

Dynamics in the inner crust: heavy impurity in Fermi superfluid

Daniel Pęczak



April 30th 2025, CAMK, Warszawa

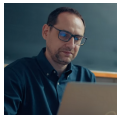


Faculty
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WARSAW UNIVERSITY OF TECHNOLOGY



Piotr Magierski



Gabriel Wlazłowski



Agata Zdanowicz



UNIVERSITÉ
LIBRE
DE BRUXELLES



Nicolas Chamel



Physical Review X
14, 041054 (2024)



NARODOWE CENTRUM NAUKI

PLGrid LUMI PLL/2022/03/016433 ★ NCN 2021/40/C/ST2/00072

Warsaw University of Technology

W-SLDA Toolkit W-BSk Toolkit

W-SLDA Toolkit

*Self-consistent solver
of mathematical problems
which have structure
formally equivalent to
Bogoliubov-de Gennes equations.*

static problems: st-wslsda

$$\begin{pmatrix} h_a(\mathbf{r}) - \mu_a & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h_b^*(\mathbf{r}) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{pmatrix}$$

time-dependent problems: td-wslsda

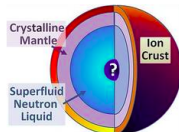
$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} u_n(\mathbf{r}, t) \\ v_n(\mathbf{r}, t) \end{pmatrix} = \begin{pmatrix} h_a(\mathbf{r}, t) - \mu_a & \Delta(\mathbf{r}, t) \\ \Delta^*(\mathbf{r}, t) & -h_b^*(\mathbf{r}, t) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r}, t) \\ v_n(\mathbf{r}, t) \end{pmatrix}$$

Extension to nuclear matter
in neutron stars

Unified solvers for static and
time-dependent problems

Dimensionalities of
problems: 3D, 2D and 1D

Extension to nuclear matter in neutron stars



The W-SLDA Toolkit has been expanded to encompass
available as the W-BSk Toolkit.

ALL FUNCTIONALITIES +

Getting the code



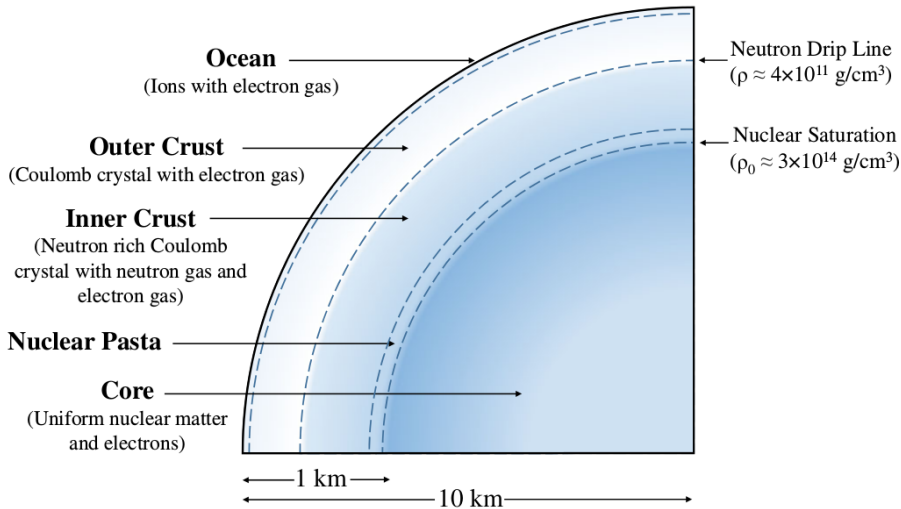
DOWNLOAD

The W-SLDA & W-BSk Toolkits are free to download. It is published as open source under GNU GPL License. In order to get W-SLDA or W-BSk Toolkit click "Read more" and follow instructions.

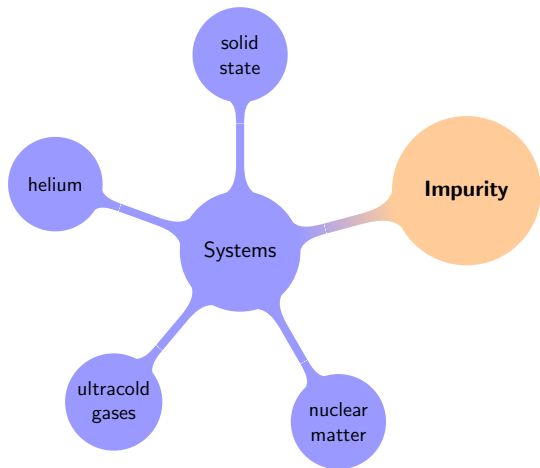
READ MORE +

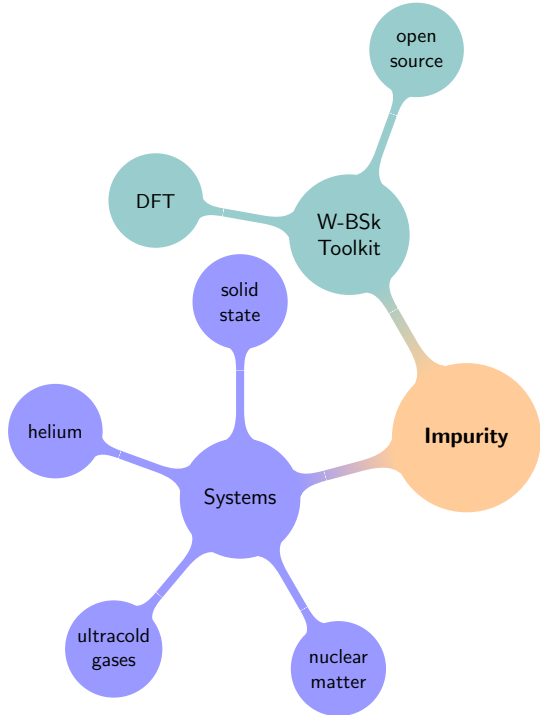
Integration with VisIt:
visualization, animation and
analysis tool

