Challenges in evolution and pulsation of Cepheids through the MESA eyepiece

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MESA

Motivation

- Problems, e.g.
  - Mass discrepancy problem (including all observables)
  - Minimum mass/period at Instability Strip (IS) entry
  - Metallicity dependence of PL relation
  - Degeneracies (overshooting - rotation - mass loss)
  - Consistency between evolutionary and pulsation calculations
- How MESA compare to other codes?

Model setup

- **MESA r-21.12.1** (+pulsation with **MESA-RSP**)
- Mass range: 2−8$M_\odot$ (varying step, 0.5 & 0.1$M_\odot$),
- Metallicity range, [Fe/H]=−1.0, +0.2 (11 values), ΔY/ΔZ=1.5
- Exponential overshoot, MS core + envelope (RGB)
  - f_H: 0.00, 0.01, 0.02, 0.03, f_{env}: 0.00, 0.02, 0.04, 0.06
- Mass Loss: Reimers, η = 0, 0.2, 0.4, 0.6
- A09 reference solar mix, Sun-calibrated MLT
- Other factors explored: $^{14}N(\alpha,\gamma)^{15}O$, solar mix (GS98), ΔY/ΔZ
- No diffusion, No rotation, No pulsation induced mass-loss

MESA

Paxton et al., 2011-2019, ApJS
Jermyn et al., 2023, ApJS

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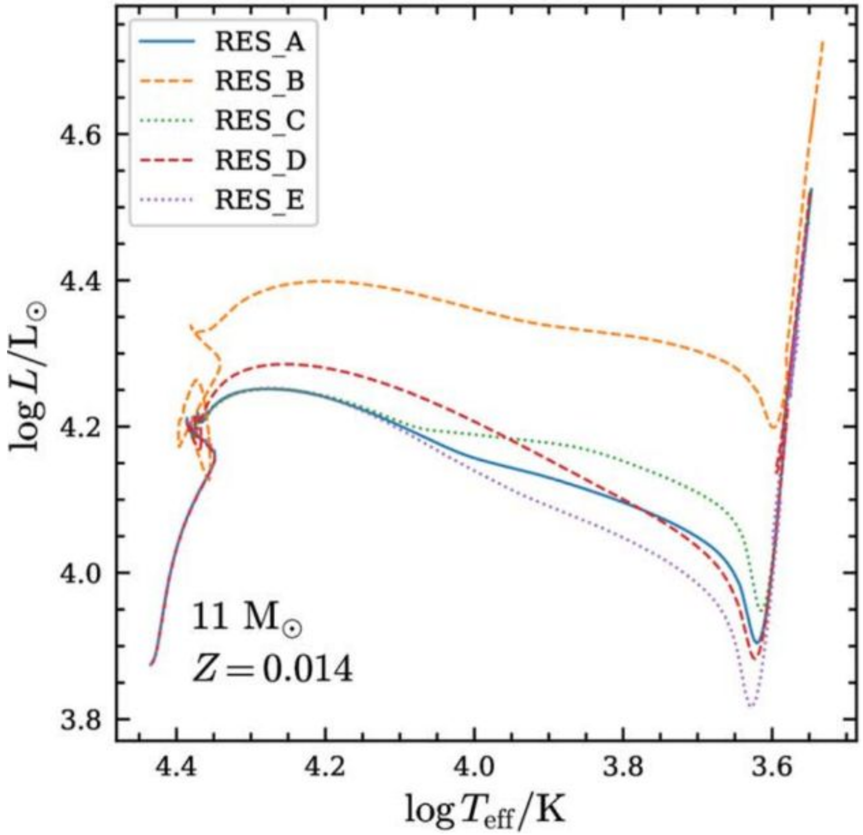
Model setup



- No numerical convergence **after the MS** for models with $M \geq 9_{\odot}$
- caused by an episode of thin convective shell formation a top of the MS core

Ziółkowska et al., 2024

Uncertainties on evolutionary tracks (Ziółkowska +, 2024)

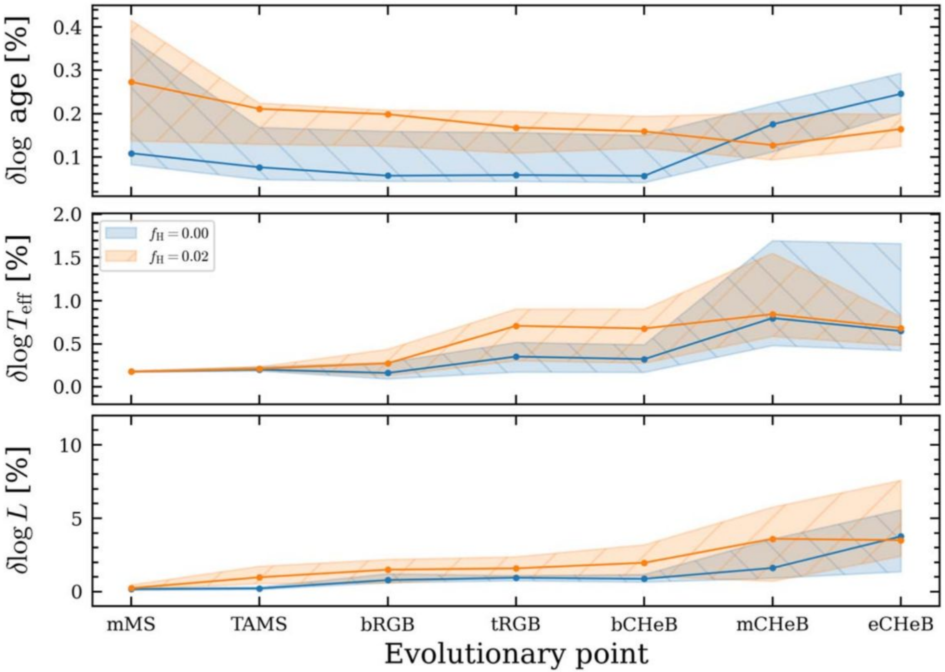


Figure 9. Similar to Figure 8, but here we present the median along with the 25th and 75th percentiles for 2–8 $M_\odot$ models and three metallicities. The two colors correspond to models with (orange right-slanted lines) and without convective core overshooting (blue left-slanted lines). Numerical data are collected in Table 6.

Table 6

25th, 50th (Median), and 75th Percentiles of the Maximum Relative Difference Distributions across All Models Computed in This Study (All Masses, Metallicities, and Parameter Sets) for the Case without Convective Core Overshooting and with Convective Core Overshooting for Eight Evolutionary Phases

	No Overshooting			MS Core Overshooting		
Evolutionary Phase	25th	Median	75th	25th	Median	75th
			$\delta \log L$			
mMS	0.1	**0.2**	0.3	0.1	**0.2**	0.5
TAMS	0.1	**0.2**	0.3	0.5	**1.0**	1.7
tRGB	0.6	**0.8**	1.2	0.6	**1.5**	2.2
bRGB	0.7	**0.9**	1.1	1.1	**1.6**	2.4
bCHeB	0.6	**0.9**	1.2	1.1	**2.0**	3.2
mCHeB	0.9	**1.6**	3.6	0.7	**3.6**	5.7
eCHeB	1.3	**3.8**	5.6	2.4	**3.5**	7.6
mIS	0.3	**0.5**	0.8	0.4	**0.6**	1.3

Overview of the track repository

- Two types of grids: coarse and fine in mass
  - GC: 2.0, 2.5, 3.0, …, 8.0 (step 0.5$M_\odot$)
    - 13 mass values
  - GF: 2.0, 2.5, 3.0, 3.1, 3.2, … 8,0 (step 0.1$M_\odot$)
    - 53 mass values
- 11 metallicities
  - 143 models in GC, 583 models in GF
- Explored effects:
  - MS core overshooting
  - RGB envelope overshooting
  - Mass loss (Reimers)
  - $^{14}N(\alpha,\gamma)^{15}O$
  - GS98 solar mixture, ΔY/ΔZ

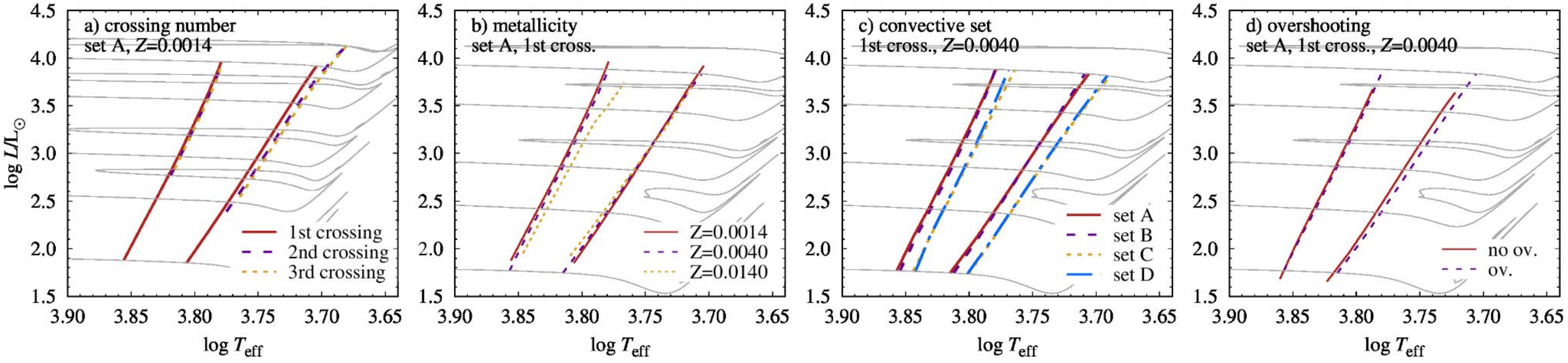
Z	Y	X	[Fe/H]
0.0200	0.2785	0.7015	0.197
0.0160	0.2725	0.7115	0.094
0.0140	0.2695	0.7165	0.033
0.0120	0.2665	0.7215	−0.037
0.0100	0.2635	0.7265	−0.119
0.0080	0.2605	0.7315	−0.219
0.0060	0.2575	0.7365	−0.347
0.0040	0.2545	0.7415	−0.526
0.0030	0.2530	0.7440	−0.652
0.0020	0.2515	0.7465	−0.830
0.0014	0.2506	0.7480	−0.985

Overview of the track repository

id	grid	$f_{\rm H}$	$f_{\rm env}$	η	
Core and envelope overshooting, no mass loss					
O00	GC, GF	0.00	0.00	0.0	
O02	GC	0.00	0.02	0.0	
O04	GC	0.00	0.04	0.0	
O06	GC	0.00	0.06	0.0	
O10	GC	0.01	0.00	0.0	
O12	GC	0.01	0.02	0.0	
O14	GC	0.01	0.04	0.0	
O16	GC	0.01	0.06	0.0	
O20	GC, GF	0.02	0.00	0.0	
O22	GC	0.02	0.02	0.0	
O24	GC, GF	0.02	0.04	0.0	
O26	GC	0.02	0.06	0.0	
O30	GC	0.03	0.00	0.0	
O32	GC	0.03	0.02	0.0	
O34	GC	0.03	0.04	0.0	
O36	GC	0.03	0.06	0.0	
with mass loss					
O20_ML2	GC	0.02	0.00	0.2	
O20_ML4	GC	0.02	0.00	0.4	
O24_ML2	GC, GF	0.02	0.04	0.2	
O24_ML4	GC, GF	0.02	0.04	0.4	
O24_ML6	GC	0.02	0.04	0.6	
nuclear reactions					
O00_AB	GC	0.00	0.00	0.0	$^{14}\rm N(p,\gamma)^{15}O$
O20_AB	GC	0.02	0.00	0.0	$^{14}\rm N(p,\gamma)^{15}O$
O22_AB	GC	0.02	0.02	0.0	$^{14}\rm N(p,\gamma)^{15}O$
O24_AB	GC	0.02	0.04	0.0	$^{14}\rm N(p,\gamma)^{15}O$
reference solar composition					
O00_AC	GC	0.00	0.00	0.0	GS98
O24_AC	GC	0.02	0.04	0.0	GS98

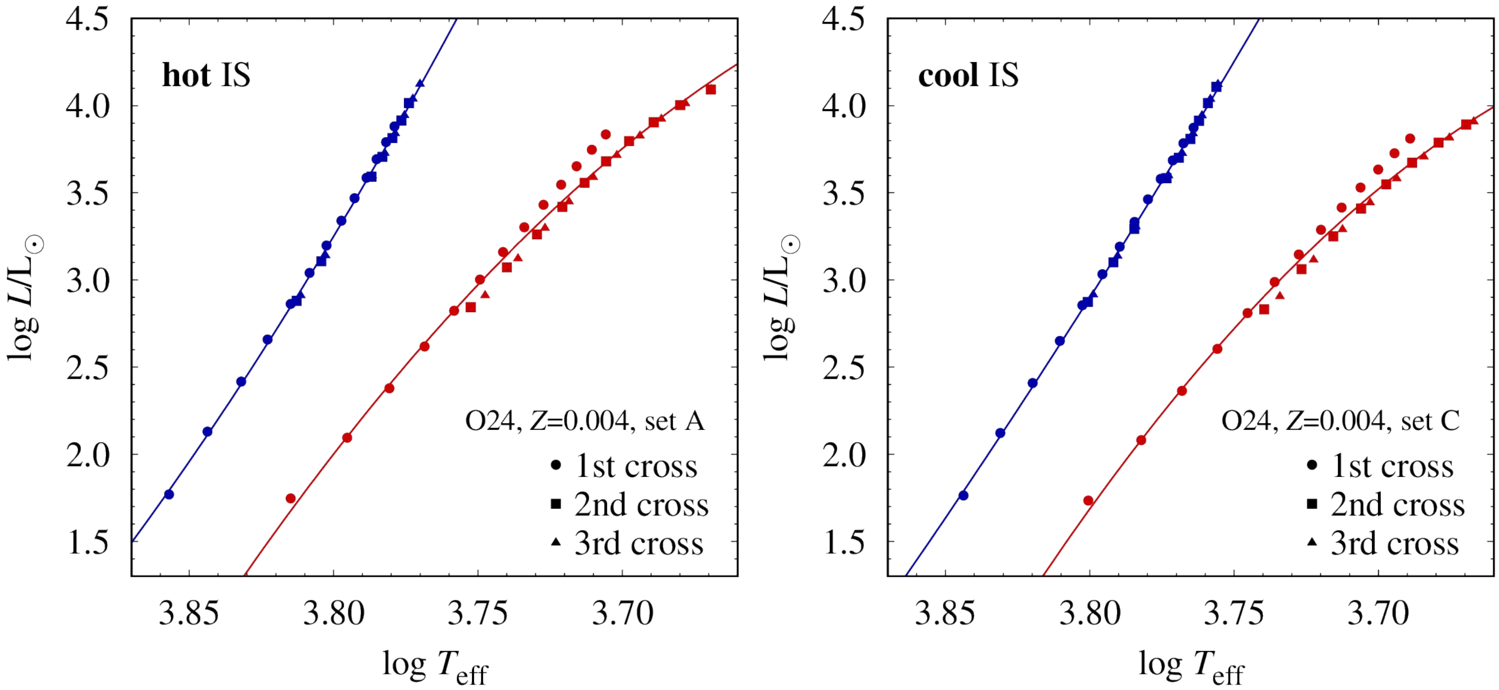
> 6000 models

Edges of the Instability Strip



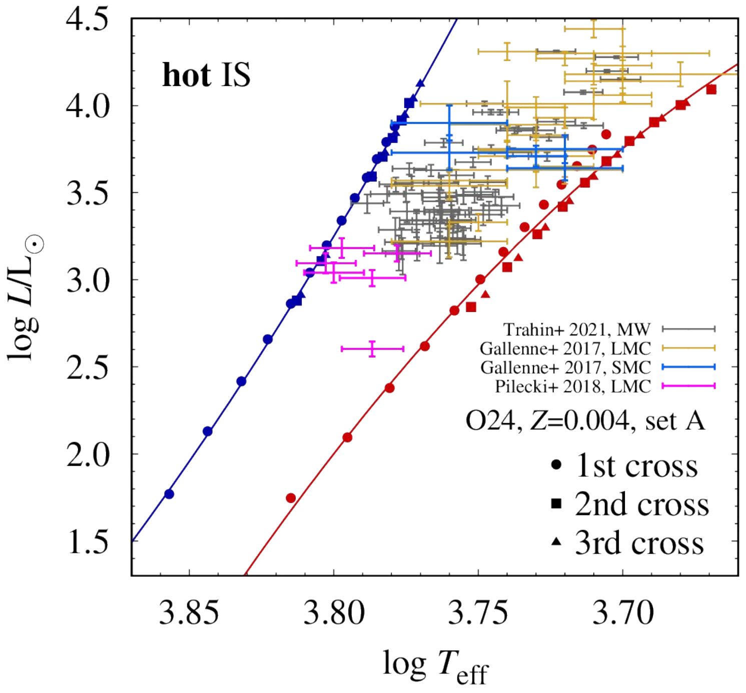
- Strongest sensitivity on convective parameter set
    - fiducial cool and hot IS (constructed based on Z=0.004, all crossings)

