Bursts and continuous waves

8.12.20

General schedule

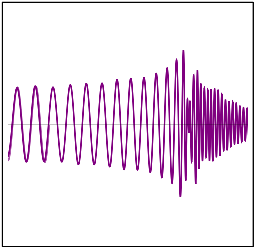
- ⋆ History
- ⋆ Introduction to general relativity
- ⋆ Detection principles
- ⋆ Detectors
- ⋆ Binary black-hole system
- ⋆ Bursts and continuous waves
  - ⋆ Taxonomy of GW sources and signal types,
  - ⋆ Burst sources: supernovæ,
  - ⋆ Continuous waves: rotating non-axisymmetric neutron stars.
- ⋆ Rates and populations & cosmology
- ⋆ Testing general relativity
- ⋆ Data analysis: waveforms and detection
- ⋆ Data analysis: parameter estimation

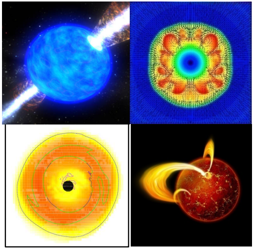
Taxonomy of GW sources

modelled

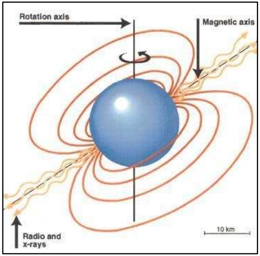
unmodelled

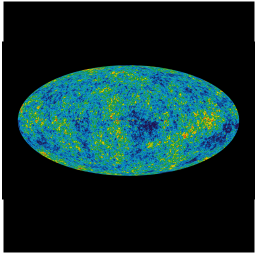
transient





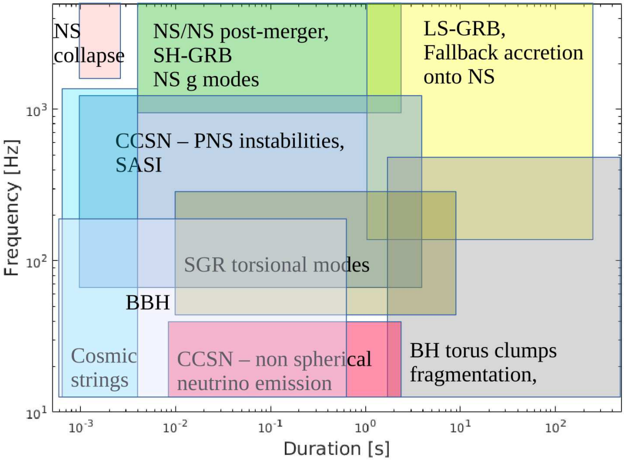
continuous





(P. Sutton, Banach Center, Warsaw, 2013/07/02)

Burst sources



”Collider” vs ”table top” experiments

- ⋆ Many potential sources, but the GW ‘engine‘ is not guaranteed
  - ⋆ ”opposite problem” to compact binary coalescences.

- ⋆ Discovery of a persistent source will be the capstone of GW astronomy:
  - ⋆ Reality of signal confirmed by need for corrections (modulation of the signal),
  - ⋆ Corrections give precise direction of source,
  - ⋆ Single interferometer can make definitive discovery,
  - ⋆ Repeatable studies,
  - → Not only NS interiors, but also
  - ⋆ Testing GR (polarizations etc.),
  - ⋆ Calibration, ”distance ladder”/cosmography.

Bursts as unmodeled signals

- Even though we don't use matched filters, the matched-filter signal-to-noise ratio (SNR) is still the natural measure of burst detectability (next lecture):

$$\rho^2 = 2\int_{-\infty}^{\infty} df\, \frac{|F_+\tilde{h}_+(f) + F_\times\tilde{h}_\times(f)|^2}{S(f)}\,. \qquad (1)$$

where $S(f)$ is the one-sided noise power spectral density (PSD):

$$\langle \tilde{n}(f)\tilde{n}^*(f')\rangle = \frac{1}{2}\delta(f-f')S(f),$$

where $\langle \ldots \rangle$ is the ensemble average over many noise $n(f)$ realisations (in reality, one noise realisation, so it is a time average for stationary stochastic noise).

(P. Sutton, Banach Center, Warsaw, 2013)

Bursts as unmodelled signals

- A standard signal measure for burst searches is the root-sum-square amplitude:

$$
\begin{aligned}
h_{\rm rss}^2 &= \int_{-\infty}^{\infty} dt \left[h_+^2(t) + h_\times^2(t) \right] &(3)\\
&= 2 \int_0^{\infty} df \left[|\tilde{h}_+(f)|^2 + |\tilde{h}_\times(f)|^2 \right] . &(4)
\end{aligned}
$$

- This has the same units as the noise spectrum, so we can use it to estimate the SNR. E.g., for unpolarised narrowband GWs we have

$$
\rho^2 \simeq [F_+^2 + F_\times^2] \frac{h_{\rm rss}^2}{S(f)} . \qquad (5)
$$

(P. Sutton, Banach Center, Warsaw, 2013)

