Superstring Theory

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- introduction
- particles
- strings
- harmonic oscillator
- overview (rather than summary)

!!!!!!!PARTY!!!!!!!

Introduction

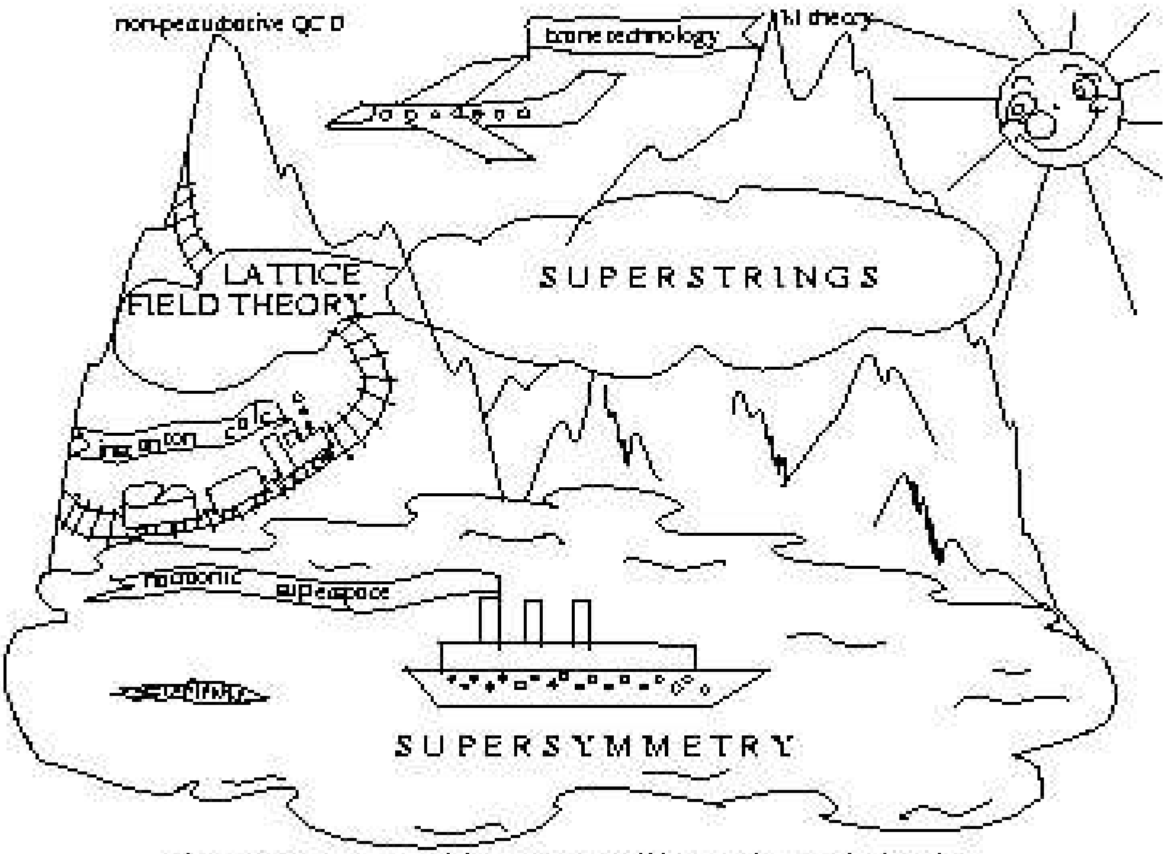
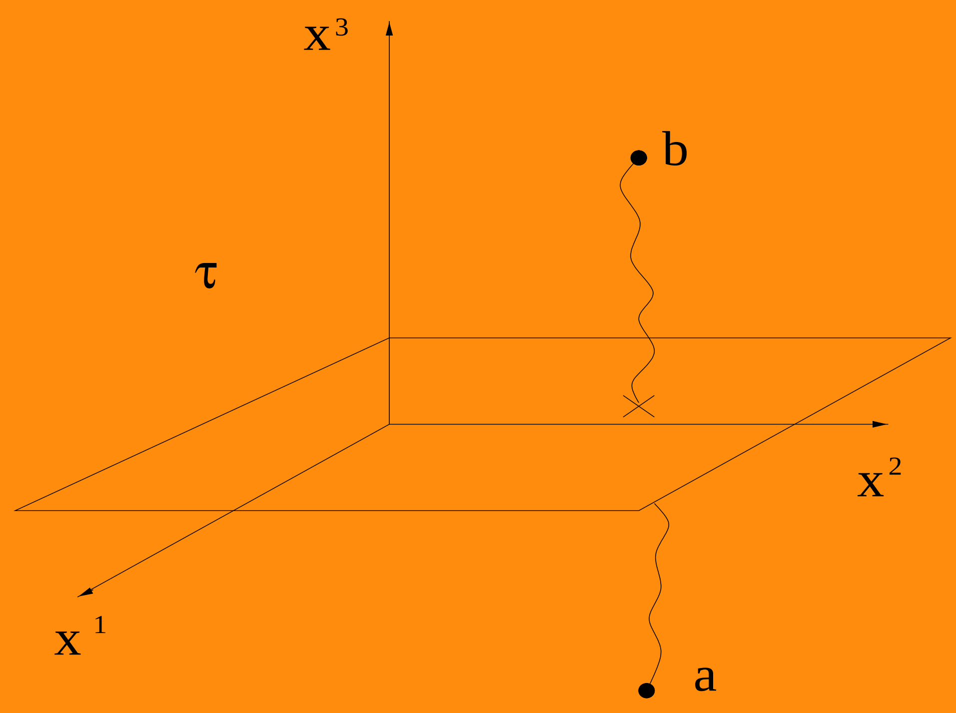