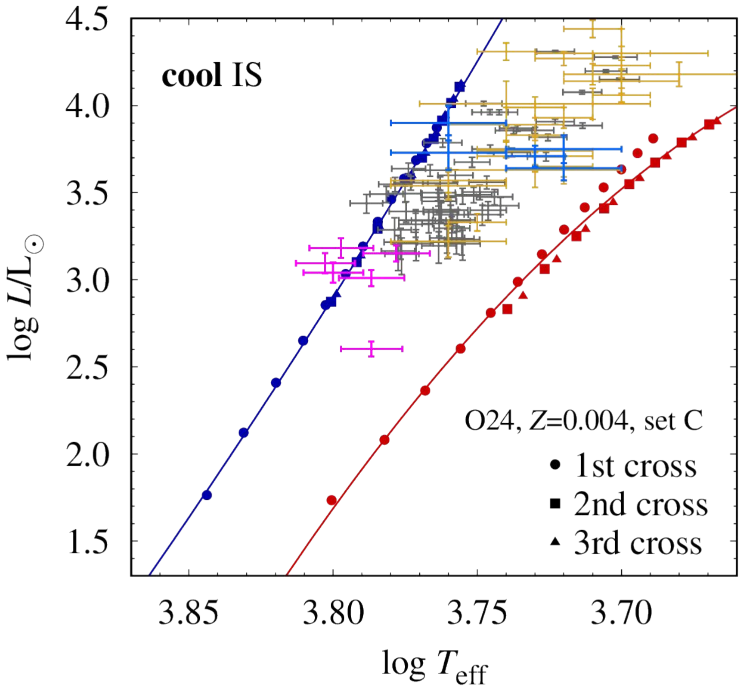
Edges of the Instability Strip



- fiducial edges hot & cool IS edges: based on O24, Z=0.004, all crossings

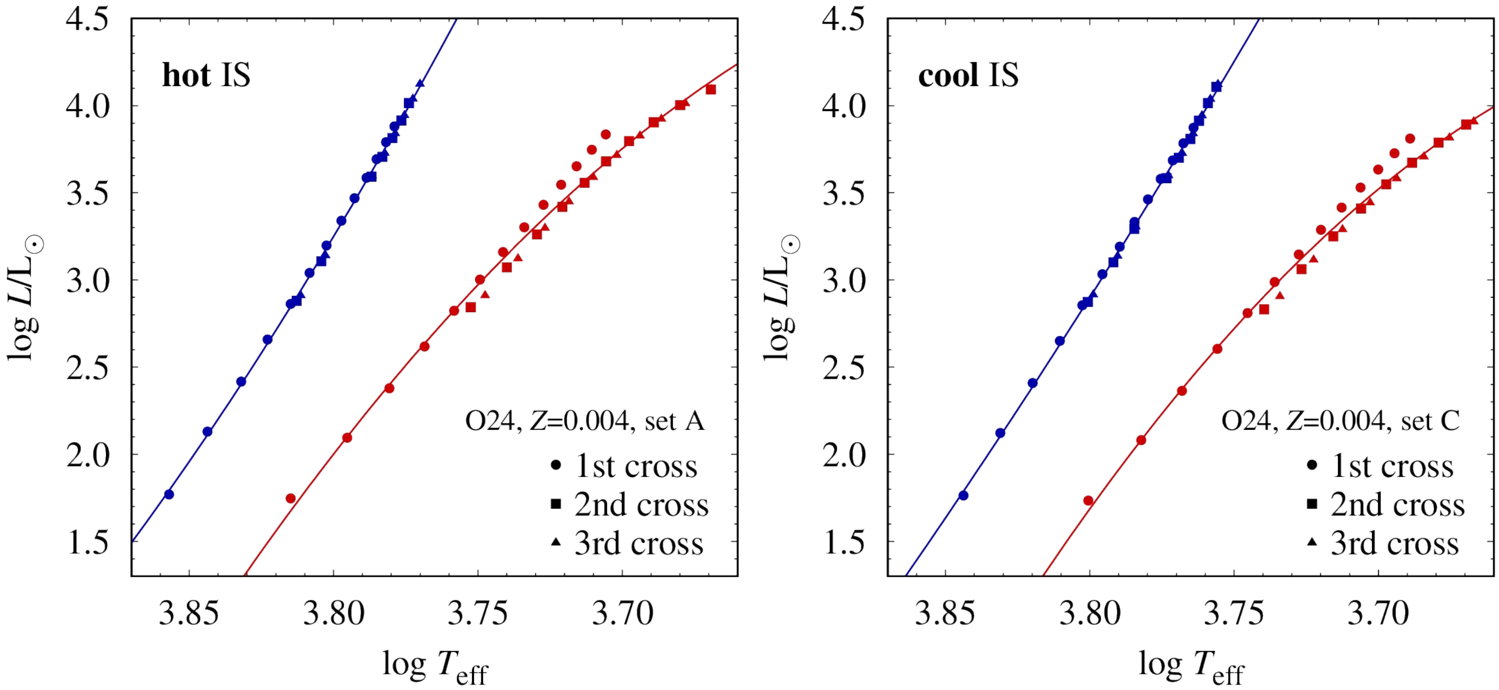
Edges of the Instability Strip





- comparison with recent SPIPS & eclipsing Cepheids data

Edges of the Instability Strip



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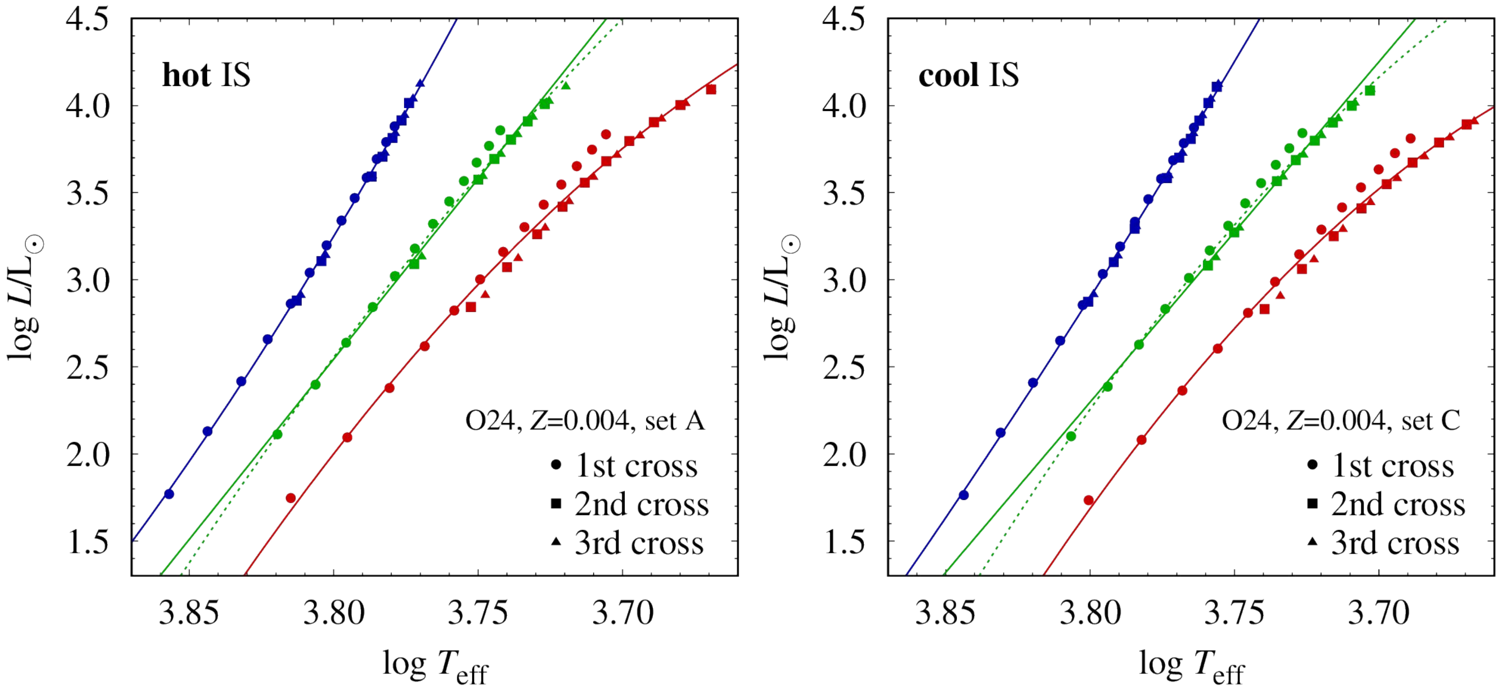
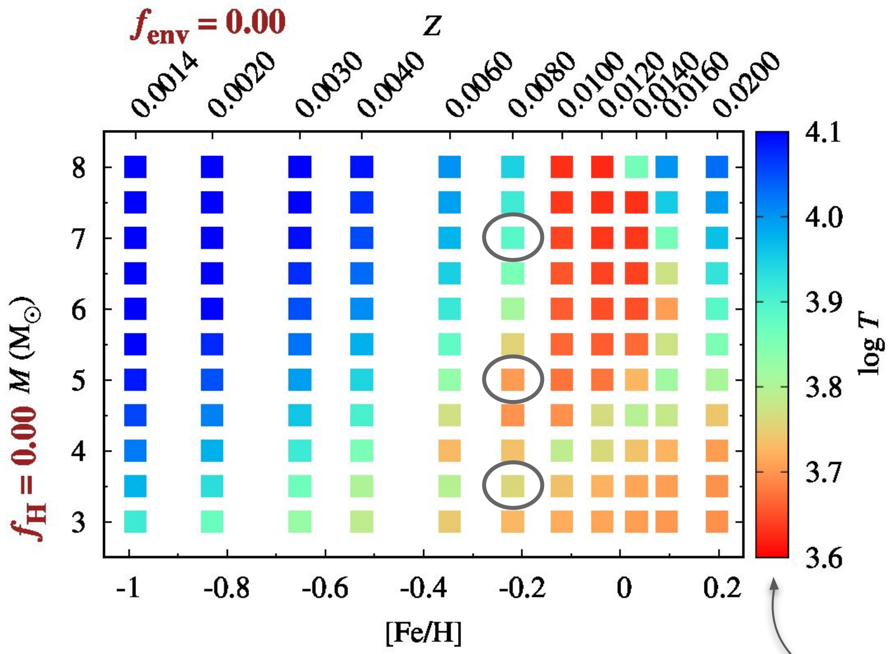
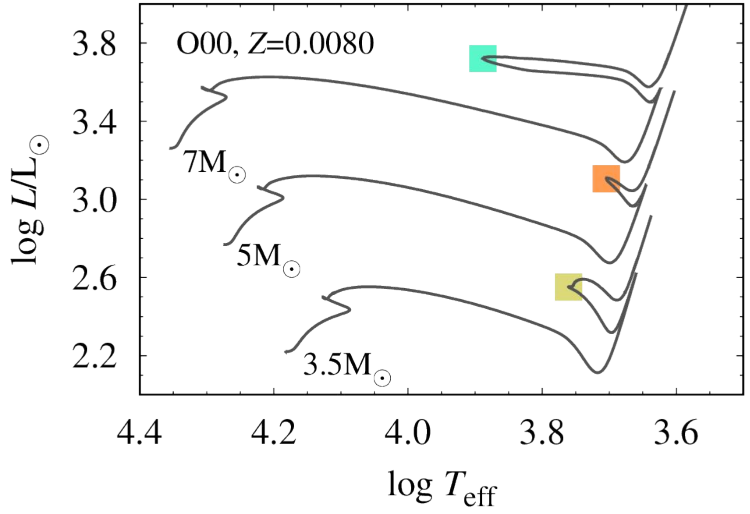
Edges of the Instability Strip



- fiducial edges hot & cool IS edges: based on O24, Z=0.004, all crossings
- mid-lines

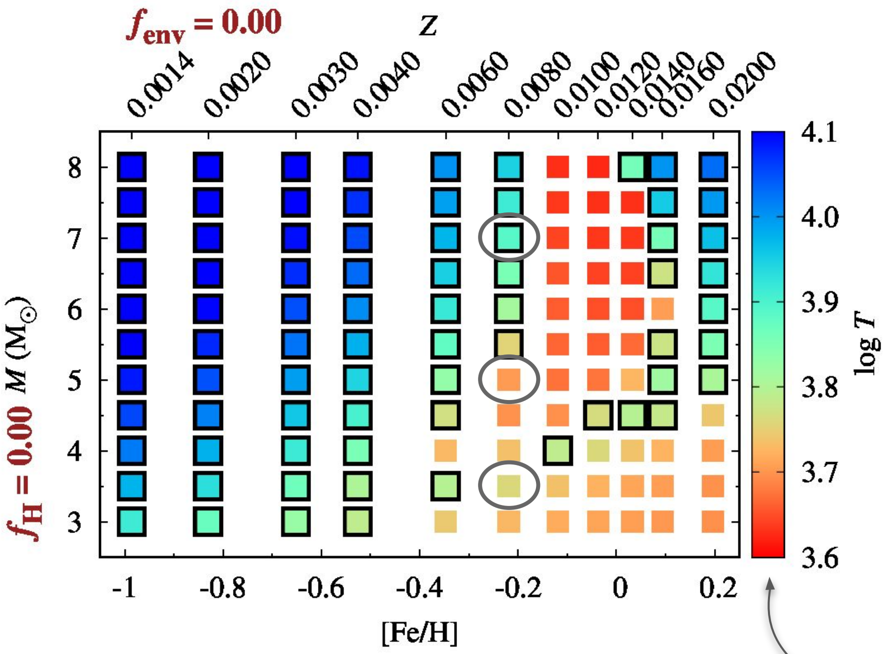
Blue loop extent across M and Z



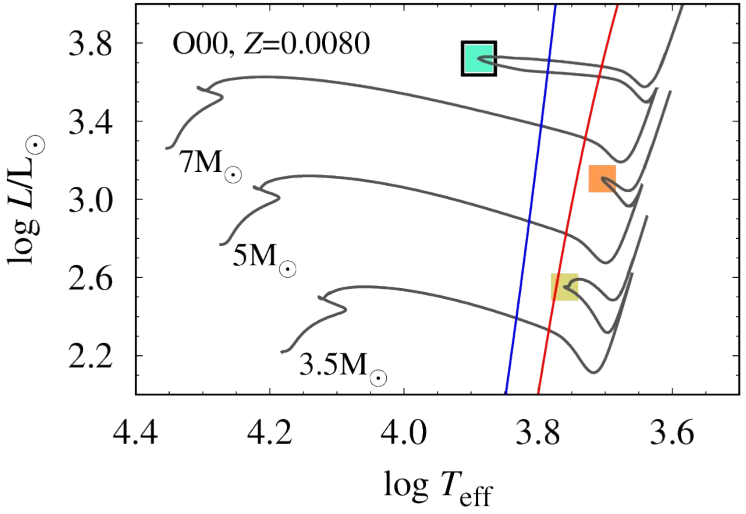


log *T* of the hottest point on the loop

Blue loop extent across M and Z

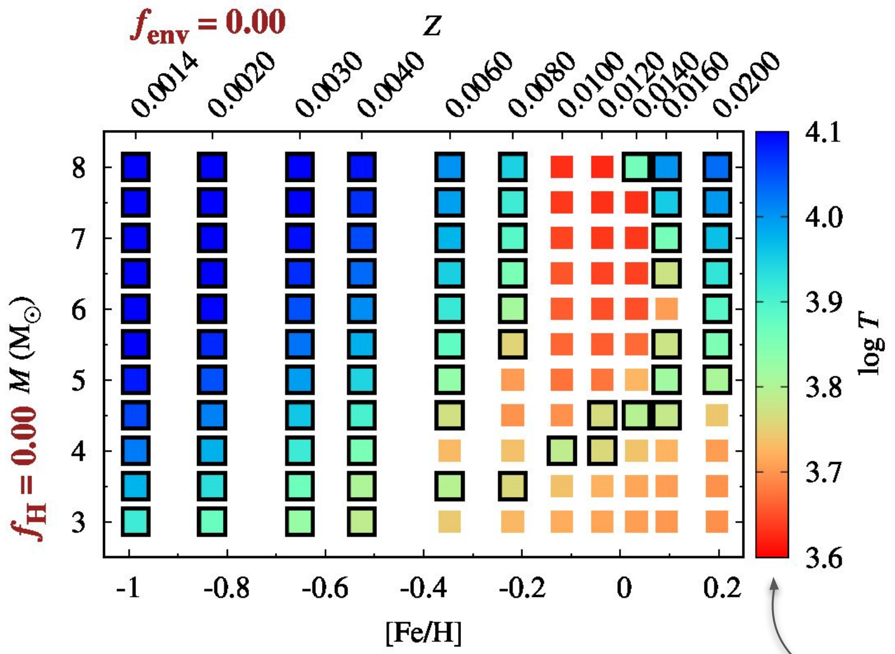


Models that enter
hot IS

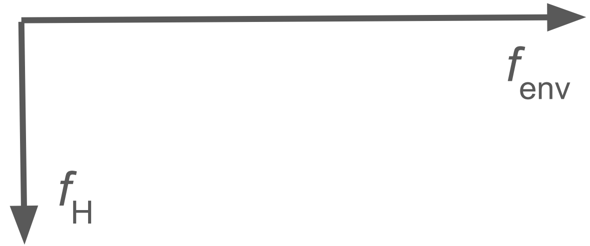


log *T* of the hottest point
on the loop

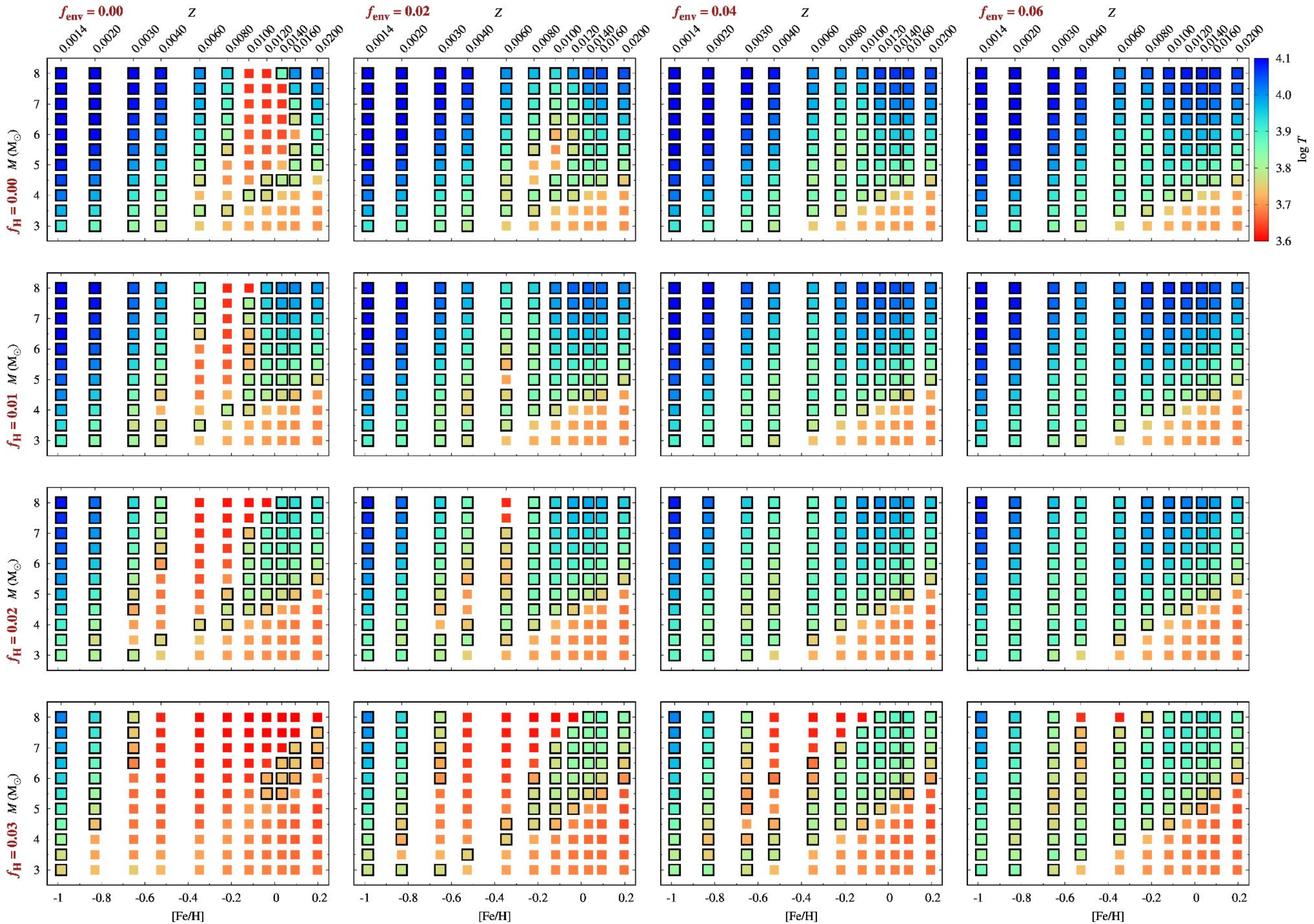
Blue loop extent across M and Z



Next slide:

