Rates and populations, cosmology

15.12.20

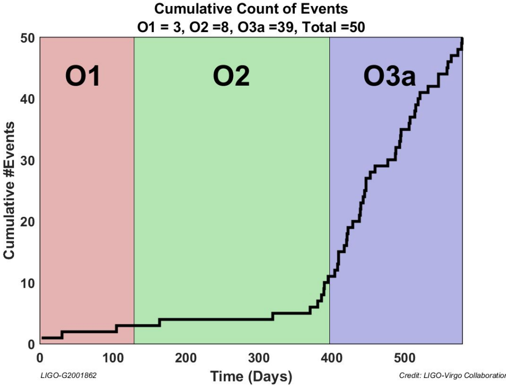
General schedule

- ⋆ History
- ⋆ Introduction to general relativity
- ⋆ Detection principles
- ⋆ Detectors
- ⋆ Binary black-hole system
- ⋆ Bursts and continuous waves
- ⋆ Rates and populations & cosmology
  - ⋆ Binary systems parameters from population studies,
  - ⋆ Standard sirens in cosmology.
- ⋆ Stochastic GW background & testing general relativity
- ⋆ Data analysis: waveforms and detection
- ⋆ Data analysis: parameter estimation

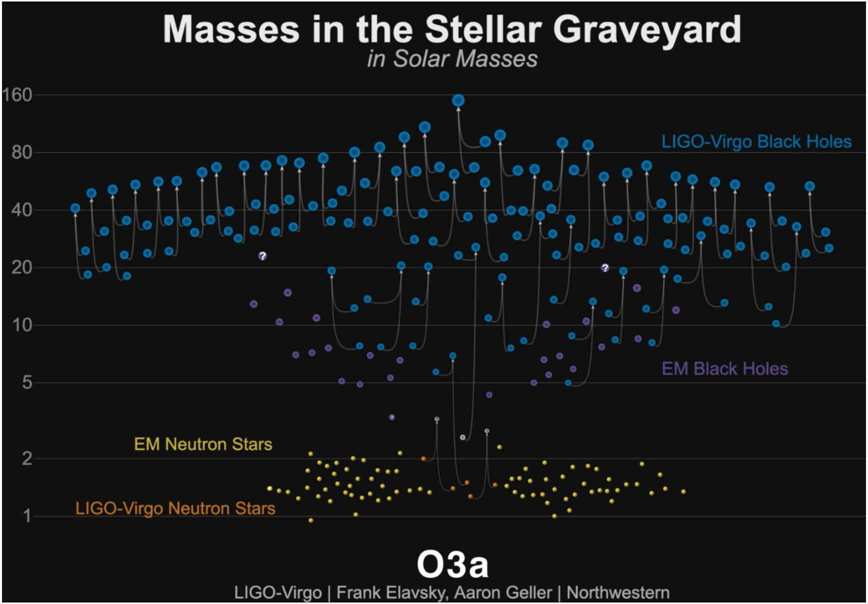
GW population of binary systems so far

As we improve our detectors we are detecting more and more GW events

- **During O1 (~4 months)**:
  - 3 confident BBHs
- **During O2 (~8 months):**
  - 7 confident BBHs
  - 1 confident BNS
- **During O3a (~6 months)**:
  - 1 consistent with BNS masses (GW190425)
  - 2 BH+lighter object (GW190814, GW190426_152155)
  - 36 consistent with BBHs



GW population of binary systems so far



Compact binary merger rate

$$\mathcal{R} = \frac{N}{V \times T}$$

where

- ⋆ N - Number of the confident detections
- ⋆ V - sensitive Volume of an assumed population ($V(\lambda)$, with λ describing parameters of the population)
- ⋆ T - observation Time

- Sensitive Volume of the population is one of the primary ingredient here (other being estimated parameters of various observations).
- Definition

$$\langle V(\lambda)\rangle \cdot T = \int \mathrm{d}z\mathrm{d}\vec{\theta}\, \frac{\mathrm{d}V_c}{\mathrm{d}z}\frac{1}{1+z}p(\vec{\theta}|\lambda)f(z,\vec{\theta}) \cdot T, \tag{4}$$

  - $\langle V(\lambda)\rangle \cdot T$ is the population averaged time-volume product. T is the observation time.
  - $\frac{\mathrm{d}V_c}{dz}$ is the differential comoving volume. Factor of $\frac{1}{1+z}$ is there to account for time dilation caused by expansion of the universe.
  - $f(z,\vec{\theta})$ is the efficiency of confidently (calling recovered now) observing a binary with parameters $(z,\vec{\theta})$

Current merger rates (LIGO-Virgo O1-O3a)

