# Stochastic GW background. Tests of GR

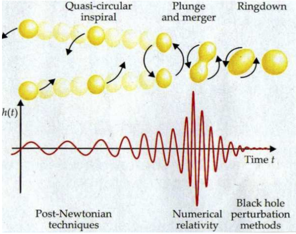
12.1.21

# General schedule

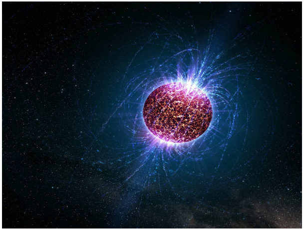
- ⋆ History
- ⋆ Introduction to general relativity
- ⋆ Detection principles
- ⋆ Detectors
- ⋆ Binary black-hole system
- ⋆ Bursts and continuous waves
- ⋆ Rates and populations & cosmology
- ⋆ Stochastic GW background. Tests of general relativity using GWs
  - ⋆ GW background emission, and its possible sources,
  - ⋆ How to use various aspects of GWs to test theories of gravity?
- ⋆ Data analysis: signal processing
- ⋆ Data analysis: parameter estimation

# Detectable astrophysical sources?

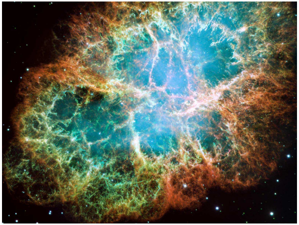
Merging neutron stars, black holes



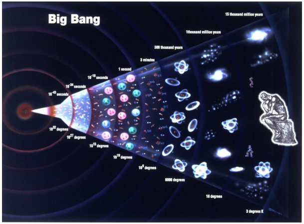

Fast-spinning neutron stars



Supernovae

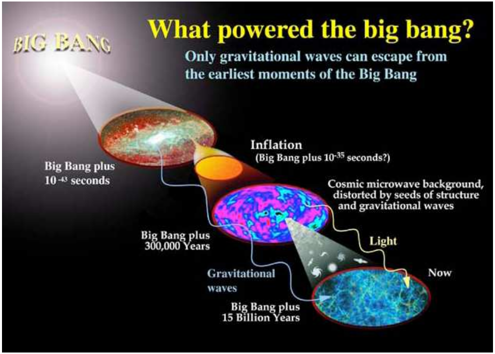


Primordial gravitational waves



# Stochastic GW background

- Incoherent superposition of many unresolved sources.
- Cosmological:
  - » Inflationary epoch, preheating, reheating
  - » Phase transitions
  - » Cosmic strings
  - » Alternative cosmologies
- Astrophysical:
  - » Supernovae
  - » Magnetars
  - » Binary black holes




Potentially could probe physics of the very-early Universe.
Typical frist approximations:

- ⋆ Gaussian: a sum of many contributions,
- ⋆ Stationary: physical time scales are much larger than observational ones,
- ⋆ Isotropic (at least for cosmological backgrounds).

Under these approximations, it is completely described by its power spectrum.

# Stochastic GW background

Stochastic GW background is usually described as energy density per logarithmic frequency interval with respect to the closure density of the universe ($\rho_c = \frac{3c^2 H_0^2}{8\pi G} \approx 7.6 \times 10^{-9}$ erg/cm$^3$):

$$\Omega_{GW}(f) = \frac{f}{\rho_c} \frac{d\rho_{GW}}{df}.$$

or over a particular frequency band:

$$\Omega_{GW} = \int d \ln(f) \Omega_{GW}(f).$$

Usually a power-law form of the energy density frequency dependence is assumed:

$$\Omega_{GW}(f) = \Omega_\alpha \left(\frac{f}{f_{ref}}\right)^\alpha.$$

# Stochastic GW background

The search uses an estimator $\hat{Y}_\alpha$

$$\hat{Y}_\alpha = \int_{-\infty}^{\infty} df \int_{-\infty}^{\infty} df' \, \delta_T(f-f')\tilde{s}_1^*(f)\tilde{s}_2(f')\tilde{Q}_\alpha(f'),$$

with a variance $\sigma_Y^2$

$$\sigma_Y^2 \approx \frac{T}{2}\int_0^{\infty} df \, P_1(f)P_2(f)|\tilde{Q}_\alpha(f)|^2,$$

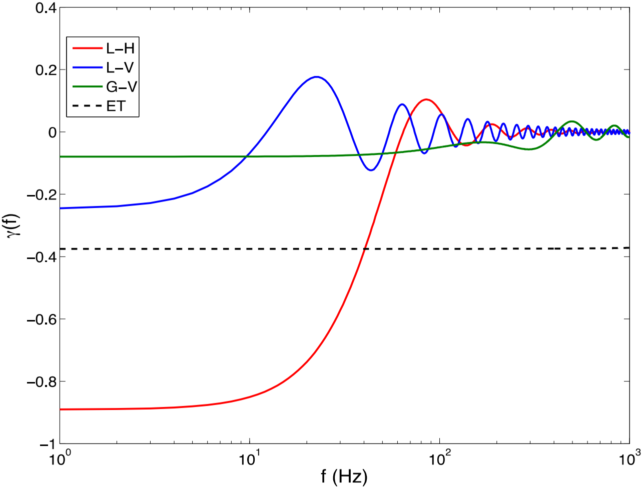
where $\delta_T(f-f')$ is a finite-time Dirac delta function, $T$ is the observation time, $P_{1,2}$ are the one-sided power spectral densities for the detectors, and $\tilde{Q}_\alpha(f)$ is a filter function to optimize the search:

$$\tilde{Q}_\alpha(f) = \lambda_\alpha \frac{\gamma(f)H_0^2}{f^3 P_1(f)P_2(f)}\left(\frac{f}{f_{\mathrm{ref}}}\right)^\alpha .$$

$\gamma(f)$ is called the overlap reduction function (measures the reduction in sensitivity due to separation and relative misalignment between the two detectors).

(see N. Christensen review for more details)

# Overlap reduction function (coherence function)




In principle, sensitive to all possible 6 polarizations ($\rightarrow$ stochastic background can be used to test theories of gravity).

# Stochastic GW background from binary mergers

• For every detected binary merger, there are many more that are too distant and too faint.

• They generate a stochastic background of gravitational waves.

