Amplitude Saturation in β Cephei Models

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Abstract

Using nonlinear hydrocodes we have calculated single mode saturation amplitudes for β Cephei models. These are systematically higher than amplitudes observed in β Cephei variables, even in monoperiodic ones. We argue that collective saturation of the pulsation instability by a dozen or so acoustic modes brings the theoretical amplitudes to the observed level. Very interesting case of double-mode behaviour in purely radiative β Cephei models is presented. Full results will be discussed in Smolec & Moskalik (2006).

Introduction

Although the driving mechanism acting in β Cephei stars is well known, several problems remain to be solved. We have adressed following questions, ilustarted in Figure 1:

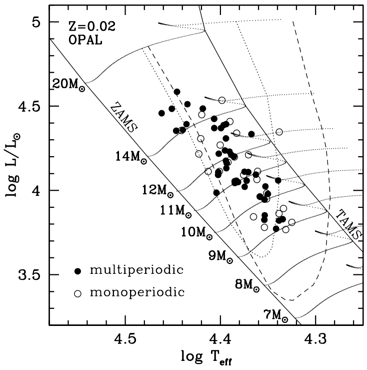

Figure 1: Theoretical HR diagram for the upper main sequence. Evolutionary tracks have been calculated with the Warsaw-New Jersey evolutionary code. Dashed and dotted lines enclose the instability strips for the radial fundamental and first overtone modes, respectively. Circles represent positions of β Cephei variables obtained using data from Stankov & Handler (2005).

- Are 38% of variables monoperiodic?
- What is the mechanism responsible for amplitude limitation?
- What is the reason for apparent lack of pulsators at high and low-mass ends of the theoretical instability strip?

Our approach

In our approach **radial modes have been treated as representative for all acoustic oscillations**. We have studied pulsation properties of several models using standard radiative lagrangian hydrocodes (Stellingwerf 1975). Nonlinear limit cycles (monoperiodic full-amplitude oscillations) have been calculated through Stellingwerf's (1974) relaxation technique. Our analysis provides stability information in terms of the growth rates both of the fundamental and first overtone modes. Definitions and modal selection rules are listed inside Frame 1.

Linear growth rates:

γ_0 – fundamental mode lin. growth rate
γ_1 – first overtone lin. growth rate

Nonlinear growth rates (switching rates):

$\gamma_{0,1}$ – switching rate toward fundamental mode
$\gamma_{1,0}$ – switching rate toward first overtone

If only one mode is lineary unstable then this mode is the only atractor of the system. If both modes are unstable, four possibilities exist:

$\gamma_{1,0} > 0, \gamma_{0,1} < 0$ – first overtone, (1O)
$\gamma_{1,0} < 0, \gamma_{0,1} > 0$ – fundamental, (F)
$\gamma_{1,0} < 0, \gamma_{0,1} < 0$ – either F or 1O, (E/O)
$\gamma_{1,0} > 0, \gamma_{0,1} > 0$ – double mode, (DM)

Frame 1. Modal selection rules (*e.g.* Dziembowski & Kovács 1984).

For each model bolometric peak-to-peak amplitude has been transformed to *V*-band through Kurucz (2006) static atmosphere models.

Results

The resulting modal selection scenario is presented in Figure 2, while resulting single mode saturation amplitudes are presented in Figure 3.

- Calculated single mode saturation amplitudes are significantly higher than amplitudes of *apparently* monoperiodic variables. **This strongly suggests that additional, undetected modes must be excited in these stars in order to account for low amplitude of the only observed mode**.

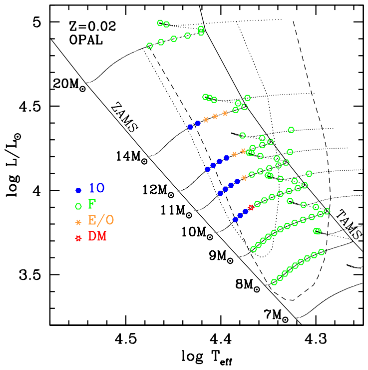

Figure 2: Theoretical HR diagram with modal selection information. Pulsation in the fundamental mode is dominant. Note one model with double-mode behaviour.

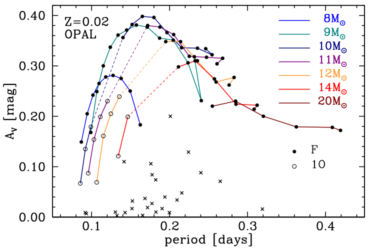

Figure 3: Theoretical single mode saturation amplitudes ploted *vs.* pulsation period. Amplitudes of monoperiodic β Cephei variables are ploted as crosses.