Binary Neutron Stars: With two confident observations of binary neutron stars in GWTC-2, we infer that the local merger rate of binary neutron stars is:

$$\mathcal{R}_{\rm BNS} = 320^{+490}_{-240}\,\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$$

Binary Black Holes: For binary black holes, we simultaneously fit for the mass, spin and merger rate. Assuming a merger rate density that is constant across cosmic time:

$$\mathcal{R}_{\rm BBH} = 23.9^{+14.9}_{-8.6}\,\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$$

(for comparison, core-collapse supernova rate is $\mathcal{R}_{SN} \simeq 10^5\ \mathrm{Gpc}^{-3}\ \mathrm{yr}^{-1}$)

An unexpected shortage of neutron-star mergers?

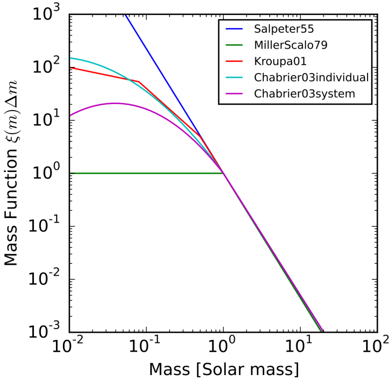
Should we be concerned that LIGO & Virgo detectors detect much more BBH than BNS signals?

Edwin Salpeter's initial mass function is

$$\xi(m)\Delta m = \xi_0 \left(\frac{m}{M_\odot}\right)^{-2.35} \left(\frac{\Delta m}{M_\odot}\right)$$

Integrated for ranges of masses for BHs and NSs progenitor stars, to get relative numbers of progenitors:

$$\frac{N(M > 80M_\odot)}{N(M > 10M_\odot)} = \left(\frac{80M_\odot}{10M_\odot}\right)^{-1.35} \simeq 0.06.$$



An unexpected shortage of neutron-star mergers?

- ⋆ Assuming the same merger rates for BBH and BNS → rates proportional to number of progenitor stars:

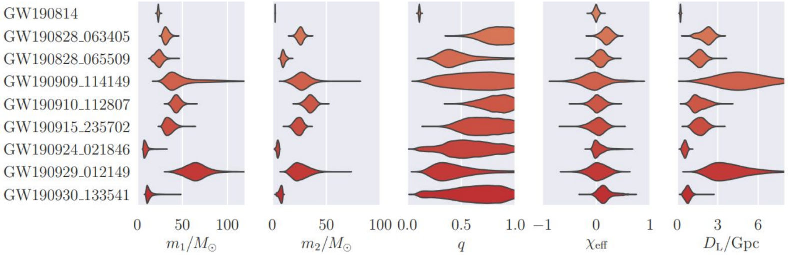
$$\frac{\mathcal{R}_{BBH}}{\mathcal{R}_{BNS}} = \left(\frac{80M_\odot}{10M_\odot}\right)^{-1.35} \simeq 0.06$$

- ⋆ But how many signals are detected? Signal-to-noise $\propto \mathcal{M}^{5/6}$, detection volume $\propto SNR^3 \propto r^3$

$$\frac{\mathcal{D}_{BBH}}{\mathcal{D}_{BNS}} = \frac{\mathcal{R}_{BBH}}{\mathcal{R}_{BNS}} \left(\frac{\mathcal{M}_{BBH}}{\mathcal{M}_{BNS}}\right)^{5/2} = \left(\frac{80M_\odot}{10M_\odot}\right)^{-1.35} \left(\frac{10M_\odot}{1.4M_\odot}\right)^{5/2} \simeq 8$$

(Phys. Usp. 44 1 2001 `[astro-ph/0008481]`)

GWTC-2: selected sources and their parameters



- ⋆ Chirp mass $\mathcal{M} = \left(\mu^3 M^2\right)^{1/5} = (m_1 m_2)^{3/5}/(m_1 + m_2)^{1/5}$,
- ⋆ Mass ratio $q = m_2/m_1$ (at 1PN), alternatively $\nu = m_1 m_2/(m_1 + m_2)^2$,
- ⋆ Spin-orbit and spin-spin coupling (at 2PN and 3PN, resp.) $\rightarrow$

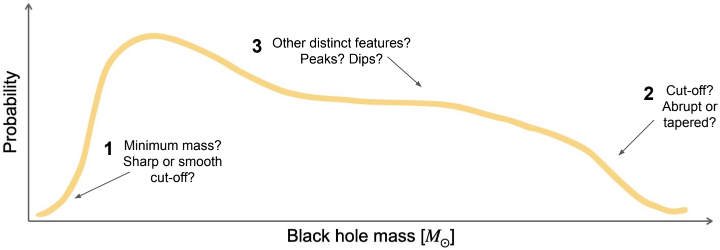
$$\chi_{eff} = (m_1\chi_{1z} + m_2\chi_{2z})/(m_1 + m_2)$$

  where χ_{iz} are spin components along system's total angular momentum,

- ⋆ Direct "luminosity" ("loudness") distance: binary systems are "standard sirens".