Source Energy Spectrum

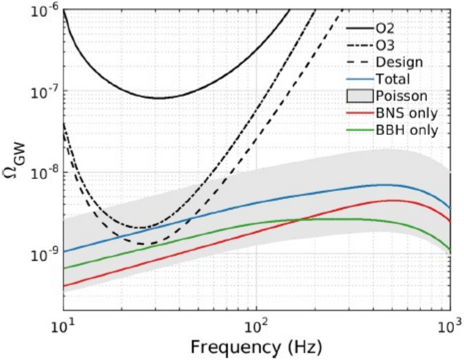
Merger Rate

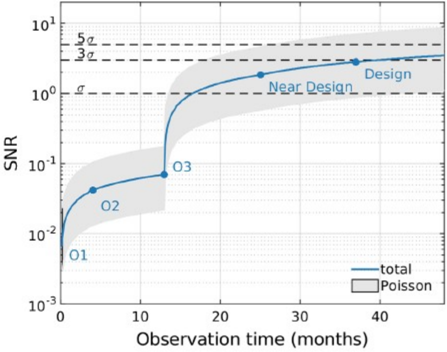
$$\Omega_{\mathrm{GW}}(f;\theta_k) = \frac{f}{\rho_c H_0} \int_0^{z_{\max}} dz \frac{R_m(z,\theta_k)\frac{dE_{\mathrm{GW}}}{df_s}(f_s,\theta_k)}{(1+z)E(\Omega_{\mathrm{M}},\Omega_{\Lambda},z)}$$

Cosmology

• Relatively high rate and large masses of observed systems implies a relatively strong stochastic background.

# Stochastic GW background from binary mergers




Based on the Field formation mechanism

# Directional stochastic GW background

- Relax assumption of isotropy and generalize the search for a stochastic signal to the case of arbitrary angular distribution.

$$\Omega_{\rm GW}(f) \equiv \frac{f}{\rho_c}\frac{d\rho_{\rm GW}}{df} = \frac{2\pi^2}{3H_0^2} f^3 H(f) \int_{S^2} d\hat{\Omega}\, \mathcal{P}(\hat{\Omega})$$

$$\boxed{\mathcal{P}(\hat{\Omega}) = \mathcal{P}_\alpha \mathbf{e}_\alpha(\hat{\Omega})}$$

**Radiometer Analysis**

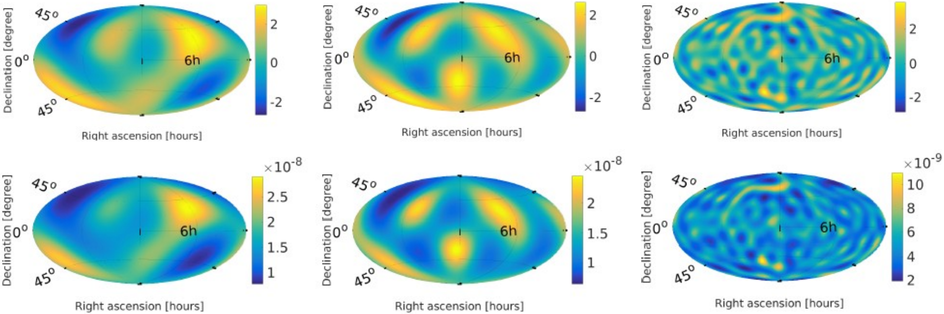
$$\mathcal{P}(\hat{\Omega}) \equiv \eta(\hat{\Omega}_0)\delta^2(\hat{\Omega}, \hat{\Omega}_0)$$

$\rightarrow$ radiometer searches.

**Spherical Harmonic Decomposition**

$$\mathcal{P}(\hat{\Omega}) \equiv \sum_{lm} \mathcal{P}_{lm} Y_{lm}(\hat{\Omega})$$

# Directional stochastic GW background



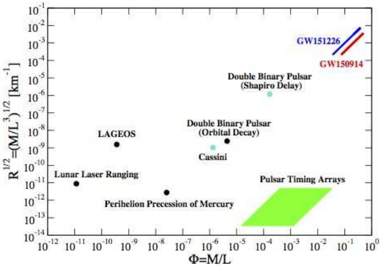
# First access to the strong-field dynamics of spacetime

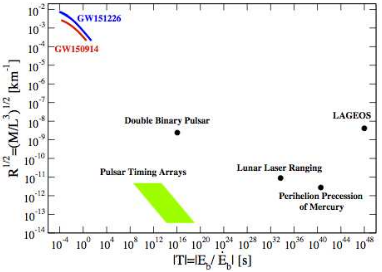
Before the direct detection of gravitational waves:

- Solar system tests:
  weak-field; dynamics of spacetime itself not being probed
- Binary neutron stars:
  relatively weak-field test of spacetime dynamics
- Cosmology:
  dark matter and dark energy may signal GR breakdown

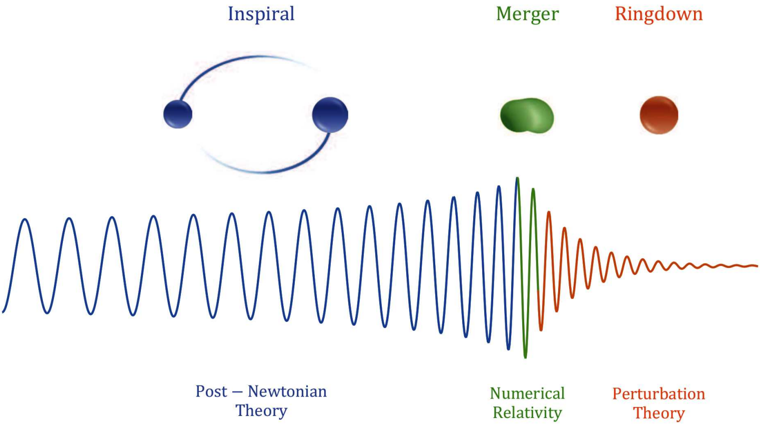
Direct detection of GW from binary black hole mergers:

- Genuinely strong-field dynamics
- (Presumed) pure spacetime events



# Evolution of a binary system



# Effect of parameters on the shape of the wave

The 'mechanics' of the GR binary system and its emitted GW waveform is actually quite well known:

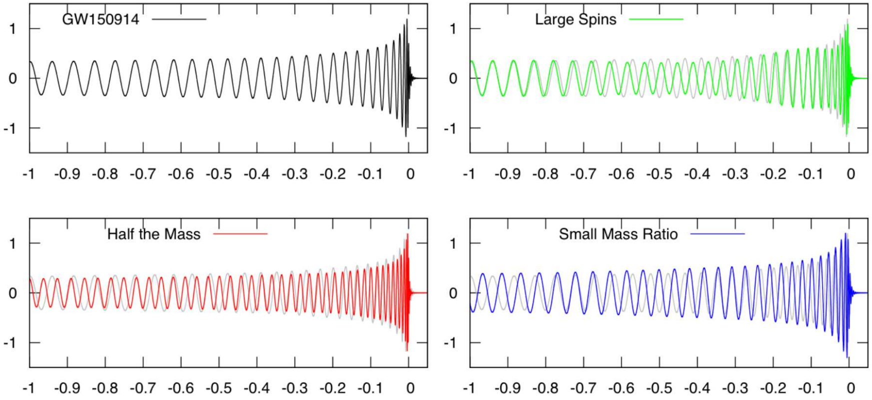
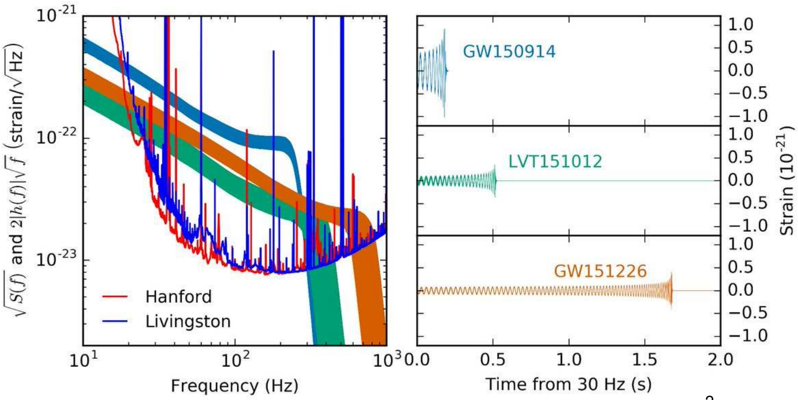