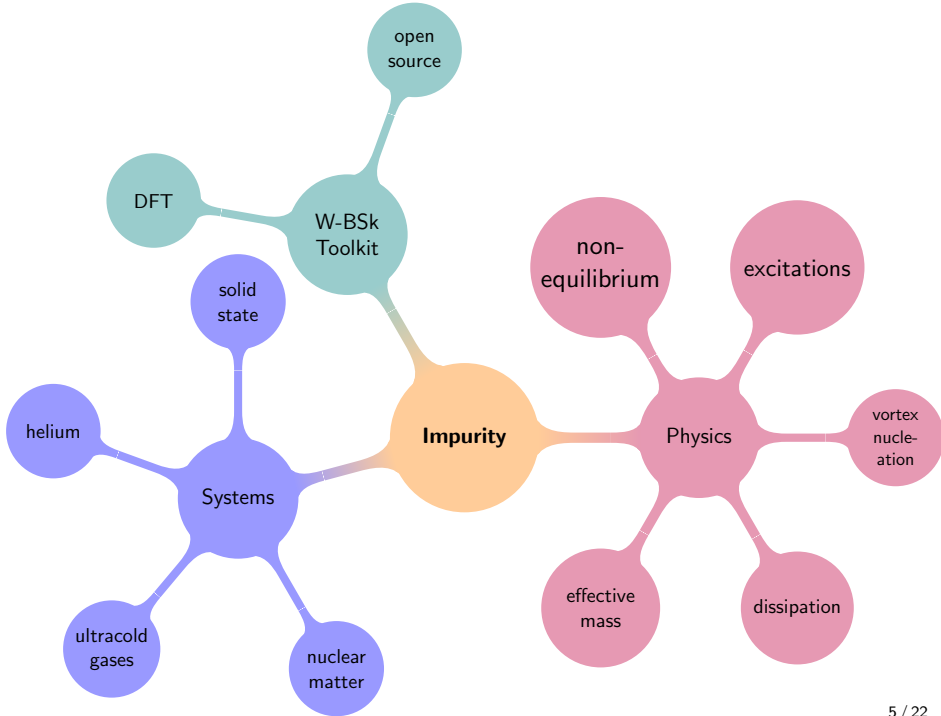


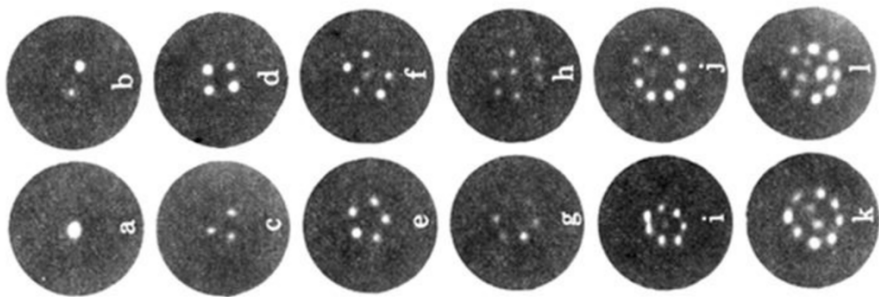


Caplan, M. E., and C. J. Horowitz, Reviews of Modern Physics 89, 041002 (2017)

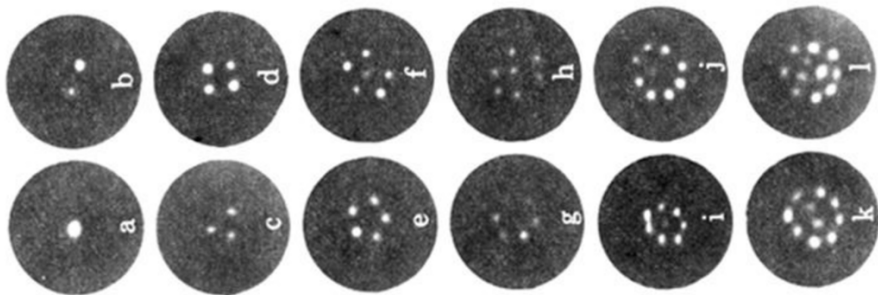




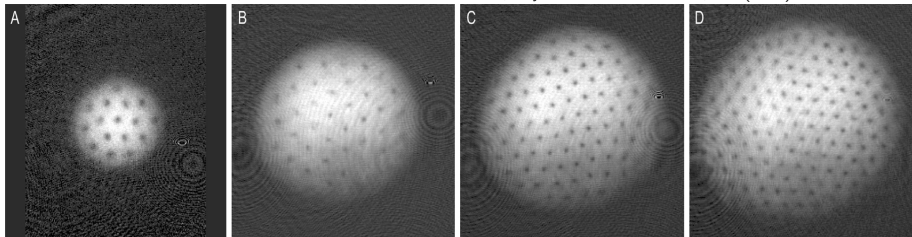




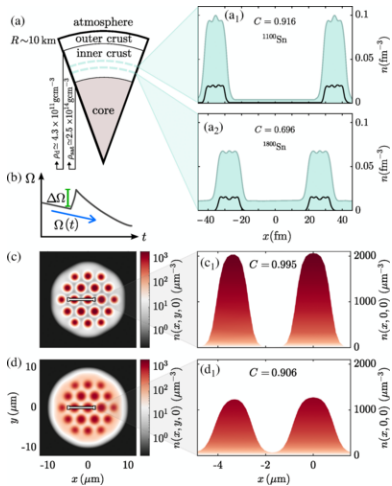
E. J. Yarmchuk, M. J. V. Gordon, and R. E. Packard, *Physical Review Letters* 43, 214 (1979)

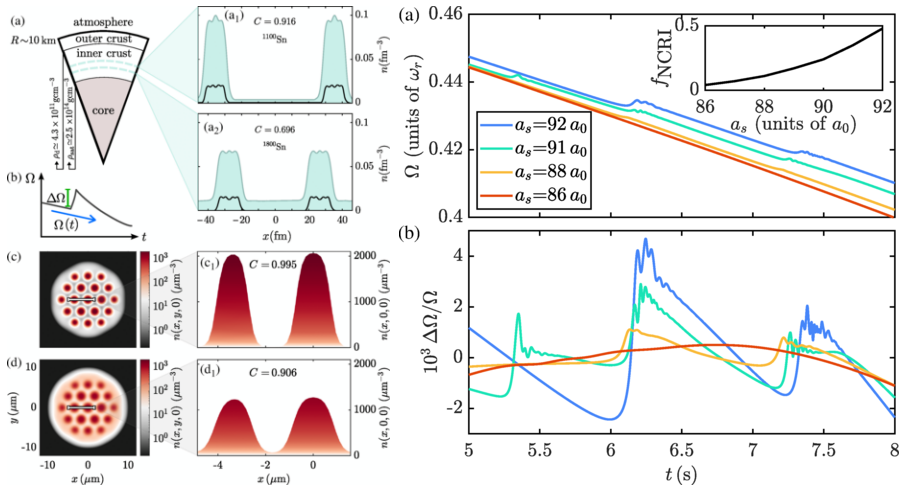


E. J. Yarmchuk, M. J. V. Gordon, and R. E. Packard, *Physical Review Letters* 43, 214 (1979)



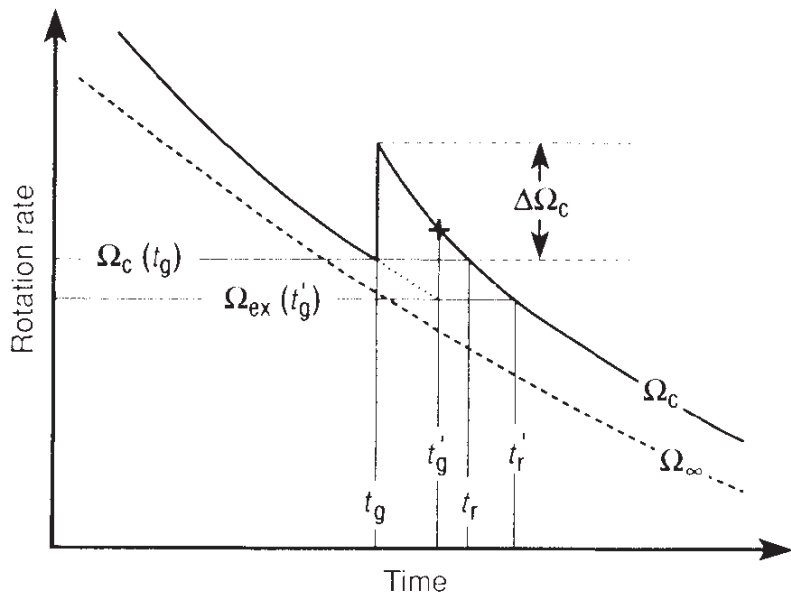
J. R. Abo-Shaeer, C. Raman, J. M. Vogels, W. Ketterle, *Science* 292, 476 (2001)





Poli, Elena, et al. "Glitches in rotating supersolids." *Physical Review Letters* 131, 223401 (2023)

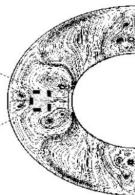
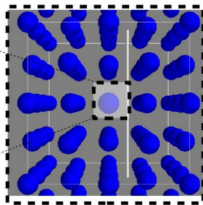
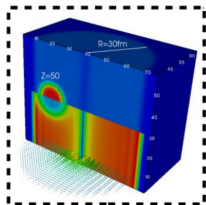
Glitch