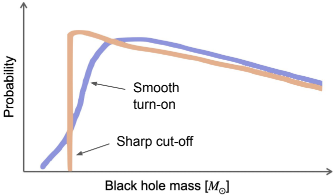
Population of binary systems: BH mass distribution

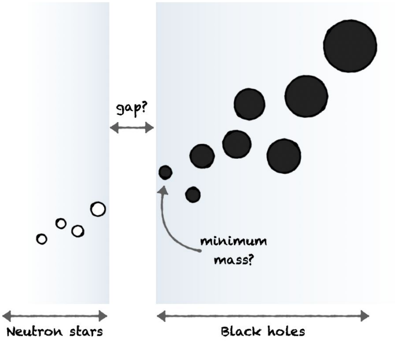
- Features in the mass distribution can help us probe how black holes formed. We can compare results to expectations from theories for stellar evolution.
- Models used in population analysis motivated by these theories.



BH mass distribution: low-mass gap

- What is the minimum mass black hole?
- Does the distribution have a smooth or sharp cut-off at low-mass end? Is there evidence for a low mass gap?

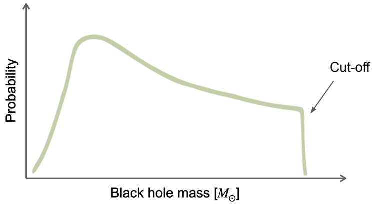


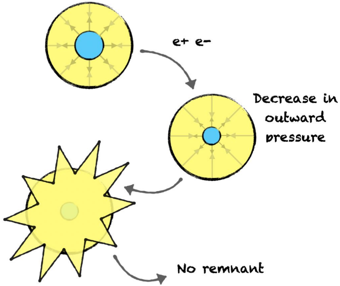


Potentially difficult to probe because of the $\mathcal{M}^{5/6}$ SNR dependence (low rate in the local Universe)

BH mass distribution: high-mass gap

- Very massive stars leave behind no remnant after a supernova.
- No black holes formed beyond a certain mass, suggests a cut-off in the mass distribution

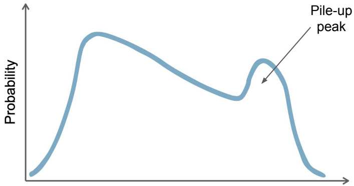


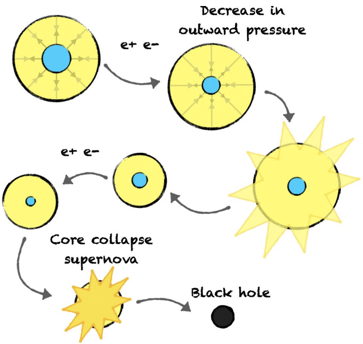


Stars of masses $> 130 M_\odot$ at ZAMS
(ZAMS = Zero Age Main Sequence ~ original mass of star)

BH mass distribution: high-mass gap

- Massive stars shed mass in ‘pulses’.
- Produce stars of similar mass, which collapse to form black holes around ~ 35 to 45 $M_\odot$

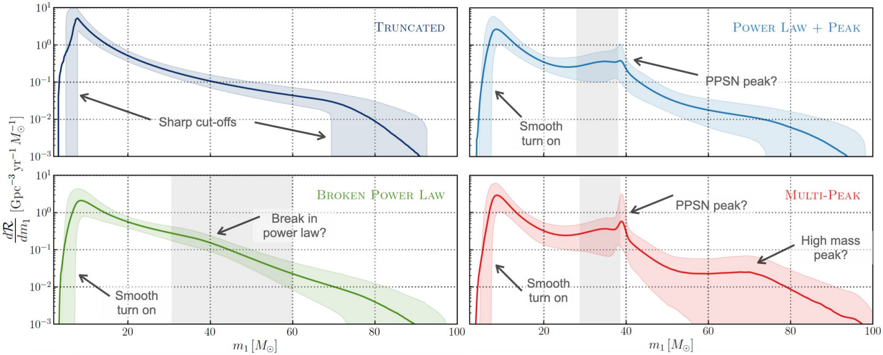




Stars of masses $\sim 80 M_\odot \rightarrow 130 M_\odot$ at ZAMS

(ZAMS = Zero Age Main Sequence ~ original mass of star)