Bursts as unmodelled signals

- We can relate h_{rss} to the energy emitted in GWs, E_{GW}, using the flux (energy per unit area per unit time):

$$F_{\mathrm{GW}} = \frac{c^3}{16\pi G}\langle \dot{h}_+^2(t) + \dot{h}_\times^2(t)\rangle \quad (6)$$

$$= \frac{\pi c^3}{4G}\frac{1}{T}\int_{-\infty}^{\infty} df\, f^2 \left(|\tilde{h}_+(f)|^2 + |\tilde{h}_\times(f)|^2\right) . \quad (7)$$

- For a narrowband signal & isotropic emission this gives

$$E_{\mathrm{GW}} = 4\pi r^2\, T\, F_{\mathrm{GW}} \quad (8)$$

$$= \frac{\pi^2 c^3}{G} r^2 f_0^2 h_{\mathrm{rss}}^2 . \quad (9)$$

(P. Sutton, Banach Center, Warsaw, 2013)

Bursts as unmodelled signals

- Combining eqns (5) and (9) relates SNR to energy:

$$\rho^2 = [F_+^2 + F_\times^2]\frac{G}{\pi^2 c^3}\frac{E_{\mathrm{GW}}}{S(f_0) r^2 f_0^2}\,. \tag{10}$$

- Averaging over angles (sky direction, source orientation) gives average range for detection with $\rho \geq \rho_0$:

$$\mathcal{R}_{\mathrm{eff}} \simeq \left(\frac{G}{2\pi^2 c^3}\frac{E_{\mathrm{GW}}}{S(f_0) f_0^2 \rho_{\mathrm{det}}^2}\right)^{1/2}\,. \tag{11}$$

- For a homogeneous isotropic population of sources of rate density $\dot{\mathcal{N}}$, the detection rate is

$$\dot{N} = \frac{4}{3}\pi \mathcal{R}_{\mathrm{eff}}^3 \dot{\mathcal{N}}\,. \tag{12}$$

(P. Sutton, Banach Center, Warsaw, 2013)

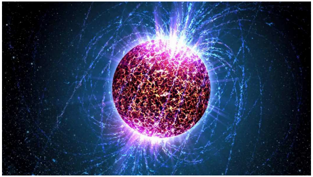
Why bursts?

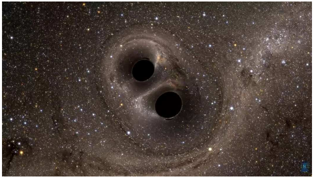
- Historically, transient signal whose waveforms are not accurately known or very complex such that templated searches are not affordable.

  Not totally true :

  - Cosmic string → templates do exist
  - Compact Binary mergers → templates do exist

- Transient : duration typically < 1s but some GW signals duration O (100s)





- Astrophysical GW sources : neutron star and black holes → core collapse supernova, black hole merger, fallback accretion onto a neutron star, neutron star instabilities (post-merger), magnetar flares, …

- Many GW sources are emitting photons and neutrinos → multi-messengers.

(Marie Anne Bizouard, Ecole de Physique des Houches, 2018)

Core-collapse supernovæ as GW burst sources

- The explosion mechanisms :

	Type I	Type Ib/Ic & type II
Progenitors	White dwarfs	Massive stars
Explosion mechanism	Matter/gas falls onto a 'dead' white dwarf raising its mass until the Chandrasekhar limit. → triggers runaway nuclear fusion explosion that destroys the star.	The core runs out of fuel to power its nuclear fusion reactions and collapses in on itself. → release gravitational potential energy in a form that blows away the star's outer layers.

OK but how ?

(Marie Anne Bizouard, Ecole de Physique des Houches, 2018)

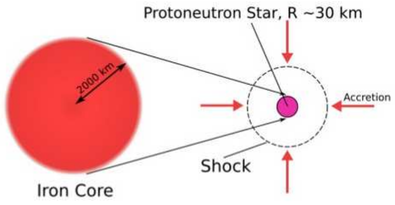
Core-collapse supernovæ as GW burst sources

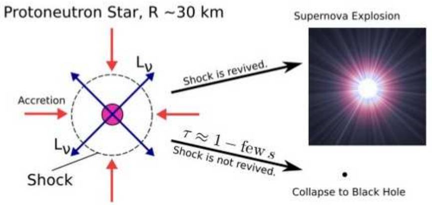
Core bounce

- Nuclear equation of state stiffens → rebound of the inner core ("core bounce").
- A hydrodynamic shock wave is launched at the outer edge of the inner core and propagates outward in mass and radius, slamming into the still infalling outer core.

After core bounce

- The shock quickly loses energy (dissociation of heavy elements + neutrino losses) and stalls.
- Without shock revival, black-hole (BH) formation is inevitable and even with a successful explosion, a BH may still form via fall-back accretion.