*Illustration by N. Cornish and T. Littenberg*

# Complementary information from different events




Matched filter signal-to-noise (SNR) $\rho$ : $$\rho^2 = \int_0^\infty \left( \frac{2|\tilde{h}(f)|\sqrt{f}}{\sqrt{S_n(f)}} \right)^2 d\ln(f)$$

- ⋆ GW150914 ($\rho \simeq 24$): merger at the most sensitive detector frequencies,
- ⋆ GW151226 ($\rho \simeq 13$): long inspiral in sensitive frequency band,
- ⋆ GW170104 ($\rho \simeq 13$): twice as far away $\rightarrow$ effects of distance on propagation

# Exploiting the phenomenology of IMR

- ⋆ Post-Newtonian description of inspiral
  - ⋆ Expansion of gravitational wave phase in powers of $v/c$,
  - ⋆ Do the coefficients depend on masses, spins as predicted by GR?
- ⋆ Consistency between inspiral and post-inspiral regimes
- ⋆ Propagation of gravitational waves over large distances
  - ⋆ Mass of the graviton,
  - ⋆ violations of local Lorentz invariance,
  - ⋆ dispersion relation,
- ⋆ Ringdown
  - ⋆ From the quasi-normal mode spectrum: (indirect) test of no-hair theorem
- ⋆ Gravitational wave echoes
- ⋆ Tidal effects during inspiral
  - ⋆ ”Black hole mimickers”: boson stars, dark matter stars, gravastars, . . . (deviations from standard GR and matter descriptions)
- ⋆ Polarization modes beyond GR

# A zoo of alternative theories of gravity

David Lovelock (1971): it's actually not straightforward to obtain a metric theory of gravity other than Einstein's GR. Specifically,

- ⋆ ”In four spacetime dimensions the only divergence-free symmetric rank-2 tensor constructed solely from the metric and its derivatives up to second order, and preserving diffeomorphism invariance, is the Einstein tensor plus a cosmological term”

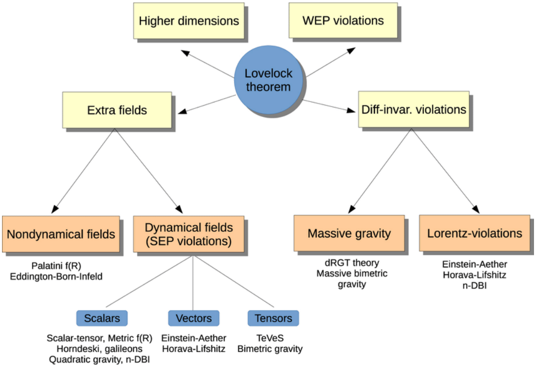
which means

- ⋆ ”If a local gravitational action contains only up to second derivatives of the four-dimensional spacetime metric, then the only possible equations of motion are the Einstein field equations.”

# A zoo of alternative theories of gravity

David Lovelock (1971): to modify Einstein's GR, one can

- ⋆ add extra fields other than metric tensor (vector, scalar fields),
- ⋆ add or remove dimensions,
- ⋆ take non-locality into account (action-at-a-distance),
- ⋆ allow breaking of GR principles (e.g. the equivalence principle).



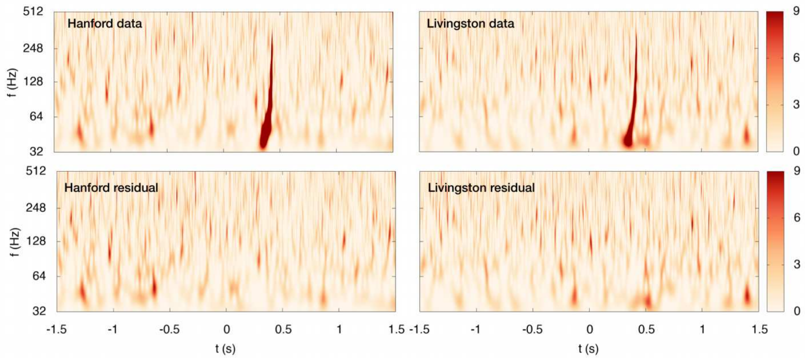
# How to check GR observationally

Karl Popper (1902-1994): falsifiability of the theory is the fundamental scientific criterion.

LIGO-Virgo O1-O2-O3a delivered 50 various detections, which were used for several kinds of tests:

- ⋆ „residual” (does the data contain anything unexpected after subtracting the signal model?)
- ⋆ „astrophysical parameters” (are the parameters consistent with each other in various regimes?)
- ⋆ „parameters of the theory” (are the coefficient values consistent with the theory?)
- ⋆ „dispersion relation” (do gravitational waves propagate like photons?)
- ⋆ „ringdown” (are we observing horizons as predicted by GR?)
- ⋆ „echoes” (are observed objects really GR black holes?)
- ⋆ „polarizations” (do gravitational waves interact with matter as GR predicts?)

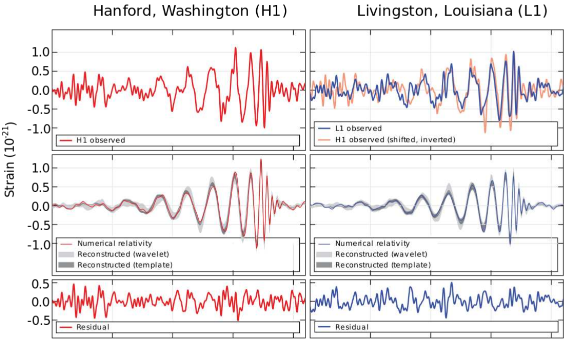
# Residuum test (left-over after removing the model)




Does the data after subtracting the model contain anything else except noise? If the waveform model reproduces the reality well, we expect:

- ⋆ No correlations between signal-to-noise ratio of the model and the residual,
- ⋆ Statistical properties of the residual consistent with data in which no signal was present (comparison with the 'background').

