Gen Relativ Gravit (2009) 41:2179–2189
DOI 10.1007/s10714-009-0868-9

GOLDEN OLDIE EDITORIAL

Editorial note to: Pascual Jordan, Jürgen Ehlers and Wolfgang Kundt, Exact solutions of the field equations of the general theory of relativity

George Ellis

Published online: 13 August 2009

Keywords General relativity · Solution of field equations · Pascual Jordan · Jürgen Ehlers · Golden Oldie

The Mainz series of papers

The series of papers now being reprinted as Golden Oldies were the product of Pascual Jordan's strong relativity group at Hamburg University in the immediate post-war period, initially comprising Jordan, Jürgen Ehlers, and Wolfgang Kundt, and later also Rainer Sachs and Manfred Trümper. Jordan made major contributions to quantum mechanics and quantum field theory[1] (Jordan algebras are named after him) before switching attention to gravitation and cosmology, mainly working on variable-G theories and authoring the book *Schwerkraft und Weltall* [1]. This group interacted strongly with Otto Heckmann and his student Engelbert Schücking at Hamburg Observatory. Heckmann directed Hamburg Observatory from 1941 to 1962, after which he became the first director of ESO (the European Southern Observatory) and won the Bruce Gold Medal.[2] He wrote the book *Theorien der Kosmologie* [2] in 1942, which laid some of the foundations for the work of Jordan's group by developing the kinematic quantities and dynamic equations for fluid flows first in the Newtonian case, then generalized to the relativistic case with Schücking [3,4].

1 See B. Schroer: http://arxiv.org/abs/hep-th/0303241.
2 see http://www.phys-astro.sonoma.edu/BruceMedalists/Heckmann/.

The republication of the original paper can be found in this issue following the editorial note and online via doi:10.1007/s10714-009-0869-8.

G. Ellis (✉)
Mathematics Department, University of Cape Town,
Rondebosch, Cape Town 7700, South Africa
e-mail: George.Ellis@uct.ac.za

Jürgen Ehlers started work with Jordan as his advisor in 1954, working on the construction and characterization of solutions of the Einstein field equations. He passed his Staatsexamen in 1955 and was awarded a doctorate in 1957. Wolfgang Kundt obtained his Diploma in theoretical physics in 1956 and Ph.D. in 1959, also with Jordan as supervisor. Schücking started his seminar work at the same time as Kundt. Rainer Sachs obtained his Ph.D. at Syracuse University with Peter Bergmann as supervisor, and joined the Hamburg group as a postdoc in late 1958, just before Kundt went to Syracuse in January 1959 (and before the official award of the PhD to Sachs in 1959). Kundt returned to Hamburg in the spring of 1960, when Sachs returned to the States. Trümper was a later Ph.D. student of Jordan.

Schücking states[3] "The first edition of Jordan's book came out in 1952 when I began to work with Jordan, and the second edition carries the subtitle 'Zweite erweiterte Auflage Bearbeitet unter Mitwirkung von E. Schücking'. This second edition contains extensive results from my Master's and PhD Theses. I was already working with Jordan when Ehlers and Kundt began to study with him. It was thus natural that Jordan brought us together because he wanted us to work on his theory. I did not believe in Jordan's scalar-tensor theory and Juergen was also not enthusiastic about it. We both felt that Einstein's 1916 theory had not been understood. Since Jordan had no longer time to actively work on his theory (he was a member of Parlament, gave numerous popular science talks and worked on his theory of skew lattices) we subverted his seminar into one on GR."

Kundt states[4] "The core of the Hamburg group was Engelbert, Juergen, and me, initiated by Jordan, and mostly attended by Jordan, rarely by Heckmann. Initially, Jordan contributed actively, later hardly. But he also attracted guests like Pauli, Joshua Goldberg, Peter Bergmann". Ehlers helped focus the group's work on the structure and interpretation of Einstein's general theory of relativity.[5] More than that, the focus was on exact solutions (*Strenge Lösungen*) of the Einstein Field Equations of General Relativity Theory, seeing what could be achieved without approximations: a crucial effort in the case of a theory as non-linear as General Relativity. It resulted in a major series of papers, the so-called Hamburg Bible [5–9], setting out the results of this work.[6] Jordan's name was on the first three of the papers, although he in fact did not take part in writing them: his name was there simply because he was the head of the group from which they came. Schücking played an important part in the seminar discussions, even though his name does not appear as an author in any of this series of papers.[7] Why were they published in the Proceedings of the Akademie der Wissenschaften und der Literatur in Mainz, when the authors were based in Hamburg? Well, Jordan was elected a member of that Academy in 1949, and was its president

[3] Private communication.