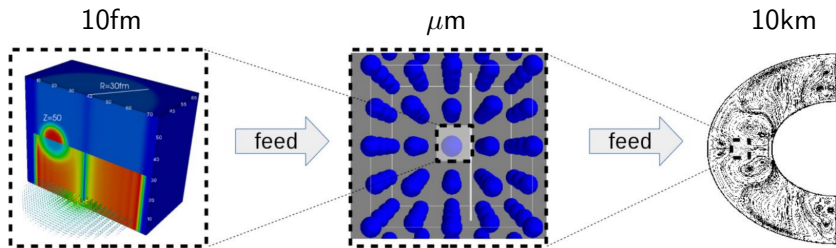


10fm

μm

10km

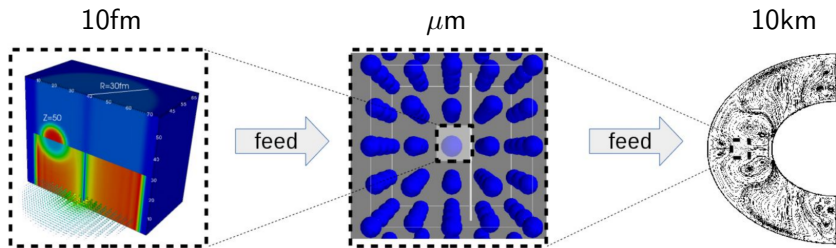




LUMI, Finland (#5 Top 500)



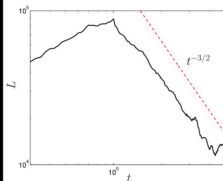
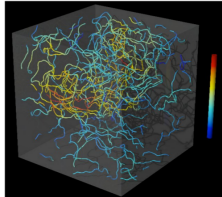
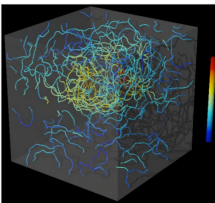
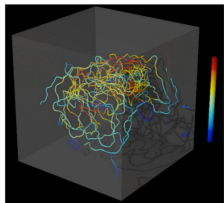
Piz Daint, Switzerland (#37 Top 500)



LUMI, Finland (#5 Top 500)



Piz Daint, Switzerland (#37 Top 500)



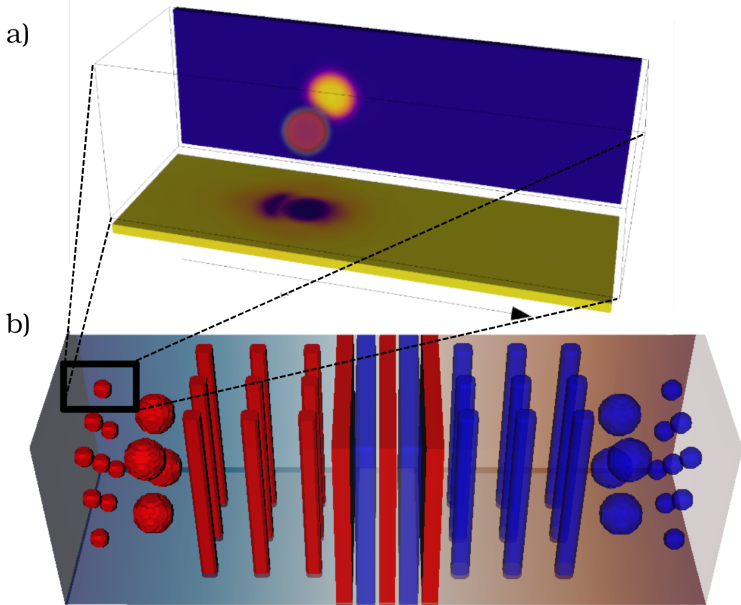
Crust composition

$\rho_{\max}$ [g cm ⁻³]	Element	Z	N	R_{cell} [fm]
8.02×10^6	⁵⁶ Fe	26	30	1404.05
2.71×10^8	⁶² Ni	28	34	449.48
1.33×10^9	⁶⁴ Ni	28	36	266.97
1.50×10^9	⁶⁶ Ni	28	38	259.26
3.09×10^9	⁸⁶ Kr	36	50	222.66
1.06×10^{10}	⁸⁴ Se	34	50	146.56
2.79×10^{10}	⁸² Ge	32	50	105.23
6.07×10^{10}	⁸⁰ Zn	30	50	80.58
8.46×10^{10}	⁸² Zn	30	52	72.77
9.67×10^{10}	¹²⁸ Pd	46	82	80.77
1.47×10^{11}	¹²⁶ Ru	44	82	69.81
2.11×10^{11}	¹²⁴ Mo	42	82	61.71
2.89×10^{11}	¹²² Zr	40	82	55.22
3.97×10^{11}	¹²⁰ Sr	38	82	49.37
4.27×10^{11}	¹¹⁸ Kr	36	82	47.92

ρ [g cm ⁻³]	Element	Z	N	R_{cell} [fm]
4.67×10^{11}	¹⁸⁰ Zr	40	140	53.60
6.69×10^{11}	²⁰⁰ Zr	40	160	49.24
1.00×10^{12}	²⁵⁰ Zr	40	210	46.33
1.47×10^{12}	³²⁰ Zr	40	280	44.30
2.66×10^{12}	⁵⁰⁰ Zr	40	460	42.16
6.24×10^{12}	⁹⁵⁰ Sn	50	900	39.32
9.65×10^{12}	¹¹⁰⁰ Sn	50	1050	35.70
1.49×10^{13}	¹³⁵⁰ Sn	50	1300	33.07
3.41×10^{13}	¹⁸⁰⁰ Sn	50	1750	27.61
7.94×10^{13}	¹⁵⁰⁰ Zr	40	1460	19.61
1.32×10^{14}	⁹⁸² Ge	32	950	14.38

Nicolas Chamel and Pawel Haensel. "Physics of neutron star crusts." Living Reviews in relativity 11.1 (2008): 1-182.

Nonequilibrium dynamics



$$\varepsilon(\rho, \vec{\nabla}\rho, \nu, \tau, \mathbf{j}) = \frac{\hbar^2}{2M}\tau + \varepsilon_\rho(\rho) + \varepsilon_\tau(\rho, \tau, \mathbf{j}) + \varepsilon_{\Delta\rho}(\rho, \vec{\nabla}\rho) + \varepsilon_\pi(\rho, \vec{\nabla}\rho, \nu)$$

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$$\rho(r) = \sum_k |v_k(r)|^2$$

$$\tau(r) = \sum_k |\nabla v_k(r)|^2$$

$$\nu(r) = \sum_k u_k(r) v_k^*(r)$$

$$\varepsilon(\rho, \vec{\nabla}\rho, \nu, \tau, \mathbf{j}) = \frac{\hbar^2}{2M}\tau + \varepsilon_\rho(\rho) + \varepsilon_\tau(\rho, \tau, \mathbf{j}) + \varepsilon_{\Delta\rho}(\rho, \vec{\nabla}\rho) + \varepsilon_\pi(\rho, \vec{\nabla}\rho, \nu)$$

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$$\tau(r) = \sum_k |\nabla v_k(r)|^2$$

$$\nu(r) = \sum_k u_k(r) v_k^*(r)$$

$$h(r) = \frac{\delta\varepsilon}{\delta\rho} - \nabla \frac{\delta\varepsilon}{\delta\tau} \nabla - \frac{i}{2} \left\{ \frac{\delta\varepsilon}{\delta\mathbf{j}}, \nabla \right\}$$

$$\Delta(r) = \frac{\delta\varepsilon}{\delta\nu}$$

Superfluid **L**ocal **D**ensity **A**pproximation

A. Bulgac, Physical Review A **76**, 040502 (2007)

$$\varepsilon(\rho, \vec{\nabla}\rho, \nu, \tau, \mathbf{j}) = \frac{\hbar^2}{2M}\tau + \varepsilon_\rho(\rho) + \varepsilon_\tau(\rho, \tau, \mathbf{j}) + \varepsilon_{\Delta\rho}(\rho, \vec{\nabla}\rho) + \varepsilon_\pi(\rho, \vec{\nabla}\rho, \nu)$$