Fig. 1. No comments (special thanks to my children, Denise(6) and Michael(8))

Particles



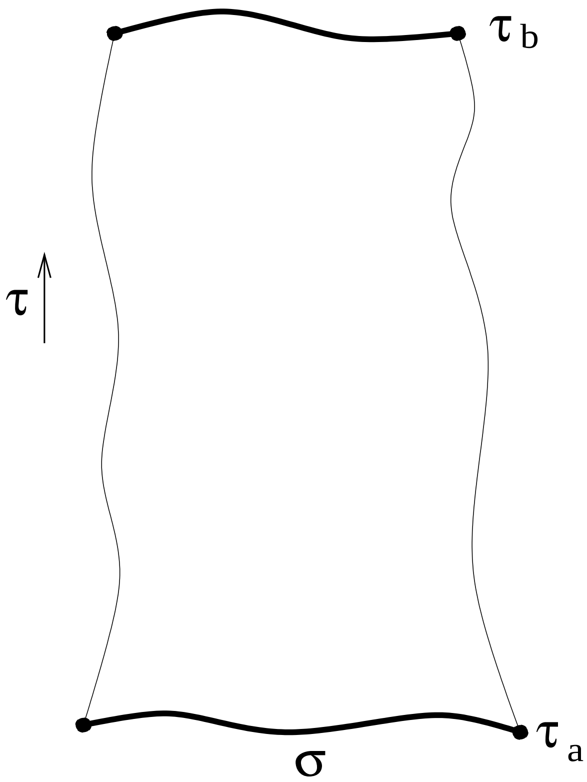
$$S = -m \int ds$$

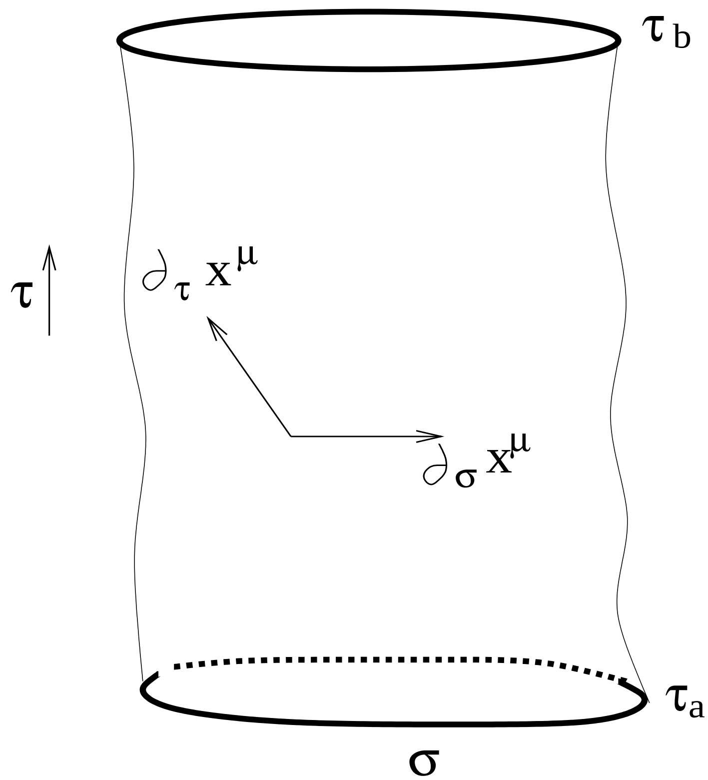
$$S \propto \text{(invariant length of the world line)}$$

$$S = -m \int_{\tau_a}^{\tau_b} \sqrt{\dot{X}^{\mu 2}} d\tau$$

$$m \frac{d^2 X^\mu}{d\tau^2} = 0$$

Strings





$$S = -\frac{T}{2}(\text{area of the string world sheet})$$

Element of the area is:

$$\sqrt{(\partial_\tau X^\mu)^2 - (\partial_\sigma X^\mu)^2}$$

Nambu-Goto Action:

$$S_{\rm N-G} = -\frac{T}{2}\int_\sigma \int_\tau \sqrt{(\partial_\tau X^\mu)^2 - (\partial_\sigma X^\mu)^2} d\sigma d\tau =$$

$$= \frac{T}{2}\int_\sigma \int_\tau \sqrt{g} d\sigma d\tau, \text{ where } g = -\det(g_{ab})$$

Harmonic oscillator

QUANTUM MECHANICS

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2$$