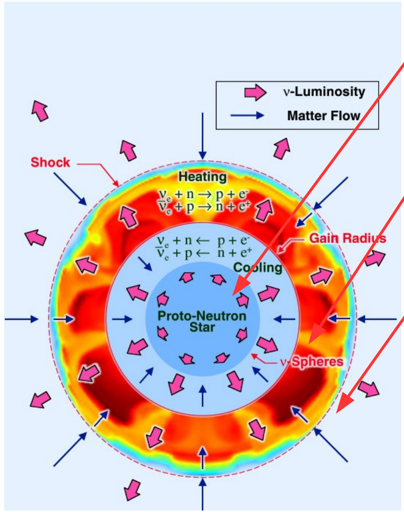




(Marie Anne Bizouard, Ecole de Physique des Houches, 2018)

CCSN engine mechanism

- The explosion mechanism current paradigm : neutrino-driven delayed explosion (Wilson 82', Bethe&Wilson 85')



1. Trapped neutrinos diffuse out ($\tau_{\nu\text{-diff}} >> 1$) of the opaque PNS

2. Neutrinos heat matter in semi-transparant ($\tau_{\nu\text{-diff}} \sim 1$) post-shock region and drive convective flow in hot bubble region between gain radius and shock

3. Neutrinos stream freely ($\tau_{\nu\text{-diff}} << 1$) through transparent stellar envelope.

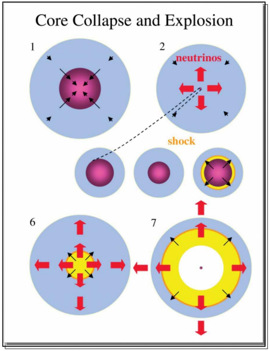
Additional key ingredients for explosion :

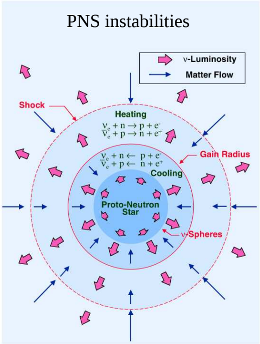
- Nuclear burning.
- Standing accretion shock instability (SASI) is an instability of the shock wave itself. SASI aids the explosion and determines the asphericity.

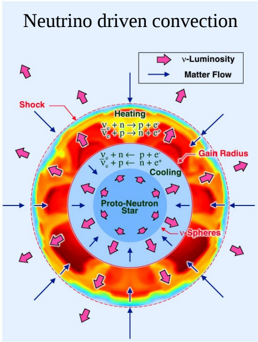
(Marie Anne Bizouard, Ecole de Physique des Houches, 2018)

CCSN engine mechanism

- GW emission mechanisms : rotating collapse and bounce, non axisymmetric rotational instabilities, postbounce convective overturn/standing accretion shock instability (SASI) and PNS pulsations.
- Neutrinos emission : A large correlation between neutrinos and GW time evolution signals is expected because of SASI (sloshing).







(Marie Anne Bizouard, Ecole de Physique des Houches, 2018)

CCSN and GW emission

P. Cerda-Duran et al, Astrophys.J. 779 (2013) L18

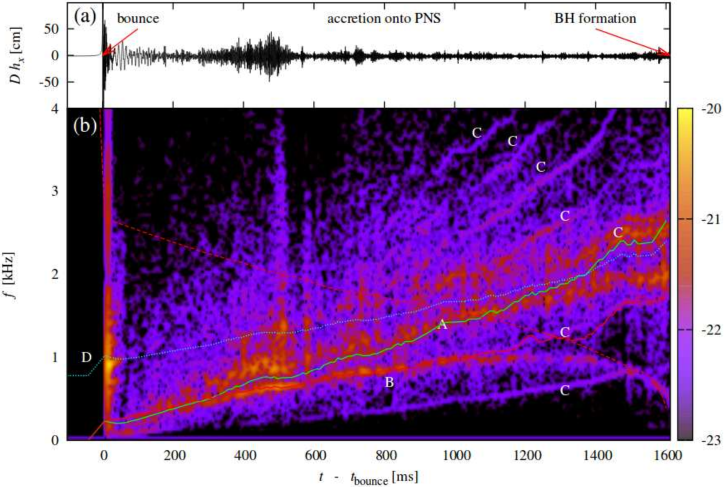
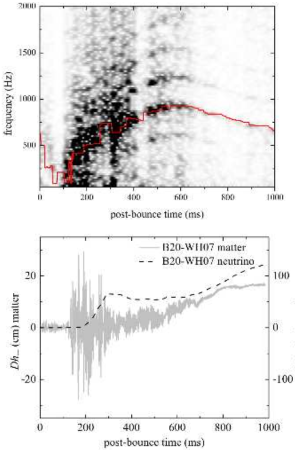


FIG. 3.— Waveform (a) and spectrogram (b) of the characteristic gravitational wave signal for the *fiducial model* at $D = 100$ kpc. We overplot estimates for the frequency evolution of g-modes at the surface of the PNS (solid-green line), g-modes in the cold inner core (solid-red line), quasi-radial mode (dashed-red line) and f-mode (dotted-blue line). Capital letters point to features described in the main text.