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Superfluid **L**ocal **D**ensity **A**pproximation

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Hartree-Fock-Bogoliubov equations

$$\begin{pmatrix} h(r) & \Delta(r) \\ \Delta^*(r) & -h^*(r) \end{pmatrix} \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix} = \epsilon_k \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix}$$

$$\varepsilon(\rho, \vec{\nabla}\rho, \nu, \tau, \mathbf{j}) = \frac{\hbar^2}{2M}\tau + \varepsilon_\rho(\rho) + \varepsilon_\tau(\rho, \tau, \mathbf{j}) + \varepsilon_{\Delta\rho}(\rho, \vec{\nabla}\rho) + \varepsilon_\pi(\rho, \vec{\nabla}\rho, \nu)$$

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A. Bulgac, Physical Review A **76**, 040502 (2007)

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$$\begin{pmatrix} h(r) & \Delta(r) \\ \Delta^*(r) & -h^*(r) \end{pmatrix} \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix} = \epsilon_k \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix}$$

Quality of results highly depends on the quality of density functional!

1. Energy density

Under the assumption of invariance under time reversal, the HFB energy is written as the integral of a purely local energy-density functional

$$E_{\text{HFB}} = \int \mathcal{E}_{\text{HFB}}(\mathbf{r}) d^3r, \quad (\text{A1})$$

where

$$\begin{aligned} \mathcal{E}_{\text{HFB}}(\mathbf{r}) = & \mathcal{E}_{\text{Sky}}[\rho_n(\mathbf{r}), \nabla \rho_n(\mathbf{r}), \tau_n(\mathbf{r}), \mathbf{J}_n(\mathbf{r}), \rho_p(\mathbf{r}), \nabla \rho_p(\mathbf{r}), \\ & \tau_p(\mathbf{r}), \mathbf{J}_p(\mathbf{r})] + \mathcal{E}_{\text{Coul}}[\rho_p(\mathbf{r})] \\ & + \mathcal{E}_{\text{pair}}[\rho_n(\mathbf{r}), \tilde{\rho}_n(\mathbf{r}), \rho_p(\mathbf{r}), \tilde{\rho}_p(\mathbf{r})]. \end{aligned} \quad (\text{A2})$$

The first term here, the energy density for the Skyrme force of this paper, is given by

$$\begin{aligned} \mathcal{E}_{\text{Sky}} = & \sum_{q=n,p} \frac{\hbar^2}{2M_q} \tau_q + \frac{1}{2} t_0 \left[\left(1 + \frac{1}{2} x_0\right) \rho^2 - \left(\frac{1}{2} + x_0\right) \sum_{q=n,p} \rho_q^2 \right] + \frac{1}{4} t_1 \left\{ \left(1 + \frac{1}{2} x_1\right) \left[\rho \tau + \frac{3}{4} (\nabla \rho)^2 \right] \right. \\ & \left. - \left(\frac{1}{2} + x_1\right) \sum_{q=n,p} \left[\rho_q \tau_q + \frac{3}{4} (\nabla \rho_q)^2 \right] \right\} + \frac{1}{4} t_2 \left\{ \left(1 + \frac{1}{2} x_2\right) \left[\rho \tau - \frac{1}{4} (\nabla \rho)^2 \right] + \left(\frac{1}{2} + x_2\right) \right. \\ & \left. \times \sum_{q=n,p} \left[\rho_q \tau_q - \frac{1}{4} (\nabla \rho_q)^2 \right] \right\} + \frac{1}{12} t_3 \rho^\alpha \left[\left(1 + \frac{1}{2} x_3\right) \rho^2 - \left(\frac{1}{2} + x_3\right) \sum_{q=n,p} \rho_q^2 \right] \\ & + \frac{1}{4} t_4 \left\{ \left(1 + \frac{1}{2} x_4\right) \left[\rho \tau + \frac{3}{4} (\nabla \rho)^2 \right] - \left(\frac{1}{2} + x_4\right) \sum_{q=n,p} \left[\rho_q \tau_q + \frac{3}{4} (\nabla \rho_q)^2 \right] \right\} \rho^\beta \\ & + \frac{\beta}{8} t_4 \left[\left(1 + \frac{1}{2} x_4\right) \rho (\nabla \rho)^2 - \left(\frac{1}{2} + x_4\right) \nabla \rho \cdot \sum_{q=n,p} \rho_q \nabla \rho_q \right] \rho^{\beta-1} + \frac{1}{4} t_5 \left\{ \left(1 + \frac{1}{2} x_5\right) \left[\rho \tau - \frac{1}{4} (\nabla \rho)^2 \right] \right. \\ & \left. + \left(\frac{1}{2} + x_5\right) \sum_{q=n,p} \left[\rho_q \tau_q - \frac{1}{4} (\nabla \rho_q)^2 \right] \right\} \rho^\gamma - \frac{1}{16} (t_1 x_1 + t_2 x_2) J^2 + \frac{1}{16} (t_1 - t_2) \sum_{q=n,p} J_q^2 \\ & - \frac{1}{16} (t_4 x_4 \rho^\beta + t_5 x_5 \rho^\gamma) J^2 + \frac{1}{16} (t_4 \rho^\beta - t_5 \rho^\gamma) \sum_{q=n,p} J_q^2 + \frac{1}{2} W_0 \left(\mathbf{J} \cdot \nabla \rho + \sum_{q=n,p} \mathbf{J}_q \cdot \nabla \rho_q \right). \end{aligned} \quad (\text{A3})$$

Brussels-Montreal (BSk) functional

Experimental data

- atomic masses
- nuclear charge radii
- symmetry energy
- incompressibility

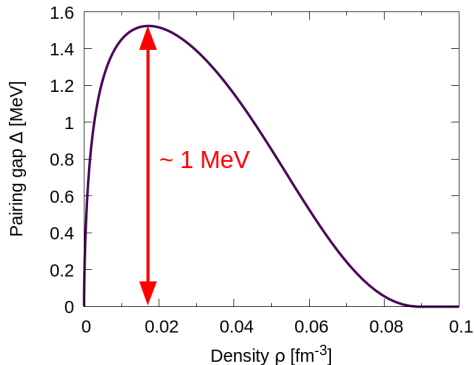
N-body calculations

- EoS of pure neutron matter
- 1S_0 pairing gaps in nuclear matter
- effective masses in nuclear matter