Theoretical amplitudes may be easily lowered to the observed level if one assumes **collective saturation of the pulsation instability**, by n similar acoustic modes. In this hypothetical multimode solution, amplitudes of individual modes are a factor of $\sim \sqrt{n}$ lower than in single mode solution. Using linear code of Dziembowski (1977) we have determined the number of lineary unstable acoustic modes for models of different masses, located in the center of the main sequence band. This number doesn't vary much along evolutionary track, but changes from track to track. Since the number of unstable modes is much higher than the number of observed modes in multiperiodic β Cephei variables, we have arbitrarily assumed that only one third of theoretically unstable modes take part in the saturation process. Rescaled saturation amplitudes are presented in Figure 4.

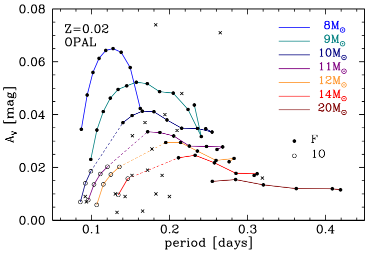

Figure 4: Theoretical amplitudes, recalculated under assumption of collective saturation of the instability mechanism by 1/3 of lineary unstable acoustic modes. Crosses as in Figure 3.

- Using only part of the lineary unstable acoustic modes, we have lowered theoretical amplitudes to the observed level. Thus, we argue that **collective instability saturation is sufficient to explain amplitude limitation in the β Cephei pulsators**.
- The strongest reduction of amplitudes occurs for 20M$_\odot$ track. However, predicted amplitudes are still above current detection limit. Under our simplifying approximations, nonlinear effects cannot explain the lack of massive and luminous β Cephei pulsators. The same conclusion is true for pulsators at the low luminosity end of the theoretical instability strip. Even under assumption of collective saturation, predicted amplitudes are well above the detection limit.

The role of opacity

To check the impact of different opacities on presented results, we have calculated nonlinear limit cycles for 11M$_\odot$ evolutionary track, using OP opacities. Comparison of amplitudes is presented in Figure 5.

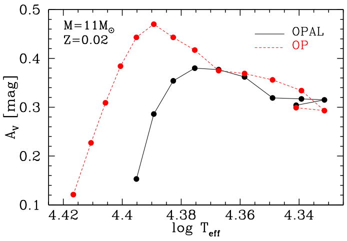

Figure 5: Comparison of *V*-band peak-to-peak theoretical amplitudes of the fundamental mode, calculated with the OPAL and OP opacities.

- The main effect of using OP opacities is a shift of the linear blue edge toward higher temperatures
- For most of the main sequence evolution, amplitudes obtained with both opacities are comparable. Thus, our conclusions are insensitive to opacities being used.

Theoretical pearl: Double-Mode behaviour

In case of classical pulsators, robust double-mode behaviour was obtained only after convection was included in hydrodynamics (*e.g.* Feuchtinger 1998). To our surprise, one of our radiative models (Figure 2) displays DM behaviour. Detailed analysis of growth rates along 10M$_\odot$ track, presented in Figure 6, reveals very interesting situation. Two narrow domains of DM behaviour are present.

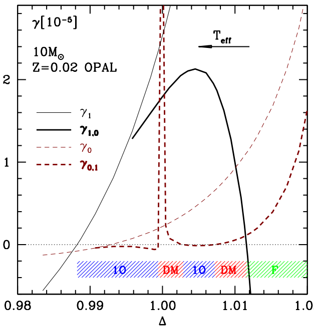

Figure 6: Linear (thin lines) and nonlinear (thick lines) growth rates along 10M$_\odot$ evolutionary track. They are ploted against Δ parameter defined as: $\Delta = 2\omega_1/(\omega_0+\omega_2)$. Δ characterizes proximity to the center of the parametric resonance: $2\omega_1 = \omega_0+\omega_2$. Modal selection rules from Frame 1 were used in construction of the coloured modal selection bar.

Two different mechanisms are responsible for DM behaviour:

- The first double-mode domain, at $\Delta \approx 1$, is clearly connected with the resonance. The main effect of the resonance is destabilization of the first overtone limit cycle through the growth of the fundamental mode perturbation, which is manifested as a very sharp peak of $\gamma_{0,1}$. As a result, double-mode *island* appears in the middle of the first overtone pulsation domain.
- The second double-mode domain, at $\Delta \approx 1.01$ is not connected with any resonance. Double-mode domain separates the first overtone and fundamental mode pulsation domains. Such behaviour is expected in case of nonresonant coupling of pulsation modes (*e.g.* Dziembowski & Kovács 1984)

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