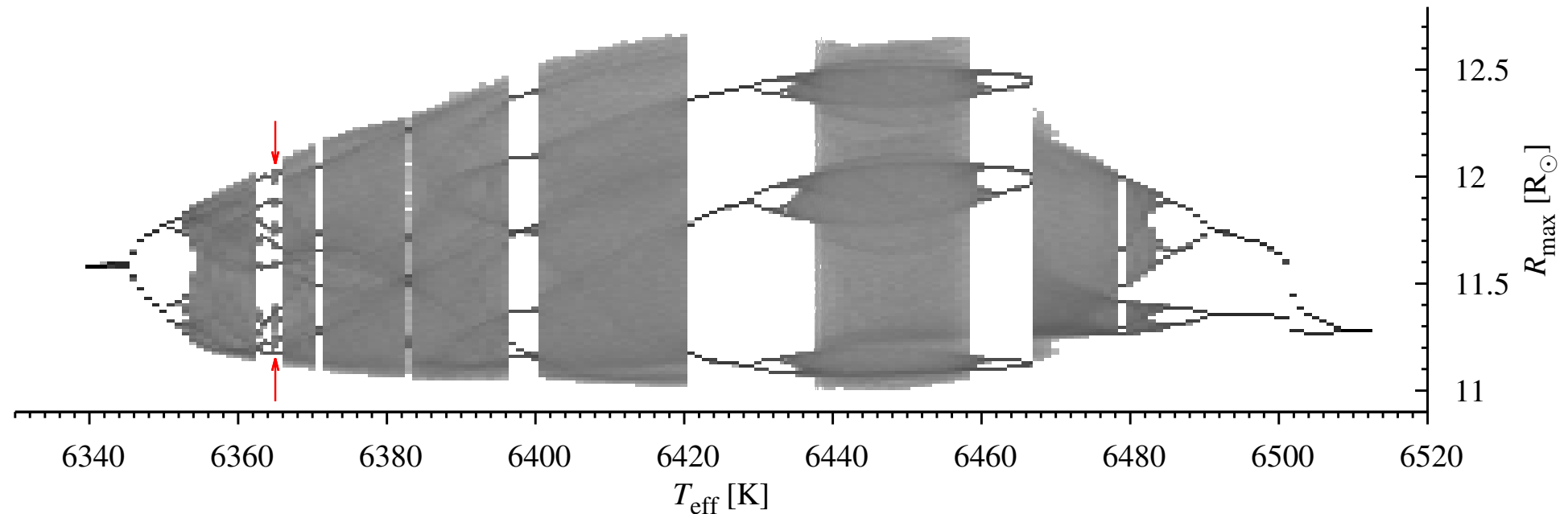
Interior crises bifurcation



Interior crises bifurcation