Beyond Sapleter: various mass functions



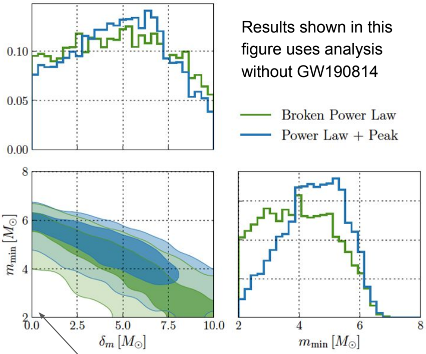
Analysis with 44 confident BBH. Primary mass distribution: Solid curve - mean; Shaded region - 90% credible interval

GWTC-2 results: mass distributions

- We rule out the combination of a small minimum black hole mass (~ 2 $M_\odot$) and a sharp low-mass cut-off.
- We are beginning to resolve the low-mass end of distribution.

- Additional study performed including GW190814. Low-mass end of distribution pulled from ~6 to ~2 $M_\odot$. GW190814 is an outlier in the BBH distribution -- only 0.02% chance of GW190814-like event in analysis with 44 confident BBH population.

GW190814 - arXiv:2006.12611

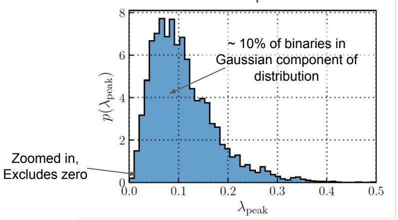


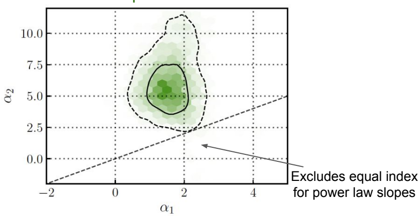
Results shown in this figure uses analysis without GW190814

Excludes low mass + sharp cut-off

GWTC-2 results: mass distributions

- Support for Gaussian component in distribution (most favoured model Power law + peak).
- Power-laws have different slopes (Broken power law slightly less favoured; by factor of 8).





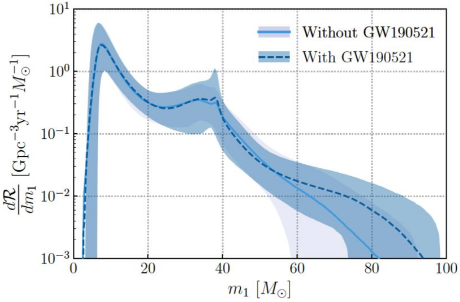
- A simple power law with sharp-cutoffs (Truncated model) is disfavoured (by factor 100 compared to Power law + peak).

(λ_{peak} - the fraction of systems that belong to the additional Gaussian component)

GWTC-2 results: mass distributions

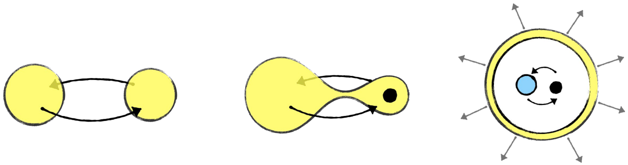
- No cut-off feature around 45 $M_\odot$
- Masses extend beyond 45 $M_\odot$ with and without GW190521. This event appears to be consistent with the population.
- Unable to conclude whether GW190521 is in the tail of the distribution, or a separate subpopulation (e.g. hierarchical mergers)

GW190521 - arXiv:2009.01075 & arXiv:2009.01190

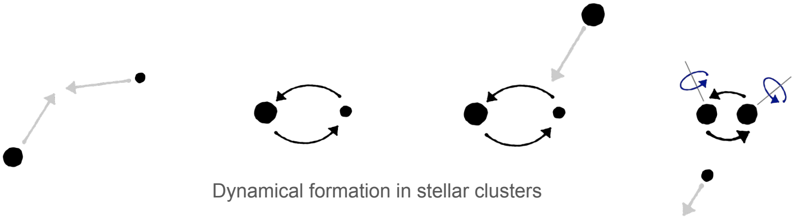


Primary mass distribution: Solid/dashed curves - mean;
Shaded region - 90% credible interval
(results from Power law + peak model)

Component spins and relation with formation channels



Isolated common envelope evolution



Dynamical formation in stellar clusters

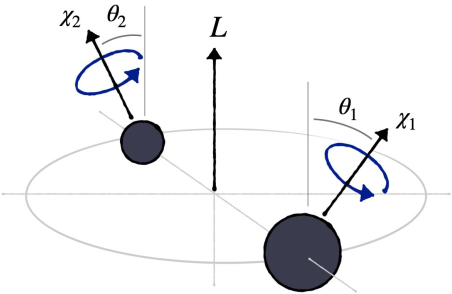
Spin-related quantities in the waveform

Effective inspiral spin quantifies total spin parallel to a binary's orbital angular momentum:

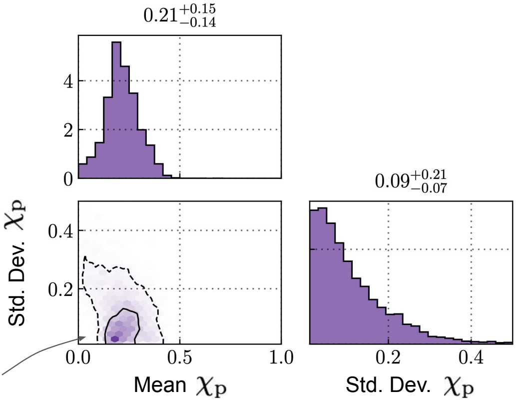
$$\chi_{\rm eff} = \frac{m_1\,\chi_1 \cos\theta_1 + m_2\,\chi_2 \cos\theta_2}{m_1 + m_2}$$

Effective precessing spin is related to degree of spin *perpendicular* to orbit:

$$\chi_{\rm p} \sim \chi_1 \sin\theta_1$$



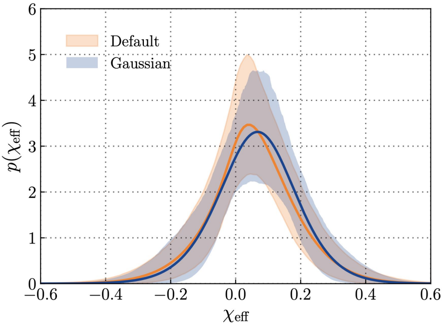
GWTC-2 results: effective precessing spin



Exclude a delta-function at $\chi_\mathrm{p} = 0$

GWTC-2 results: effective inspiral spin

- Negative $\chi_{\rm eff}$ implies **spins tilted by more than 90°** relative to their orbital angular momentum
- Between 12% and 44% of BBHs have negative effective spins
- If we attribute negative $\chi_{\rm eff}$ to dynamics, then **between 25% and 93% of events originate in dynamical channels**

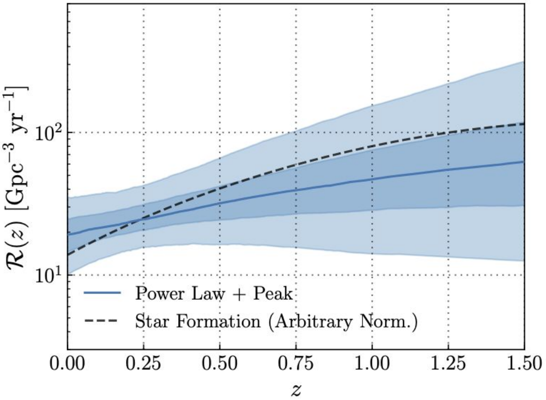


Default: measuring of physical spin magnitude and spin tilt distrubutions,
Gaussian: measuring the distribution of phenomenological parameters (χ_{eff} and χ_p).

Merger rate vs redshift

With GWTC-2, we now know:

- Today ($z = 0$), the binary black hole merger rate is between [10, 35] Gpc^{-3} yr^{-1}
- 8 billion years ago ($z = 1$), the binary black hole merger rate was between 0.6 and 10 times its present rate

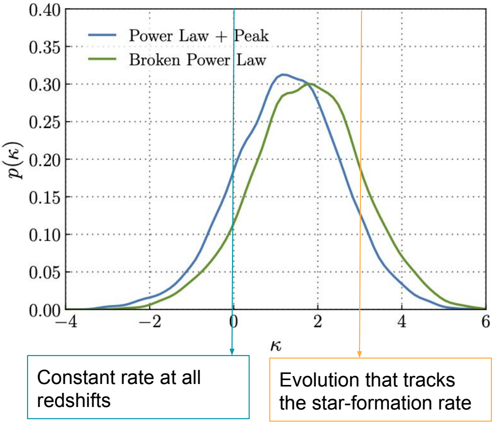


Merger rate evolution with redshift

Assume that the rate ***R*** as a function of redshift *z* is described by $R(z) = (1+z)^K$

Measure the slope *K*

The most likely values are between 0 (no evolution) and 2.7 (approximating the star-formation rate)



GWTC-2: summary

- **The black hole mass spectrum *does not* terminate abruptly at 45 solar masses**, but *does* show a feature at ~40 solar masses, which can be represented by a *break* in the power law or a Gaussian *peak*.
- **There is a dearth of low-mass black holes** between 2.6 solar masses and ~6 solar masses.
- **The distribution of mass ratios is broad** in the range ~0.3-1, with a mild preference for equal-mass pairings. (GW190814 is an outlier.)