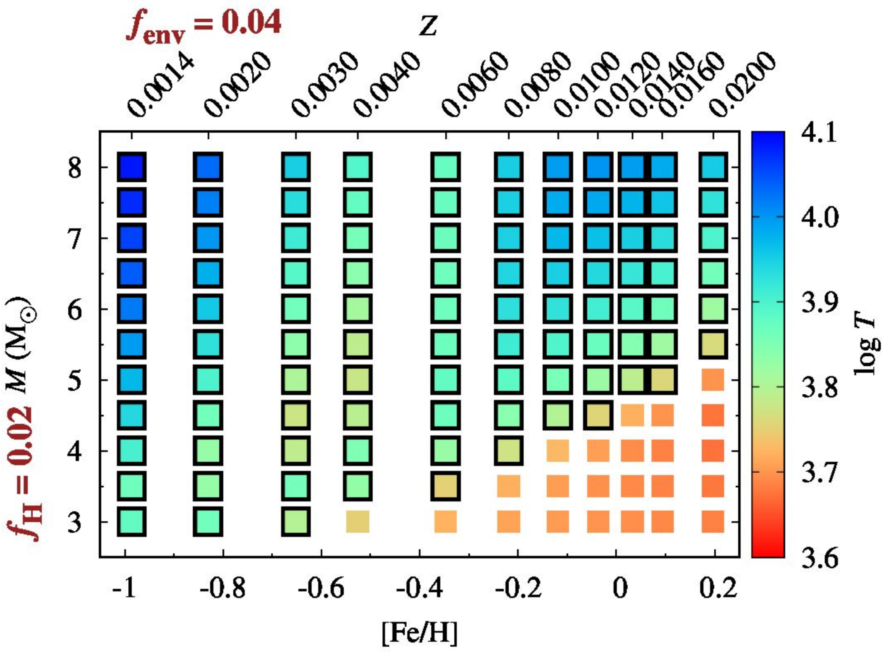


log *T* of the hottest point on the loop



f_{env} = 0.00
f_{env} = 0.02
f_{env} = 0.04
f_{env} = 0.06
Z
0.0014 0.0020 0.0030 0.0040 0.0060 0.0080 0.0100 0.0120 0.0140 0.0160 0.0200
f_H = 0.00
f_H = 0.01
f_H = 0.02
f_H = 0.03
M (M$_\odot$)
[Fe/H]
log T
4.1
4.0
3.9
3.8
3.7
3.6

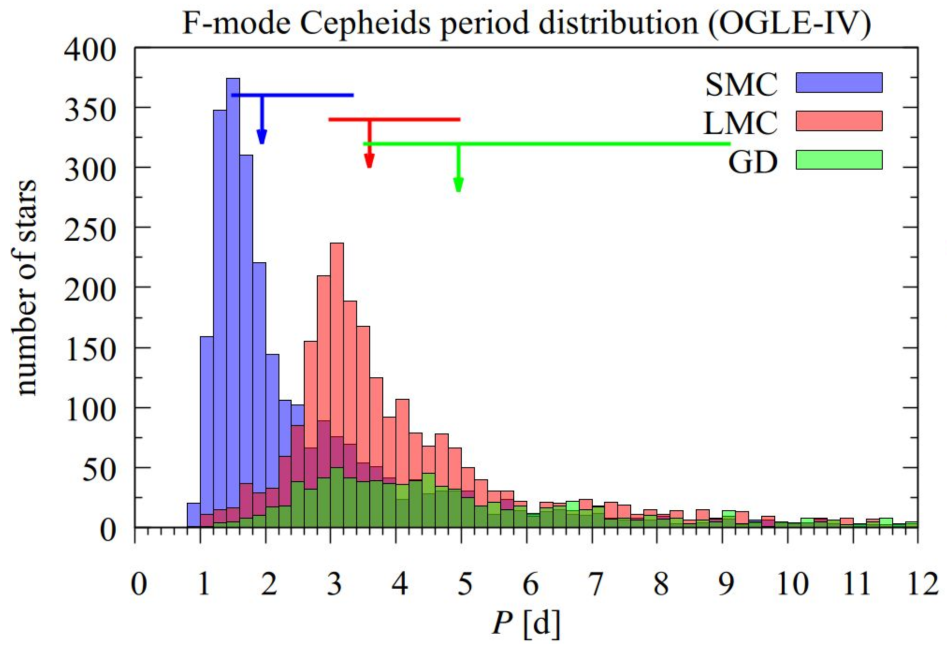
Blue loop extent across M and Z

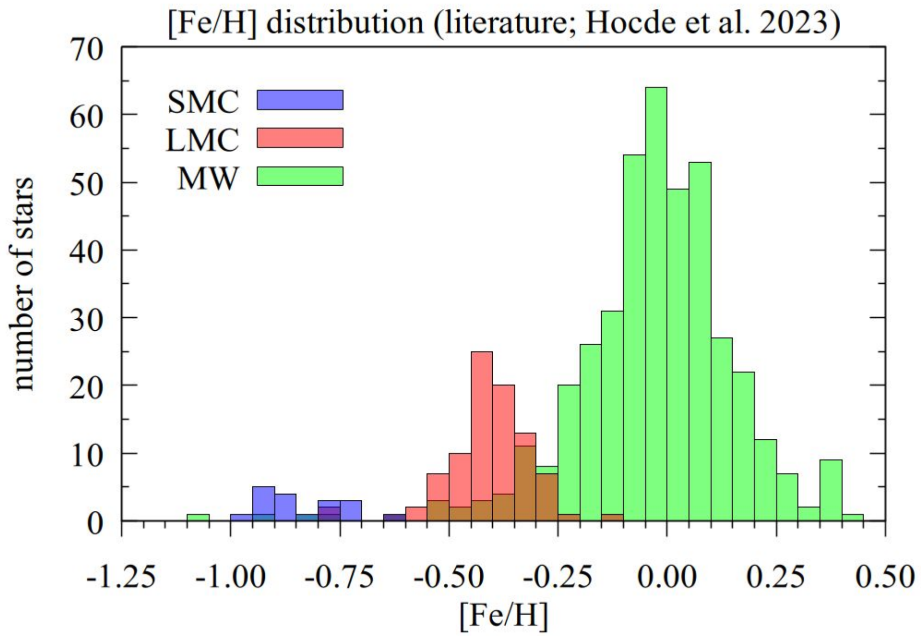


f_H=0.00, f_{env}=0.00 and
f_H=0.02, f_{env}=0.04

- selected as a reference

Minimum pulsation periods at 2nd crossing



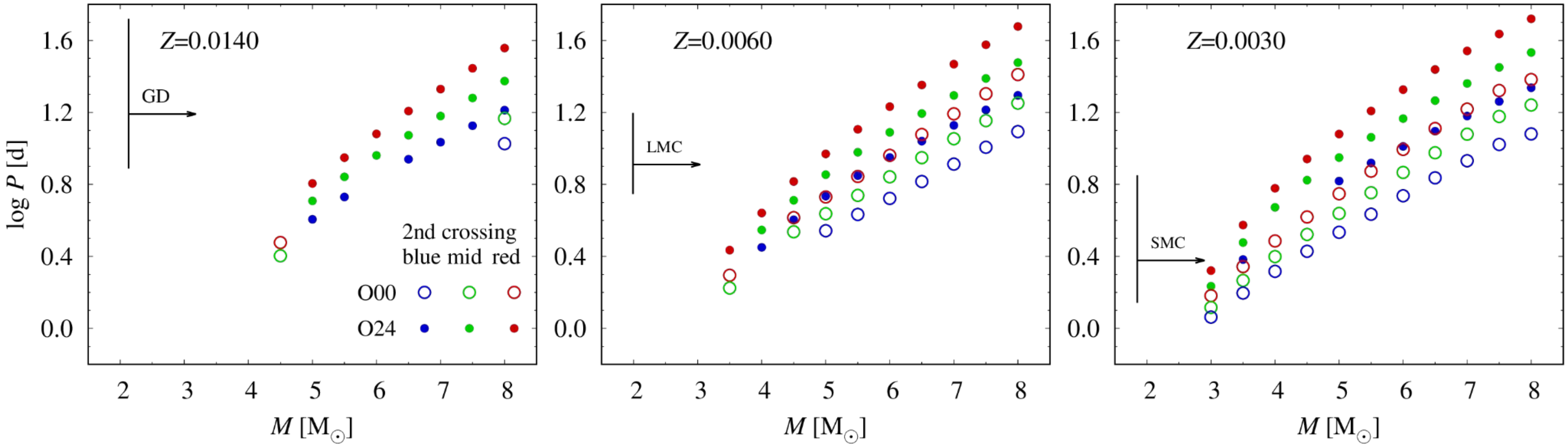


Median periods:
SMC: 1.94d, LMC: 3.59d, GD: 4.96d

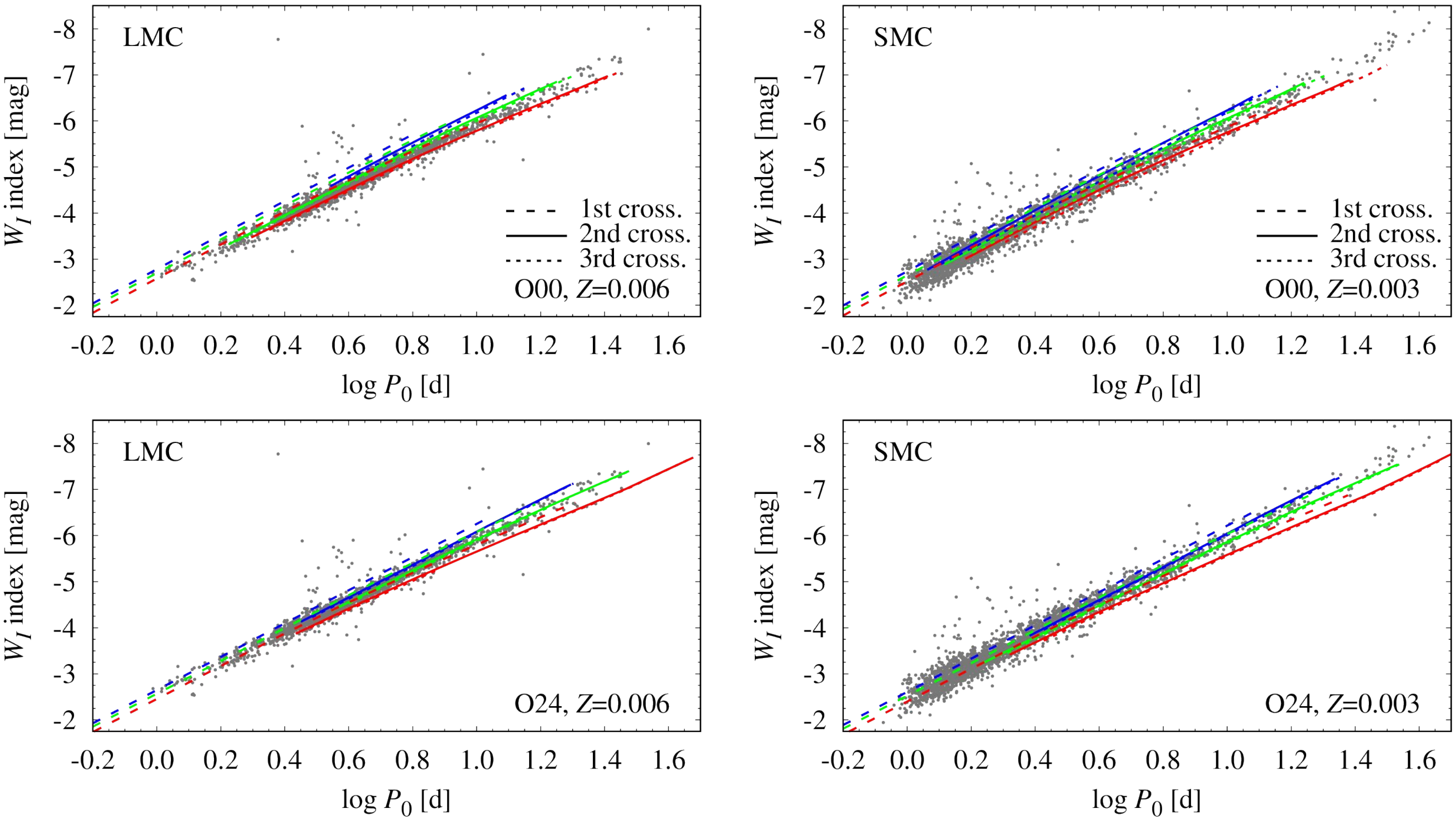
Mean [Fe/H]:
LMC: −0.35 (**Z=0.006**)
SMC: −0.70 (**Z=0.003**, Z=0.002)

Soszyński et al., 2015, AcA
Hocde et al, 2023, A&A

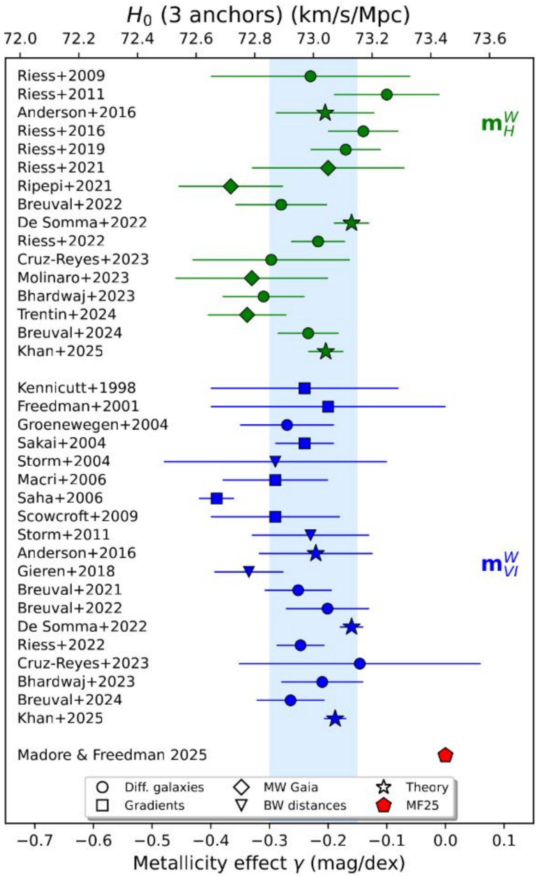
Minimum pulsation periods at 2nd crossing



Period - luminosity relation (@LMC & SMC; OGLE data)



Period - luminosity relation: metallicity dependence

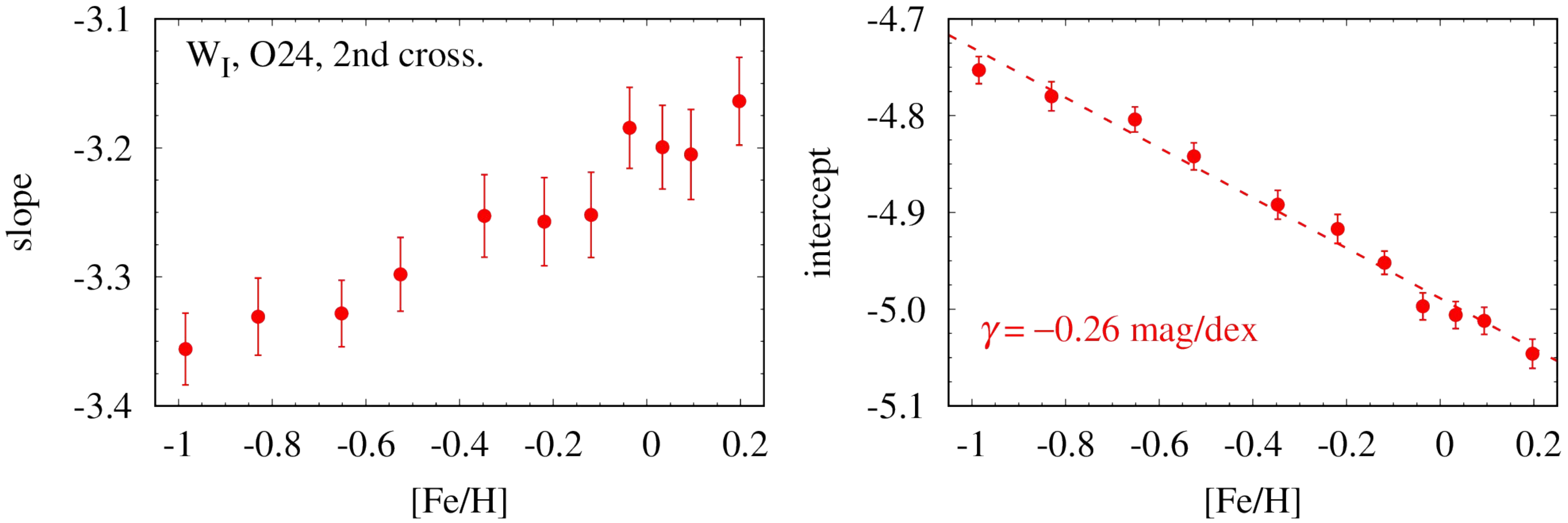


- Observational studies converge on metallicity effect of order of γ = −0.15,…, −0.30 mag/dex
- Theoretical studies are more scarce, but recent agree with observations

Breuval et al. 2025, ApJ, subm.