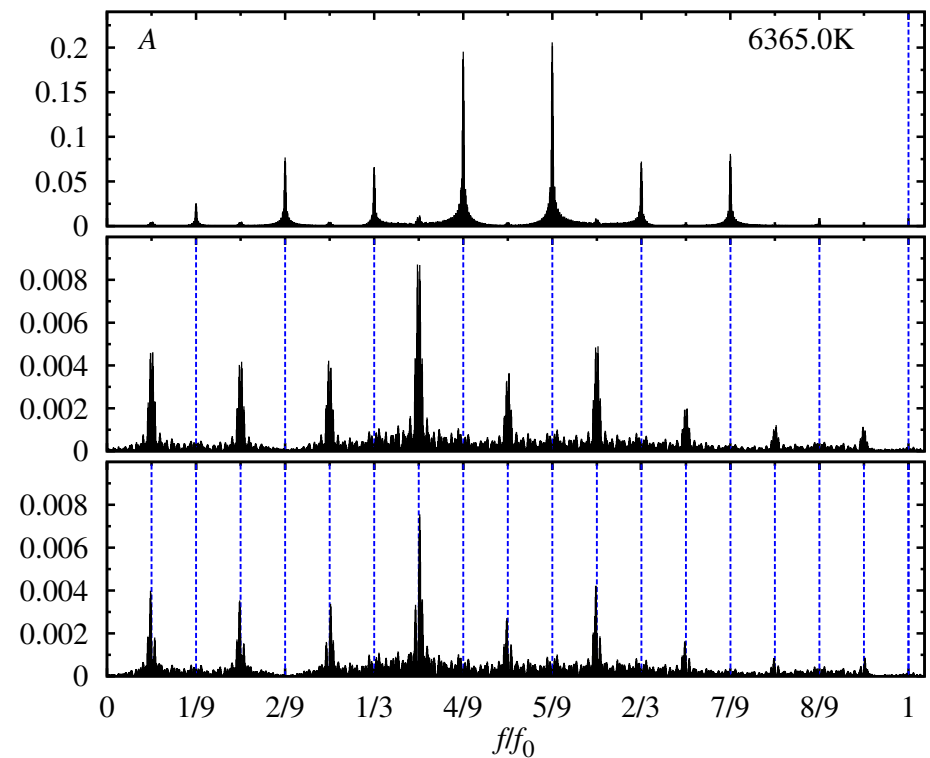
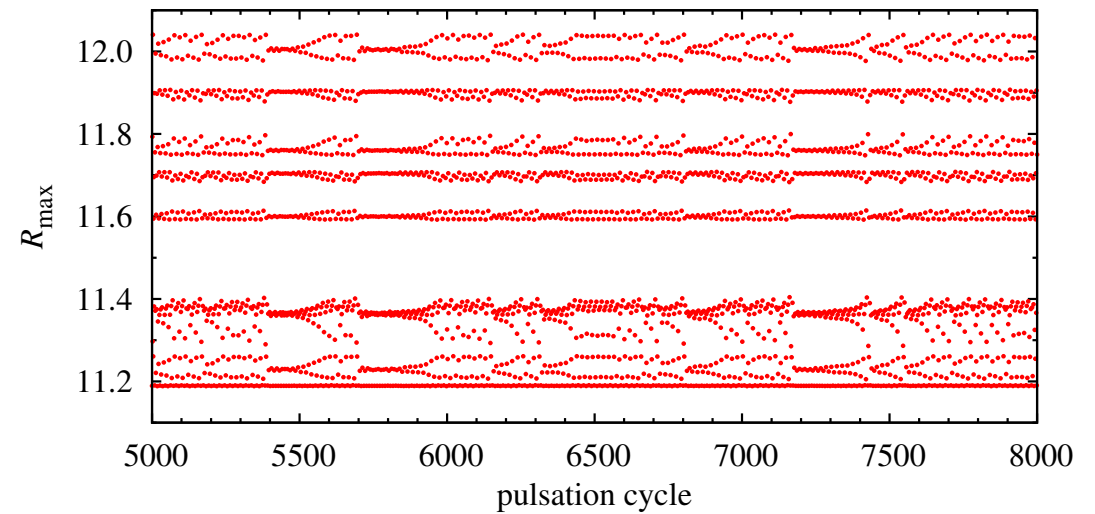
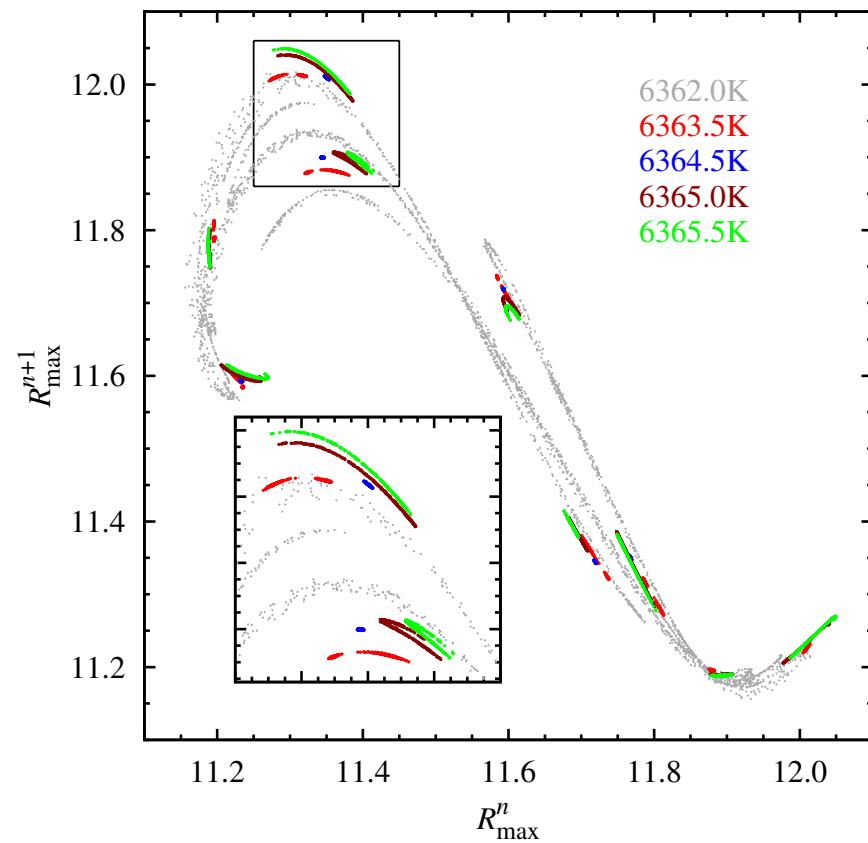


