Order and chaos in hydrodynamic BL Her models

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Deterministic chaos. Many dynamical systems of different complexity, e.g. 1D logistic map discussed here, the Lorentz system of three differential equations, or real phenomena, like turbulent convection, show chaotic behaviour. Despite huge differences, the dynamical scenarios for these systems are strikingly similar: chaotic bands are born through the series of *period doubling bifurcations* and merge through *interior crises*. Within chaotic bands *periodic windows* are born through the *tangent bifurcations*, preceded by the *intermittent* behaviour. We talk about **universal behaviour** of dynamical systems (Feigenbaum 1983). For the first time we demonstrate such behaviour in models of pulsating stars.

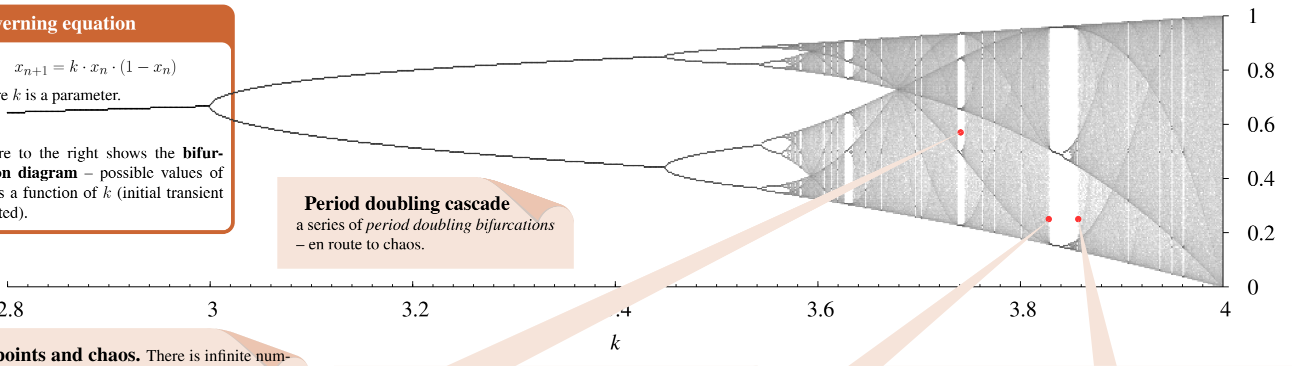
Chaos in the logistic map

Governing equation

$$x_{n+1} = k \cdot x_n \cdot (1 - x_n)$$

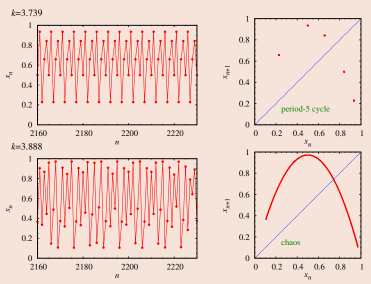
where k is a parameter.

Figure to the right shows the **bifurcation diagram** – possible values of x_n as a function of k (initial transient omitted).

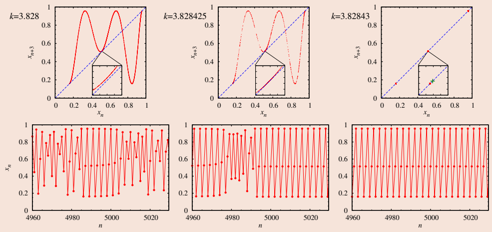


Period doubling cascade
a series of *period doubling bifurcations* – en route to chaos.

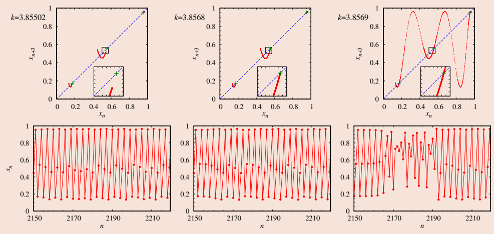
Periodic points and chaos. There is infinite number of stable period-l cycles, for any integer l. *Top:* Period-5 cycle and its first return map, i.e. a plot of x_{n+1} vs. x_n. *Bottom:* Chaotic behaviour and its first return map.



Intermittency. Evolution of the system is characterized by long phases of almost periodic behaviour interrupted with shorter bursts of chaos (*bottom*; see Pomeau & Manneville 1980). As control parameter is increased the almost periodic phases become longer up to a critical value of k for which stable period-3 cycle is born through the tangent bifurcation (*top*, illustrated with third return map). A set of three unstable fixed points appears as well that will soon cause *crises*.



Crises. The bifurcation in which the volume occupied by chaotic attractor suddenly changes is called crises (*bottom*; see Grebogi, Ott & Yorke 1982). We demonstrate the interior crises – three chaotic bands born through the period doubling cascade of period-3 cycle merge as they collide with the unstable period-3 fixed points (*top*; +) to form one large chaotic band.