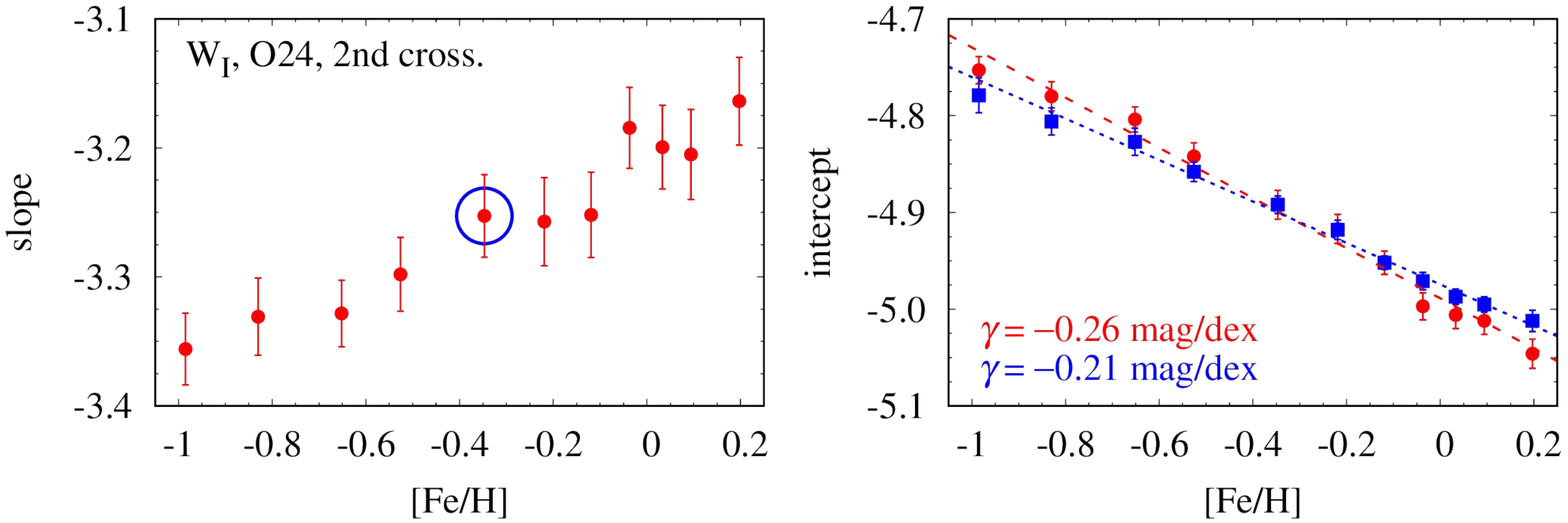
Period - luminosity relation: metallicity dependence

- fit $W_I = a \cdot (\log P - 0.7) + b$, separately for each [Fe/H]

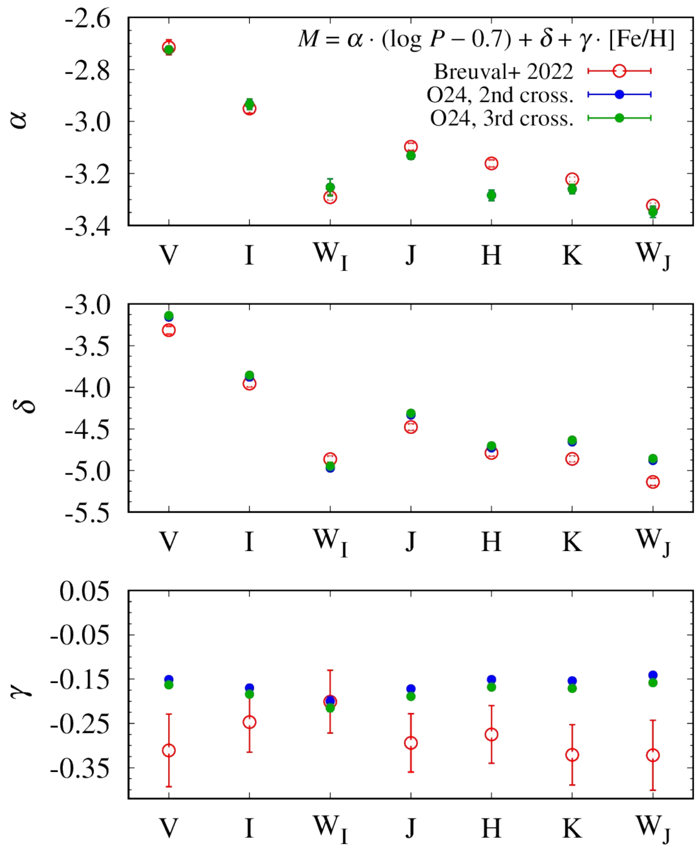


Period - luminosity relation: metallicity dependence

- fit $W_I = a \cdot (\log P - 0.7) + b$, separately for each [Fe/H]
- fixed slope ([Fe/H]=−0.35; LMC)



Period - luminosity relation: metallicity dependence



Comparison with Breuval+ 2022

Evolutionary and pulsational relations: [Fe/H] dependence

- fit $Y = a \cdot (\log X - \log X_0) + b$, separately for each [Fe/H]
- a([Fe/H]) and b([Fe/H]) dependence

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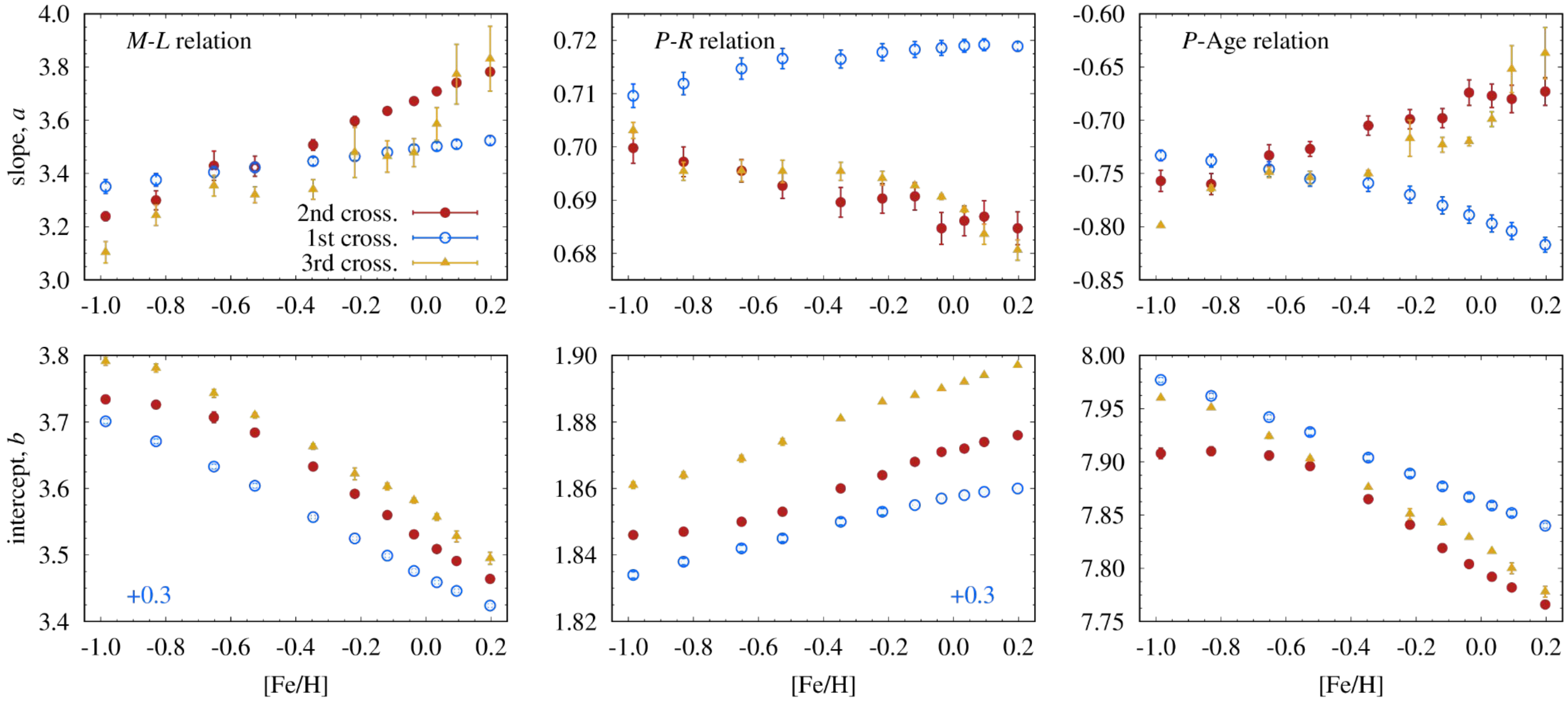
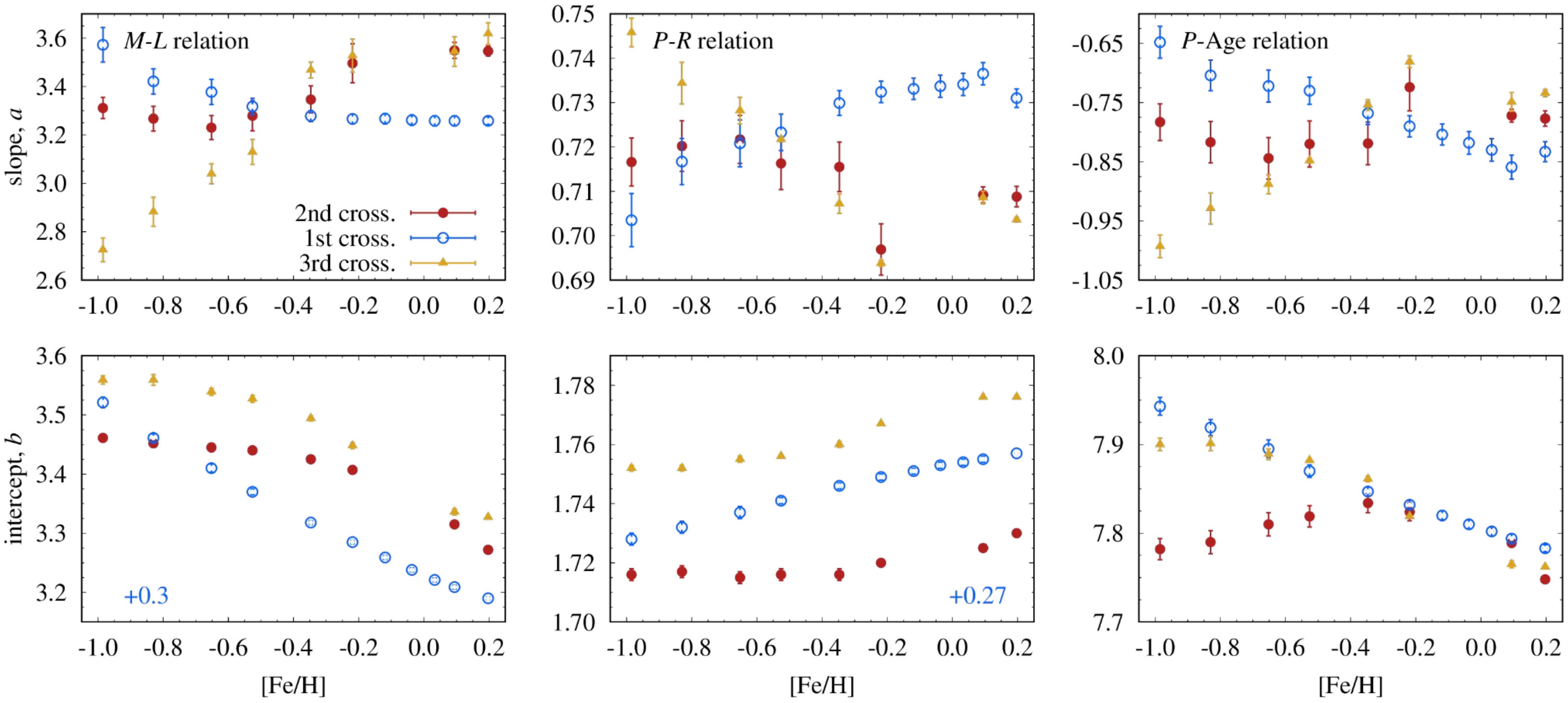
O24, overshooting



Evolutionary and pulsational relations: [Fe/H] dependence

- fit $Y = a \cdot (\log X - \log X_0) + b$, separately for each [Fe/H]
- a([Fe/H]) and b([Fe/H]) dependence

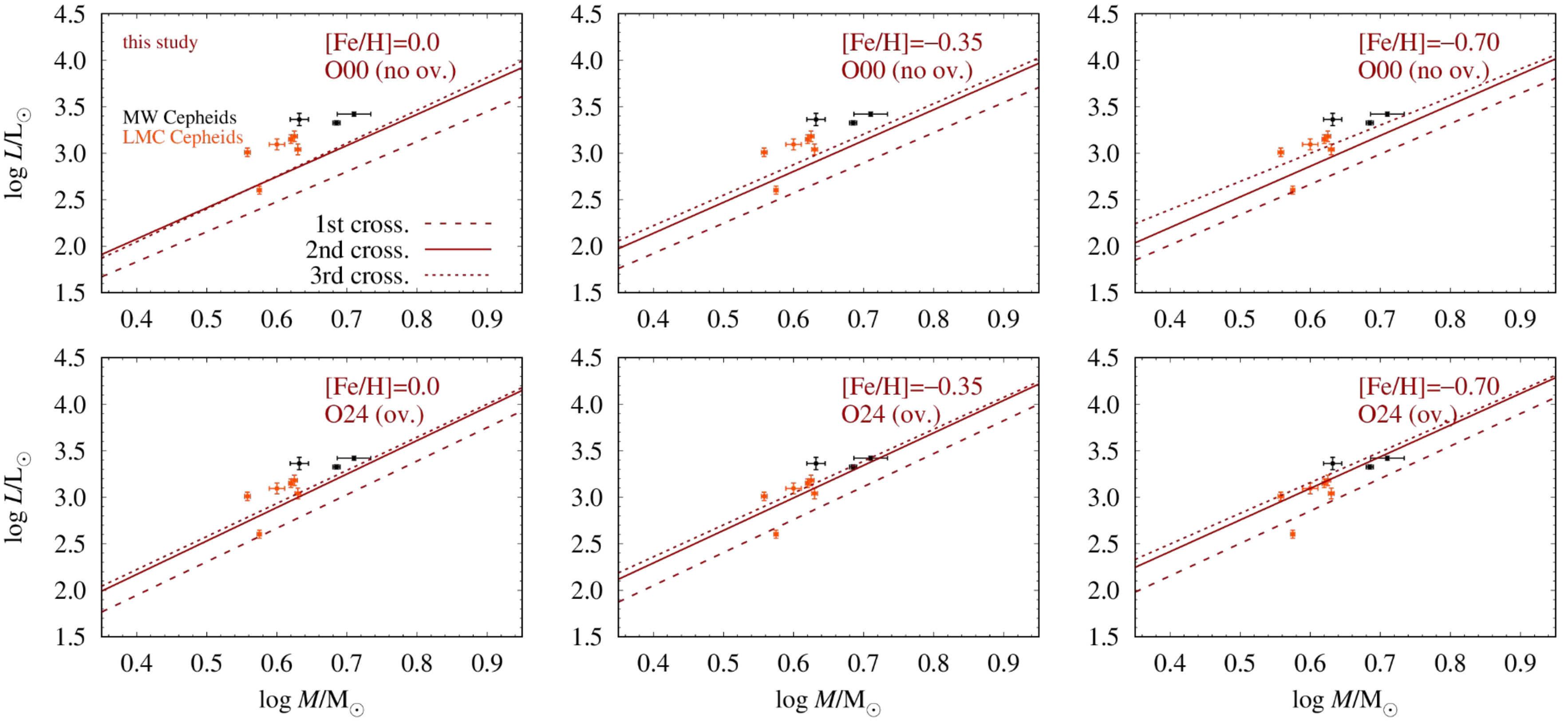
O00, no overshooting



Evolutionary and pulsational relations: [Fe/H] dependence

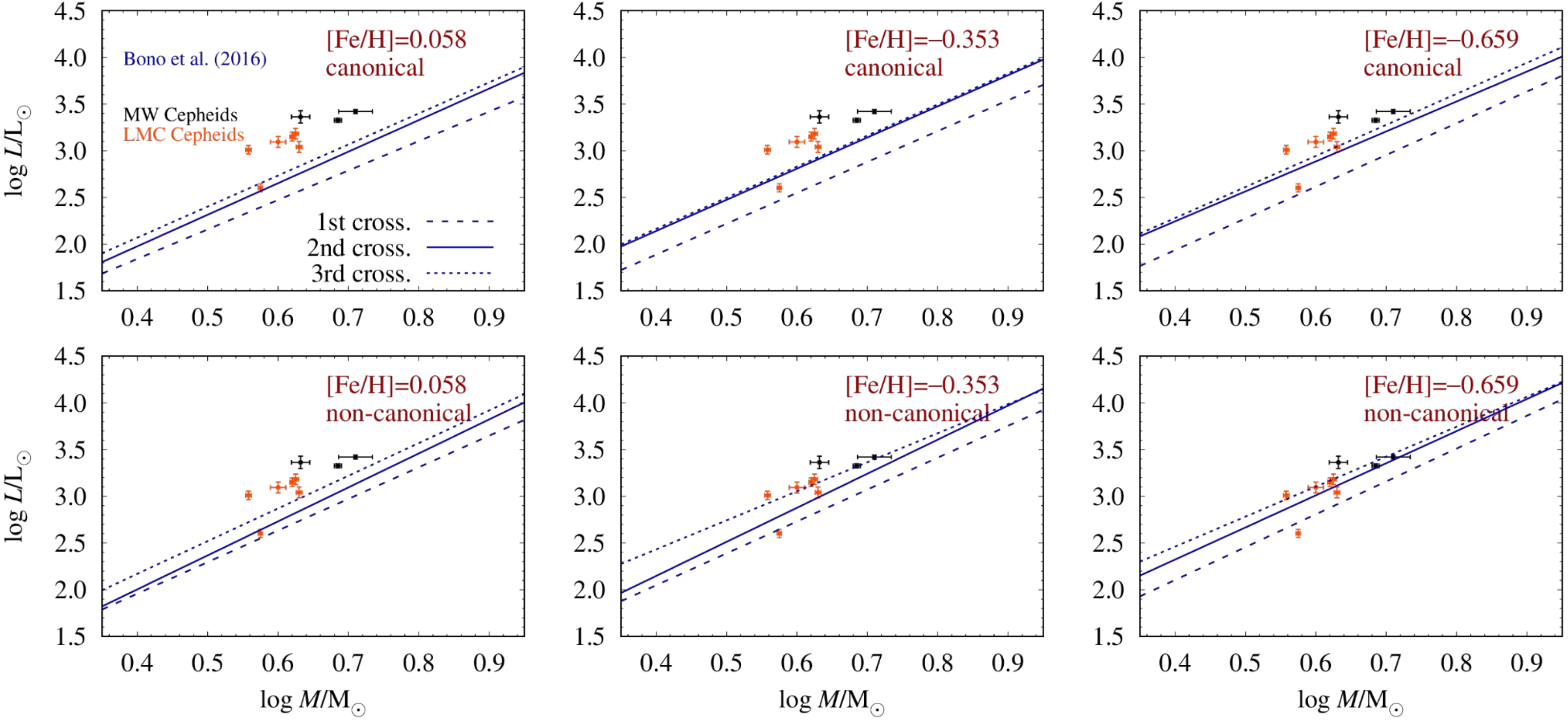
- fit $Y = a \cdot (\log X - \log X_0) + b$, separately for each [Fe/H]
- a([Fe/H]) and b([Fe/H]) dependence
- fit $Y = (a + d \cdot [\mathrm{Fe/H}]) \cdot \log X + b + c \cdot [\mathrm{Fe/H}]$
  - **good for O24**
  - **for O00 - more complex - specific [Fe/H] relations will be plotted**