# Residual data after subtraction of best-fitting waveform



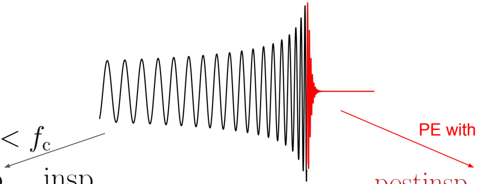

After subtraction of best-fitting waveform, is residual data consistent with noise?

$$SNR_{res}^2 = \frac{1 - FF^2}{FF^2} SNR_{det}^2$$

In case of GW150914, $FF \geq 0.96$; GR violations limited to 4%, at least for effects that can not be absorbed into redefinition of physical parameters.

# Inspiral-merger-ringdown (IMR) consistency

- If GR is correct, the final state of a BBH merger is a Kerr BH.
- The final mass and spin of the BH inferred from high and low frequency regimes must be consistent.
- Use parameter estimation on full signal and NR-calibrated fits to infer the final masses and spins, and obtain the cutoff frequency $f_c$ splitting signal into inspiral and merger-ringdown regimes.



PE with $f < f_c$

$$M_f^{\rm insp}, \chi_f^{\rm insp}$$

PE with $f > f_c$

$$M_f^{\rm postinsp}, \chi_f^{\rm postinsp}$$

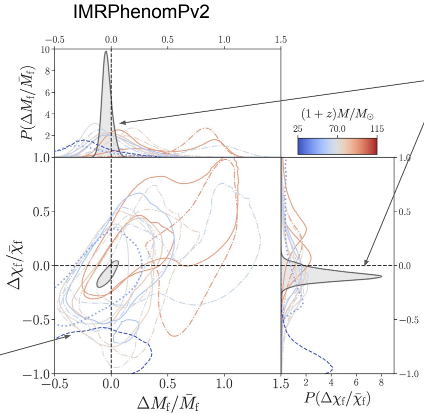
$$\frac{\Delta M_f}{\bar{M}_f} = 2\frac{M_f^{\rm insp} - M_f^{\rm postinsp}}{M_f^{\rm insp} + M_f^{\rm postinsp}}$$

$$\frac{\Delta \chi_f}{\bar{\chi}_f} = 2\frac{\chi_f^{\rm insp} - \chi_f^{\rm postinsp}}{\chi_f^{\rm insp} + \chi_f^{\rm postinsp}}$$

- For this test to be applicable, the inspiral and merger-ringdown regions of the signal must be informative.
- Impose a cut on SNR: both inspiral and merger ringdown regimes must have optimal SNR > 6.
- Additionally, demand that the detector frame total mass is below 100 solar masses.
- Reweight the posteriors to a prior uniform in deviation parameters.

# Inspiral-merger-ringdown (IMR) consistency

Shown are 90% credible regions




Assuming deviations are the same for all events

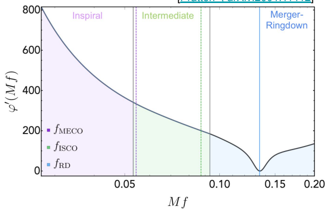
GW190814

# Parameterized tests of the coalescence process

- Inspiral-Merger-Ringdown (IMR) waveform model written as frequency-dependent amplitude and phase

$$\tilde{h}(f) = \mathcal{A}(f)\, e^{i\varphi(f)}$$

[Pratten+, arXiv:2001.11412]




- Parameterize phase corrections in 3 distinct regions:
  - Inspiral

$$\varphi_{\rm Ins}(f) = \varphi_{\rm ref} + 2\pi f t_{\rm ref} + \varphi_{\rm Newt}(Mf)^{-5/3} + \varphi_{0.5\rm PN}(Mf)^{-4/3} + \varphi_{1\rm PN}(Mf)^{-1} + \varphi_{1.5\rm PN}(Mf)^{-2/3} + \cdots$$

*Coefficients analytically known in GR*

- *Phenom*enological Coefficients
  - Intermediate

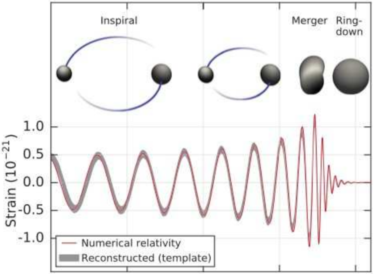
$$\varphi_{\rm Int} = \frac{1}{\eta}\left(\beta_0 + \beta_1 f + \beta_2 \log(f) - \frac{\beta_3}{3} f^{-3}\right)$$

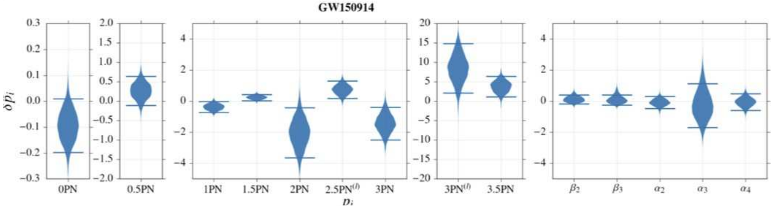
  - Merger-Ringdown

$$\varphi_{\rm MR} = \frac{1}{\eta}\left\{\alpha_0 + \alpha_1 f - \alpha_2 f^{-1} + \frac{4}{3}\alpha_3 f^{3/4} + \alpha_4 \tan^{-1}\left(\frac{f - \alpha_5 f_{\rm RD}}{f_{\rm damp}}\right)\right\}$$

*Caution:* Coefficients calibrated against NR but are not expressed in parameters relevant to GR or modified theories of gravity...

# Parameterized tests of the coalescence process




LSC+Virgo, Phys. Rev. X **6**, 041015 (2016)

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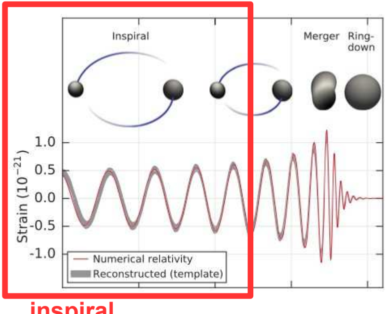


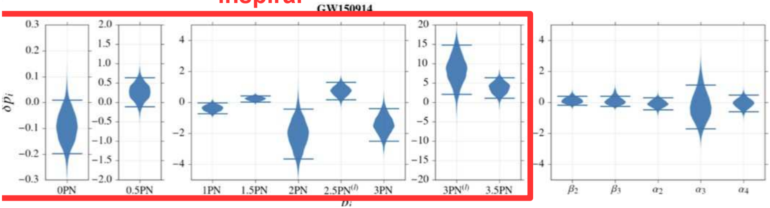


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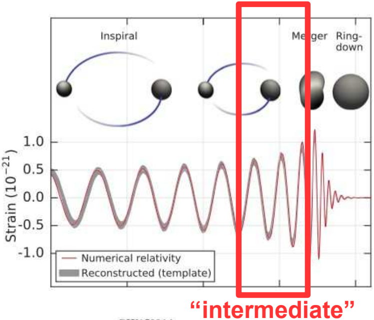