Diversity of dynamic phenomena

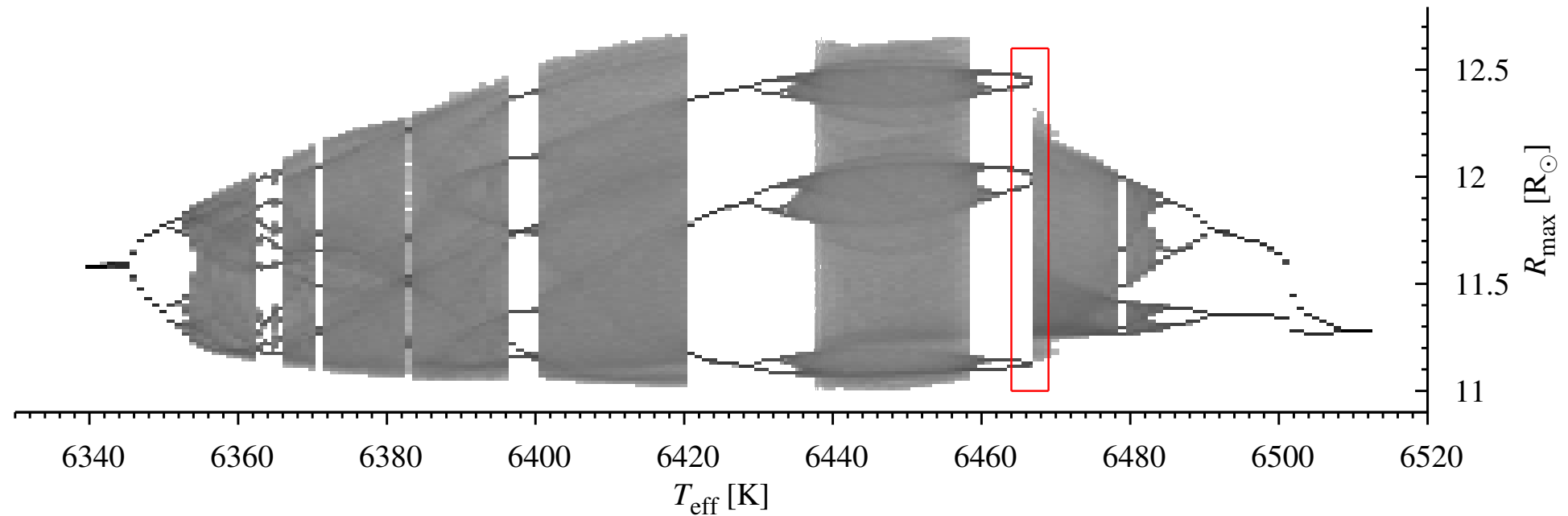


- ★ period doubling cascades
- ★ chaotic domains and periodic windows
- ★ type-I intermittency
- ★ crises bifurcation
- ★ **type-III intermittency**