M-L Relation



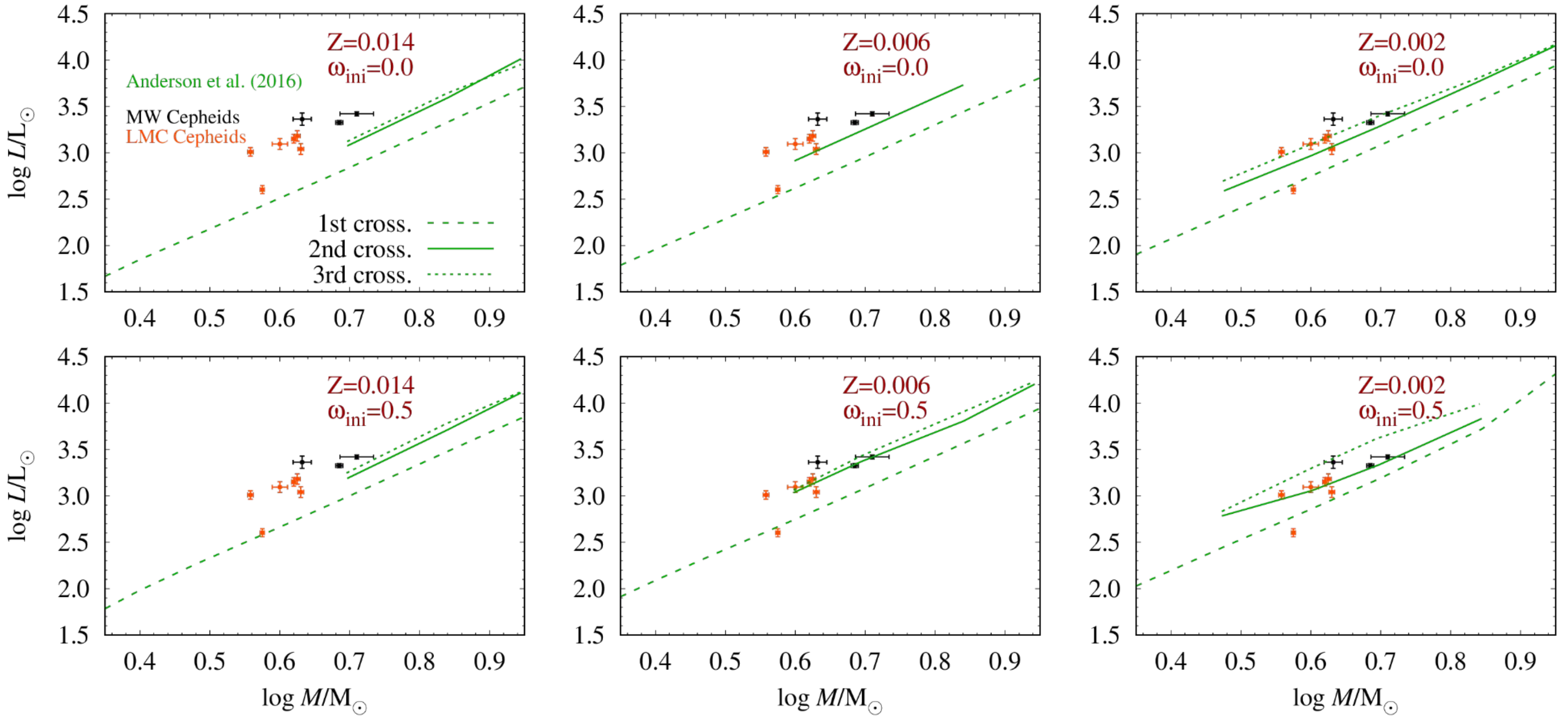
data: Pilecki+, Gallenne+, Evans+

M-L Relation



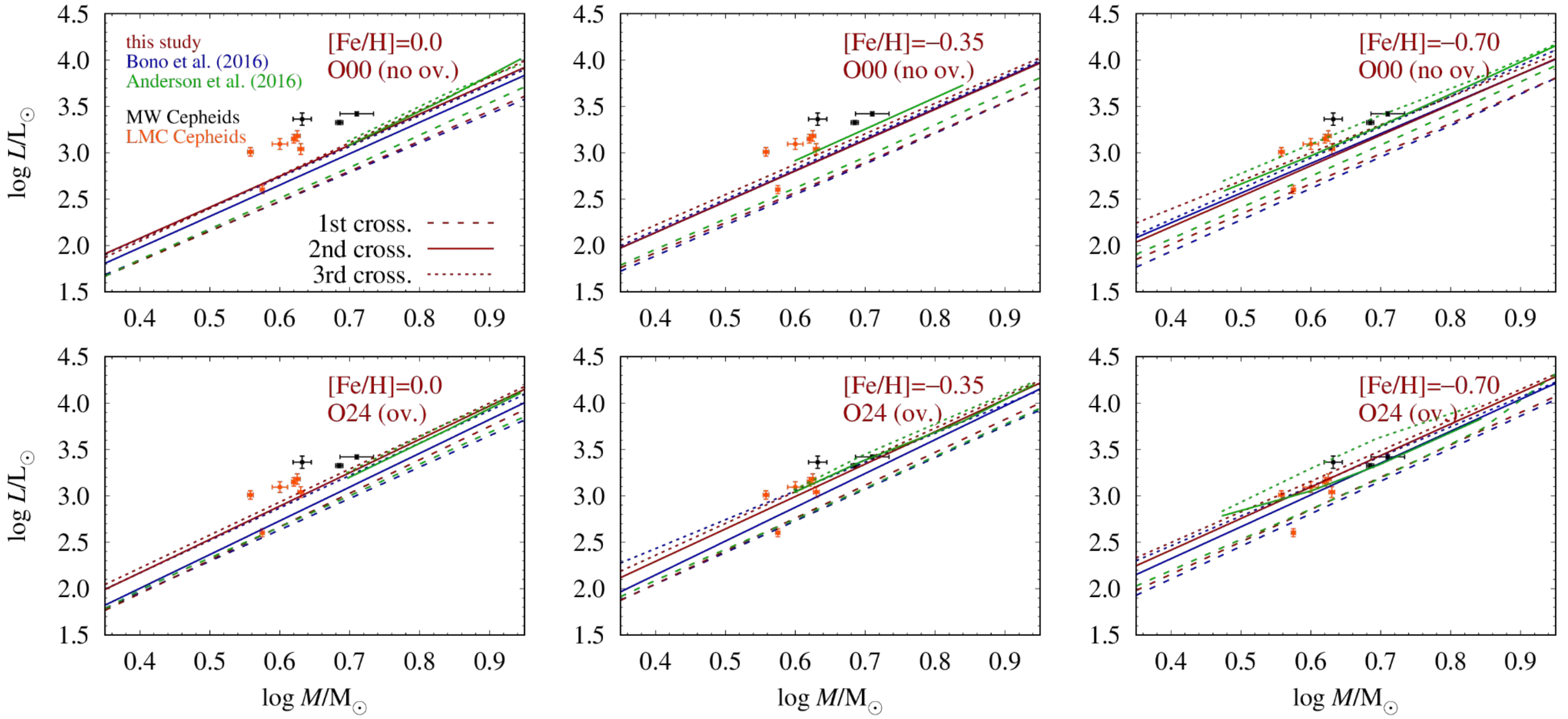
data: Pilecki+, Gallenne+, Evans+

M-L Relation



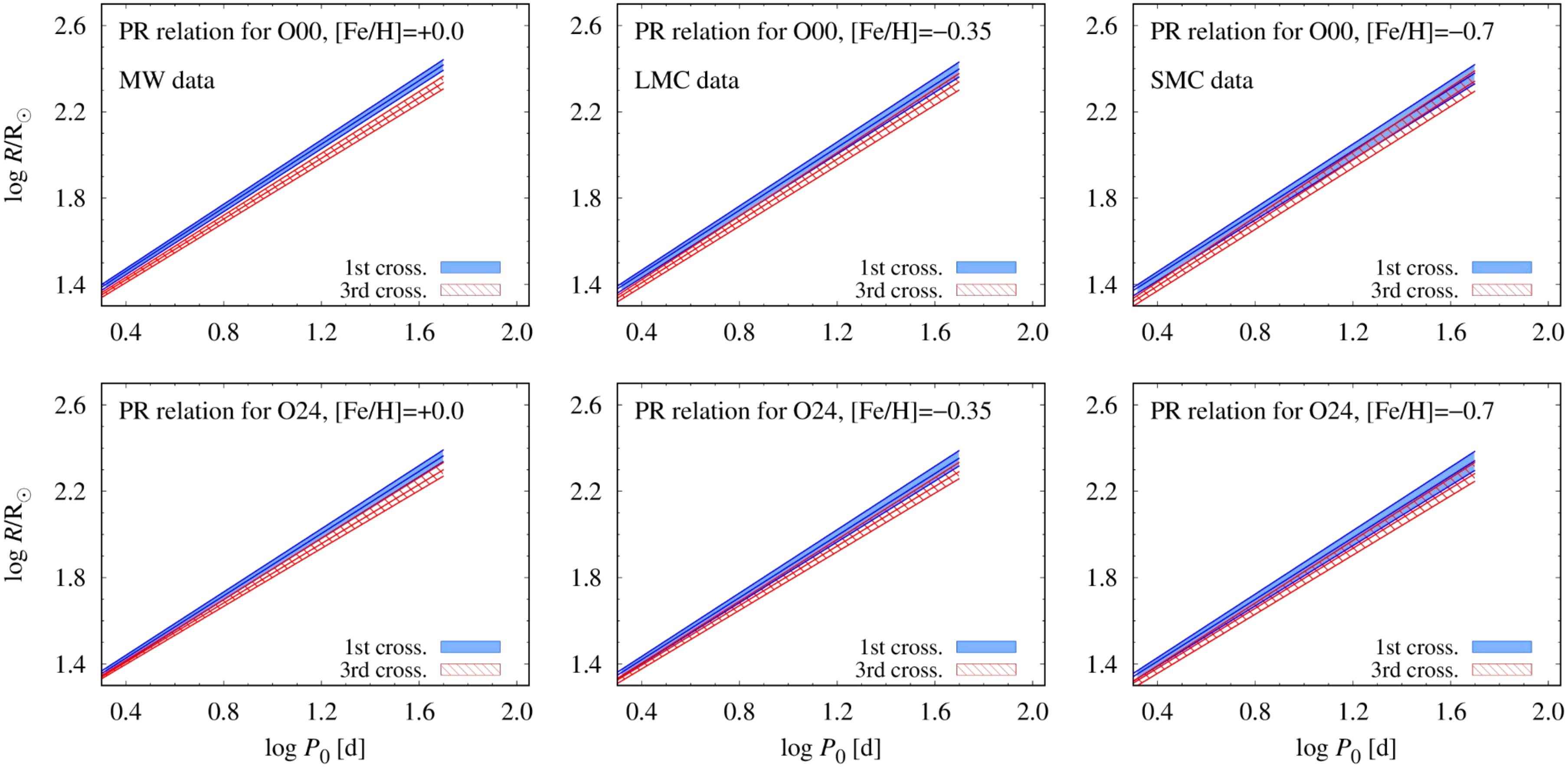
data: Pilecki+, Gallenne+, Evans+

M-L Relation

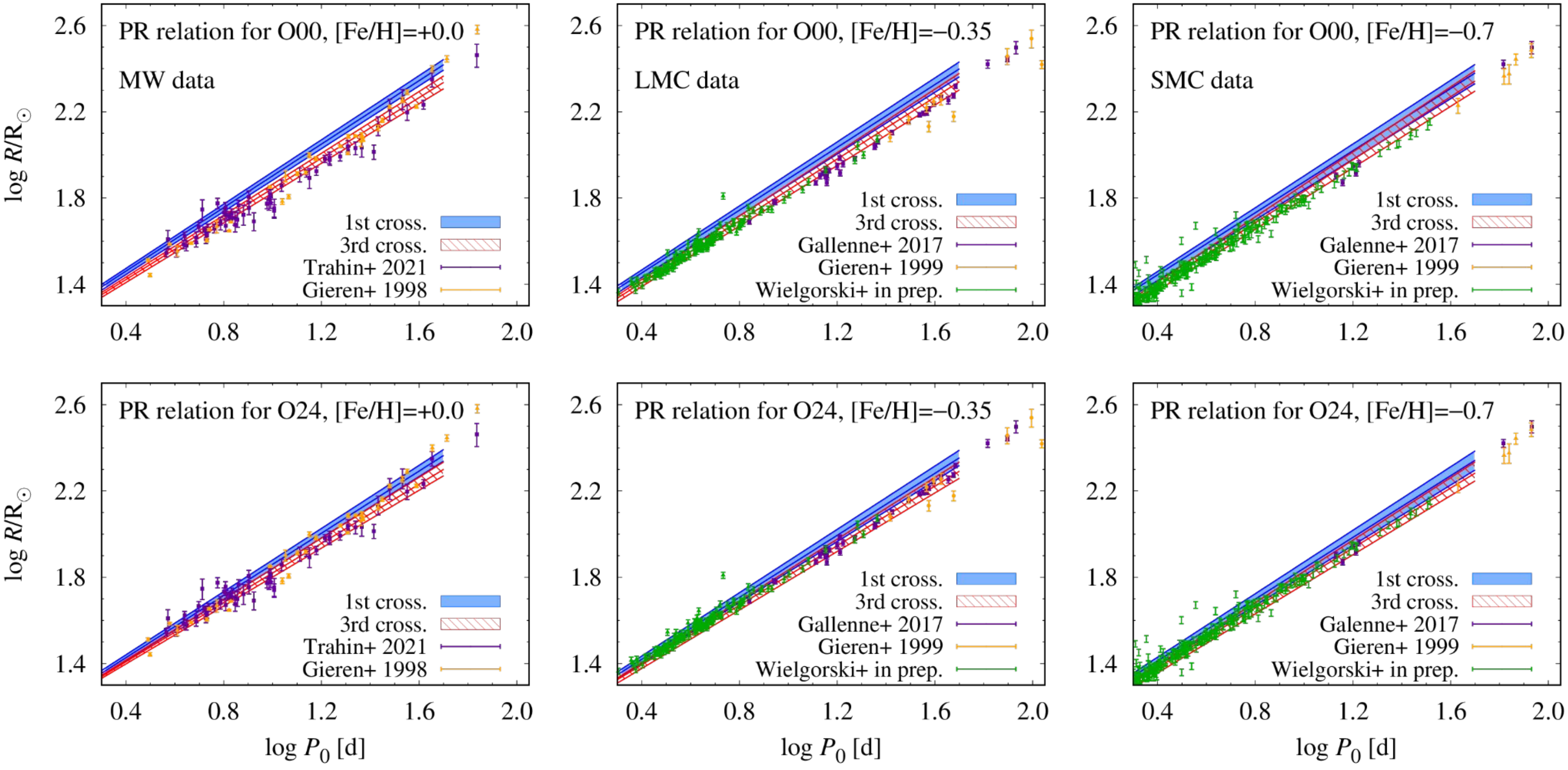


data: Pilecki+, Gallenne+, Evans+

P-R Relation

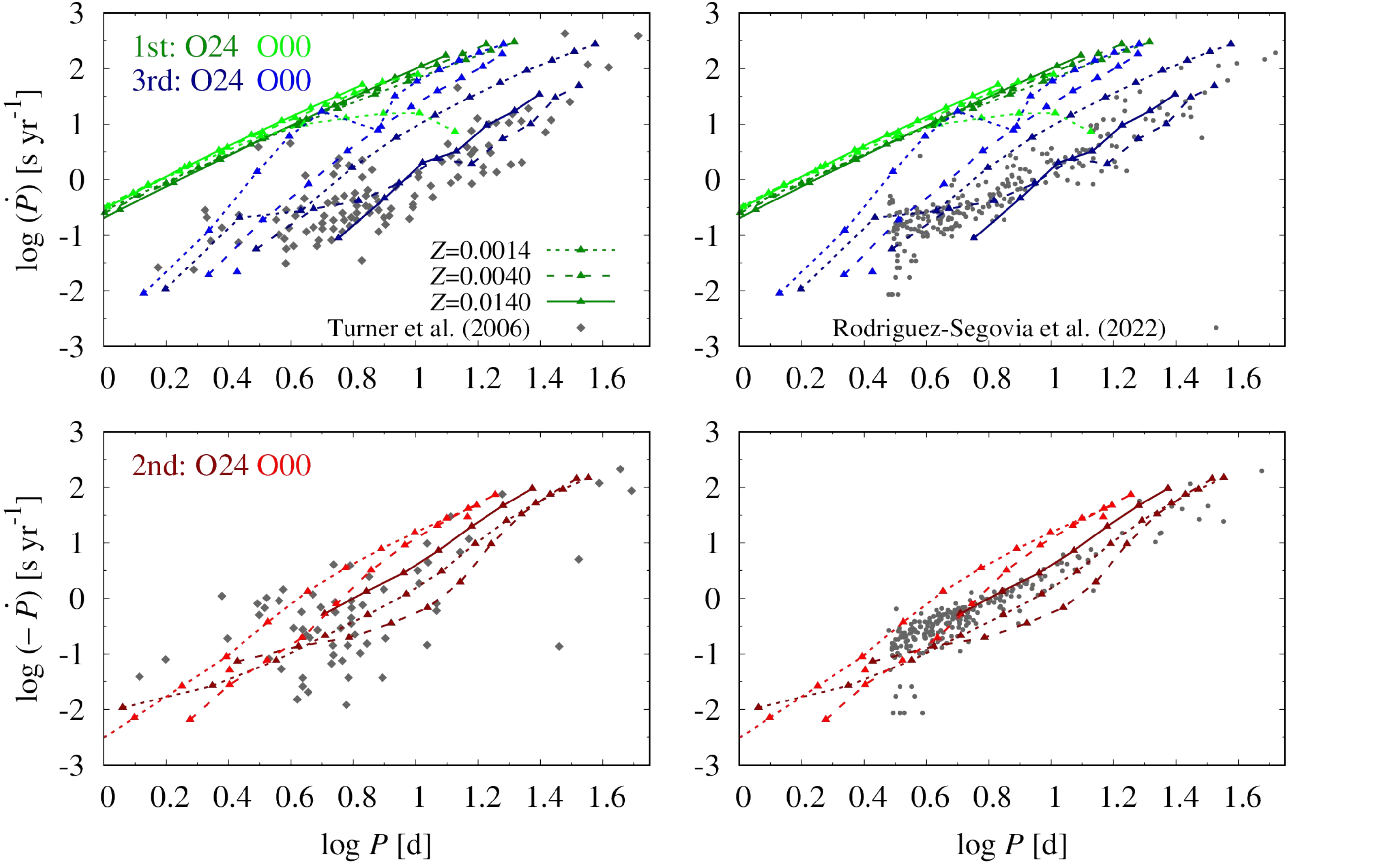

PR relation for O00, [Fe/H]=+0.0
MW data
PR relation for O00, [Fe/H]=−0.35
LMC data
PR relation for O00, [Fe/H]=−0.7
SMC data
PR relation for O24, [Fe/H]=+0.0
PR relation for O24, [Fe/H]=−0.35
PR relation for O24, [Fe/H]=−0.7
1st cross.
3rd cross.
log R/R⊙
log P0 [d]
2.6
2.2
1.8
1.4
0.4
0.8
1.2
1.6
2.0

P-R Relation

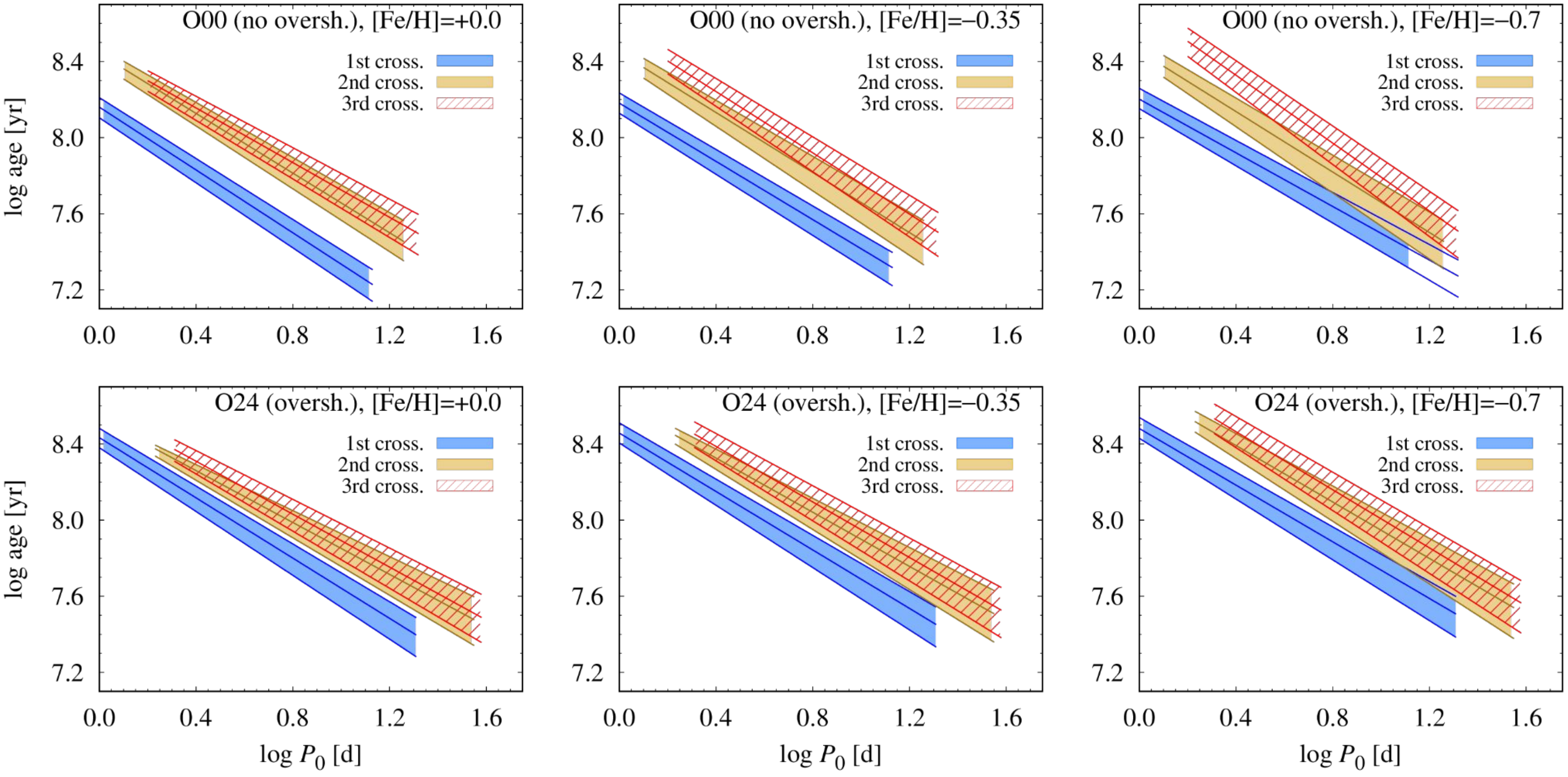


PR relation for O00, [Fe/H]=+0.0
MW data
1st cross.
3rd cross.
Trahin+ 2021
Gieren+ 1998
PR relation for O00, [Fe/H]=−0.35
LMC data
1st cross.
3rd cross.
Gallenne+ 2017
Gieren+ 1999
Wielgorski+ in prep.
PR relation for O00, [Fe/H]=−0.7
SMC data
1st cross.
3rd cross.
Galenne+ 2017
Gieren+ 1999
Wielgorski+ in prep.
PR relation for O24, [Fe/H]=+0.0
PR relation for O24, [Fe/H]=−0.35
PR relation for O24, [Fe/H]=−0.7
log R/R☉
log P0 [d]