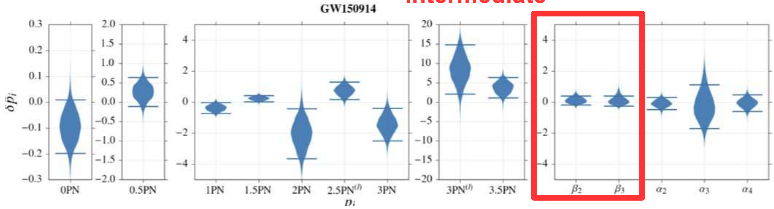


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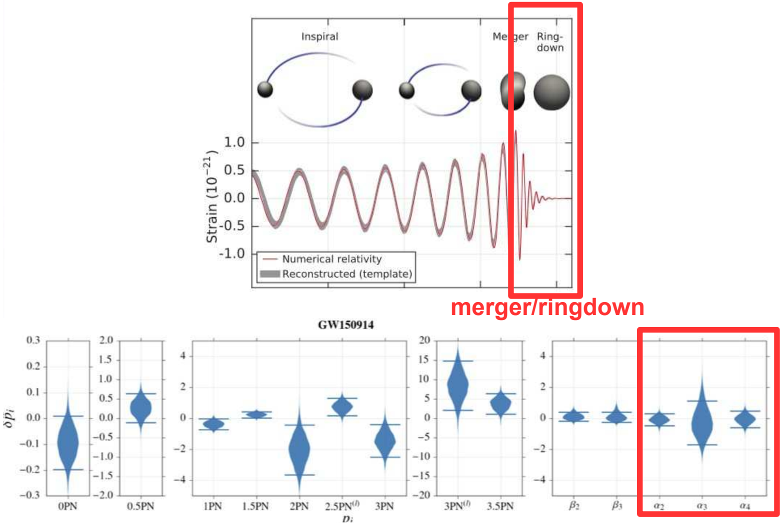
# Parameterized tests of the coalescence process

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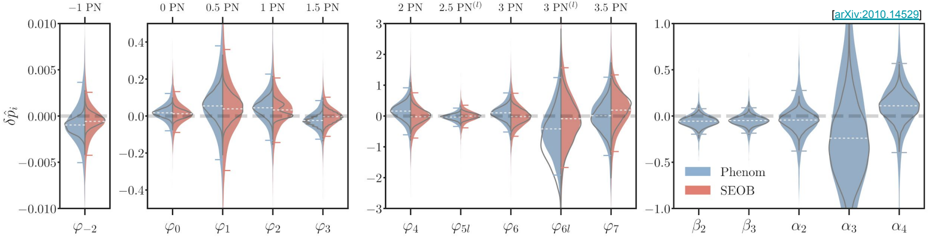
# Parameterized tests of the coalescence process

- Construct joint posteriors using two approaches:
  - Shared common value of deviation parameter
  - Hierarchical analysis
- Distributions and hyperparameters must be consistent with GR: $\delta\hat{p}_i = 0, \mu = \sigma = 0$

- Dashed horizontal line is GR limit (vanishing deformations)
- Shaded regions: population-marginalized expectations from hierarchical analysis for *Phenom* and *SEOB*
- Black distributions: events share common value of parameter

[arXiv:2010.14529]




★ Parameters of the theory: $p_i \rightarrow (1+\delta\hat{p}_i)\, p_i,\ p_i \in \{\phi_i, \beta_i, \alpha_i\}$,

★ Similarily, also astrophysical parameters may be studied like this, e.g. the spin-induced quadrupole moment $Q = -(1+\delta\kappa)\,\chi^2 m^3$.

# Spin-induced quadrupole moment

- Spinning motion of a compact object creates a distortion in the mass distribution
- Induces a distortion in the gravitational field measured by the quadrupole-moment tensor $Q$
- Effect imprinted in emitted GW radiation at specific PN orders - specialized variant of parameterized test
  - Include leading order correction at 2PN and a correction at 3PN
- For a compact object of mass $m$ and spin $\chi$

$$Q = -(1 + \delta\kappa)\,\chi_A^2 m_A^3$$

Coefficient depends on the equation of state, mass and spin of compact object...

Enables us to test the black hole nature of the compact object!

| | |
|---|---|
| Black Holes (*no-hair conjecture*) | $\delta\kappa$ = 0 [Poisson '98] |
| Neutron Stars | $\delta\kappa$ ~ 1 - 13 [Laarakkers '97, Pappas '12] |
| Boson Stars | $\delta\kappa$ ~ 10 - 150 [Ryan '97] |

# Spin-induced quadrupole moment

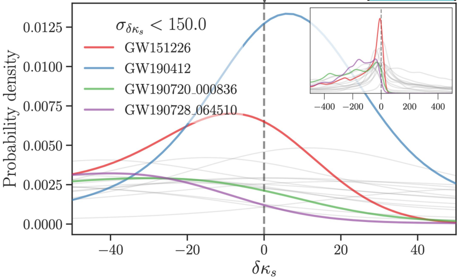
- Highly correlated with masses and spins, adopt an alternative parameterisation (cf $\chi_{\rm eff}$)

$$\kappa_s = (\kappa_1 + \kappa_2)/2$$
$$\kappa_a = (\kappa_1 - \kappa_2)/2$$

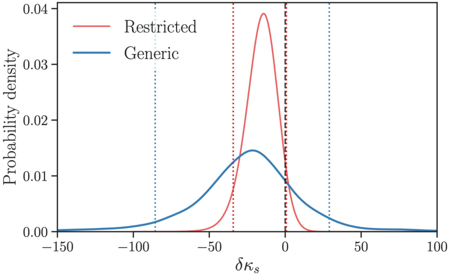
For black holes the coefficients are $\kappa = 1$, so we set this term to zero

Black holes in GR have $\delta\kappa_s = 0$

*individual results* [arXiv:2010.14529]




*combined results* [arXiv:2010.14529]



# Propagation tests. Dispersion relation

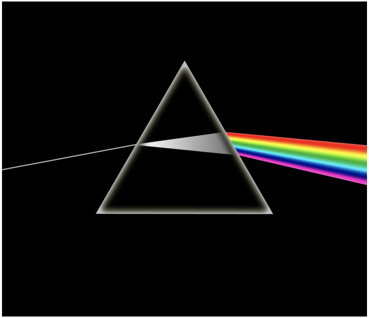
Generalized dispersion relation:

$$\boxed{E^2 = p^2\, c^2 + A_\alpha\, p^\alpha\, c^\alpha}$$

$E$ = Energy
$p$ = momentum
$c$ = speed of light
$A_\alpha, \alpha$ = phenomenological parameters

| **GR** | **Extensions of GR** |
|---|---|
| $A_\alpha = 0,\ \text{for all}\, \alpha$ | Massive gravity theory: |
| $m_g = 0$ | $\alpha = 0,\ A_\alpha > 0,\ m_g = A_0^{1/2} c^{-2}$ |