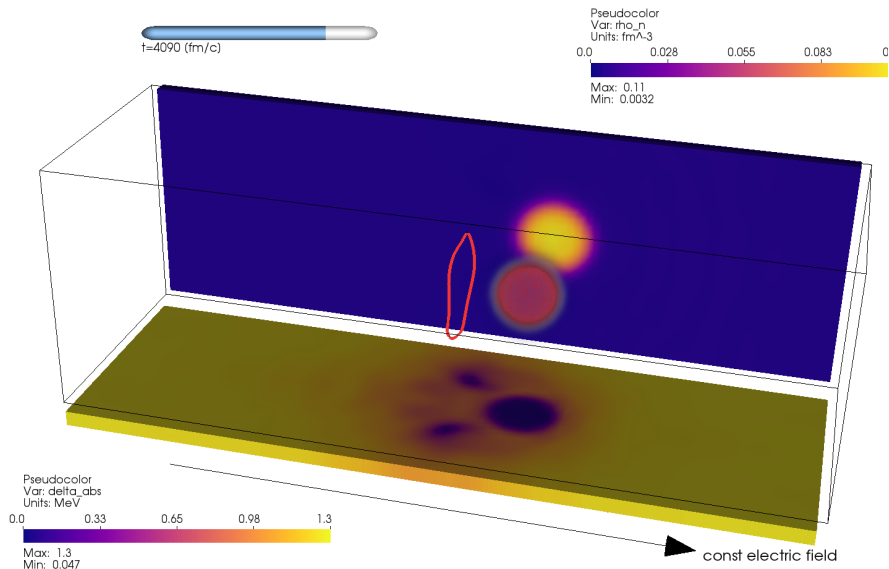
Chamel et al., Phys. Rev. C **80**, 065804 (2009)

Goriely et al., Phys. Rev. Lett. **102**, 152503 (2009)

Goriely et al., Phys. Rev. C **93**, 034337 (2016)



$$\varepsilon(\rho_q, \vec{\nabla} \rho_q, \nu_q, \tau_q, \mathbf{j}_q)$$







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By Robert D Anderson - Own work, CC BY-SA 3.0

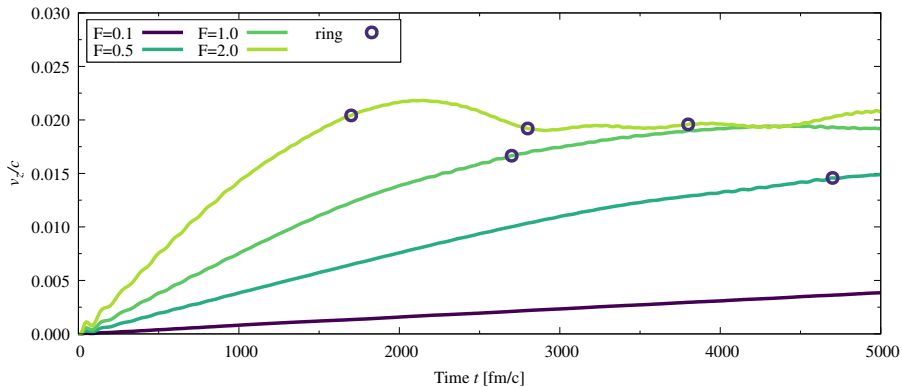


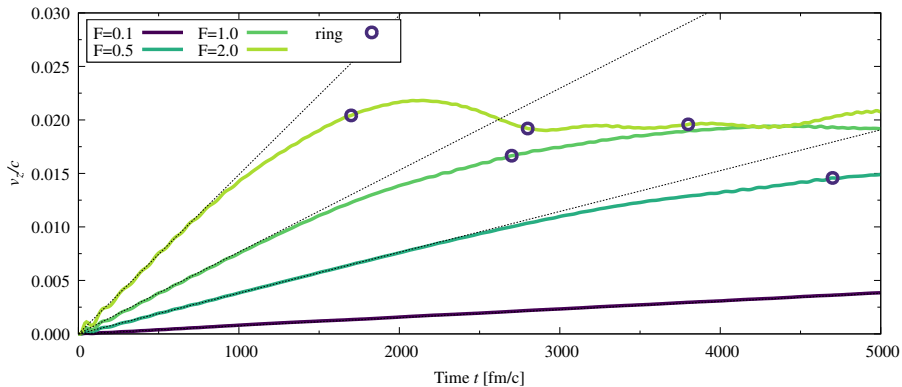
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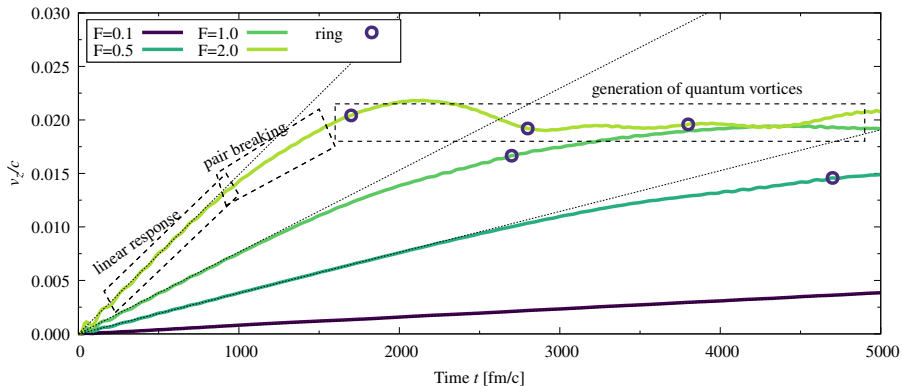






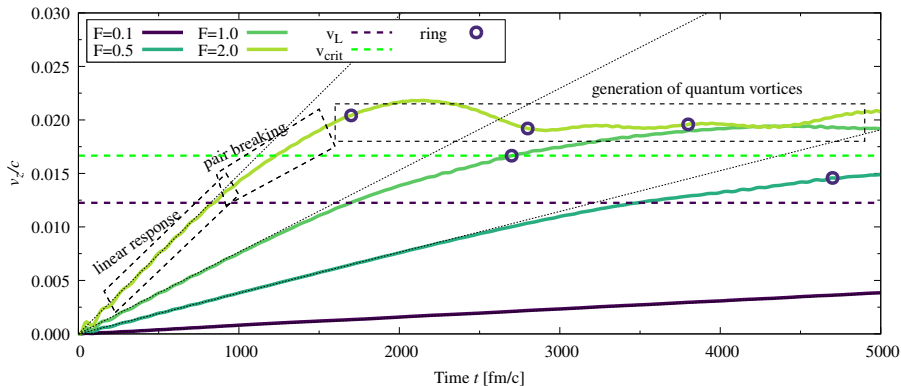
$$v_z = a_z t$$

$$M_{eff}^{(d)} = \frac{F}{a_z}$$



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$$M_{eff}^{(d)} = \frac{F}{a_z}$$

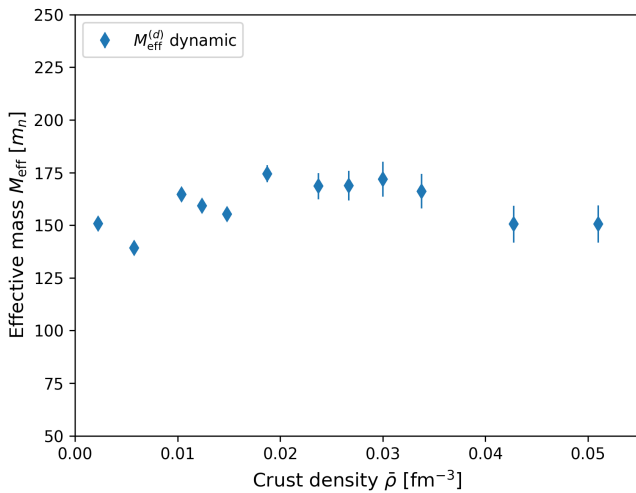


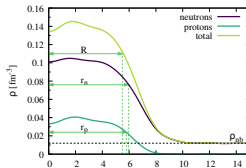
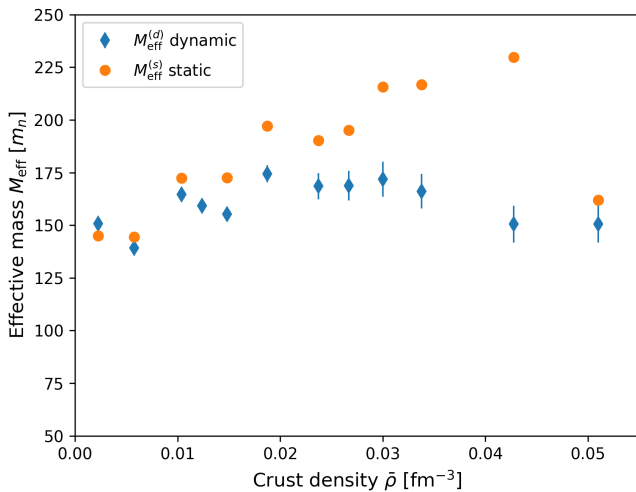
$$v_z = a_z t$$