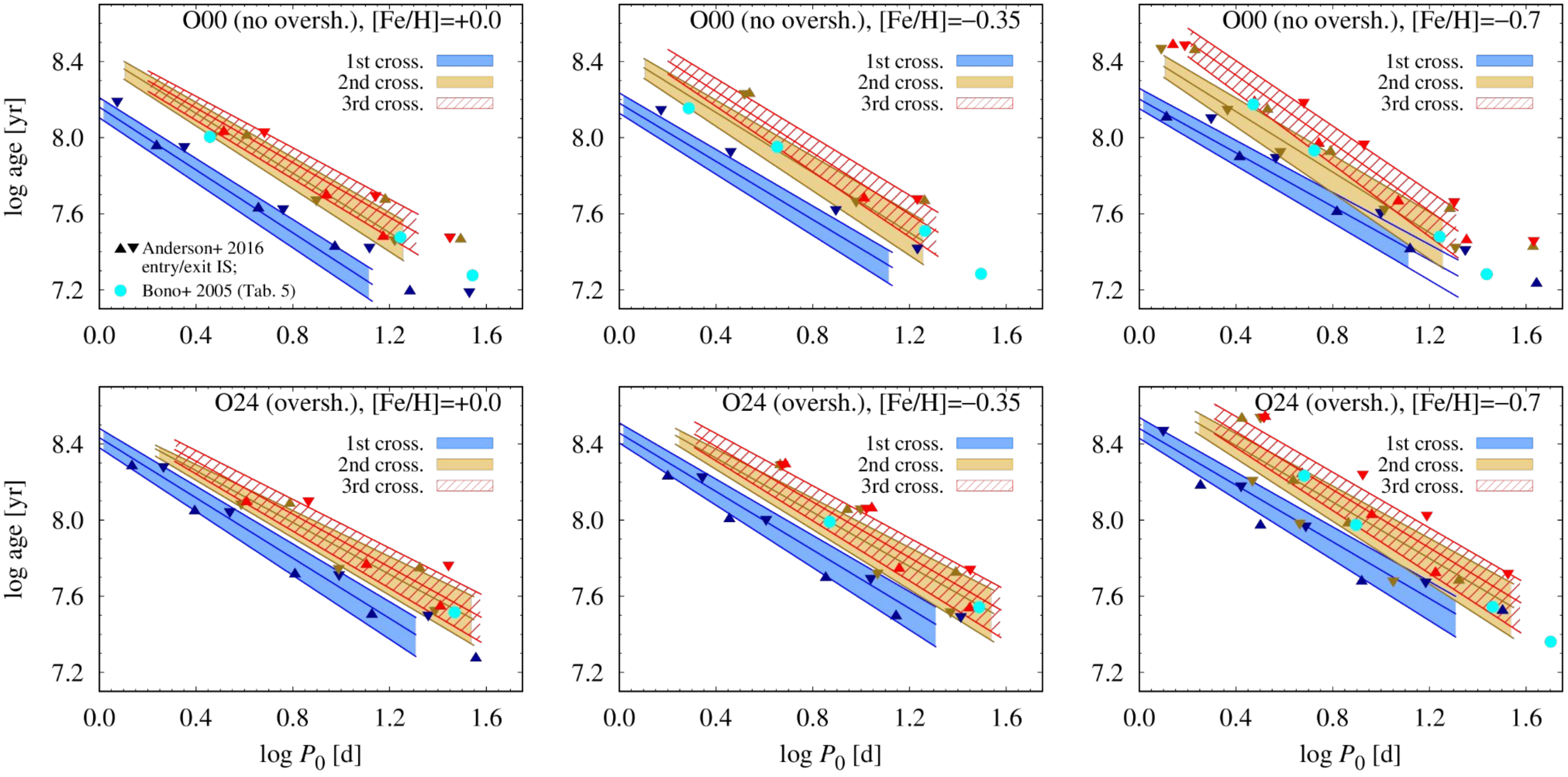
Period Change Rates



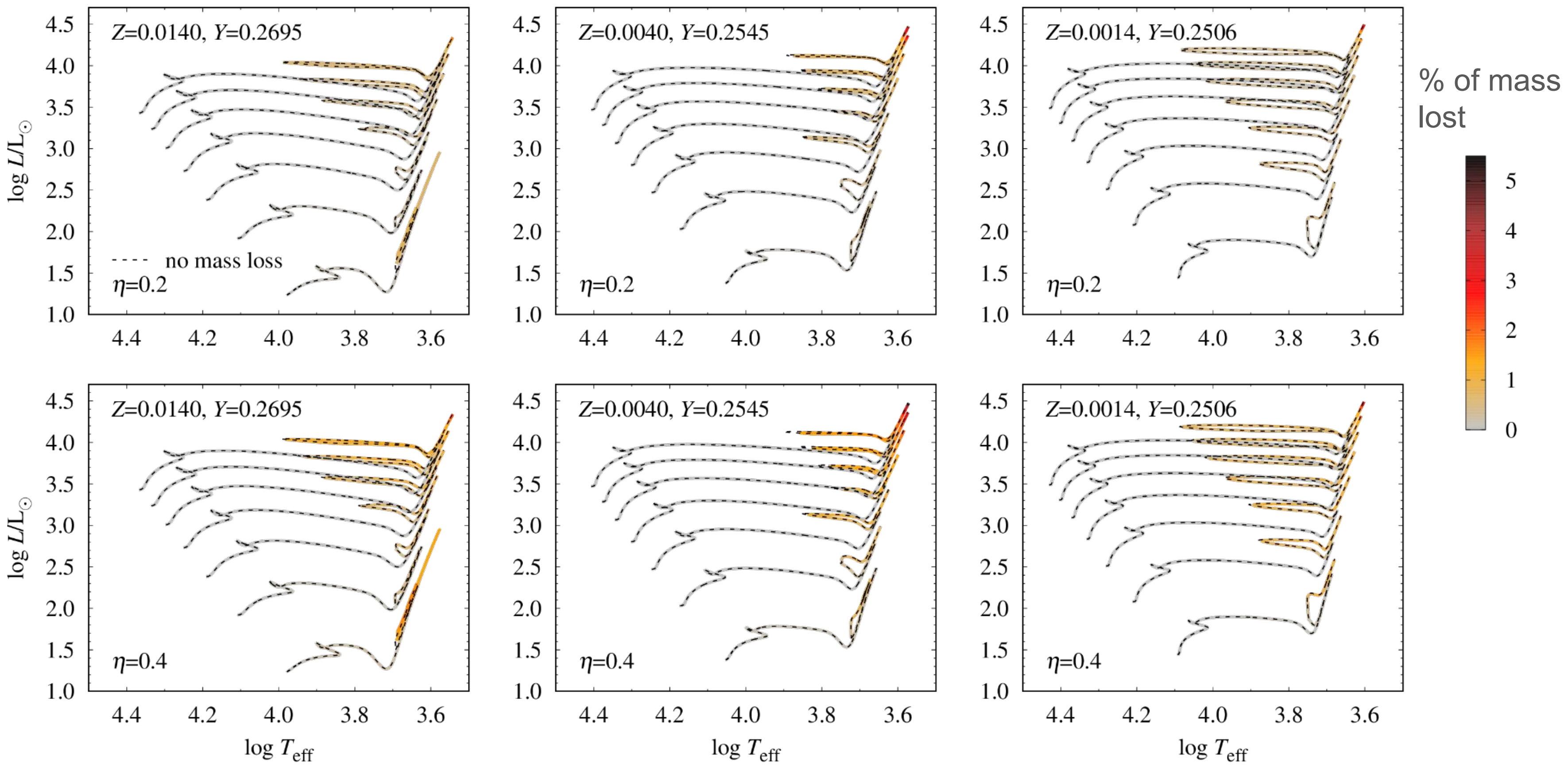
P-Age Relation



P-Age Relation



The Effects of Mass Loss, $^{14}N(\alpha,\gamma)^{15}O$, Solar mix, ΔY/ΔZ…



Summary & Future Work

- Paper to be submitted soon
- Challenges
  - Minimum pulsation period at IS entry
  - More massive models (convective boundaries)
- Inclusion of rotation
  - The effect on M-L relation
- Population synthesis
  - A comprehensive comparison with observations
- Even more consistent pulsation and evolution?

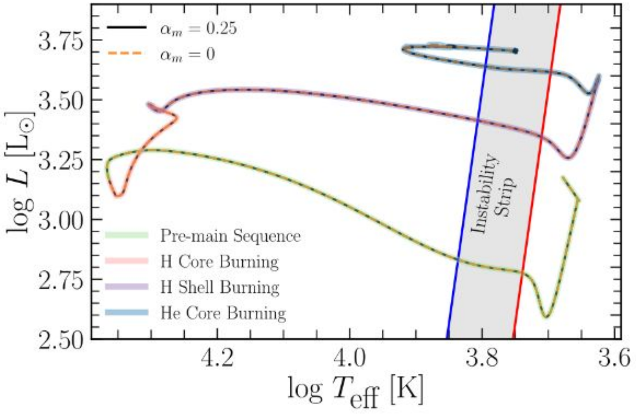
Towards fully consistent evolution and pulsation in MESA

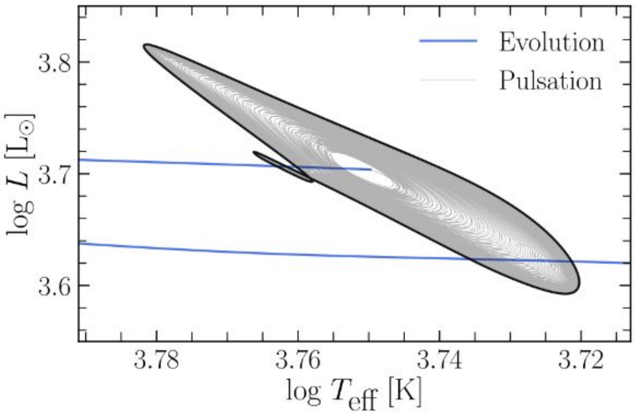
- Present work:
  - RSP uses the same microphysical setup (EOS, opacities) as MESA
  - but it uses **envelope models** (bound to evolutionary tracks, M, T_{eff}, L, [Fe/H])
  - Pretty good for Cepheids!
- Is better consistency posible?
  - Development of **TDC** in MESA, **Farag et al., in prep.**

Self-Consistent Nonlinear Cepheid Pulsations During Stellar Evolution with MESA

Ebraheem Farag et al.,

- TDC: implementation of Kuhfuss model into MESA (may replace MLT)
- now extended with eddy-viscous terms





Farag et al., in prep.

Self-Consistent Nonlinear Cepheid Pulsations During Stellar Evolution with MESA

Ebraheem Farag et al.,

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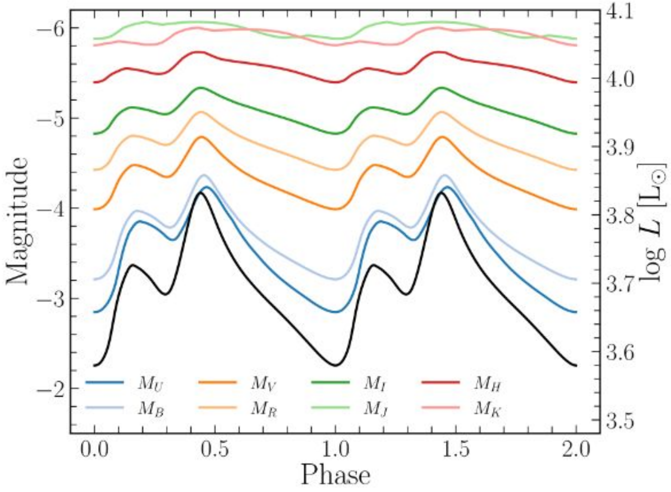

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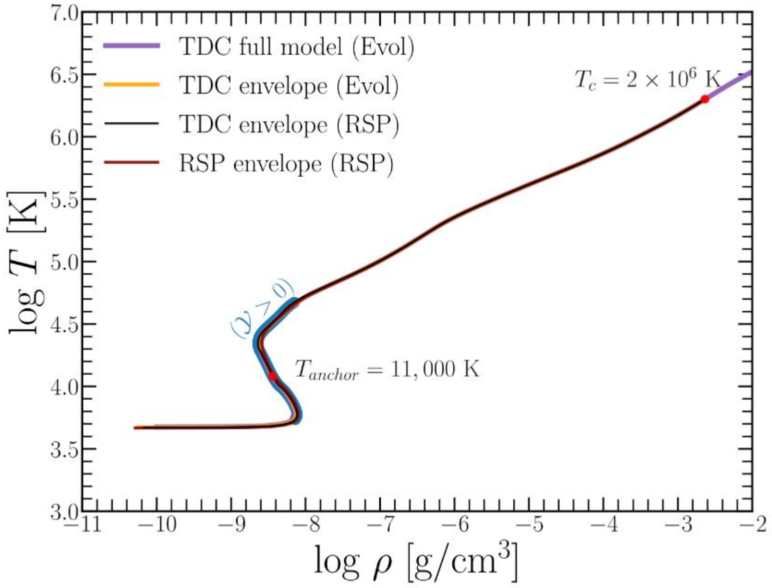

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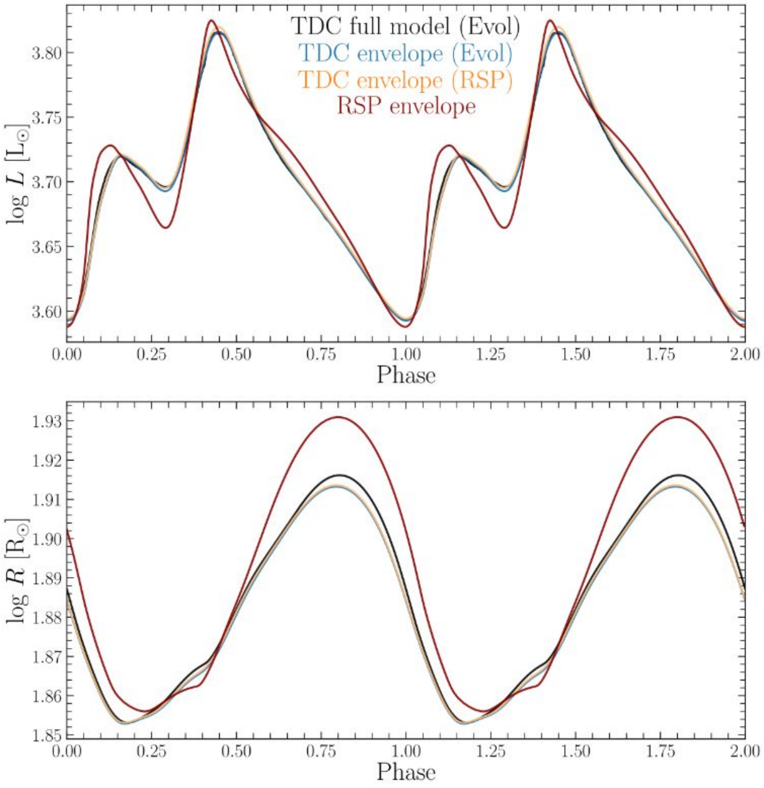
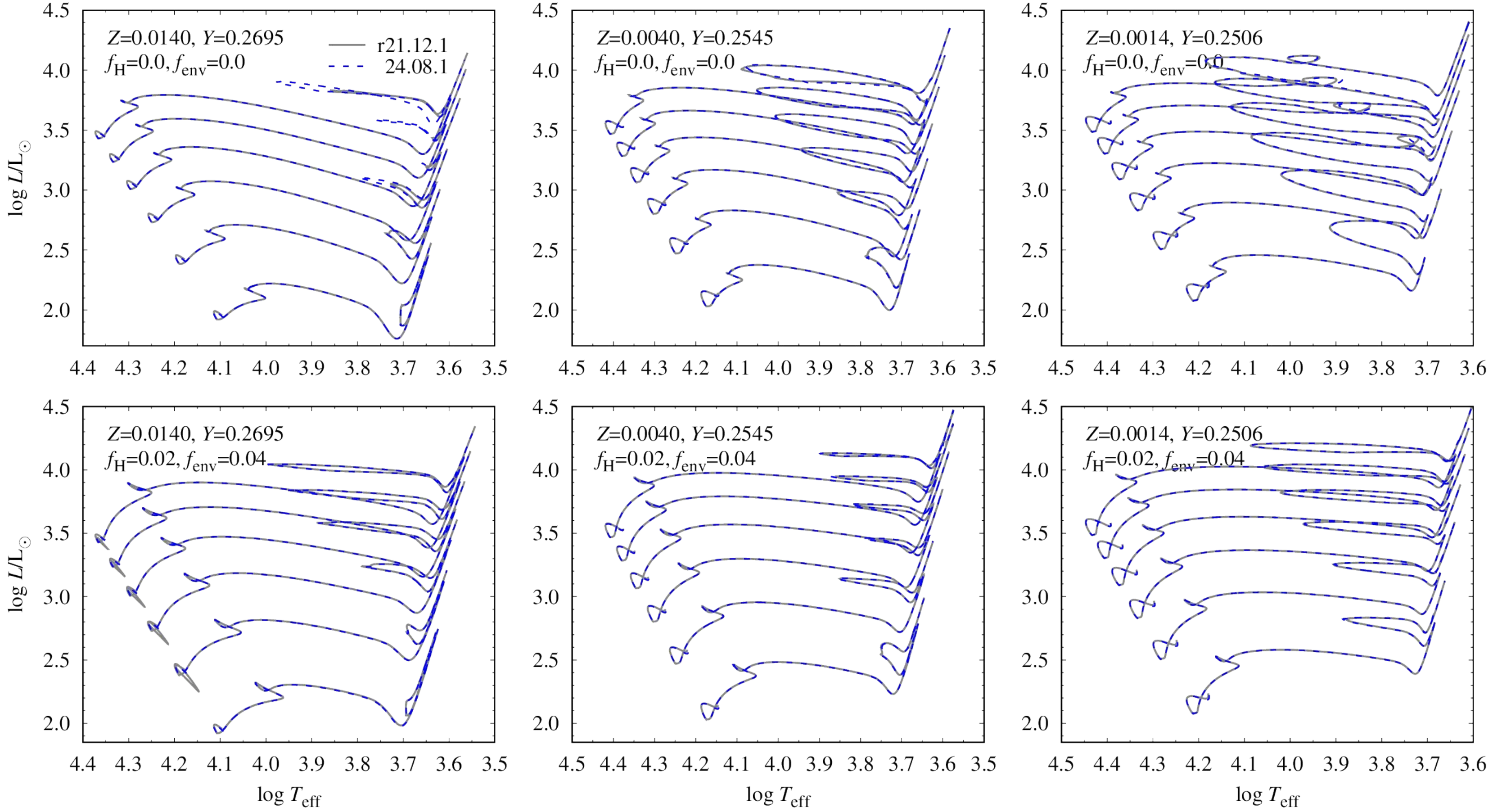
- TDC: implementation of Kuhfuss model into MESA (may replace MLT)
- now extended with eddy-viscous terms
- single model (5.91$M_\odot$, 5056$L_\odot$, 5620K, Z=0.003032) integrated in 4 versions
- Excellent agreement of TDC models, good agreement with RSP
- Differences traced to different TDC/RSP solvers