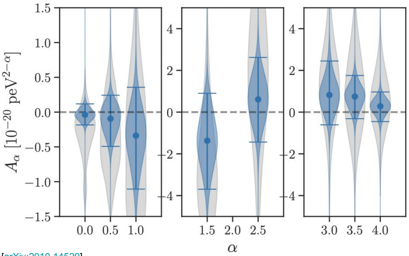
$m_g$ = graviton mass



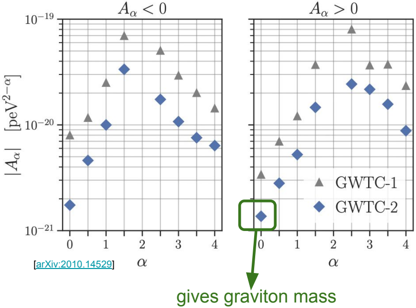
https://commons.wikimedia.org/wiki/File:Optical-dispersion_(1).png

**Dispersion of light wave**

# Propagation tests. Dispersion relation




[arXiv:2010.14529]




[arXiv:2010.14529]

- Noticeable improvement in the upper bound of $|A_\alpha|$ as compared to GWTC-1
- A factor of ~2.6 improvement, consistent with the increase in number of events from GWTC-1 to GWTC-2
- $m_g \leq 1.76 \times 10^{-23}\,\mathrm{eV/c^2}$, with 90% credibility
- A factor of ~2.7 improvement as compared to GWTC-1
- 1.8 times more stringent than the recent Solar System bound of $3.16 \times 10^{-23}\,\mathrm{eV/c^2}$ with 90% credibility [Phys. Rev. D 102, 021501 (2020)]

# GW170817: speed of gravitation

Relative speed difference between GWs and photons:

$$\frac{v_{GW} - c}{c} = \frac{\Delta v}{c} \approx \frac{c \Delta t}{d}.$$

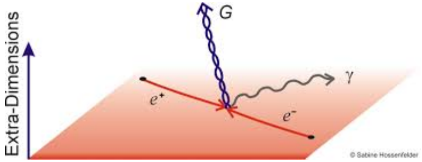
Assuming very conservative values:

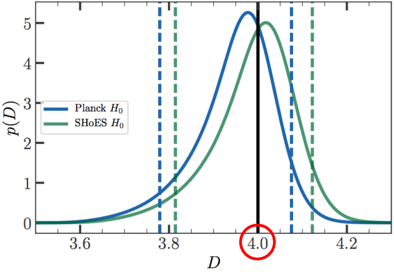
- ⋆ Distance $d = 26$ Mpc (lower bound from 90% credible interval on luminosity distance derived from the GW signal),
- ⋆ Time delay $\Delta t = 10$ s (actual delay between GW170817 and GRB170817A was $\simeq 1.7$ s)

$$-3 \times 10^{-15} \leq \frac{\Delta v}{c} \leq 7 \times 10^{-16}$$

$$\boxed{v_{GW} = 299792458^{+0.000001}_{-0.000006}\ \text{m/s} = c^{+0.000001}_{-0.000006}\ \text{m/s}}$$

# Constraining large extra dimensions



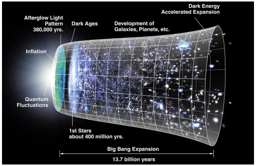




- Braneworld models
  - Standard model physics confined to the brane
  - Gravity can propagate into the bulk
  - "Leakage" of gravitational radiation into large extra dimensions
    - Gravitational wave strength drops off as $1/d^{(D-2)/2}$
- Compare distance inferred from GW signal with the distance to host galaxy

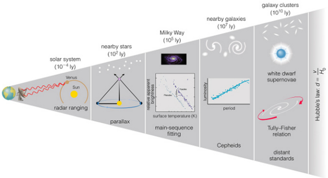
  **D = 4.02 ± 0.1**

Pardo *et al.*, arXiv:1801.08160

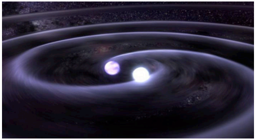
# A new cosmic distance marker




- Mapping out the large-scale structure and evolution of spacetime by comparing:
  - Distance
  - Redshift

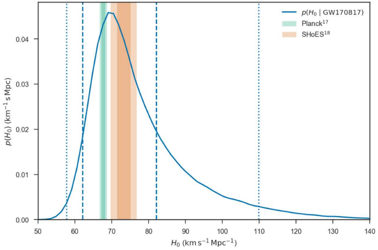



- Current measurements depend on *cosmic distance ladder*
  - Intrinsic brightness of e.g. supernovae determined by comparison with different, closer-by objects
  - Possibility of systematic errors at every “rung” of the ladder

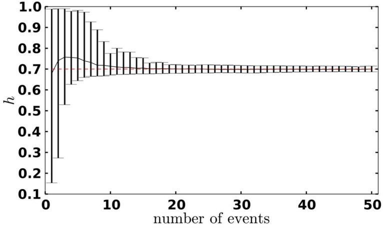


- Gravitational waves from binary mergers: *Distance can be measured directly from the gravitational wave signal!*

# A new cosmic distance marker




LIGO+Virgo *et al.*, Nature **551**, 85 (2017)

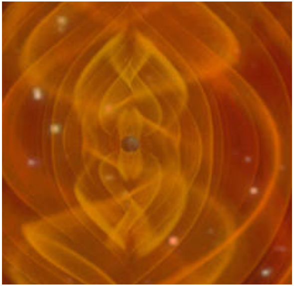



Del Pozzo, PRD **86**, 043011 (2012)

- Measurement of the local expansion of the Universe: The Hubble constant
  - Distance from GW signal
  - Redshift from EM counterpart (galaxy NGC 4993)

- One detection: limited accuracy
- Few tens of detections: O(1%) accuracy after few tens of detections

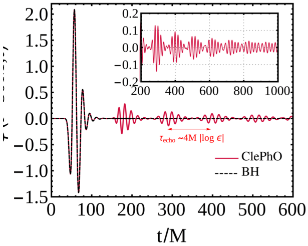
# What is the true nature of black holes?