CCSN and GW emission

Yakunin et al, Phys.Rev. D92 (2015) no.8, 084040



Morozova et al, Astrophys.J. 861 (2018) no.1, 10

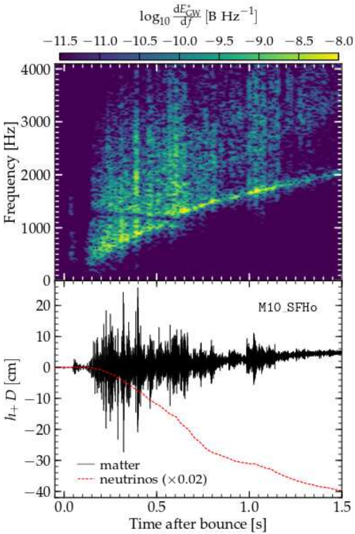
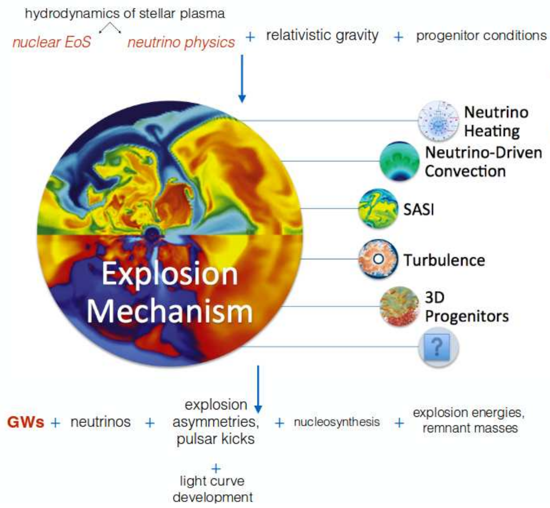


Figure 1. Spectrogram (top) and the corresponding waveform (bottom) of the GW signal from the model M10_SFHo.

(Marie Anne Bizouard, Ecole de Physique des Houches, 2018)

CCSN - what can we learn?



What can we learn from CCSN GW signatures?

Find explosion mechanism

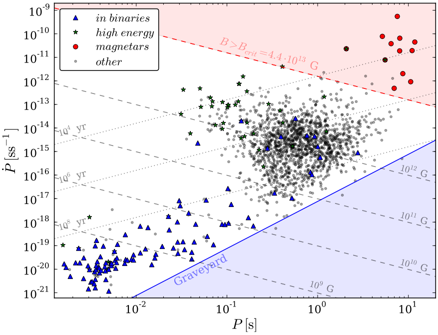
- Nuclear Equation of State,
- evolution of ProtoNeutron Star,
- asymmetry of explosion
- Set *upper limits on energy emitted by supernova*
- Measure angular momentum and rotational rate of ProtoNeutron Star
- Spot formation of Black Hole at its birth

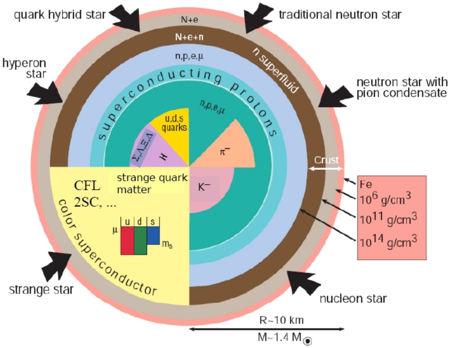
Credit : T. Mezzacappa

(Marie Anne Bizouard, Ecole de Physique des Houches, 2018)

Neutron stars = very dense, magnetized stars

The most relativistic **material** objects in the Universe: compactness $M/R \simeq 0.5$, observed in all EM spectrum as pulsars, magnetars, in supernovæ remnants, in accreting systems, in double neutron star binaries...





About 2500 NS observed to date, $\sim 10^8 - 10^9$ in the Galaxy.

Continuous GWs from spinning neutron stars

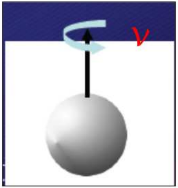
Characteristics:

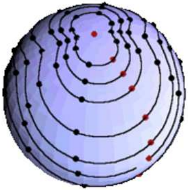
- ⋆ Long-lived: $T > T_{obs}$,
- ⋆ Nearly periodic: $f_{GW} \propto \; f_{rot}$

Mechanisms that can create time-varying quadrupole moment:

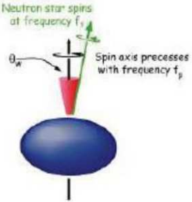
- ⋆ "Mountains" (elastic and/or magnetic stresses, $f_{GW} = 2f_{rot}$),
- ⋆ Oscillations (r-modes, $f_{GW} = 4/3f_{rot}$ + GR corr.),
- ⋆ Free precession ($f_{GW} \propto f_{rot} + f_{prec}$)
- ⋆ Accretion (drives deformations from r-modes, thermal gradients, magnetic fields, $fGW \simeq f_{rot}$)

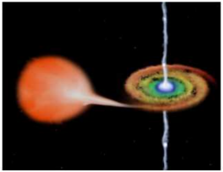
(see PASA 2015 **32**, 34 or Universe 2019, **5(11)**, 217)





Courtesy: B. J.Owen





Courtesy: McGill U.