- Some binary black holes have measurable in-plane spin components, leading to **precession of the orbital plane**.
- Some binary black holes have spins **misaligned by more than 90 degrees**, but the distribution of spin tilts is not perfectly isotropic.
- There are hints, but **no clear evidence that the spin distribution varies with mass.**

GWTC-2: summary

- In the local universe, the average **binary black hole merger rate is between 15 and 40 $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$**
- The binary black hole merger rate **probably evolves with redshift, but slower than the star-formation rate**, increasing by a factor of ~2.5 between $z = 0$ and $z = 1$.

Open questions:

- What is the physical origin for the feature at ~40 solar masses?
- What is the origin of black holes with masses above 45 solar masses?
- Is there a mass gap between neutron stars and black holes?
- What is the nature of the 2.6 solar mass object in GW190814?
- Are the systems with misaligned spins the result of dynamical assembly?
- Are we observing binary black holes from multiple formation channels?

Binary systems as standard sirens

Binaries are *clean* systems: we have accurate models even in full general relativity.

Loss of energy to GWs causes orbit to decay, orbital frequency to go up. So the GWs will chirp up in frequency. Chirp time $t_{\text{chirp}} \sim f\,/\,[\mathrm{d}f\,/\,\mathrm{d}t\,]$.

Signal contains both apparent brightness (from h and f) and intrinsic luminosity (from t_{chirp}), from which we can compute the distance to the source:

$$\text{Distance} \propto c\,\frac{}{\text{frequency}^2 \times t_{\text{chirp}}}$$

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GW Astronomy and cosmology

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Standard sirens

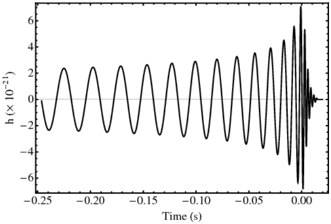
The luminosity distance can be inferred directly from the measured waveform produced by a binary system

$$\boxed{h_\times = \frac{4}{d_L}\left(\frac{G\mathcal{M}_c}{c^2}\right)^{\frac{5}{3}}\left(\frac{\pi f}{c}\right)^{\frac{2}{3}}\cos\iota\,\sin[\Phi(t)]}$$

$\Rightarrow$ GW sources are standard distance indicator (standard sirens)

The problem with GW is to obtain the redshift of the source through the detection of an EM counterpart such as

- EM emission at merger
- Hosting galaxy



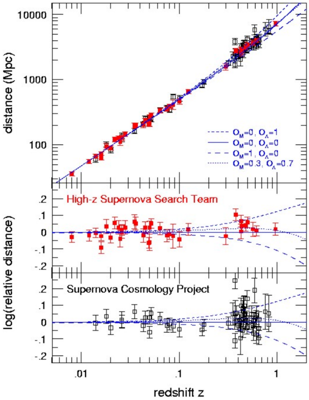
Standard sirens

$$d_L(z) = \frac{c}{H_0}\frac{1+z}{\sqrt{\Omega_k}} \sinh\left[\sqrt{\Omega_k}\int_0^z \frac{H_0}{H(z')}dz'\right]$$

The distance-redshift relation connects the luminosity distance (d_L) to the redshift (z) at any point in the universe and depends on the cosmological parameters

$\Rightarrow$ if for some astrophysical object both d_L and z are known, one can fit the distance-redshift relation and obtain constraints on the cosmological parameters

Example: Supernovae type-Ia (standard candles)



Standard sirens

With EM waves:

- Measuring redshift is easy: compare EM spectra
- Measuring distance is hard: need objects of known luminosity (SNIa $\rightarrow$ **standard candles**)

With GW:

- Measuring distance is easy: directly from the waveform (**standard sirens**)
- Measuring redshift is hard:
  - Degeneracy with masses in the waveform (GR is scale-free)
  - Need to identify an **EM counterpart**:
    - Optical, Radio, X-rays, γ-rays, ...
  - Need good sky location accuracy from GW detection to pinpoint the source or its hosting galaxy

Hubble plot ($v_H = H_0 d$)