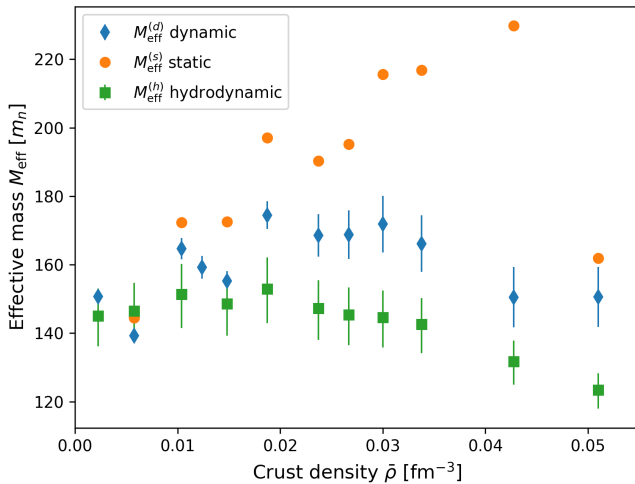
$$M_{eff}^{(d)} = \frac{F}{a_z}$$

$$v_L = \frac{\Delta}{\hbar k_F}$$

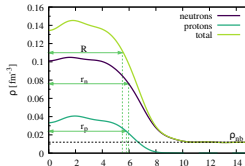
$$v_{crit} = \frac{e}{2} \frac{\Delta}{\hbar k_F}$$



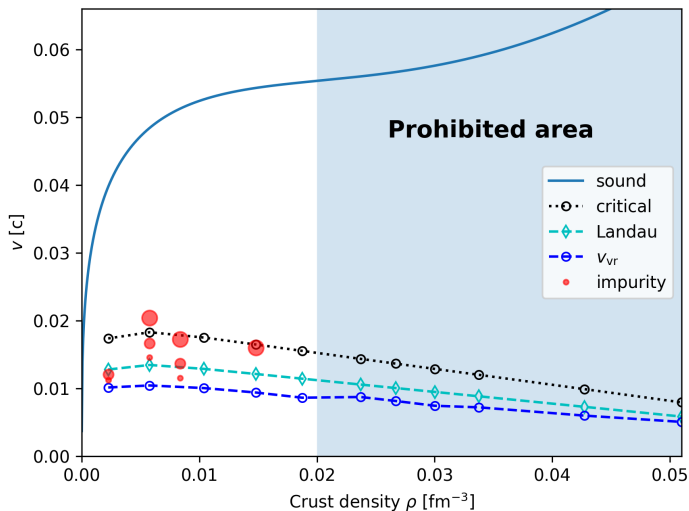


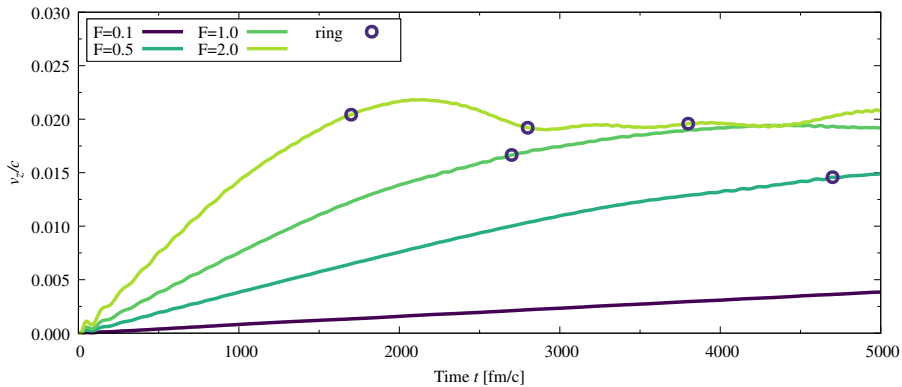


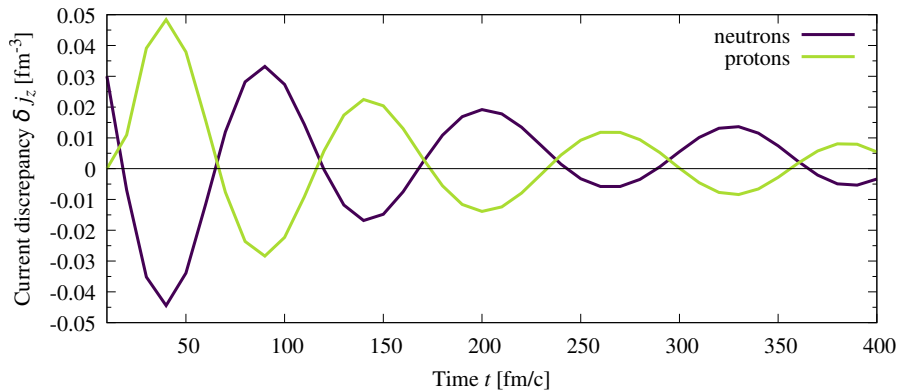
$$M_h = \frac{4}{3} \pi R^3 M_n \frac{(\rho_{\text{in}} - \rho_{\text{out}})^2}{\rho_{\text{in}} + 2\rho_{\text{out}}}.$$



$$v_{vr} = \frac{1}{2\pi R} \frac{\hbar c}{M_n c^2} \left(\ln \frac{8R}{a_{core}} - \alpha \right)$$

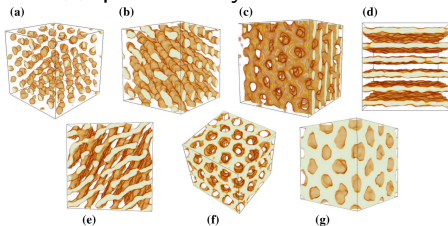




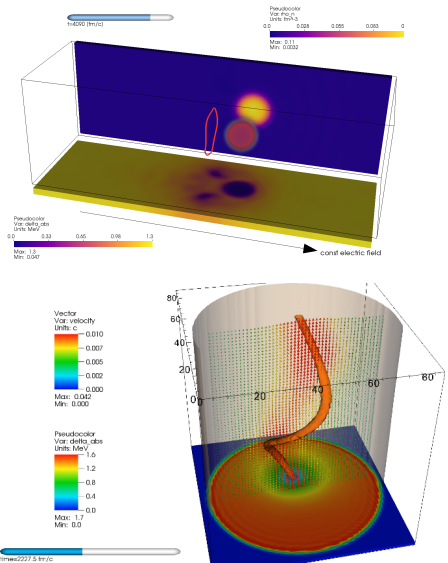


Outlook

- ★ dynamics
- ★ impurities
- ★ nuclear pasta
- ★ vortex-nucleus interaction
- ★ polarized systems

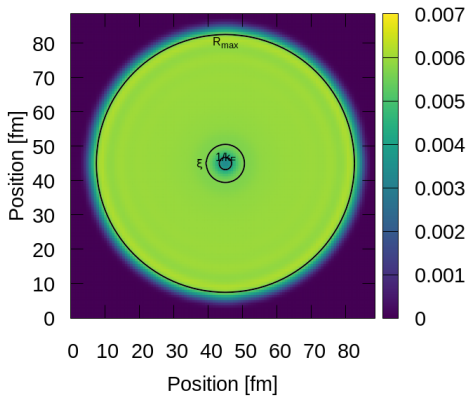


Caplan, M. E., and C. J. Horowitz. 2017. "Colloquium: Astromaterial Science and Nuclear Pasta." *Reviews of Modern Physics* 89 (4): 041002.

