Nonlinear hydrodynamic models of type II Cepheids

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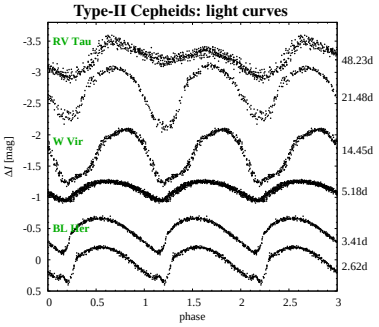
Type-II Cepheids and their modelling

The light curves

Type-II Cepheids are low mass, population II stars, pulsating in the fundamental mode. They are divided into 3 classes. BL Her stars, with $1\,\mathrm{d} < P_\mathrm{F} \lesssim 4\,\mathrm{d}$ and W Vir stars, with $4\,\mathrm{d} \lesssim P_\mathrm{F} \lesssim 20\,\mathrm{d}$, have fairly regular light curves. Characteristic feature of RV Tau stars, with $P_\mathrm{F} \gtrsim 20\,\mathrm{d}$ is period doubling effect. As period increases, RV Tau stars display more and more irregular behavior, likely they follow the period doubling route to deterministic chaos.

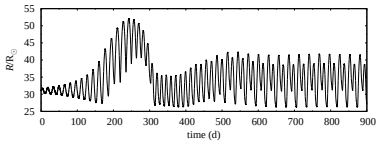
Modelling of type-II Cepheids, in particular of the most bright W Vir and RV Tau stars is scarce. The last model survey was published by Kovács & Buchler (1988) with purely radiative code. As effective temperature in their models is lowered, they detect period-doubling transition to chaos, at periods around 10 days, much too low as compared with observation. Unfortunately, their models, and models reported here, are limited to low luminosities as dynamical instability prevents computation of more luminous models.

Type-II Cepheids: light curves



Typical, I-band light curves of type II Cepheids. Period doubling is clear in the top two RV Tau-type curves. Data from OGLE catalog (Soszyński et al., 2011).

Type-II Cepheid models: dynamical instability



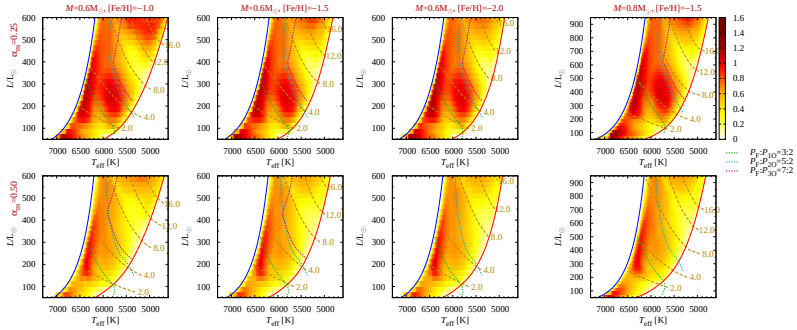
The above Figure shows the variation of the radius of the outermost model shell during the initial phase of non-linear model integration. The model rapidly expands; depending on the model the maximum radius excursion can easily reach $100-200\,\%$ of the static radius within just a few pulsation cycles. Then, the model contracts and finally reaches a stable limit cycle – period doubled for the presented model ($t > 600\,\mathrm{d}$). For the most luminous models, i.e. for $L > 600\,\mathrm{L}_\odot$ ($M = 0.6\,\mathrm{M}_\odot$) and $L > 1000\,\mathrm{L}_\odot$ ($M = 0.8\,\mathrm{M}_\odot$) the model breaks down in the initial phase. This is a physical effect not numerical one. The pulsation of the models is driven much too strongly. Similar phenomenon was described for red giant models (see a review by Wood 2007). Physically it may corresponds to mass loss, which is strong in stars during the AGB phase (e.g. Zijlstra, 2006), and can be augmented by the pulsations.

Model properties

Model grid and model properties

We have computed an extensive grid of type-II Cepheid models assuming $[\mathrm{Fe/H}] = -1.0,\ -1.5,\ -2.0$ ($M = 0.6\,\mathrm{M}_\odot$, left to right panels in the Figure to the right) and $[\mathrm{Fe/H}] = -1.5$ ($M = 0.8\,\mathrm{M}_\odot$, right-most panels). We used non-linear convective codes of Smolec & Moskalik (2008) and considered models with two different levels of eddy-viscous dissipation, which is the most important amplitude limitation factor. The models assuming $\alpha_\mathrm{m} = 0.25$ fairly well reproduce the amplitudes of the observed BL Her stars (Smolec et al. 2012). For larger eddy viscosity parameter, $\alpha_\mathrm{m} = 0.50$, the amplitudes are indeed much lower. The level of eddy-viscous dissipation has large impact on the computed form of pulsation, in particular on the appearance of period doubling.

An interesting property of the discussed models, well known in convective models of red giants, is apparently peculiar behaviour of non-linear period. In case of RR Lyr and classical Cepheid models the non-linear period is typically longer than the linear one by up to a few per cent. Here we observe much larger differences, that can reach up to 20%. In addition, the non-linear period can be significantly shorter than that predicted for the static equilibrium models. This is because of the significant rearrangement of the mean structure of the envelope due to large amplitude, non-linear pulsation (see e.g., Wood 2007).