Type-III intermittency



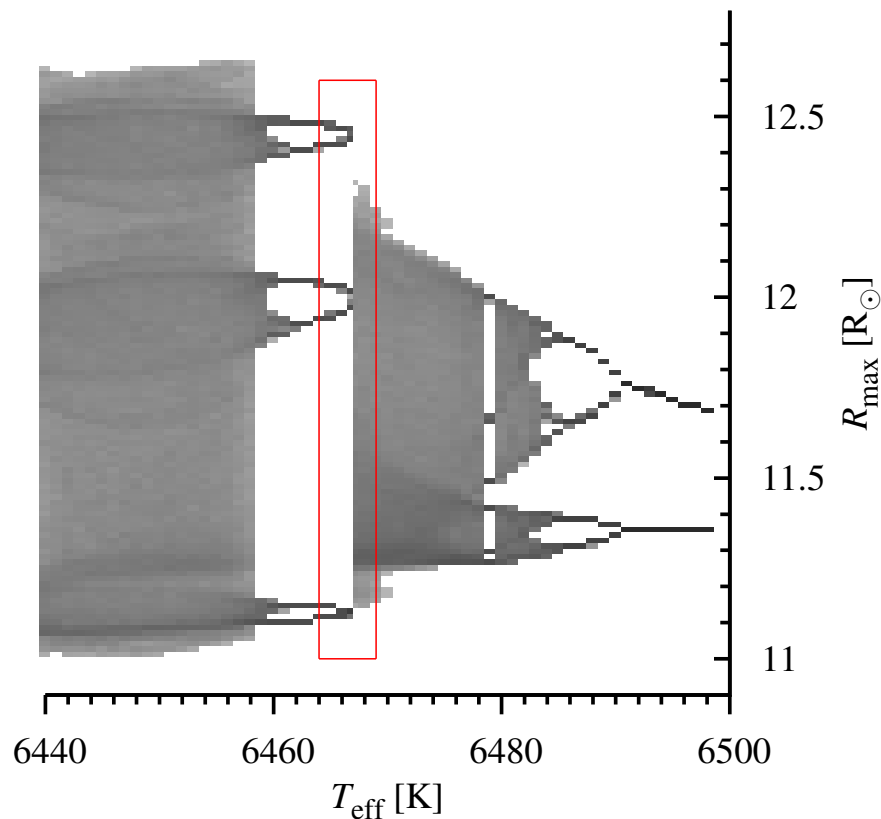
Diversity of dynamic phenomena



- ★ period doubling cascades
- ★ chaotic domains and periodic windows
- ★ type-I intermittency
- ★ crises bifurcation
- ★ type-III intermittency

★ ?

Bifurcation at 6468 K?



- ▶ a stable period-6 cycle, which looks like two stable period-3 cycles born very close to each other, emerges from the chaos
- ▶ a very narrow domain of period-3 cycle? (*must be narrower than 0.1K*)
- ▶ two attractors? (*not detected*)

Conclusions

- ★ a full wealth of chaotic dynamics demonstrated in the models of pulsating stars for the first time
- ★ chaos was not yet detected in BL Her stars
- ★ the models are helpful in understanding more luminous irregular pulsators
- ★ the models motivate search for dynamic phenomena: periodic cycles, intermittency, etc. in type II Cepheids



Ministry of Science
and Higher Education
Republic of Poland

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