[4] Private communication.

[5] His influence is described nicely in the Wikipedia entry on Ehlers, which has substantially helped formulate the following.

[6] There were a couple of others published somewhat later by I. Ozsváth, but they were really a separate series.

[7] For an example of his influence, see [10].

from 1963 till 1967. He regularly published in their proceedings, probably because they would readily take lengthy survey articles.

The series was important not just for the systematic compendium of useful information they provided, but also for the 'exact solutions' approach to the subject that they (together with other workers such as André Lichnerowicz and John Lighton Synge) fostered. The fourth one (by Ehlers alone) has already been published as a Golden Oldie, translated into English [8]; the others in the series will now be so published, beginning with the classic paper by Ehlers and Kundt [5].

Mainz 1

The first paper in the series [5], written by Ehlers and Kundt, is a systematic exposition of the properties and characteristics of exact vacuum solutions of the Einstein field equations, utilizing the Petrov classification of the Weyl tensor, isometry groups, and conformal transformations. This work gives the first definition and classification of pp-gravitational waves. The main results contained here were later published (in English) in a chapter in the 1962 survey book edited by Louis Witten [11] (which also contains an excellent cosmology survey by Heckmann and Schücking, as well as the original ADM paper, recently published as a Golden Oldie).

Mainz 2

The second paper in the series [6], written by Ehlers and Sachs, was a treatise on electromagnetic and gravitational radiation. It studies vacuum solutions with special algebraic properties, using the 2-component spinor formalism recently introduced into GR by Roger Penrose [12]. It also gives a systematic exposition of the geometric properties of congruences of null geodesics in terms of their expansion, twist, and shear, in particular giving the null version of the Raychaudhuri equation. This work develops from the previous work on timelike congruences by Heckmann and Schücking [3,4] and by Gödel [13]. One of the results is the Ehlers-Sachs theorem describing the properties of shadows produced on a screen by a light beam passing an opaque object. The methods developed here were important later in such results as the Goldberg-Sachs theorem [14], and the derivation of exact solutions such as the Kerr solution.

Mainz 3

In the third paper in the series [7], Kundt examined solutions of the Einstein–Maxwell equations with a null electromagnetic field, that is a field represented by a null bivector. The paper hinges on proving that the associated null rays are shearfree geodesics.

Mainz 4

The fourth paper [8], by Ehlers, dealt with the general-relativistic treatment of the mechanics of continuous media. It particularly developed the use of the kinematic quantities for timelike curves to characterise fluid solutions, and gives a particularly clear covariant derivation of the fundamentally important Raychaudhuri equation [15], generalized to non-geodesic curves and arbitrary matter. It also includes a brief introduction to general relativistic kinetic theory. In terms of its clarity, conciseness, and focus, I regard this as one of the best papers ever written in General Relativity Theory.

Mainz 5

The final one in this series [9] is by Kundt and Trümper, and dealt with the propagation of gravitational radiation when matter is present. It is noteworthy for the systematic use of the covariant full Bianchi identities to determine exact properties of such solutions with null (type N) Weyl tensors, proving interesting non-existence theorems for the matter flows in these cases and examining properties of pure radiation fields.

The present paper

The first paper in the series is the paper [5] published here. The later ones will be published in subsequent Golden Oldies.

This paper[8] is in four parts, the first two parts deriving useful mathematical tools, and the second two applying them. The first part analyses the geometry of bivectors and the algebraic classification of the Weyl tensor, giving what is in effect a 1 + 3 decomposition of that tensor and the associated normal forms. This develops out of work by A Z Petrov [16], its importance for gravitational theory having been recognized by Felix Pirani [17]. This part also summarizes properties of isometry groups and the nature of the equivalence problem. The second part presents ways of calculating the curvature tensor, giving reduction formulae for curvature of subspaces, product spaces, and for spaces with abelian isometry groups.

The third part is a systematic summary of exact properties of axially symmetric static solutions of the vacuum gravitational field equations, in particular characterising all degenerate subcases, including the Schwarzschild family of vacuum solutions and the Weyl solutions. The last part examines plane-fronted gravitational waves in detail, giving their line elements, physical interpretation, and symmetry properties of the various subclasses, such as plane fronted waves with parallel rays. It closes with an examination of homogeneous plane-fronted waves. The work in these two parts formed a sound basis for further examinations of exact vacuum solutions. As stated above, a shorter presentation is contained in the book edited by Louis Witten [11]; it is now as difficult to get hold of that book as it is to get copies of the Mainz papers.