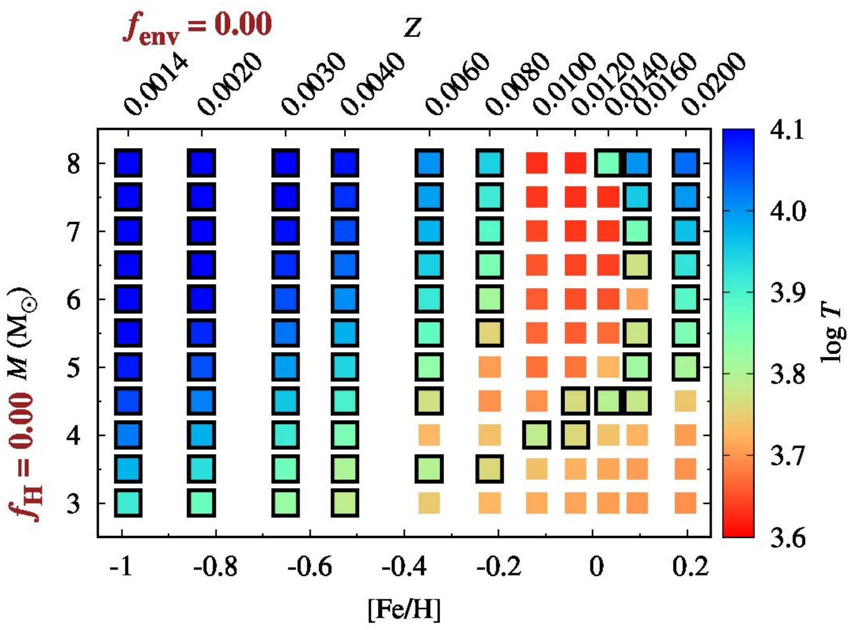
Farag et al., in prep.

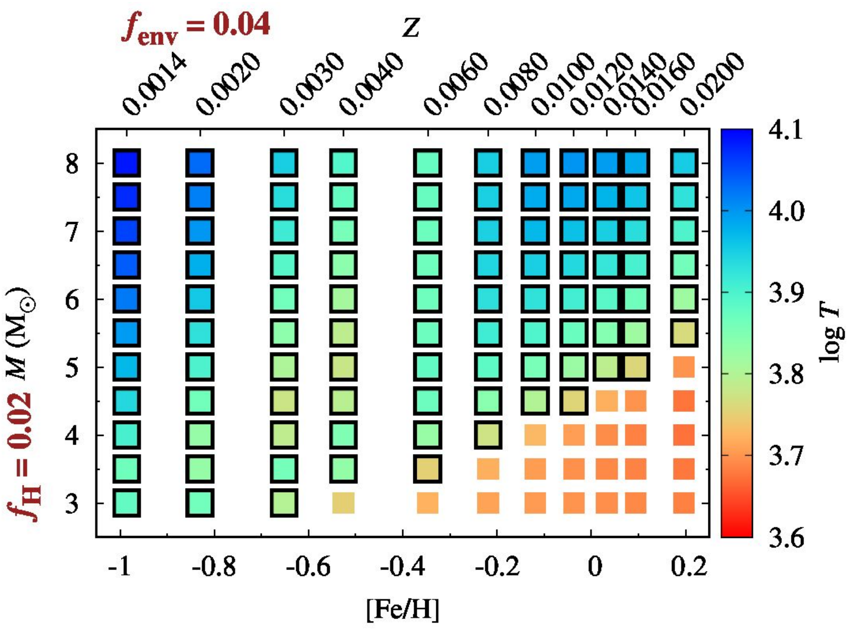
Extra slides

MESA r-21.12.1 vs. last MESA release (24.08.1)



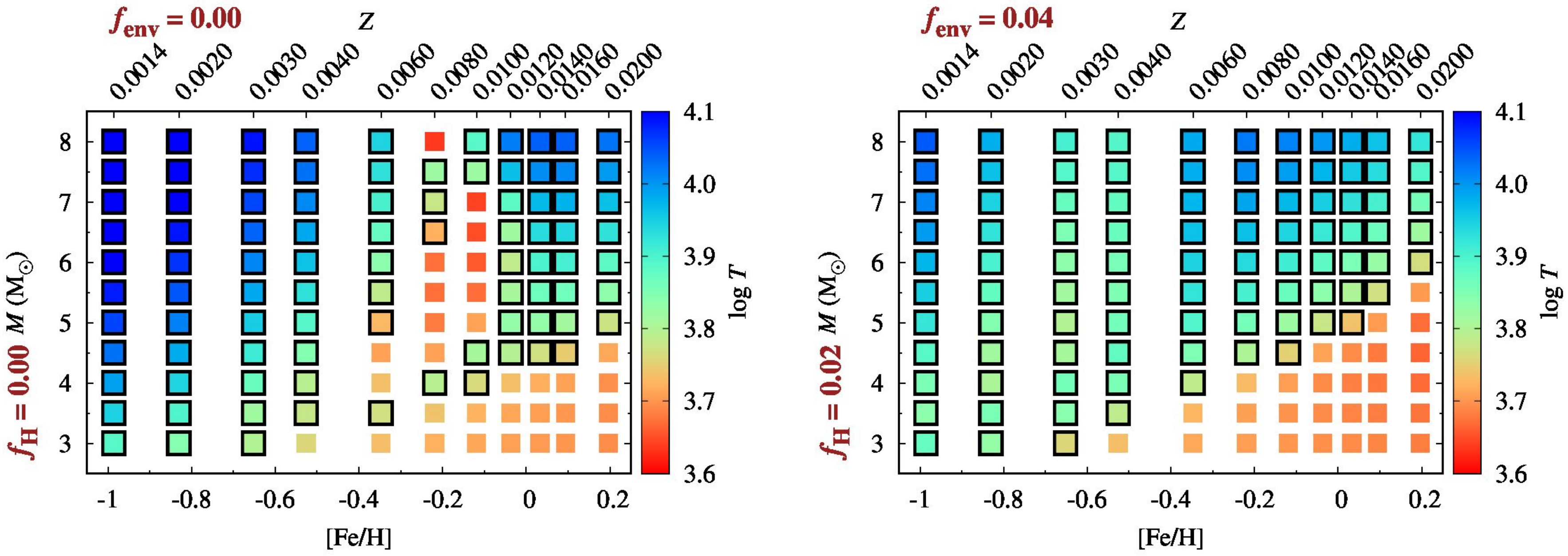
Blue loop extent across M and Z: effect of $^{14}N(\alpha,\gamma)^{15}O$





