Survey of hydrodynamic RR Lyrae models

Radosław Smolec
Nicolaus Copernicus Astronomical Center, PAS, Warsaw, Poland
smolec@camk.edu.pl

Model grid, light curves, Fourier decomposition parameters

Hydrodynamic models

All models were computed with the Warsaw nonlinear, convective pulsation codes (Smolec & Moskalik 2008a). Radiation is treated in the diffusion approximation and turbulent convection is treated with the model of Kuhfuß (1986).

Each of the Lagrangian models comprise 150 mass shells extending down to 2×10^6 K. Each pulsation cycle is covered by at least 600 time-steps.

All models adopt OPAL opacities (Iglesias & Rogers 1996) and Asplund et al. (2009) solar mixture. Colors are computed using Kurucz (2005) model atmospheres.

The basic grid of physical parameters for non-linear models is the following:

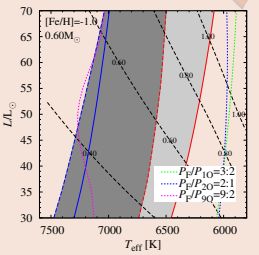
parameter	values
[Fe/H]	$-2.5, -2.0, -1.5, -1.0, -0.5, 0.0$
$M/M_\odot$	$0.50, 0.55, 0.60, 0.65, 0.70$
$L/L_\odot$	$40.0, \ldots, 60.0$, step 2.5
$T_{\rm eff}$	step 50 K (within instability strip)

Linear stability analysis is conducted in a much wider parameter range.

Several sets of convective parameters that enter the turbulent convection model are considered.

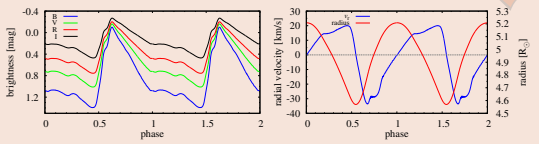
The full grid of models, with more than 10 000 light curves, will be published as an on-line database at the end of 2014. Exemplary content of the database is illustrated in the figures.

The HR diagram



Light and dark grey-shaded areas correspond to fundamental (F) and first overtone (1O) instability strips. Lines of constant period and loci of some radial mode resonances are also plotted.

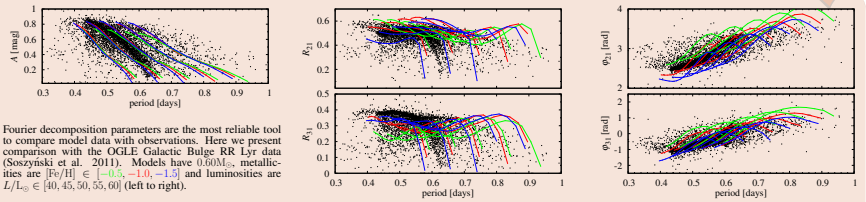
Light and radial velocity curves



Light curves are available in U, B, V, R and I bands together with radial velocity curves, and radius variation curve. The presented model has $M = 0.6M_\odot$, $L = 50L_\odot$, [Fe/H] $= -1.0$ and $T_{\rm eff} = 6900$ K.

The on-line database will be released by the end of 2014.

Fourier decomposition parameters



Fourier decomposition parameters are the most reliable tool to compare model data with observations. Here we present comparison with the OGLE Galactic Bulge RR Lyr data (Soszyński et al. 2011). Models have $0.60M_\odot$, metallicities are [Fe/H] $\in [-0.5, -1.0, -1.5]$ and luminosities are $L/L_\odot \in [40, 45, 50, 55, 60]$ (left to right).

Mode selection in RR Lyrae models

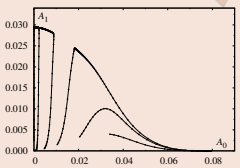
The long-standing problem of double-mode pulsators

With radiative calculations stable double-mode pulsation cannot be found (eg. Buchler 2009). Inclusion of the turbulent convection in the Florida-Budapest code led to success for both Cepheid and RR Lyr models (Kolláth et al. 1998, 2002). In Smolec & Moskalik (2008b) we showed however, that the double-mode Cepheid pulsation in these models is caused by unphysical neglect of negative buoyancy in convectively stable regions. Once buoyancy is properly treated, stable, non-resonant double-mode Cepheid pulsation cannot be found.

In this study we analyse the problem for RR Lyr models. Mode selection analysis is illustrated in the figures. Each model is integrated with different initial conditions and growth of the F and 1O mode amplitudes is followed (**1**). Next, we fit the hydrodynamic trajectories with amplitude equations (AEs) to get saturation coefficients (**2**). Then, AEs can be solved yielding all possible fixed points and their stability (**3**). The procedure is repeated along a model sequence and derived saturation coefficients allow to deduce the mode selection for arbitrary effective temperature (**4**) and finally in the full HR diagram (**5**).