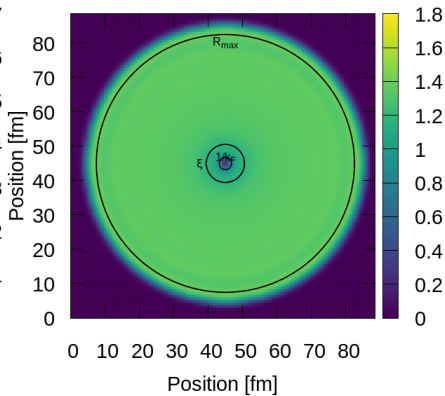


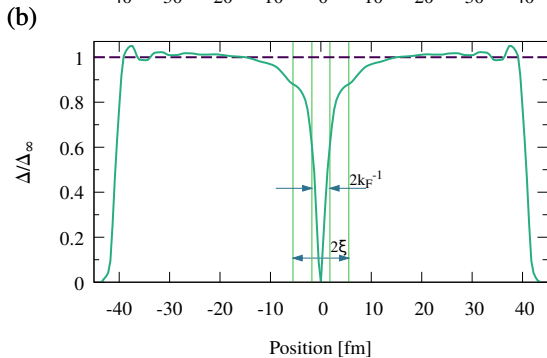
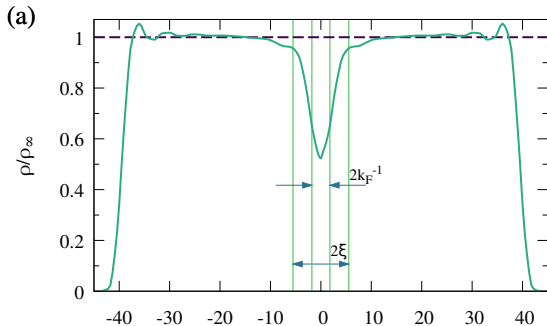
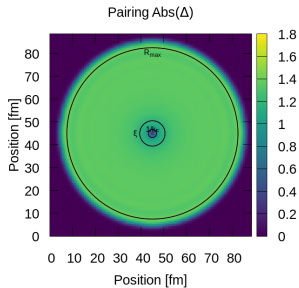
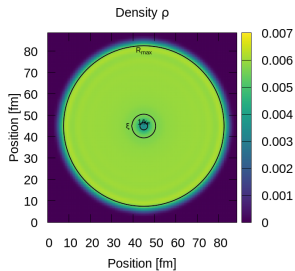
Density: $\rho=0.00590$ [fm $^{-3}$] $\Delta=1.33394$ [MeV]

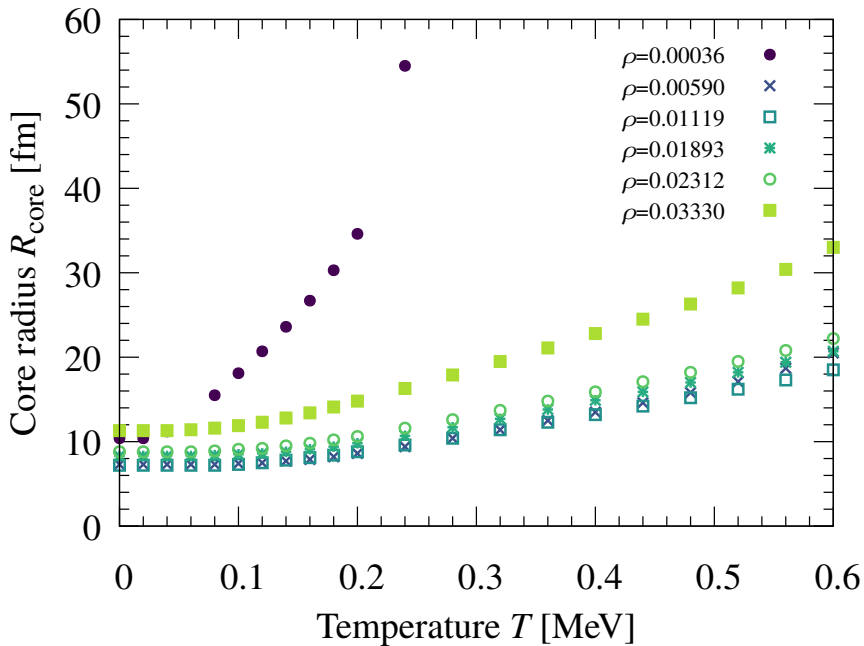
Density ρ



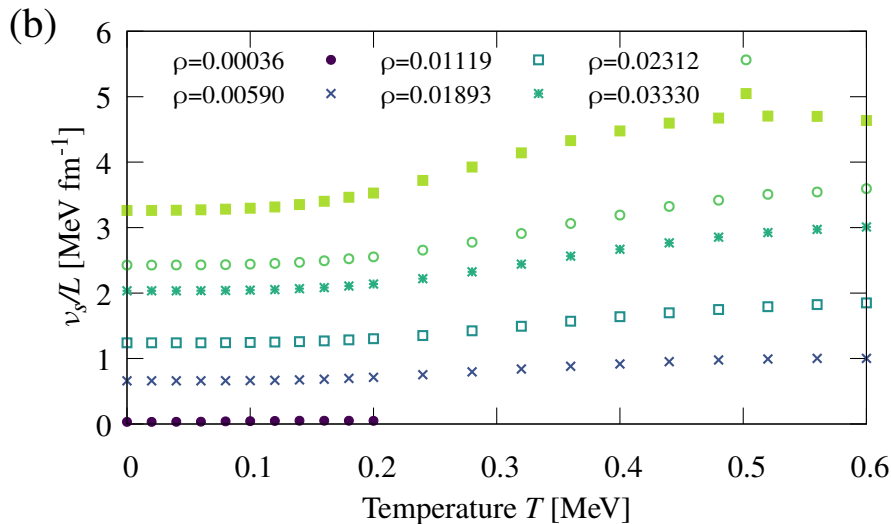
Pairing Abs(Δ)