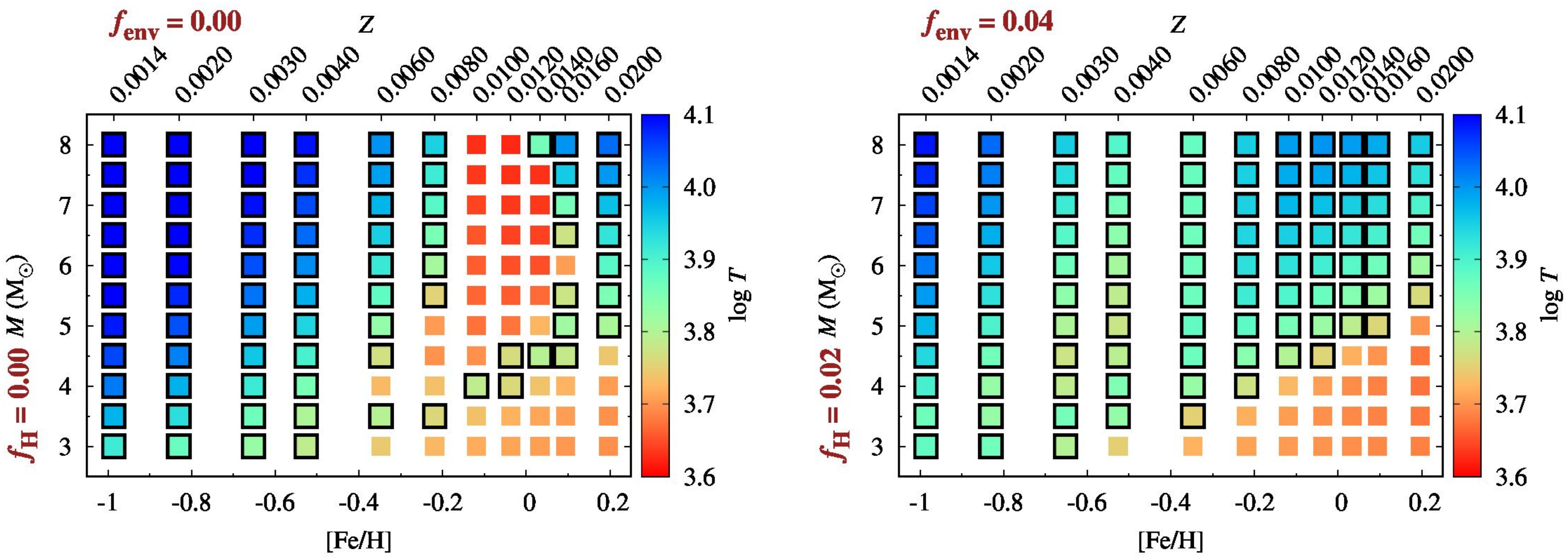
JINA, Cyburt+ 2010

Blue loop extent across M and Z: effect of $^{14}N(\alpha,\gamma)^{15}O$



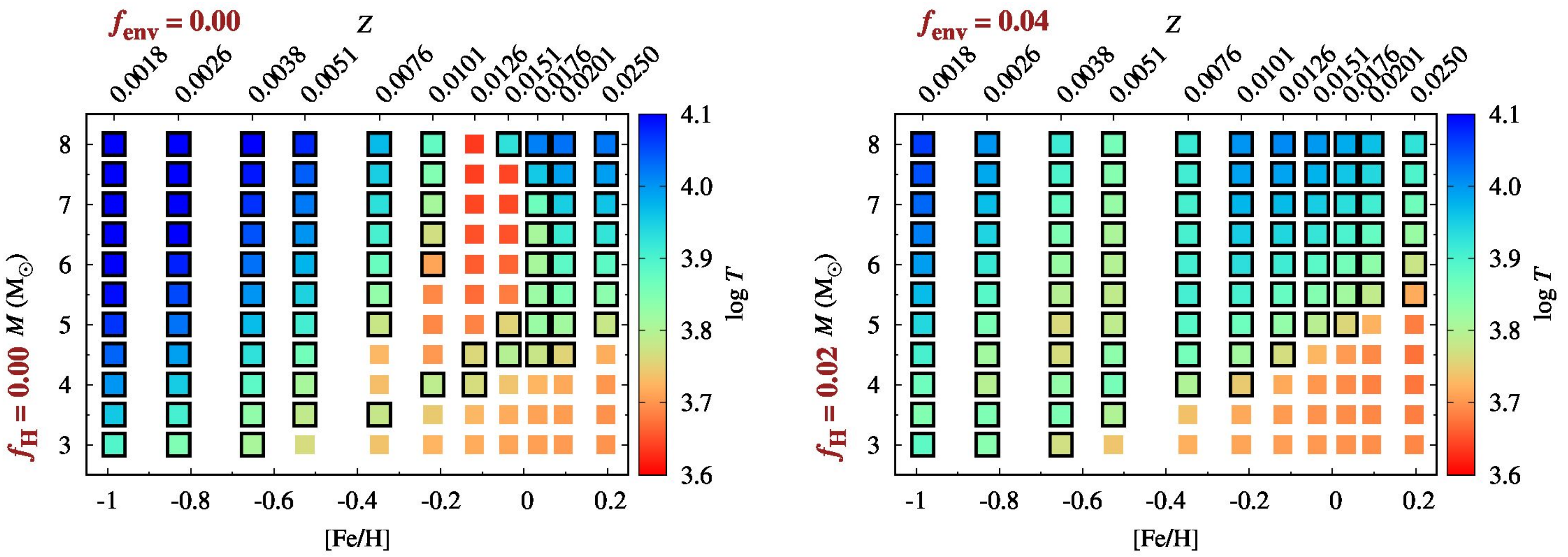
NACRE, Angulo+ 1999

Blue loop extent across M and Z: effect of Solar Mix



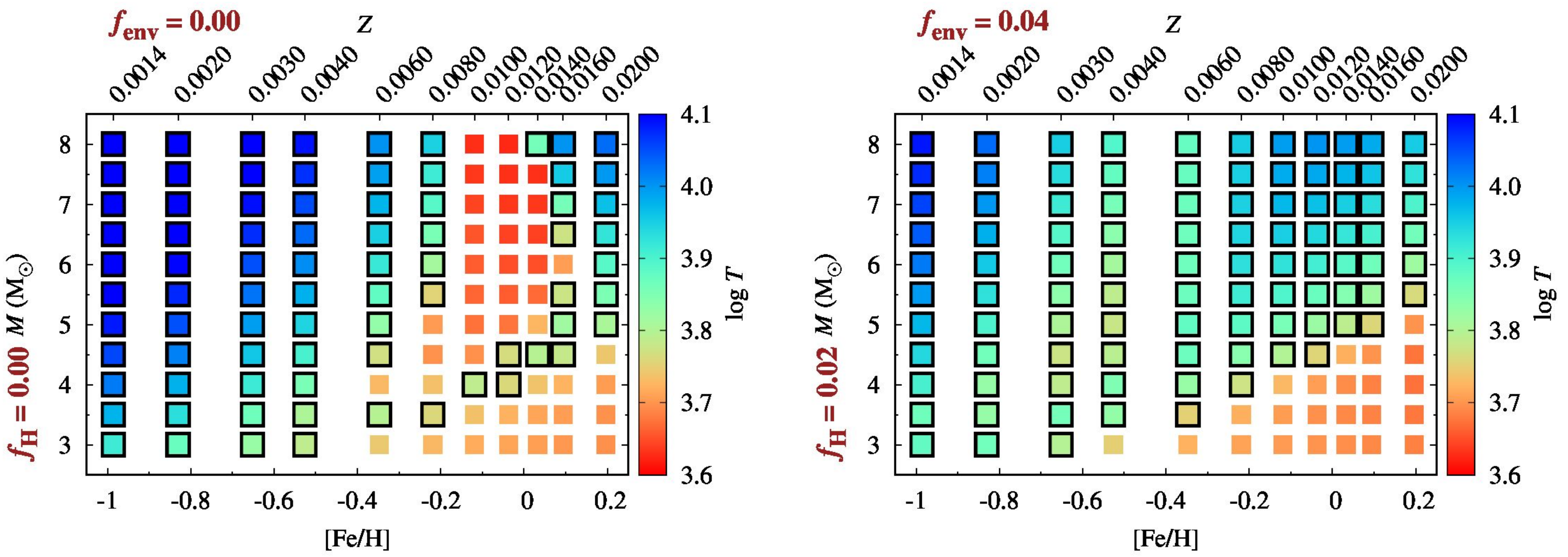
A09, Asplund+ 2009

Blue loop extent across M and Z: effect of Solar Mix



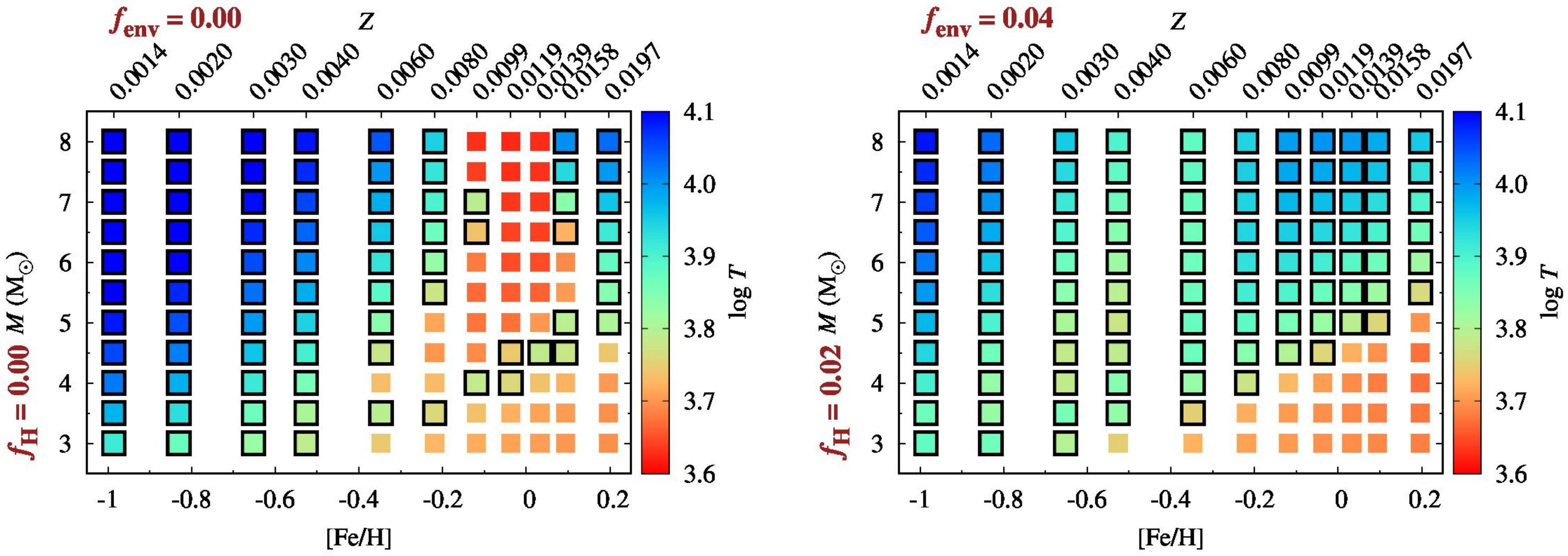
GS98, Grevesse & Sauval 1998

Blue loop extent across M and Z: effect of ΔY/ΔZ



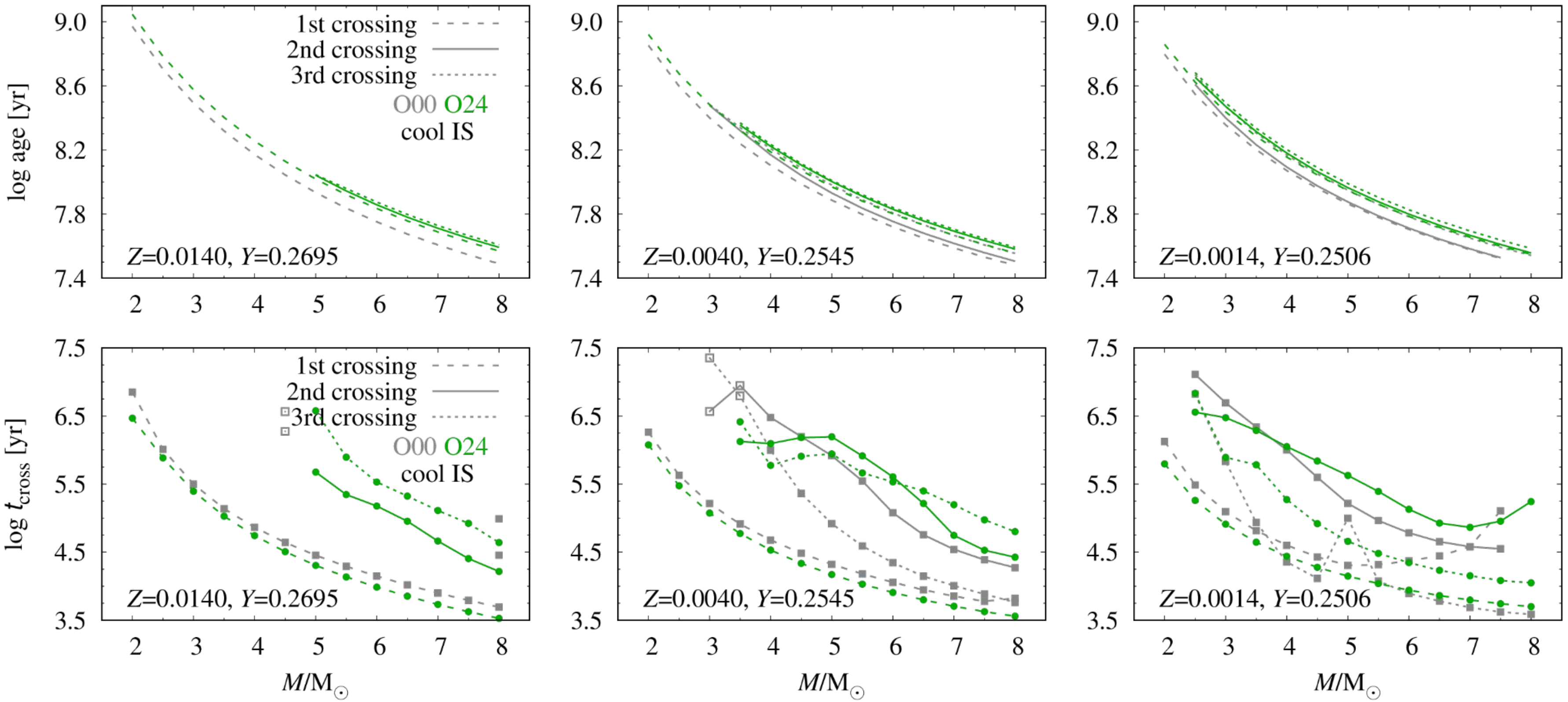
ΔY/ΔZ=1.5

Blue loop extent across M and Z: effect of ΔY/ΔZ

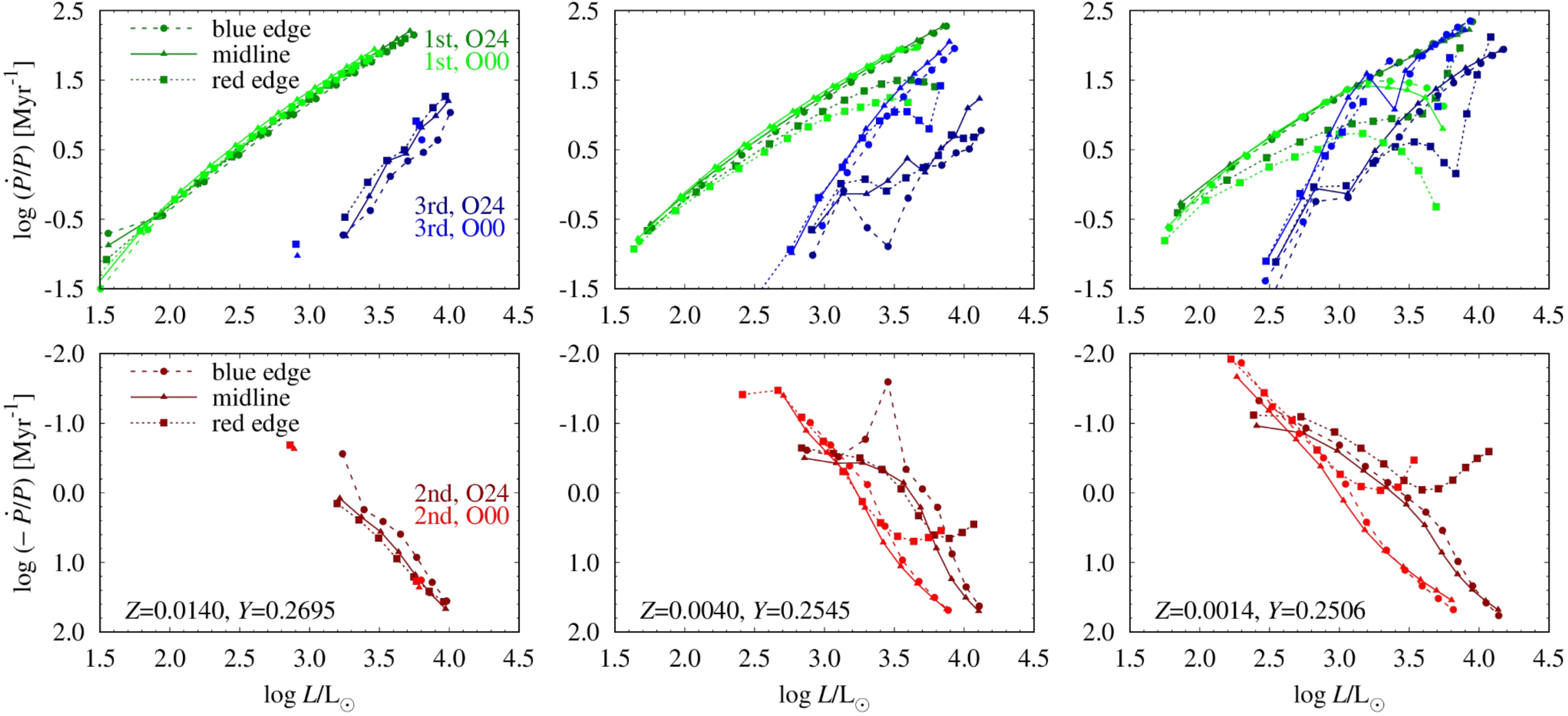


ΔY/ΔZ=2.0

Ages and crossing times



Period Change Rates



Uncertainties on evolutionary tracks (Ziółkowska +, 2024)

Table 2
Characteristics of Various Sets of Models Considered in This Paper: INT, MIX, DIFF, NET, ATM, MLT, CONV, and RES

Set	Varied Options
INT_A	cubic X/Z interpolation of opacity tables
INT_B	linear X/Z interpolation of opacity tables
MIX_A	scaled solar mixture based on Asplund et al. (2009)
MIX_B	scaled solar mixture based on Grevesse & Sauval (1998)
MIX_C	scaled solar mixture based on Grevesse & Noels (1993)
DIFF_A	atomic diffusion neglected
DIFF_B	atomic diffusion included
NET_A	$^{12}C(\alpha, \gamma)^{16}O$ from Kunz et al. (2002) + $^{14}N(\alpha, \gamma)^{15}O$ from Cyburt et al. (2010) + pp_and_cno_extras.net
NET_B	$^{12}C(\alpha, \gamma)^{16}O$ from Kunz et al. (2002) + $^{14}N(\alpha, \gamma)^{15}O$ from Cyburt et al. (2010) + MESA49.net
NET_C	$^{12}C(\alpha, \gamma)^{16}O$ from Angulo et al. (1999) + $^{14}N(\alpha, \gamma)^{15}O$ from Cyburt et al. (2010) + pp_and_cno_extras.net
NET_D	$^{12}C(\alpha, \gamma)^{16}O$ from Kunz et al. (2002) + $^{14}N(\alpha, \gamma)^{15}O$ from Angulo et al. (1999) + pp_and_cno_extras.net
NET_E	$^{12}C(\alpha, \gamma)^{16}O$ from Angulo et al. (1999) + $^{14}N(\alpha, \gamma)^{15}O$ from Angulo et al. (1999) + pp_and_cno_extras.net
ATM_A	model atmosphere tables (Hauschildt et al. 1999a, 1999b; Castelli & Kurucz 2003)
ATM_B	T–τ relation Eddington
ATM_C	T–τ relation Krishna_Swamy (Krishna Swamy 1966)
ATM_D	T–τ relation solar_Hopf (Paxton et al. 2013)
ATM_E	T–τ relation Trampedach_solar (Ball 2021; Trampedach et al. 2014)
MLT_A	Henyey (Henyey et al. 1965)
MLT_B	ML1 (Böhm-Vitense 1958)
MLT_C	Cox (Cox & Giuli 1968)
MLT_D	Mihalas (Mihalas 1978)
CONV_A	predictive mixing + Schwarzschild criterion
CONV_B	predictive mixing + Ledoux criterion
CONV_C	sign change algorithm + Schwarzschild criterion
CONV_D	predictive mixing + Schwarzschild criterion + including predictive mixing in the envelope
RES_A	time_delta_coeff=0.50+ mesh_delta_coeff=0.50
RES_B	time_delta_coeff=0.25+ mesh_delta_coeff=0.50
RES_C	time_delta_coeff=0.50+ mesh_delta_coeff=0.25
RES_D	time_delta_coeff=0.25 + mesh_delta_coeff=0.25
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RES_D	time_delta_coeff=0.25 + mesh_delta_coeff=0.25
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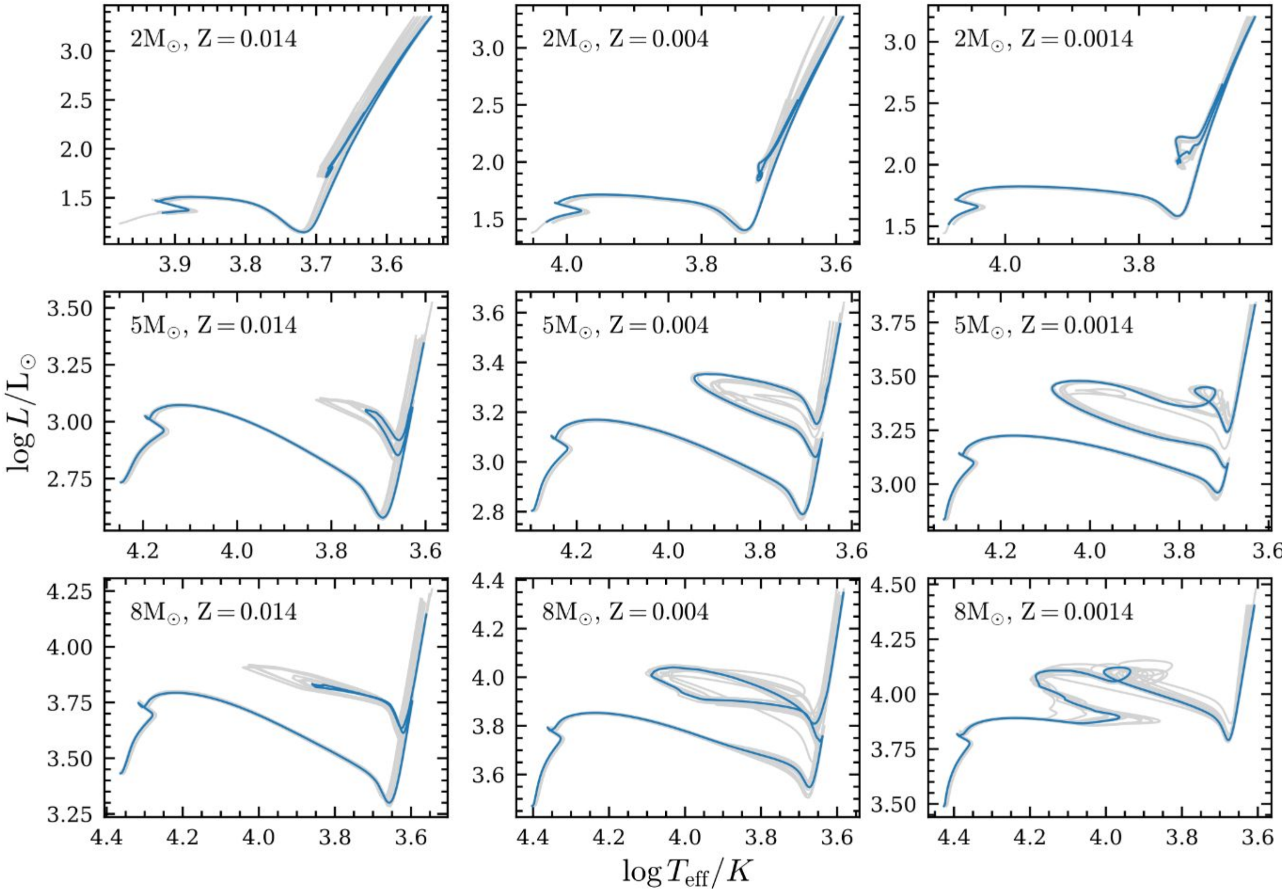
Reference model + 22 variants

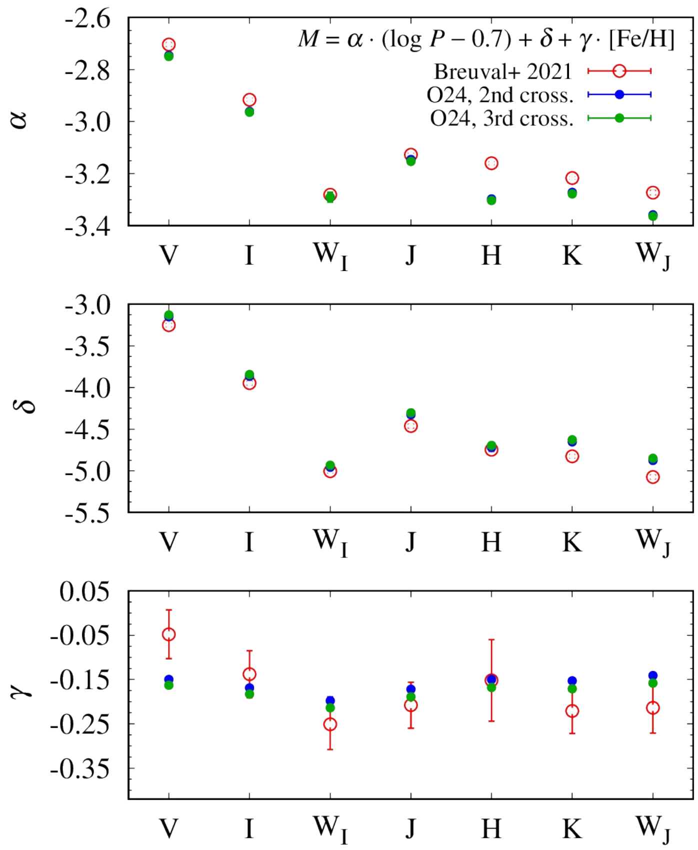
Uncertainties on evolutionary tracks (Ziółkowska +, 2024)



Reference model + 22 variants

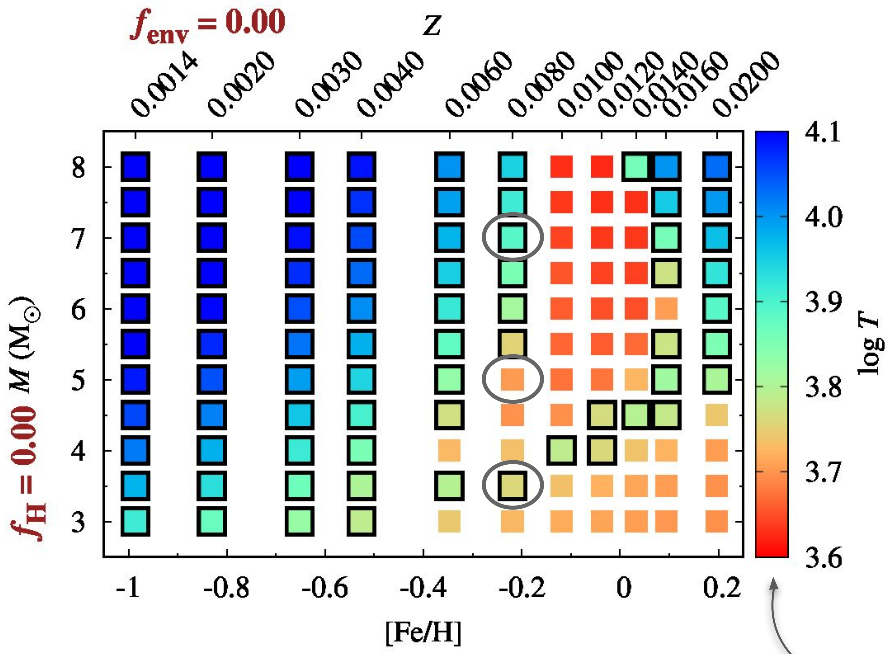
- 8 benchmark points
- differences with respect to a reference

Period - luminosity relation: metallicity dependence

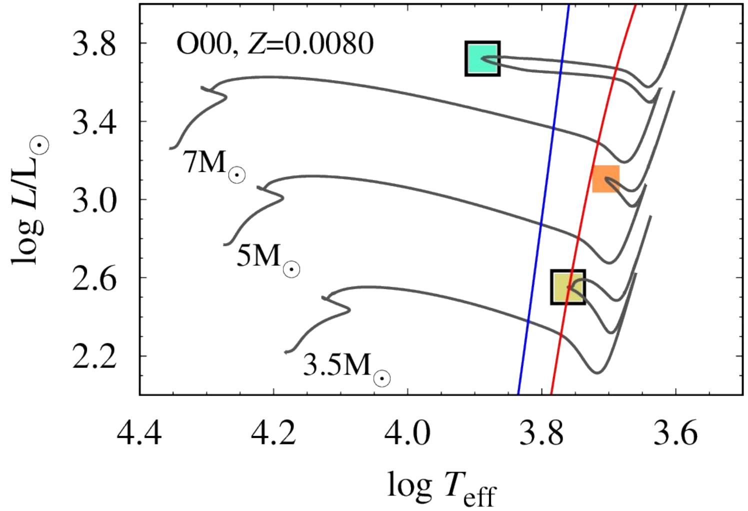


Comparison with Breuval+ 2021

Blue loop extent across M and Z



Models that enter ***cool*** IS



log *T* of the hottest point on the loop