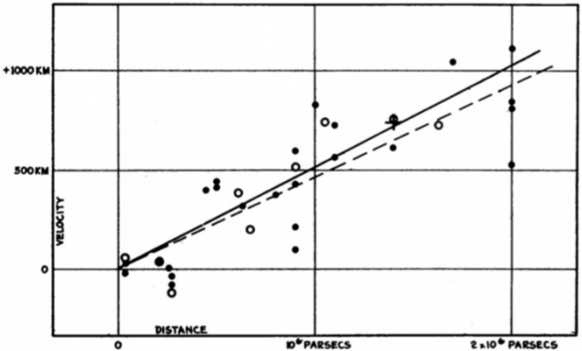
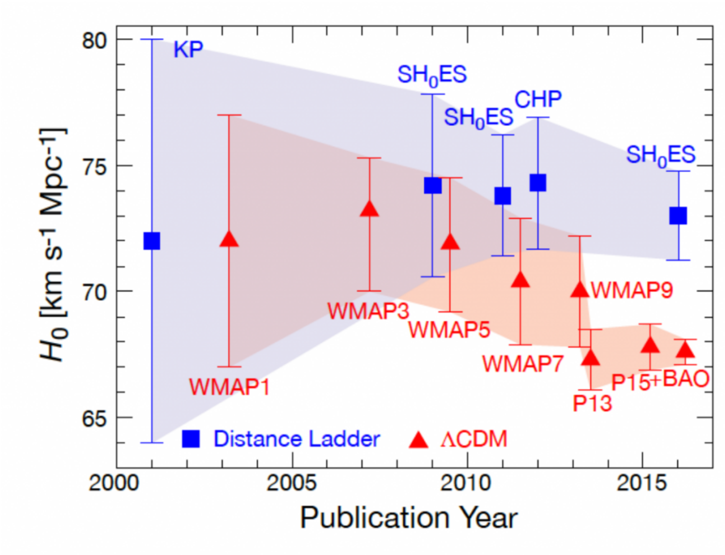


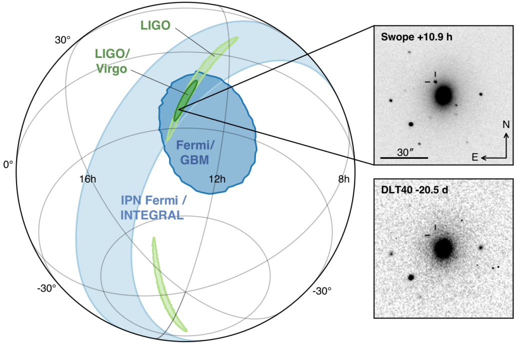
FIGURE 1
Velocity-Distance Relation among Extra-Galactic Nebulae.

Hubble's law tension



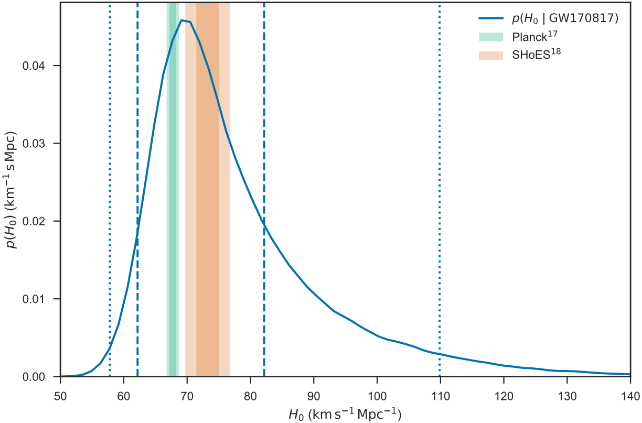
Freedman (2017)

GW170817: 17 August 2017, 14:41:04 CEST



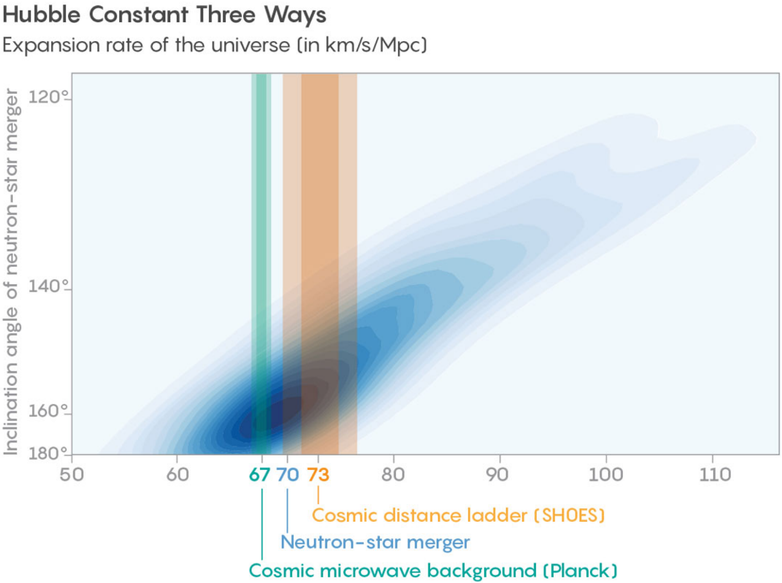
- ⋆ Combined LIGO-Virgo signal-to-noise ratio: SNR=32.4 (strongest signal so far!),
- ⋆ False alarm rate: less than one in 80000 years,
- ⋆ Chirp mass $\mathcal{M} = 1.188^{+0.004}_{-0.002}\ M_{\odot} \rightarrow$ a very light system!
- ⋆ New EM source in NGC 4993, consistent with GW distance 40^{+8}_{-14} Mpc,
- ⋆ Chance of temporal-spatial coincidence $< 5 \times 10^{-8}$.