Pascual Jordan: a brief biography

By A. Krasiński compiled from Refs. [18] and [19]

Pascual Ernst Jordan was born on 18 October 1902 in Hannover. His parents were Ernst Jordan and Eva Jordan b. Fischer.

[8] From the frontispiece: “Presented to the Academy by Mr. Jordan at the Plenary Sitting of 24 October 1959, accepted for printing the same day, published on 20 August 1960.”

He began his studies in 1921 at the Technical University in Hannover. In 1923 he moved to the University of Göttingen, where he graduated in 1924 as a student of Max Born, and then got his habilitation in 1926. While there, he attended mathematics courses by Richard Courant, and later became his assistant. He provided help on the “Methods of mathematical physics” by Courant and Hilbert. In 1927 he became a *Privatdozent* at the University in Hamburg, and in 1928 a professor at the University in Rostock.

In 1944 he became professor at the Institute of Theoretical Physics of the University in Berlin, as a successor of Max von Laue. In 1947–1953 he was a guest professor, and then in 1953–1971 an ordinary professor of theoretical physics at the University in Hamburg. He died in Hamburg on 31 July 1980.

Pascual Jordan, together with his teacher Max Born, contributed substantially to the formulation of Heisenberg’s matrix mechanics, in particular he found a proof of the $[p, q]$ commutation relation proposed by Born. He was also the first to discover what is now known as the Fermi–Dirac statistics, but his priority was lost because of an accidental disaster. Max Born, then the editor of Zeitschrift für Physik, buried Jordan’s manuscript in his suitcase when preparing for a long trip to the USA, and discovered it only half a year later. At that time, the papers by Fermi and Dirac were already in press. As the author of Ref. [18] said, the fermions might have been called “jordanons”, were it not for Born’s negligence.

In the years 1926/27, Jordan became one of three authors who independently proved the equivalence of Heisenberg’s and Schrödinger’s formulations of quantum mechanics; the other two were Dirac and Fritz London. Jordan also introduced the “quantization of wave fields”, which became the starting point for quantum field theory.

The list of Jordan’s selected publications, given in Ref. [19], contains also contributions to physics in general, relativity, astronomy and cosmology, geophysics (among other things, the now-abandoned theory of an expanding Earth), mathematics, biology, philosophy and history of culture, and biographies of scientists. More than 20 entries in this list are books, some of them had up to 6 editions. His full scientific output is stored in the State Library of the Prussian Culture Heritage (Staatsbibliothek Preußischer Kulturbesitz) in Berlin.

In 1933 Jordan joined the National-Socialist German Workers’ Party (NSDAP) and its semi-military wing, the Sturm Abteilung (Storm Troopers, SA), known as the “brown shirts”. He was a devoted active member and wrote several propaganda articles in the national-socialist newspapers. His underlying psychological problems and his behaviour are partly explained in Ref. [18]. Reportedly, he believed that he would be able to use his fame to tame the Hitler regime.[9] This caused serious difficulties for him after the war, when, despite his great scientific achievements, he was occasionally considered an embarrassing presence. In particular, he was unemployed for two years, and later, until 1953, was not allowed to carry out some of his professor’s duties, for example he could not advise PhD candidates. His pre-war political activity may

[9] Such a belief was later popular among a group of cultural and scientific celebrities of Eastern European countries that were enslaved in the communist camp. However, practice proved that the influence was rather in the opposite direction.

have also cost him his Nobel Prize, for which he was nominated a few times, always unsuccessfully. Nevertheless, he did not follow the advice of his colleagues to stay out of politics. He again alienated himself from them by joining the Christian Democratic Union (CDU), then ruling West Germany, and writing an article that supported chancellor Adenauer's controversial policy on re-armament of Germany.

Perhaps as a result of these turbulences in his life, after world war II he did not continue his highly successful research in quantum theory and mathematical physics. His greatest achievement after the war was creating (and securing material support for) the conspicuously successful group of researchers in relativity at the Hamburg University. Its (in some cases temporary) members were, among others, Jürgen Ehlers, Wolfgang Kundt, Rainer Sachs, Engelbert Schücking and Manfred Trümper.

Jordan died on 31 July 1980 while working on the theory of gravitation with a time-dependent gravitational coupling (its refinement was the once semi-successful Brans–Dicke theory).