Mountains and NS oscillation modes

NS instabilities and their possible driving mechanisms:

- ⋆ Pressure modes: driven by pressure,
- ⋆ Fundamental mode: (aka “Kelvin mode”) the first (nodeless) p-mode,
- ⋆ Gravity modes: driven by buoyancy (thermal/composition gradients),
- ⋆ Inertial modes: driven by rotation (Coriolis force),
- ⋆ Magnetic (Alfven) modes: driven by the magnetic force,
- ⋆ Spacetime modes: like BH QNMs, need dynamical spacetime,
- ⋆ Shear modes: driven by elastic forces in the crust,
- ⋆ Superfluidity-related modes (e.g. Tkachenko modes: driven by tension of superfluid vortex array)

GW amplitude and the spindown limit

In general

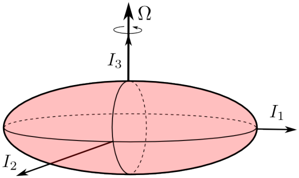
$$h_{\mu\nu} = \frac{2G}{dc^4}\frac{d^2 Q_{\mu\nu}}{dt^2}$$

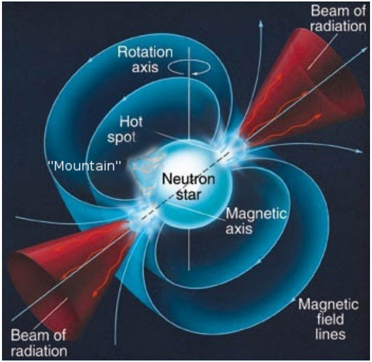
with $Q \propto I$ we get

$$\text{GW strain } h_0 = \frac{4\pi^2 G}{c^4}\frac{I_3 \epsilon f_{GW}^2}{d}$$

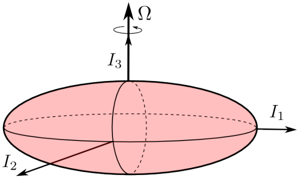
where d is the distance to the source and $\epsilon = (I_1 - I_2)/I_3$ is the deformation (non-axisymmetry).

Depending on the dense matter model, $\epsilon_{max} = 10^{-3} - 10^{-6}$.





GW amplitude and the spindown limit



Rotational energy: $E_{rot} \propto f^2$

Rotational energy loss: $\dot{E}_{rot} \propto f\dot{f}$

Energy emitted in GWs: $\dot{E}_{GW} \propto f^6 I_3^2 \epsilon^2$

Spindown upper limit: a gravitar, in which the observed spindown is fully due to GWs

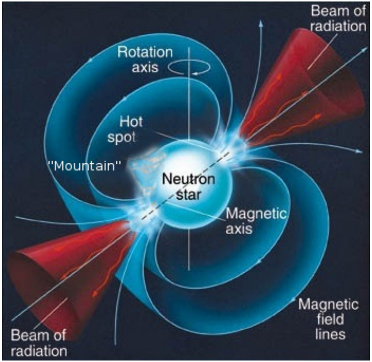
$$\dot{E}_{rot} = \dot{E}_{GW}$$

Assuming the knowledge of I_3 and d

$$\rightarrow \text{upper limit } h_0^{sd} = \frac{1}{d}\sqrt{\frac{5G}{2c^3}\frac{|\dot{f}|}{f} I_3}$$

$$h_{\rm sd} = 8.06 \times 10^{-19} I_{38}^{1/2} \left[\frac{1\rm kpc}{d}\right] \left[\frac{\dot{f}_{\rm rot}}{\rm Hz/s}\right]^{1/2} \left[\frac{\rm Hz}{f_{\rm rot}}\right]^{1/2}$$

$$\epsilon_{\rm sd} = 0.237\, I_{38}^{-1} \left[\frac{h_{\rm sd}}{10^{-24}}\right] \left[\frac{\rm Hz}{f_{\rm rot}}\right]^2 \left[\frac{d}{1\rm kpc}\right]$$



Direct spindown limit

It is useful to define the "direct spindown limit" for a known pulsar, under the assumption that it is a "gravitar", i.e., a star spinning down due to gravitational wave energy loss

Unrealistic for known stars, but serves as a useful benchmark

Equating "measured" rotational energy loss (from measured period increase and reasonable moment of inertia) to GW emission gives:

$$h_{SD} = 2.5\times 10^{-25}\left[\frac{kpc}{d}\right]\sqrt{\left[\frac{1kHz}{f_{\mathrm{GW}}}\right]\left[\frac{-df_{\mathrm{GW}}/dt}{10^{-10}\,Hz/s}\right]\left[\frac{I}{10^{45}\,g\cdot cm^2}\right]}$$

Example:

Crab → $\mathbf{h_{SD} = 1.4 \times 10^{-24}}$

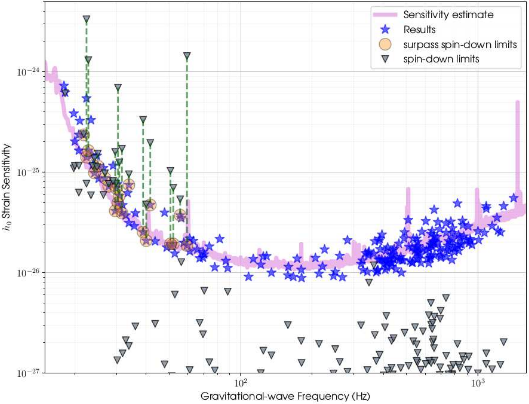
(d=2 kpc, f_{GW} = 59.5 Hz, $df_{GW}/dt = -7.4\times10^{-10}$ Hz/s)

(K. Riles, Boston A.A.S. Summer Meeting 2011)

O1/O2 CW search for known pulsars (arXiv:1902.08507)

222 known pulsars analyzed (55 new sources), at $l = m = 2$ and $l = 2, m = 1$ mode.

- ⋆ 20 targets surpass the spin-down limit:
  - ⋆ Crab: less than $0.017\% \dot{E}_{rot}$ in GW,
  - ⋆ Vela: less than $0.18\% \dot{E}_{rot}$.



⋆ PSR J0711-6830 millisecond recycled pulsar ($f_{rot} \simeq 182$ Hz) → ellipticity $\epsilon < 1.2 \times 10^{-8}$.

Indirect spindown limit

If a star's age is known (e.g., historical SNR), but its spin is unknown, one can still define an <u>indirect</u> spindown upper limit by assuming gravitar behavior has dominated its lifetime:

$$\tau = \frac{f}{4\,(df\,/\,dt)}$$

And substitute into h_{SD} to obtain
[K. Wette, B. Owen,… CQG 25 (2008) 235011]

$$h_{ISD} = 2.2\times10^{-24}\left[\frac{kpc}{d}\right]\sqrt{\left[\frac{1000\,yr}{\tau}\right]\left[\frac{I}{10^{45}\,g\cdot cm^2}\right]}$$

Example:

Cassiopeia A → **h_{ISD} = 1.2 × 10^{-24}**

(d=3.4 kpc, **τ**=328 yr**)**

(K. Riles, Boston A.A.S. Summer Meeting 2011)

Accretion, X-ray flux torque vs GW emission

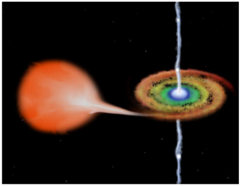
For an LMXB, equating accretion rate torque (inferred from X-ray luminosity) to gravitational wave angular momentum loss (steady state) gives: **[R.V. Wagoner ApJ 278 (1984) 345; J. Papaloizou & J.E. Pringle MNRAS 184 (1978) 501; L. Bildsten ApJ 501 (1998) L89]**

$$h_{X-ray} \approx 5\times10^{-27}\sqrt{\left[\frac{600Hz}{f_{sig}}\right]\left[\frac{F_x}{10^{-8}\,erg\cdot cm^{-2}\cdot s^{-1}}\right]}$$

Example: Scorpius X-1

→ **$h_{X\text{-}ray} \approx 3 \times 10^{-26}$** $[600\text{ Hz} / f_{sig}]^{1/2}$

($F_x = 2.5 \times 10^{-7}\ erg\cdot cm^{-2}\cdot s^{-1}$)



Courtesy: McGill U.

(K. Riles, Boston A.A.S. Summer Meeting 2011)

Torque balance: accretion vs GW emission

1. GW strain from mass quadrupole is $h_{ij} = \frac{2G}{dc^4}\frac{d^2 I_{ij}}{dt^2}$,

   For a rotating deformed NS: $h_0 = \frac{2G}{dc^4}\omega_{\rm gw}^2 \epsilon I_0$, with

   - ⋆ $\omega_{\rm gw} = 2\pi f_{\rm gw} = 4\pi f_{rot}$,
   - ⋆ $f_{\rm gw} = 2 f_{rot}$ is the signal frequency, f_{rot} is the spin frequency, and I_0 is the unperturbed moment of inertia,

   $\rightarrow$ $h_0 = \frac{8\pi^2 G}{dc^4} f_{\rm gw}^2 \epsilon I_0$

2. GW torque $N_{\rm gw} = L_{\rm gw}/\omega_{rot}$, because $\frac{d(I_0\omega_{rot}^2/2)}{dt} = I_0\omega_{rot}\dot{\omega}_{rot} = N_{\rm gw}\omega_{rot}$.