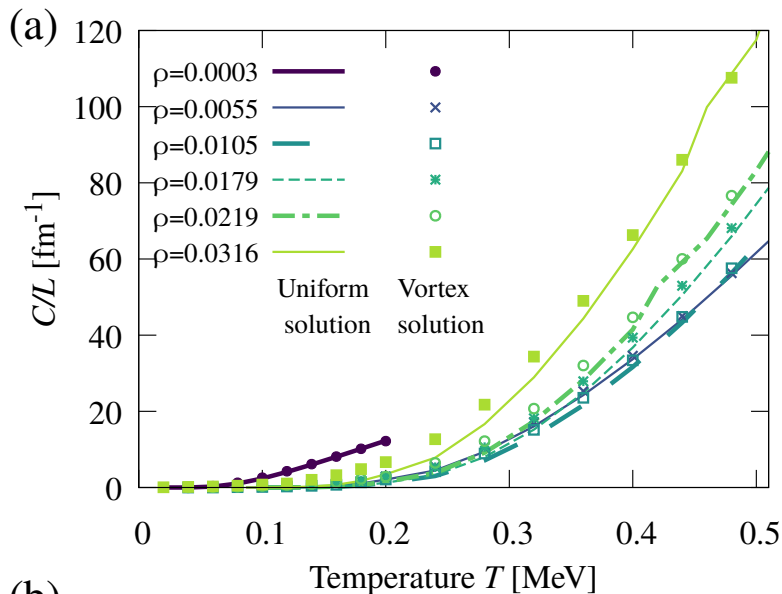




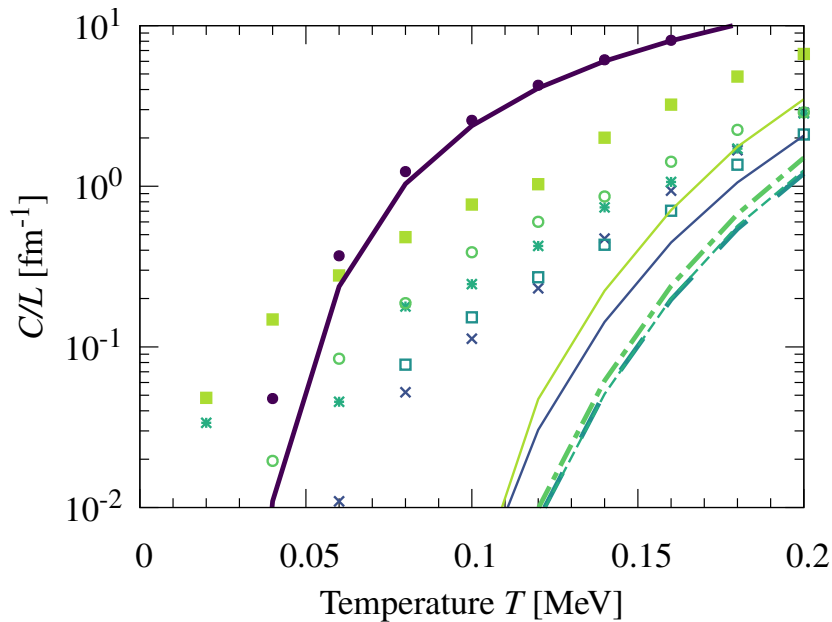
Tension



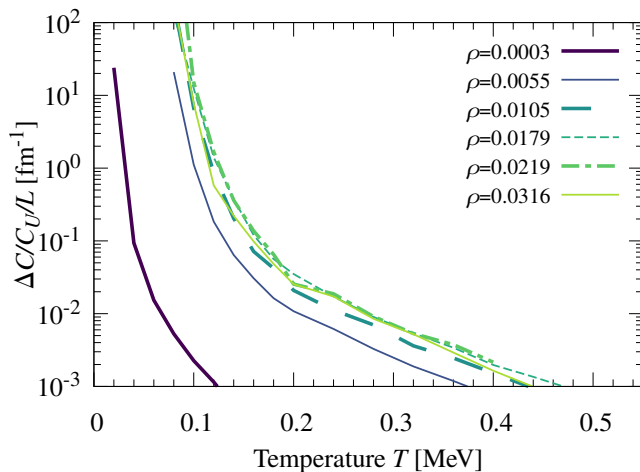
Heat capacity



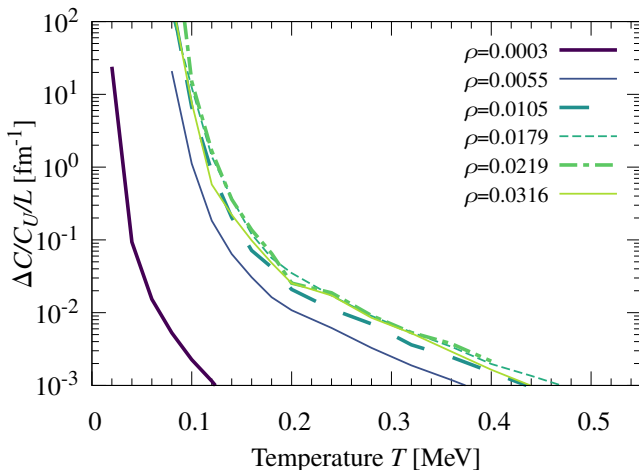
Heat capacity



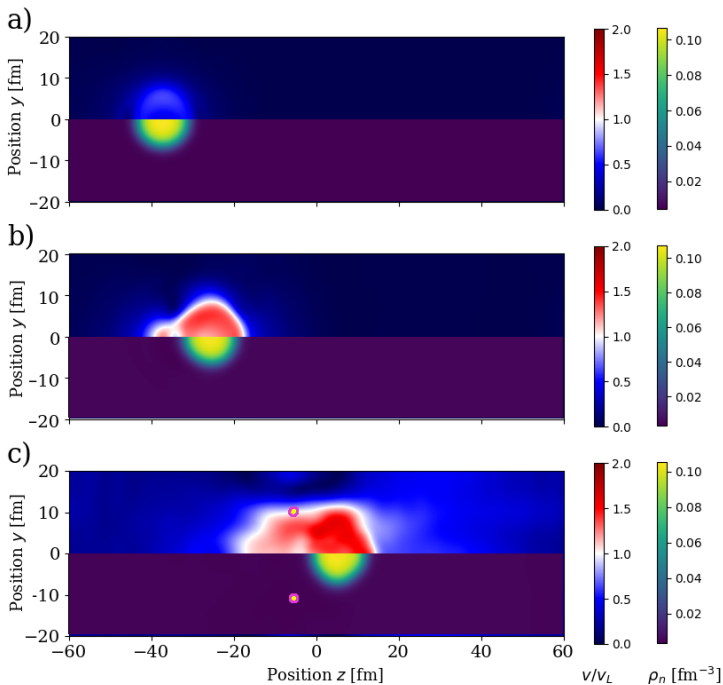
Heat capacity



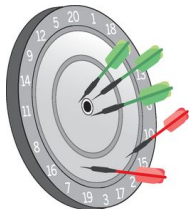
Heat capacity



$$\frac{\Delta C_V^{flow}}{L} = \frac{\hbar^2}{24M\Delta} \pi \ln \left(\frac{R_{out}}{R_{in}} \right) \frac{\mu}{\Delta} \left(\left(\frac{\Delta}{T} \right)^2 - 4 \left(\frac{\Delta}{T} \right) + 2 \right) C_V^{uniform}$$



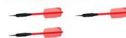




New methods
Mutual agreement



Old methods
Different values



Scorecard

	0.3	0.3	0.6	1.0	0.9	0.3	1.5	0.6	0.9	0.4	1.0	0.4	0.4	6.3	13.5	1.1	2.1	0.7	1.4
0.3		0.1	0.5	0.9	0.8	0.2	1.5	0.6	0.8	0.4	1.0	0.5	0.3	6.3	13.4	1.1	2.1	0.7	1.4
0.3	0.1		0.5	0.9	0.8	0.2	1.5	0.6	0.8	0.4	0.9	0.5	0.3	6.3	13.4	1.1	2.1	0.7	1.4
0.6	0.5	0.5		0.8	0.6	0.4	1.5	0.6	0.8	0.6	1.0	0.7	0.5	6.3	13.2	1.0	1.9	0.6	1.3
1.0	0.9	0.9	0.8		0.9	0.9	1.8	0.9	1.3	1.0	1.4	1.0	0.9	6.4	13.0	1.2	1.8	1.0	1.6
0.9	0.8	0.8	0.6	0.9		0.8	1.7	0.7	1.1	0.8	1.3	1.0	0.8	6.5	13.2	1.1	1.8	0.8	1.5
0.3	0.2	0.2	0.4	0.9	0.8		1.5	0.5	0.8	0.3	1.0	0.5	0.3	6.2	13.4	1.0	2.0	0.6	1.4

Lejaeghere, Kurt, et al. "Reproducibility in density functional theory calculations of solids." *Science* 351, 6280 (2016)