Ringdown of newly formed black hole

- Not yet observed in detail
- Will enable indirect test of *no hair theorem*: *"Stationary, neutral black holes only characterized by mass and spin"*
- Requires further factor 3-4 improvement of detectors for test at the few percent level

Carullo *et al.*, arXiv:1805.04760
Brito *et al.,* arXiv:1805.00293




Gravitational wave echoes

- Alternatives to standard black holes, e.g. "firewalls" prompted by Hawking's information paradox
- Even after ringdown, black hole will continue to emit gravitational wave bursts: *echoes*
- Macroscopic signature of quantum gravity

Cardoso *et al.*, PRD **94**, 084031 (2016)
Tsang *et al.*, PRD **98**, 024023 (2018)

# Ringdown of newly formed black hole

- Ringdown regime: Kerr metric + linear perturbations
  - Ringdown signal is a superposition of quasi-normal modes

$$h(t) = \sum_{nlm} \mathcal{A}_{nlm} e^{-t/\tau_{nlm}} \cos(\omega_{nlm} t + \phi_{nlm})$$

  - Characteristic frequencies $\omega_{nlm}$ and damping times $\tau_{nlm}$

- No-hair conjecture: stationary, electrically neutral black hole completely characterized by mass $M_f$, spin $a_f$
  - Linearized Einstein equations around Kerr background enforce specific dependences:

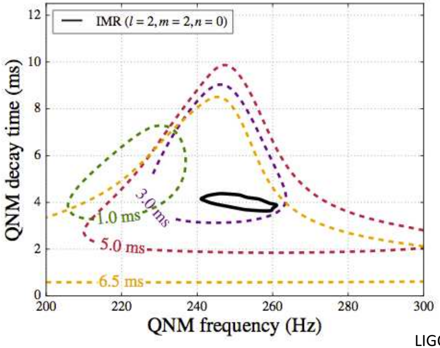
$$\omega_{nlm} = \omega_{nlm}(M_f, a_f)$$
$$\tau_{nlm} = \tau_{nlm}(M_f, a_f)$$

  - *Empirically checking these dependences would constitute an indirect test of the no-hair conjecture*

# Ringdown of GW150914?

- Numerical relativity simulations: linearized regime valid no earlier than ~10 M after merger
  - For GW150914: 10 M corresponds to ~3 milliseconds

- Evidence for a least-damped quasi-normal mode in GW150914 from fitting a single damped sinusoid:




LIGO + Virgo, PRL **116**, 221101 (2017)

# Testing the black hole no-hair conjecture

- GW50914: ringdown part had signal-to-noise ratio of ~8.5
  - *Would have been 3 times louder in Advanced LIGO/Virgo at design sensitivity*

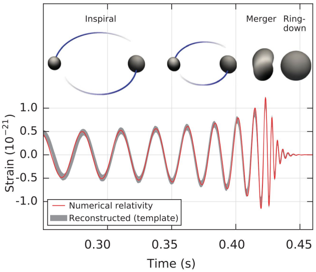
- Should a similar signal be seen after final detector upgrades, will we be able to test the no-hair conjecture?
  - Assume availability of accurate ringdown signal model for "standard" black holes
  - Find out at what time after merger the linearized regime is valid
  - Allow deviations from expressions for frequencies, damping times:

$$\omega_{lmn}(M_f, a_f) \;\to\; (1 + \delta\hat{\omega}_{lmn})\,\omega_{lmn}(M_f, a_f)$$
$$\tau_{lmn}(M_f, a_f) \;\to\; (1 + \delta\hat{\tau}_{lmn})\,\tau_{lmn}(M_f, a_f)$$

  - If no-hair conjecture valid then measured deviations should be consistent with zero

# Ringdown tests in GWTC-2

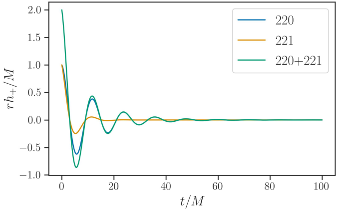
- Ringdown: quasi-normal modes (QNMs) with set frequencies and damping times
  - **Infer final mass and final spin** independent of inspiral
  - **Constrain deviations from GR** predictions of the frequencies and damping times
- Key results (qualitatively):
  - Measurements of the final mass and the spin **consistent** with the measurements using the full IMR signals
  - Inferred QNM frequencies and damping times **consistent** with BH perturbation theory calculations




Credit: Phys. Rev. Lett. 116, 061102 (2016)

$$h_+(t) - ih_\times(t) = \sum_{\ell=2}^{+\infty}\sum_{m=-\ell}^{\ell}\sum_{n=0}^{+\infty} \mathcal{A}_{\ell mn} \exp\left[-\frac{t-t_0}{(1+z)\tau_{\ell mn}}\right] \exp\left[\frac{2\pi i f_{\ell mn}(t-t_0)}{1+z}\right] {}_{-2}S_{\ell mn}(\theta,\phi,\chi_f)$$

- Mass and spin measurement using 3 different ringdown-only waveforms
  - $\mathrm{Kerr}_{220}$: include only $\ell = 2, |m| = 2, n = 0$ mode
  - $\mathrm{Kerr}_{221}$: include both $\ell = 2, |m| = 2, n = 0, 1$ mode
  - $\mathrm{Kerr}_{\mathrm{HM}}$: include all the fundamental modes ( $n = 0$) for $\ell \leq 4$



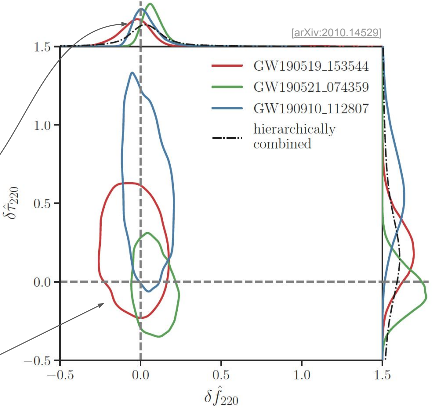
# Ringdown tests in GWTC-2

**Mass and spin** of the final BH *inferred from the inspiral part of the signal*

$\delta\hat{f}_{220} = 0.03^{+0.38}_{-0.35}$

constrained using hierarchical analysis with a set of BBHs

contour for 90% credible region




220 mode, i.e.

$$(\ell = 2, |m| = 2, n = 0)$$

Fundamental mode

$$f_{220} = f_{220}^{\rm GR}(1 + \delta\hat{f}_{220})$$

$$\tau_{220} = \tau_{220}^{\rm GR}(1 + \delta\hat{\tau}_{220})$$

$\delta\hat{\tau}_{220} = 0.16^{+0.98}_{-0.98}$

with a wider uncertainty

# Zoo of possible exotic compact objects

"Black hole mimickers":

- Boson stars
- Dark matter stars
- Gravastars
- Firewalls, fuzzballs
- ...