   $\rightarrow$ GW luminosity is $L_{\rm gw} = \frac{G}{5c^5}\epsilon^2 I_0^2 \omega_{\rm gw}^6$, so $N_{\rm gw} = \frac{32\pi^5 G}{5c^5}\epsilon^2 I_0^2 f_{\rm gw}^5$,

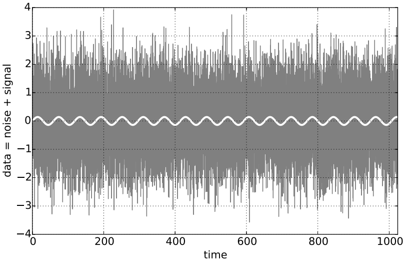
3. Accretion torque $N_{\rm a} = \dot{M}\sqrt{GMr_A}$, where r_A is Alfven radius,

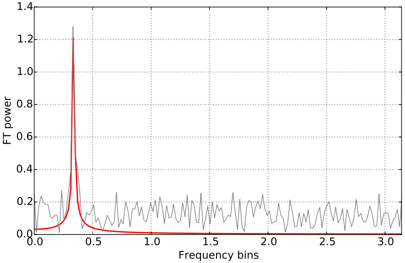
   Accretion rate is related to X-ray flux via $\dot{M} = 4\pi d^2 F_X R/XGM$, where R is stellar radius, M is stellar mass, and X is efficiency.

4. Balance equation: $N_{\rm a} = N_{\rm gw}$ to get ϵ, and estimate h_0:

$$h_0 = \frac{20^{1/2} G^{1/4}}{c^{3/2}} \left(\frac{F_X}{X}\right)^{1/2} R^{1/2} f_{\rm gw}^{-1/2} r_A^{1/4} M^{-1/4}$$

Matched filtering: a monochromatic signal





In this case a Fourier transform is sufficient to detect the signal (simplest matched filter method):

$$F = \int_0^{T_0} x(t) \exp(-i\omega t) dt$$

Signal-to-noise $SNR = h_0 \sqrt{\frac{T_0}{S_0}}$

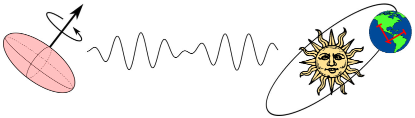
T_0 - time series duration, S_0 - spectral density of the data.

(see

`users.camk.edu.pl/bejger/snr-periodic-signal`)

In reality: signal is modulated

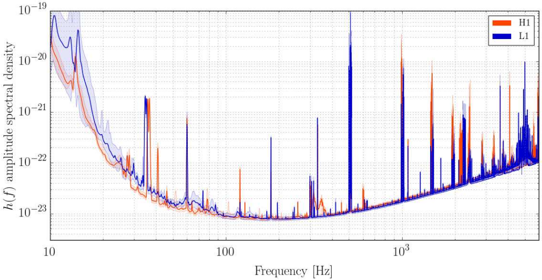
Since the detector is on Earth, planets and Earth's rotation influences signal's amplitude and phase.



- ⋆ Signal is almost monochromatic: sources may slow down/spin up,
- ⋆ it has to demodulated (detector is moving),

→ precise ephemerides of the Solar System needed.

Detector movement distinguishes a real signal from detector's spectral artifacts ("lines").



Example: the $\mathcal{F}$-statistic

$\mathcal{F}$-statistic estimates how well the amplitude and phase modulated model matches the data $x(t)$

$$\mathcal{F} = \frac{2}{S_0 T_0} \left(\frac{|F_a|^2}{\langle a^2 \rangle} + \frac{|F_b|^2}{\langle b^2 \rangle} \right)$$

where S_0 is the spectral density, T_0 is the observation time, and

$$F_a = \int_0^{T_0} x(t) a(t) \exp(-i\phi(t)) dt, \qquad F_b = \ldots$$

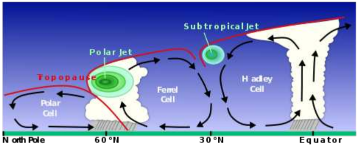
$a(t)$, $b(t)$ - amplitude modulation functions that depend on the sources' sky position $(\alpha,\ \delta)$,

$\phi(t)$ - phase modulation function that depends on $(f, \dot{f}, \alpha, \delta)$

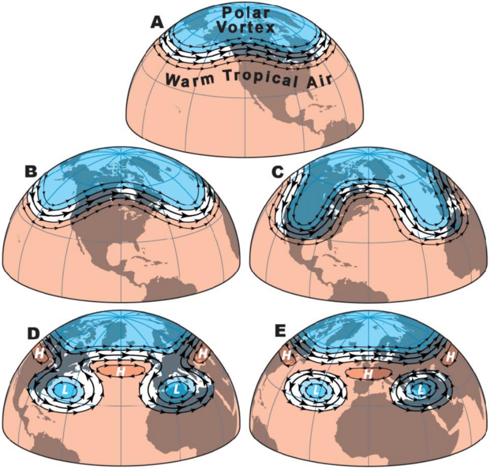
(PRD **58**, 063001, 1998)

Rossby waves in planetary atmospheres

- ⋆ Carl-Gustaf Rossby (1898 - 1957),
- ⋆ A type of inertial planetary wave, driven by Coriolis force (→ present in rotating systems),
- ⋆ On Earth, associated with high-altitude winds (→ jet stream),