Universality in action. Qualitatively the same types of behaviour as computed for simple 1D map are apparent in hydrodynamic models of BL Her stars.

Chaos in the BL Her hydrodynamic models

Governing equations

$$\frac{dU}{dt} = -\frac{1}{\rho}\nabla(P + P_t + P_\nu) - \nabla\phi$$

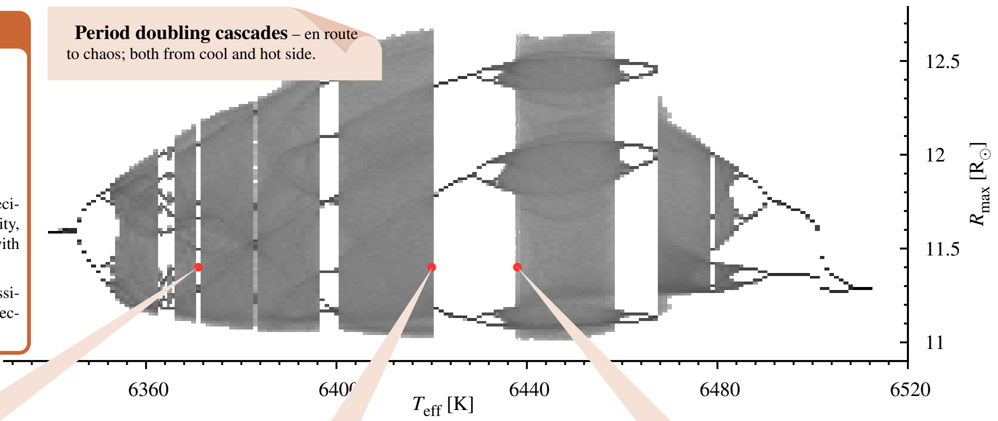
$$\frac{de}{dt} + P\frac{dV}{dt} = -\frac{1}{\rho}\nabla(F_r + F_c) - (S - D)$$

$$\frac{de_t}{dt} + (P_t + P_\nu)\frac{dV}{dt} = -\frac{1}{\rho}\nabla F_t + (S - D)$$

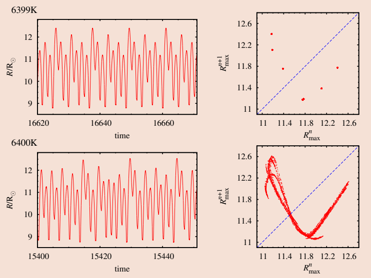
solved in 180 zones of stellar BL Her-type model specified by constant mass, $M = 0.55\mathrm{M}_\odot$, constant luminosity, $L = 136\mathrm{L}_\odot$, and varying effective temperature, $T_{\rm eff}$, with the pulsation codes of Smolec & Moskalik (2008).

Figure to the right shows the **bifurcation diagram** – possible values of maximum radii as a function of model's effective temperature.

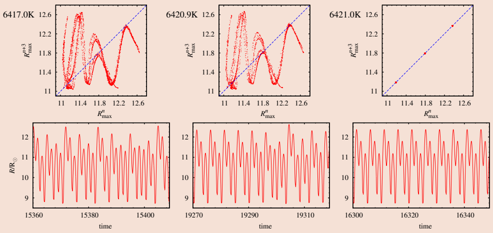
Period doubling cascades – en route to chaos; both from cool and hot side.



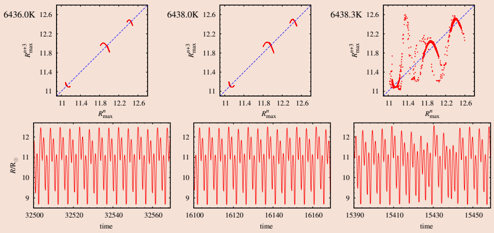
Periodic points and chaos. We find stable period-2, 3, 5, 7 (*top*) and 9 cycles, most of them undergo a period doubling bifurcation. *Bottom:* Example of chaotic model.



Intermittency. Stable period-3 cycle is born through the tangent bifurcation. Before the bifurcation, intermittent behaviour is apparent.



Crises. The three chaotic bands merge as they collide with the unstable period-3 cycle fixed points (located on the diagonal).



Summary. Except of period doubling effect (Smolec et al. 2012), the behaviour computed in our models is not observed in any BL Her star. Nevertheless the described chaotic dynamics may be crucial in pulsation of luminous semi-regular and irregular pulsators. We also stress, contrary to recent claims (Plachy et al. 2013), that stable periodic cycles are not necessarily caused by the resonances among pulsation modes, but may be an intrinsic property of the chaotic systems.

References
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