The 208-page volume that contains Ref. [18] is a comprehensive description and evaluation of the whole of Jordan's scientific legacy. In particular, Ref. [20] in the same volume contains a probably complete listing of Jordan's publications in all areas, including those he wrote under a pseudonym. Ref. [21] contains some personal recollections of the author about Jordan.

Acknowledgment I am grateful to Wolfgang Kundt for very valuable corrections to this note.

The biography of **Jürgen Ehlers** is printed elsewhere in this issue.

Wolfgang Kundt—a brief autobiography

By W. Kundt

Wolfgang H. Kundt, Professor Emeritus of Physics and Astrophysics at Bonn University, Auf dem Hügel 71, D-53913 Bonn, Germany

E-mail: wkundt@astro.uni-bonn.de

Born: 3 June 1931 in Hamburg. Lost his father by car accident in same year.

Spent the war years 1940-45 in Dresden.

Diploma, Ph.D., and Habilitation at Hamburg, in 1956, 1959, 1965.

Married with Ulrike since 1966, two children.

Visiting Prof. at Kiel in 1965/66; Scientific Counselor & Prof.(H2)at Hamburg since 1971.

C3 Prof. at Bonn since 1977, retired in 1996.

Supervisors and distinguished colleagues: Pascual Jordan, Erich Bagge, Ernst Witt, Werner Böge, Jürgen Ehlers, Engelbert Schücking, Klaus Hasselmann, Hans-Jürgen Borchers, Ted Newman, Wolf Priester, Eckhard Krotscheck, Felix Pirani, Hermann Bondi, Thomas Gold, Peter Scheuer, David Branch, Werner Israel.

Extended and/or repeated invitations to Syracuse (N.Y.), London, Pittsburgh (Pa), Edmonton (Alberta), CERN, Bielefeld, Cambridge (GB), Kyoto, Bangalore, Boston, Maribor (Slovenia), Moscow, Linz, Gorakhpur, Taiwan, Pune, Çanakkale, Washington, Rio.

Main scientific interests shifted and extended from mathematics and fundamental problems in General Relativity, Quantum Field Theory, and Statistical Mechanics through cosmology towards astrophysics and planetary physics, geophysics, biophysics, and astrobiology. In this process, awareness emerged of how difficult physics can be, and how easily physical insight can be mislead by hasty conclusions, enhanced by the Gold effect. The list of ‘alternatives’ in his 2005 Springer book ‘Astrophysics, A New Approach’ offers 100 entries, and has grown since to a present 125.

Notes regarding the Mainz 1 paper by Jürgen Ehlers

Introductory remarks by translator Manfred Trümper

The notes consist of 6 pages, handwritten by Ehlers, mostly in German language. A copy of the notes had been mailed to the translator by Mrs. Anita Ehlers. The notes appear to have been gathered by Ehlers in view of the edition of an English language version of the Mainz I paper. The notes are dealing mainly with the subjects of differential invariants, groups of isometries, and the equivalence problem, corresponding to sections 6, 7, 8 of chapter 1. Those parts which had been written in English language are enclosed by ≪;... ≫. In the English language reproduction of the notes, line breaks have generally been respected. Where the original notes had a page break, the line of asterisks is inserted in the present text. Terms which could not be recognized are marked <illegible>. Annotations by translator e.g. <annotation>.

Uzès, France, 2009-03-06
Manfred Trümper

Comment by the Golden Oldie Editor

Jürgen Ehlers probably intended to rework these notes into a readable comment, to be added to the translated article. Having nothing better at our disposal, we decided to publish them as they are in the hope that they will be intelligible at least to some readers, which we think is better than to let them be lost for ever.

Andrzej Krasiński

On the Notion of Differential Invariants.

$$f(g_{ij(r)}) = f(g_{ij},\ g_{ij,k},\ g_{ij,kl}, \ldots) \tag{1}$$

indices in alphabetic order,
with $f[g_{ij}](x^r)$ a functional, ultralocal.

$$g_{ij}(x^r) \mapsto f[...](x^r) \tag{2}$$

The <u>function</u> $\mathbb{R}^N \to \mathbb{R}$ in (1) "should be" independent of the coordinates.

$$\begin{array}{ccc} & x^{s'}(x^p) & \\ g_{ij(r)}(x^s) & \longleftrightarrow & g_{i'j'(r)}(x^{s'}) \\ \downarrow & & \downarrow \\ f(\ldots) & \longleftrightarrow & f(g_{i'j'(r)}(x^{r'})) \end{array}$$

f equivariant with respect to pull-back mappings;
Needed: "set" $g_{ij(r)}$, group of linear
transformations $G(n,r)$,
In tensor space $(O|2)_{symm.} \oplus (O|3)_{symm.} \oplus \cdots$

$$g_{i'j'} = g_{ij}L^i{}_{i'}L^j{}_{j'},\ g_{i'j',k'} = g_{ij,k}L\cdots L^k{}_{k'} + g_{ij}LLL^i{}_{j',k'}$$

With respect to geodesic normal coordinates:
Signature and dimension fixed.

$$f(\ldots) = g(R_{ijkl}, R_{ijkl;m})$$

Arguments are tensors, transformed according to tensor representations of $GL(n,\mathbb{R})$. Or "spinors".