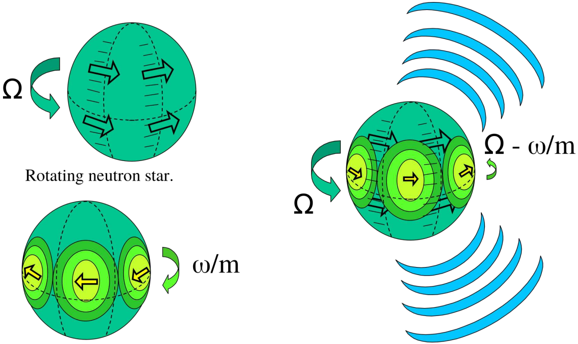
(Wikipedia)



(S. Harris, AIMS Environmental Science 6(1):14-40 2019)

Rossby waves on neutron stars: r-modes

GW emission from time-varying current quadrupole:



Rotating neutron star.

Perturbations in rotating frame.

Star drags perturbations in opposite direction. GR drives mode instability.

- ⋆ r-mode frequency in the rotating frame: $\omega_r = \kappa\Omega$, with $\kappa = \frac{2m}{l(l+1)}$.
- → for $l = m = 2$, $\omega_r = 2/3\Omega$,
- ⋆ In the inertial frame: $\omega_i = \omega_r - m\Omega = -4/3\Omega$.

Rossby waves on neutron stars: r-modes

- ⋆ r-modes belong to a subset of inertial modes supported by rotation (Coriolis force as a restoring mechanism),
- ⋆ retrograde in frame co-rotating with the star, prograde in inertial frame $\rightarrow$ unstable to Chandrasekhar-Friedman-Schutz (CFS) instability,
- ⋆ the amplitude of the mode evolves $\propto \exp(i\omega_i t - t/\tau)$,
- ⋆ modes damped by viscous processes (shear and bulk viscosity), depend on dense matter equation of state,

$$\frac{1}{\tau} = -\frac{1}{\tau_{GR}} + \frac{1}{\tau_S} + \frac{1}{\tau_B},$$

- ⋆ $l = m = 2$ mode frequency in Newtonian approximation is $\omega_i = 4/3\Omega$, in GR corrections related to NS mass and radius ($\rightarrow$ EOS):

$$\omega_i = \frac{4}{3}\Omega\left(1 + C_1\frac{GM}{Rc^2} - C_2\left(\frac{GM}{Rc^2}\right)^2\right),$$

where the $(1 + \dots)$ corrections are specifically due to GR, rapid rotation, NS crust, matter stratification, magnetic fields...

(see Idrisy et al. 2014, `arXiv:1410.7360`)

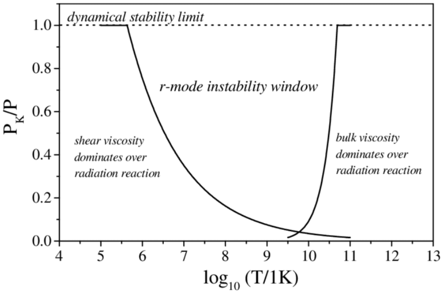
Key early r-mode references: Papaloizou & Pringle (1978); Andersson (1998); Friedman & Morsink (1998); Lindblom et al. (1998); Owen et al. (1998); Andersson et al. (1999); Andersson & Kokkotas (2001)

R-modes instability window

R-modes in newborn NSs (Owen et al. 1998), characteristic strain is:

$$h_0 = 4.4 \times 10^{-24} \alpha \left(\frac{\omega}{\sqrt{\pi G \bar{\rho}}} \right)^3 \left(\frac{20\ Mpc}{d} \right)$$

where $\bar{\rho}$ is the mean NS density, and α is dimensionless r-mode amplitude (see Sect. 4 in `arXiv:1909.12600` for a review).



$$\frac{1}{\tau} = -\frac{1}{\tau_{GR}} + \frac{1}{\tau_S} + \frac{1}{\tau_B}$$

NS (astro)physics questions

- ⋆ Magneto-elastic 'mountains': elastic properties of the crust, braking strain,
- ⋆ Thermally induced quadrupole: accretion processes, heating reactions in the crust,
- ⋆ Instabilities (r-modes): heating & cooling, rotational evolution,
- ⋆ Superfluidity.

- ⋆ Conditions at birth: SN $\leftrightarrow$ NS deformation connection,
- ⋆ Long-term evolution of NS asymmetry,

- ⋆ Available populations $\leftrightarrow$ search strategies.

Literature

- ★ ”CFS instability”, `http://www.personal.soton.ac.uk/dij/cfs.html`
- ★ ”Gravitational Wave Astronomy”, K. D. Kokkotas, `arXiv:0809.1602`
- ★ ”Gravitational wave physics", K. D. Kokkotas, `www.tat.physik.uni-tuebingen.de/~kokkotas/Teaching/NS.BH.GW_files/GW_Physics.pdf`
- ★ ”Continuous gravitational waves from neutron stars: current status and prospects”, M. Sieniawska, MB, `arXiv:1909.12600`
- ★ ”Gravitational Waves from Neutron Stars: A Review”, P. Lasky, `arXiv:1508.06643`