$$q \to \sqrt{\frac{\hbar}{m\omega}}Q, \; p \to \sqrt{m\omega\hbar}P$$

$$H = \frac{\omega}{2}(P^2 + Q^2)$$

$$[P,Q] = [Q,Q] = 0, \quad [P,Q] = -i$$

$$a = \frac{1}{\sqrt{2}}(Q + iP)$$

$$a^* = a^+ = \frac{1}{\sqrt{2}}(Q - iP)$$

$$[a, a^+] = 1, \quad [a,a] = [a^+, a^+] = 0$$

$$H = \omega(aa^+ + \frac{1}{2})$$

$$a|0> = 0, \; a^+|0> = |1>$$

$$a^+|n> \propto |n+1>$$

$$a|n> \propto |n-1>$$

STRING THEORY

$$\mathcal{H} = \frac{T}{2}(\dot{X}^{\mu^2} + \dot{X}'^{\mu^2})$$

$$H = \frac{T}{2}\int_\sigma(\dot{X}^{\mu^2} + \dot{X}'^{\mu^2})d\sigma =$$

$$= \frac{T}{2} + \sum_{n\neq 0}^{\infty}(a_{-n}a_n + \bar{a}_{-n}a_n) - P^{\mu^2}$$

$$M^2 = 4\pi T[\sum_{n=1}^{\infty} n(a_n^{\mu^+} a_n^{\mu} + \tilde{a}_n^{\mu^+}\tilde{a}_n^{\mu}) - \frac{d-2}{24}]$$

$$|a^{\mu^+}>, \; a^{-\mu^+}|0> \quad (M^2 < 0)$$

$$a^{\mu^+}\tilde{a}^{\nu^+}|0> \;\; \text{tensor :}$$

$$G^{\mu\nu}, \; B^{\mu\nu}, \; \Phi$$

Overview (no Summary)