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Number of "independent" invariants $\leq n$

Worst case:

$\mu = 2$ 0 Inv.
$\mu = 3$ 1 Inv.
$\mu = 4$ 2 Inv.

$\mu = n+2$ n Inv. $N = n+2$
Problem: Def. 1) Algebr. inv.
2) Independence with respect to x^α
3) Proof that, if the $(\mu+1)$-invariants depend on the μ-invariants then there are no independent invariants for all orders $> \mu+1$.
4) Restriction to "regular" points and their neigborhoods.
5) Def. "regular" points if "all" alg. inv. are to be considered.

<u>Equivalence problem, Invariants</u>

Book on solutions (McCallum et al), Ehlers-Kundt

$\ll$ Cartan invariants: Let $\{e_a\}$ be an intrinsically defined frame field (section of ??), unique or unique up to isometrics of fixed groups; then all tensor components of Ricci, Riemann, ? are scalars: Cartan inv. $\gg$

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<First entries: partly illegible, incomprehensible.>

V_4 solution, const. curvature K (deSitter) $\underline{N=1}$ or $\underline{N=2}$?
Equivalent with V_4^c : $K' = K$, <u>one</u> algebr. inv. (K) of order $\mu = 2$.
There is no f of order 1 which would have to be taken into account.
Theorem 1.8.3 $\underline{\underline{\text{wrong}}}$ $\underline{\underline{\nexists N}}$

If there is no invariant which is independent with respect to the coordinates $\Longleftrightarrow$ K =const. then $N = 1$ has the first two properties of Theorem 1.8.3, but not the third one. There is no N. This special case should be excluded.

Let there be no invariant of order 2 which is not a constant, but let there be some of order 3. Then it is $N \geq 3$.
$\mu = 4$, no other invariants exist. $N = 3$.
$\mu = 4$, other independent invariants exist $\rightarrow N \geq 4$
$\mu = 5$, nothing new $\rightarrow N = 4$
The first time no new invariants turn up in the transition from μ to $\mu + 1$, then $\Rightarrow \underline{\mu = N}$.

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"Application": Lagrange density of metric,
$\rightarrow$ Hilbert etc.
<u>Survey?</u>
$\rightarrow$ Rund, Lovelock,
Functional independence? Criteria? Number?

Lie derivative $Xf = \frac{\partial f}{\partial R_{ijkl}} \pounds_X R_{ijkl} + \cdots \Leftarrow$ useful?

At a GR-Conference, Géhéniau's 14 general invariants for $n = 4$, $\mu = 2$ have been critisized as "dependent". (?)

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$\ll$ <u>Conventions</u> $c \equiv 1$, $G = \frac{1}{8\pi}$.
Riemann $\rightarrow$ -Riemann, Ricci $\rightarrow$ -Ricci, $R \rightarrow -R$
$\eta \rightarrow -\eta$, η tensor. Changes compared to MTW, W???, myself.

normal hyperbolic Riemannian $\rightarrow$ Lorentzian $\gg$

$\ll$ order, rank, degree... for tensors? $\gg$

Pseudo tensor can be avoided by choice of an orientation
= tensor η. - The operation * in W is a complex structure.

It should be avoided that "algebraic invariants" is used here without definition. What is meant are coordinate-independent numbers or number-valued functions which are determined by a tensor or a tensor field.

≪ Terminology; "metric invariant" := a scalar determined by the metric and its derivatives up to a finite order, not necessarily a "general" diff. invariant.≫

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Remarks.
Submitted prior to Robinson & Trautman ( ),
Kruskal (1960), Penrose (1960),

Not taken into account: Synge (1950),<illegible>(despite JE 58)

Continuations: EK (1962), book with solutions, JE about equivalence problem,
Cosmology: Ozsvath (Schücking).

Later parts: JES II Spinors; (parallel Robinson+Trautman)
Matter: E IV, K & Tr. V
Einstein-Maxwell: JK III,
→ Airforce Reports

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