Observational Astrophysics
4. Introduction to Geometrical Optics

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1 Introduction

The optical principles needed to understand and use optical instruments can be divided into three categories: geometrical (or ray) optics, physical (or wave) optics, and quantum optics. Which one to use depends on the scales involved[1].

For geometrical optics, we treat light as rays and are concerned with the path taken by these rays as they pass through a system of lenses and/or mirrors. Rays essentially represent the direction of the energy flow. The simplification of treating light as rays is appropriate when the wavelength is small in comparison to the size of the relevant optical elements.

For dealing with light rays, we mostly need to apply the simple laws of refraction and reflection. However, a much more powerful method to deal with geometrical optics is given by the so-called Fermat's Principle[2]. With it, we can recover the laws of reflection and refraction but also treat deviations like aberrations.

There are whole textbooks on geometrical optics but, again, our goal is not to dive deep into all details of the theory. We just need to remember the basic concepts. Later, we will also discuss the basics of wave optics, which are needed for topics like interference, diffraction, and polarization.

2 Read these texts

Here, I am perhaps recommending too much reading ($\sim$ 60 pages) for topics that we are going to touch just very quickly. Nevertheless, these are some classical texts that present things in summarized but useful ways.

The first one is a simple introduction to the concepts of light rays, reflection, refraction, and Fermat's principle. If you are familiar with the topics, you might want to skip it. Otherwise, it is

[1]Although one can indeed use wave optics to describe all the same phenomena that are treated with geometrical optics, the latter approach tends to be simpler and gives answers with sufficient accuracy.

[2]Pierre de Fermat was a French mathematician. In addition to optics, he is also known for his work on number theory and for developing a mathematical method that is similar to differential calculus. See a biography here: https://mathshistory.st-andrews.ac.uk/Biographies/Fermat/.

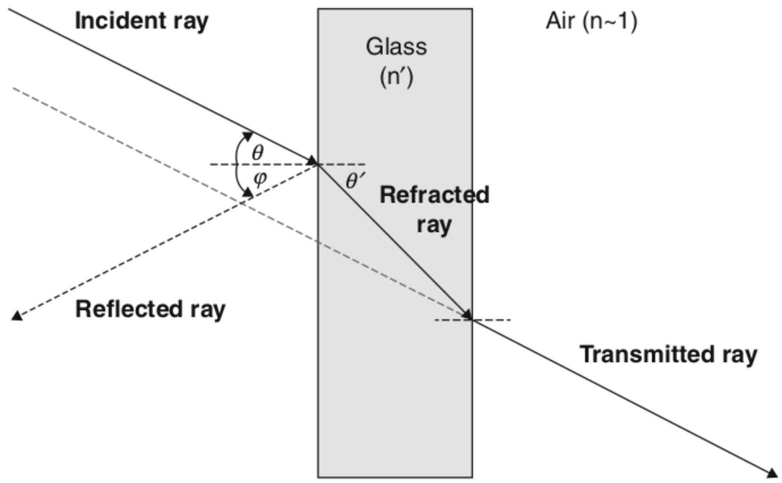

Figure 1: Reflection and refraction. Credit: Fig. 1.4 of Silva & McLean (2013), see `https://link.springer.com/referenceworkentry/10.1007%2F978-94-007-5621-2_1`.

quite useful to read this before the next two texts (where things are shown in more mathematical detail). Chapter 1 (pages 3 to 20), called "Properties of light" of the book "Fundamentals of Optics" by Jenkins & White (1976)[3].

Then, for the same concepts and other useful definitions, see Chapter 2 and 3 of the book "Astronomical Optics" by Schroeder (2000)[4]. In these Chapters, you will find also a brief discussion of lenses, prisms, and mirrors.

3 Summary of concepts

Reflection and refraction of a light ray are shown in Fig. 1. Of course, part of the energy of the light ray can also be absorbed by the material. This energy will later be reemitted at longer wavelengths.

- As we saw before, the refractive index (or index of refraction) of a medium is the ratio between the speed of light in vacuum and the speed of light in that medium; $n = c/v$.
- Refraction depends on wavelength (the refractive index is higher towards the blue than towards the red). One can define the dispersive power (V) as the difference between the refractive indices at two extreme wavelengths (n_1 and n_2) relative to the difference between the average refractive index in that medium and the one in vacuum: $V = n_2 - n_1/n_{\text{mean}} - 1$.
- The paraxial approximation is used when the ray is close enough to the optical axis that we can write, for the angle of incidence θ, that $\sin(\theta) = \theta$ (in radians).

[3] `https://archive.org/details/fundamentalsofop00jenk`

[4] `https://www.sciencedirect.com/book/9780126298109/astronomical-optics`

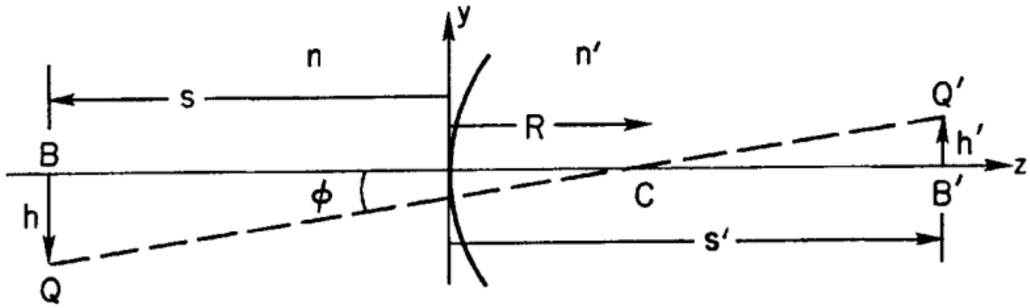

Figure 2: Conjugate points (image and object) in a paraxial region with some definitions of quantities useful for the summary below. Credit: Fig. 2.2 of Schroeder (2000).