So far, our search for stable double-mode pulsation yielded negative result. The HR diagram to the right (**5**) illustrates the typical mode selection scenario. We find either-or domain, in which the model pulsates either in the F mode or in the 1O mode, depending on initial conditions (direction of evolution). Since double-mode pulsators exist and are numerous, we conclude that the modelling of double-mode RR Lyr pulsation remains an open issue (just as in case of classical Cepheids).

1. Hydrodynamic models



The same model is integrated with different initial conditions. Hydrodynamic trajectories are extracted with the analytic signal method.

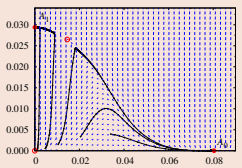
2. Amplitude equations

The long-term evolution of mode amplitudes may be described with AEs. In the simplest case, non-resonant, cubic AEs are:

$$\dot{A}_0 = (\kappa_0 + q_{00}A_0^2 + q_{01}A_1^2)A_0,$$
$$\dot{A}_1 = (\kappa_1 + q_{10}A_0^2 + q_{11}A_1^2)A_1$$

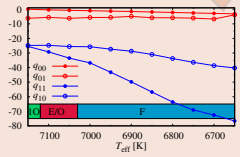
Having (A_0, A_1) and ($\dot{A}_0$, $\dot{A}_1$) from hydrodynamic trajectories, and linear growth rates from linear stability analysis, we may determine the saturation coefficients, q_{ij} through simple linear fit.

3. Fixed points and their stability



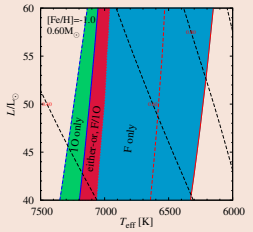
Solution of AEs allows to find the fixed points ($\dot{A}_0 = \dot{A}_1 = 0$, red symbols) and determine their stability (filled circle – stable, open circle – unstable). The resulting flow field is visualised with short blue line segments.

4. Mode selection



The above outlined procedure is repeated along a sequence of models which allows to find the edges of mode selection domains through interpolation.

5. Mode selection – HR diagram



On the hot side of the HR diagram 1O-only domain extends, while on the cool side F-only domain is present. In between there is a narrow either-or domain, in which pulsation mode depends on the direction of evolution. No double-mode domain is present.

The problem of modelling the double-mode RR Lyrae pulsation is not solved yet!

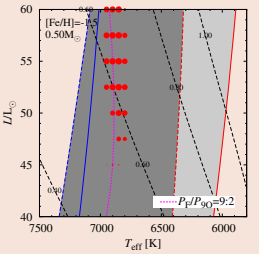
Search for pulsation modulation (Blazhko Effect) in RR Lyrae models

The Blazhko Effect Mystery

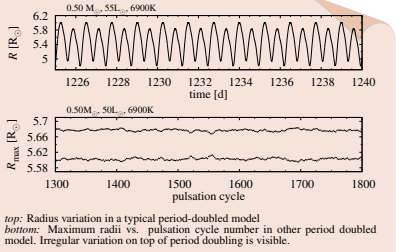
The Blazhko Effect is a quasi-periodic modulation of pulsation amplitude and phase which affects nearly 50% of stars pulsating in the fundamental mode. Despite being discovered more than 100 years ago, its physical origin remains unclear. Among many models proposed to explain the phenomenon, the model of Buchler & Kolláth (2011), in which modulation is caused by half-integer resonance between the radial modes (likely $P_{\rm F}/P_{9{\rm O}} = 9:2$) is the most promising one. It is supported by the analysis of amplitude equations and recent *Kepler* observations, but lacks confirmation from hydrodynamic modelling.

We searched for Blazhko-like modulation in our RR Lyr models, unfortunately, it was not found so far. In the models with decreased eddy-viscous dissipation, we find period doubling effect, which is also caused by half-integer resonance, but not modulation akin to Blazhko. This interesting dynamical behaviour is illustrated in the figures.

Period doubling in the HR diagram



Red dots mark the location of period-doubled models. Larger the dot, larger the amplitude of alternations. The period doubling domain clearly coincide with the loci of the $P_{\rm F}/P_{9{\rm O}} = 9:2$ resonance.



top: Radius variation in a typical period-doubled model
bottom: Maximum radii vs. pulsation cycle number in other period doubled model. Irregular variation on top of period doubling is visible.

The lack of pulsation modulation in hydrodynamic models does not invalidate the radial resonance model. Likely our pulsation codes are too simple and more involved treatment of the turbulent convection and/or 3D modelling is needed.

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