Giudice et al., JCAP **1610**, 001 (2016)

Find through:

- Anomalous tidal effects during inspiral

  Cardoso et al., arXiv:1701.01116

- Anomalous ringdown spectrum

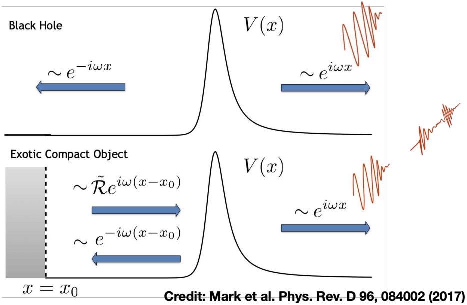
  Meidam et al., Phys. Rev. D **90**, 064009 (2014)

- Gravitational wave "echoes" after ringdown

  Cardoso et al., Phys. Rev. D **94**, 084021 (2016)

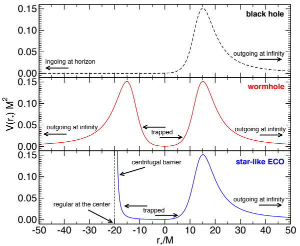
# Echoes

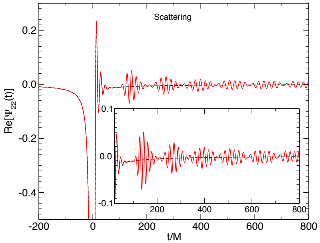
- What if the remnant compact object is ***not*** a classical BH?
- **Exotic compact objects (ECOs)**: event horizon replaced by a reflective surface
- GWs reflecting back and forth between the surface and the light ring ⇒ **GW echoes**
- Inspiral + Merger + Ringdown + Echoes (IMRE)
- Smoking-gun evidence for a BH mimicker if detected




**Credit: Mark et al. Phys. Rev. D 96, 084002 (2017)**

# Gravitational wave echoes




Cardoso et al., PRL **116**, 171101 (2016)
Cardoso et al., PRD **94**, 084031 (2016)

- Exotic objects with corrections near horizon: inner potential barrier for radial motion

- After formation/ringdown: continuing bursts of radiation called *echoes*

- If microscopic horizon modification $\ell \ll M$ then time between successive echoes

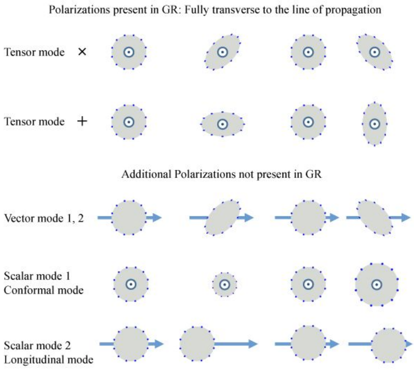
$$\Delta t \sim -nM \log\left(\frac{\ell}{M}\right)$$

where *n* set by nature of object:
  - *n* = 8 for wormholes
  - *n* = 6 for thin-shell gravastars
  - *n* = 4 for empty shell

- For GW150914 ($M$ = 65 $M_{sun}$), taking $\ell = \ell_{Planck}$, and $n$ = 4: **Δ**t = 117 ms

# Searching for alternative polarizations

- Generic metric theories of gravity allow up to *six* GW polarizations
  - two tensor modes (helicity ±2), allowed in GR
  - two vector modes (helicity ±1)
  - two scalar modes (helicity 0)
- Polarization content is imprinted in the relative amplitudes of the output at different detectors
- Used to reconstruct the GW polarization content in the data
- Five-detector network would be ideal for this test
- We used three-detector network to distinguish between specific subsets of all the possible polarization combinations




**Credit: Claudia de Rham, LRR, 17 (2014).**

# Searching for alternative polarizations

- Extreme polarization hypotheses:
  - full-tensor vs full-vector
  - full-tensor vs full-scalar

- Null-stream based polarization test, does not rely on specific waveform models

  - *Null-stream*: linear combination of data streams from different detectors

    - Free of true GW signal with a given helicity and sky-location

    - Marginalized over sky-location

  - Any excess power in the *null stream* must be produced by a different helicity and sky-location

  - Quantify the excess power by *null energy*

# Searching for alternative polarizations

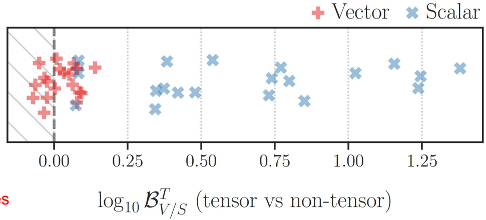
| Event | $\log_{10} \mathcal{B}^T_V$ | $\log_{10} \mathcal{B}^T_S$ |
|---|---|---|
| GW170809 | 0.078 | 0.421 |
| GW170814 | −0.032 | 0.740 |
| GW170818 | 0.002 | 0.344 |
| GW190408_181802 | 0.076 | 0.480 |
| GW190412 | 0.079 | 0.539 |
| GW190503_185404 | −0.072 | 1.245 |
| GW190512_180714 | −0.024 | 0.346 |
| GW190513_205428 | 0.139 | 1.380 |
| GW190517_055101 | 0.008 | 0.730 |
| GW190519_153544 | 0.067 | 0.799 |
| GW190521 | 0.093 | 1.156 |
| GW190602_175927 | −0.064 | 0.373 |
| GW190706_222641 | 0.052 | 0.771 |
| GW190720_000836 | 0.034 | 0.074 |
| GW190727_060333 | 0.087 | 1.024 |
| GW190728_064510 | −0.024 | 0.083 |
| GW190828_063405 | 0.063 | 0.851 |
| GW190828_065509 | −0.034 | 0.084 |
| GW190915_235702 | 0.020 | 1.238 |
| GW190924_021846 | −0.051 | 0.384 |

highest Bayes factor

lowest Bayes factors

$\mathcal{B}^T_V$ = Bayes factor for full-tensor vs full-vector hypotheses

$\mathcal{B}^T_S$ = Bayes factor for full-tensor vs full-scalar hypotheses




[arXiv:2010.14529]

# Summary

- ⋆ Population tests of the genuinely strong-field dynamics of pure spacetime:
  - ⋆ No evidence for violations of GR.
- ⋆ Tests of coalescence dynamics
  - ⋆ Parameterized tests in inspiral and merger/ringdown regimes,
  - ⋆ Consistency of masses and spins between inspiral and post-inspiral.
- ⋆ Tests of gravitational wave propagation:
  - ⋆ Bound on graviton mass and speed of gravity,
  - ⋆ Bounds on violation of local Lorentz invariance.
- ⋆ First tests of non-GR polarizations
- ⋆ Tests of the black hole nature of the component and remnant objects,
- ⋆ Ringdown and no-hair theorem tests, GW echoes,
- ⋆ 'Real' cosmology/cosmography with GWs.

# Literature


- ⋆ N. Christensen, ”Stochastic Gravitational Wave Backgrounds”, Reports on Progress in Physics, Vol. 82, 016903 (2019) (arXiv:1811.08797)
- ⋆ ”Tests of General Relativity with Binary Black Holes from the second LIGO–Virgo Gravitational-Wave Transient Catalog” (dcc.ligo.org/LIGO-P2000091/public)
- ⋆ Z. Carson, K. Yagi, ”Testing General Relativity with Gravitational Waves” (arXiv:2011.02938)
- ⋆ F. Dyson, ”Is a Graviton Detectable?” publications.ias.edu/sites/default/files/poincare2012.pdf