GW170817: First "standard siren" H_0 measurement



- ⋆ $70.0^{+12.0}_{-8.0}$ km s^{-1} Mpc^{-1} (maximum a posteriori and 68% credible interval) = $\sim$14% at 1σ:
  - ⋆ $\sim$11% because of GW luminosity distance,
  - ⋆ The rest from the peculiar velocity of the galaxy.
- ⋆ Planck: 67.74 ± 0.46, SHoES: 73.24 ± 1.74 km s^{-1} Mpc^{-1}

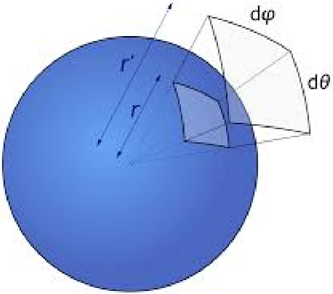
GW170817: Distance-binary inclination study



Statistical standard sirens (Schutz 1986)

Even without a counterpart BHB inspirals can still be used to extract cosmological information statistically [Schutz, 1986]

The idea is the following: consider each galaxy within the volume error box ($d\Omega \times dz$) of the GW source to have a non-zero probability of being the hosting galaxy and then statistically add up the information coming from all the galaxies in all boxes, with enough GW events the true value of cosmological parameters will emerge



(Serious problem: completeness of galaxy catalogues for far-away galaxies!)

Ground-based detectors: present and future

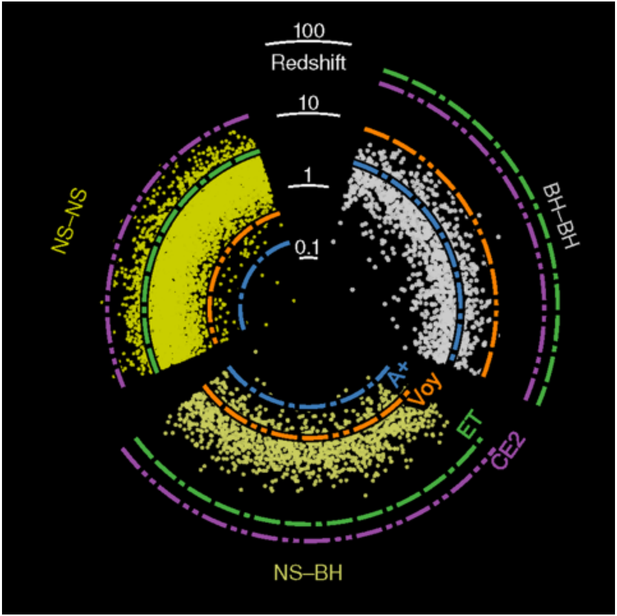
Cosmological forecasts for LIGO/VIRGO: [1612.06060,1710.06424]

- few % constraints on H_0 can be obtained either
  - with ~ 50 standard sirens with EM counterpart (NSBs)
  - with ~ 100 standard sirens without EM counterpart (BHBs)
- This accuracy will be achieved in the next years, but probably not with O3
- No estimates with NS-BH binary mergers yet

Cosmological forecasts for ET: [0906.4151]

- ET will detect thousands of NSB and BHB mergers up to $z \sim 3$
- Precise probe of the cosmic expansion at large redshifts
- Accurate measurement of the cosmological parameters

Future: Voyager, Cosmic Explorer, Einstein Telescope



Literature

- ⋆ ”Population Properties of Compact Objects from the Second LIGO-Virgo Gravitational-Wave Transient Catalog”, `arXiv:2010.14533`
- ⋆ ”Gravitational-Wave Astronomy Still in Its Infancy”, B. S. Sathyaprakash, M. Evans, `physics.aps.org/articles/v13/113`
- ⋆ ”Determining the Hubble constant from gravitational wave observations”, B. F. Schutz, Nature **323**, 310 (1986)