PROBLEM 8: BLACK HOLE FIELD

- Estimate (order of magnitude) the magnetic field strength in the immediate vicinity of a black hole of mass (1) $10M_\odot$, (2) $10^9 M_\odot$ sufficient to drive a Poynting flux (through the $4\pi R_{\rm Sch}^2$ cross section) equal to the Eddington luminosity.

This problem is worth 5 points. Solutions should be sent as 1-page PDF files to knalew@camk.edu.pl before the next lecture.

TOTAL POYNTING FLUX

- Poynting flux (density): $\vec{S} = \frac{c}{4\pi}\left(\vec{E} \times \vec{B}\right)$

- electric field strength: $E \sim \beta B \simeq B$

- Poynting flux (total): $\mathcal{S} = c\frac{B^2}{4\pi}A = cB^2R_{\rm Sch}^2$,

  where $A = 4\pi R_{\rm Sch}^2$ is the cross section,

  and $R_{\rm Sch} = \frac{2GM}{c^2} \simeq 3\ {\rm km} \times \frac{M}{M_\odot}$ is the Schwarzschild radius.

EDDINGTON LUMINOSITY

- $$L_{\rm Edd} = \frac{4\pi G M m_{\rm p} c}{\sigma_{\rm T}} = \frac{2\pi R_{\rm Sch} m_{\rm p} c^3}{\sigma_{\rm T}}$$

- $\mathcal{S} \sim L_{\rm Edd}$: $$cB^2 R_{\rm Sch}^2 \sim \frac{2\pi R_{\rm Sch} m_{\rm p} c^3}{\sigma_{\rm T}}$$

- $$B^2 \sim \frac{2\pi m_{\rm p} c^2}{\sigma_{\rm T} R_{\rm Sch}}$$

RESULTS

- $B \sim \dfrac{2.2 \times 10^8 \text{ G}}{\sqrt{M/M_\odot}}$

- For $M = 10 M_\odot$: $B \sim 7 \times 10^7$ G

- For $M = 10^9 M_\odot$: $B \sim 7 \times 10^3$ G

BLANDFORD-ZNAJEK POWER

- $P_{\rm BZ} \propto \dfrac{k a^2 \Phi_{\rm BH}^2}{R_{\rm Sch}^2} c$ for $a < 0.5$

  with a proportionality constant

- $k = \dfrac{1}{6\pi}$ for split-monopole field geometry

  (Tchekhovskoy et al. 2010)

- $\Phi_{\rm BH} \sim 2\pi R_{\rm Sch}^2 B$

- $P_{\rm BZ} \propto a^2 \mathcal{S}$