- The law of reflection simply states that the angle of incidence of the light ray is equal to the angle of reflection[5]. In addition, the incident and reflected rays are in a plane that contains the normal to the reflecting surface.
- The law of refraction, also called Snell's law[6], relates the angle of incidence (θ) to the angle of refraction (θ') through the refractive indexes of the media, $\sin(\theta)/\sin(\theta') = n'/n$.
- Optical path or optical path length (OPL) is the distance that light would travel in vacuum for the same time that it took to travel through a given distance within another medium. This is an important concept because, for example, it is the OPL that gives the change in phase of light travelling through a medium, not the effective distance d that it travelled inside that medium. The optical path is given by the refractive index times the distance travelled; OPL $= nd$, if the refractive index is a constant. If n itself is a function of the distance, then we can define the OPL as the integral of the refractive index over the path S:

$$OPL = \int_S n(s)\,\mathrm{d}s \tag{1}$$

- Fermat's principle, in its modern version, states that the actual path that a light ray follows between two points, through any set of media, "is such that the time of travel between two

[5]The law of reflection was already known by the ancient Greeks. The earliest surviving reference appears in the works of Hero (or Heron) of Alexandria, who probably lived in the first century AD (Smith 1999) in `https://www.jstor.org/stable/3185879`. See a summary of what is known (and questioned) about him in `https://mathshistory.st-andrews.ac.uk/Biographies/Heron/`. Among the things described in his works are a coin-operated machine and a steam-powered engine. His works seem to be in the form of lecture notes and so it is believed he was a teacher at the Museum in Alexandria.

[6]Named after Willebrord Snell, sometimes spelled Snel or Snellius. He was a Dutch astronomer and mathematician. For a biography see `https://mathshistory.st-andrews.ac.uk/Biographies/Snell/`. He seems to have arrived to the refraction law sometime around 1621, but it remained unpublished. This only became known in 1703, when Christiaan Huygens (another Dutch mathematician and astronomer, the first to propose that Saturn's rings were actually rings and the discoverer of Titan) published Snell's result in his book *Dioptrica* (Huygens 1703), see `https://play.google.com/books/reader?id=rZpGAAAAcAAJ&pg=GBS.PP24&hl=en` for the original in Latin. It seems that Isaac Vossius (sometimes the name is spelled Isaak and the surname Voss), a Dutch scholar and natural philosopher, also mentions this in his book *De lucis natura et proprietate* (Vossius 1662), see `https://archive.org/details/delucisnaturaetp00voss` for the book, in Latin, of course. Before that, the law of refraction was attributed to René Descartes, the famous French philosopher and mathematician, who published it in 1637 (Descartes 1637) as an appendix to his *Discourse on the Method*. In any case, to be less Eurocentric, it seems the first written register of the refraction law belongs to the Persian scientist Ibn Sahl, working in Baghdad, in 984 (Rashed 1990), see `https://www.jstor.org/stable/233423`.

fixed points has a stationary value with respect to small changes of that path", i.e. $\delta(OPL) = 0$. The quote is from Schroeder (2000).

- Fermat's principle is akin to a principle of least action in classical mechanics, where the "optical Lagrangian" integrated on the light physical path gives the optical light path. One can actually develop a Hamiltonian version of geometrical optics (see e.g. Buchdahl 1993)[7].

- The transverse or lateral magnification (m) is defined as the ratio between the image height (h') and the object height (h), which in paraxial approximation can be written as $m = ns'/n's$ (see Fig. 2 for the graphical definition of the variables).

- The angular magnification (M) can be defined in terms of the slope angles that the rays make with the optical axis. If we call the angle of the incident ray as θ and of the refracted ray as θ' (and the slopes are the tangent of the angles), then $M = \tan\,\theta'/\tan\,\theta = s/s' = nh/n'h'$.

- The product $nh \tan\,\theta$ which is equal to $n'h' \tan\,\theta'$, is called the Langrage invariant (H). This is a quantity that is invariant regardless of the number of reflections and refractions suffered by the ray.

- The focal length (f) is the distance from the optical element to the point where the image is formed.

- The optical power (P) is defined as n/f and is a measure of how much an optical element converges or diverges the incident ray (it is positive for a converging element and negative for a diverging one).

- The focal ratio (f-ratio or f-number), N, is the ratio between the focal length and the aperture of your telescope (e.g., the diameter of the mirror), $N = f/D$. You will hear/read about telescopes that are f/2 or f/3.5, meaning that $N = 2$ or 3.5, respectively.

- Optical elements that have small focal ratios are said to be "fast". This is because it takes less time to collect a certain amount of light, as the smaller the f/number, the lower the magnification, the wider the field, and the brighter the image will be. Smaller f-numbers also result in shorter telescopes and smaller enclosures, reducing the cost of these structures.

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