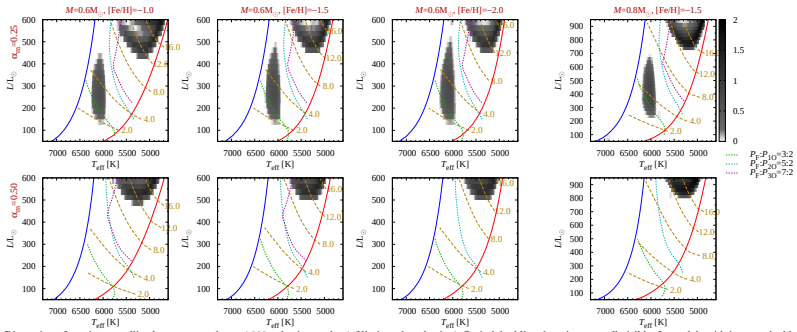


Peak-to-peak bolometric light amplitude of the models in limiting cycle pulsations. Over-plotted are lines of constant period and loci of some half-integer resonances. The amplitudes are lower in models assuming larger eddy-viscous dissipation

Period doubling effect

Grey-shaded figure to the right shows the dispersion of maximum radii values over 1 000 pulsation cycles. In white areas we calculate single-periodic fundamental mode pulsation. The two grey shaded domains correspond to more complex pulsation scenarios. In the nearly vertical domain, centered at $T_\mathrm{eff} \approx 6100\,\mathrm{K}$, and existing only for models with lower eddy viscous dissipation, period doubling is detected. This domain was revealed by Buchler & Moskalik (1992) who traced its origin to the 3:2 resonance between the fundamental mode and first overtone. They predicted the existence of period doubled BL Her stars, and indeed, first such star was discovered 20 years later (Soszyński et al., 2011, Smolec et al., 2012).

The extent of the domain in the top right part of the HR diagram cannot be calculated because of the dynamical instability preventing computation of the more luminous models. Majority of the models in this domain display a period doubled pulsation, in some period-4 pulsation is detected. This domain overlaps with the model sequences of Kovács & Buchler (1988), in which they detected deterministic chaos while moving towards lower effective temperatures. We do not detect such behaviour. When we decrease the effective temperature of the models, the pulsation amplitude drops, period doubling domain ceases and we finally reach the red edge. The radiative calculations of Kovács & Buchler lack the pulsation quenching mechanism at the cool side of the instability strip.



Fractional difference between non-linear period and linear period of the fundamental mode. The non-linear period can differ by as much as 20% from the linear period and can be both longer (yellow-red) and shorter (blue). The models in the bottom left corner of the plots are of RR Lyr type and switched the pulsation to first overtone.

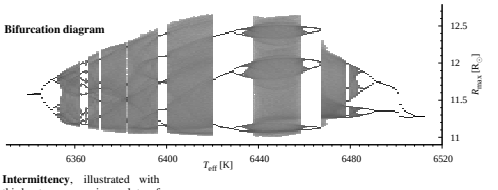
Dispersion of maximum radii values computed over 1 000 pulsation cycles (of limit cycle pulsation). Period doubling domains are well visible. In models with increased eddy viscosity, one of them disappears, the other shrinks.

Decreasing eddy viscosity

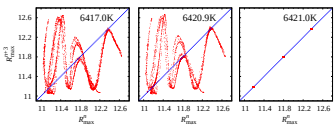
Deterministic chaos in hydrodynamic BL Her models

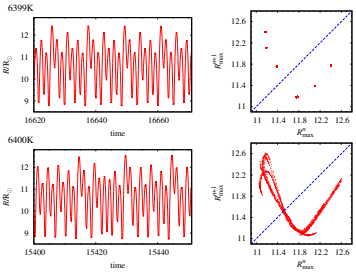
When eddy viscosity is strongly reduced, to $\alpha_m = 0.05$, the models display a wealth of dynamical behaviours characteristic to deterministic chaos. We have studied such behaviour along a sequence with $L = 136\,\mathrm{L}_\odot$, so the models are of BL Her type. Bifurcation diagram to the right shows the possible values of maximum radii during the pulsation, versus the model's effective temperature. Grey bands correspond to deterministic chaos. They are reached through the period doubling cascades, well visible both from the cool and the hot side of the computation domain. They are separated by the domains of stable, periodic, period-k pulsation. We detected many phenomena characteristic to chaotic systems, eg. intermittency or crises bifurcation. Read more in Smolec & Moskalik (2014).

Bifurcation diagram



Intermittency, illustrated with third return maps, i.e. plots of R^{n+3}_max vs. R^n_max. Stable period-3 cycle is born through the tangent bifurcation. Before the bifurcation, intermittent behaviour is apparent, i.e. evolution of the system is characterized by long phases of almost periodic (period-3) behaviour interrupted with shorter bursts of chaos.





Periodic points and chaos. Time series (left) and first return maps, i.e. plots of R^{n+1}_max vs. R^n_max (right). We find stable period-2, 3, 5, 7 (*top*) and 9 cycles, most of them undergo a period doubling bifurcation. *Bottom:* Example of chaotic model.

References

Buchler J.R., Moskalik P., 1992, *ApJ*, **391**, 736
Kovács G., Buchler J.R., 1988, *ApJ*, **334**, 971
Smolec R., Moskalik P., 2008, *Acta Astron.*, **58**, 193
Smolec R., Moskalik P., 2014, *MNRAS*, **441**, 101
Smolec R., Soszyński I., Moskalik P., et al., 2012, *MNRAS*, **419**, 2407
Soszyński I., et al., 2011, *Acta Astron.*, **61**, 285
Wood P.R., 2007, IAU Symp., **239**, 343
Zijlstra A.A., 2006, IAU Symp., **234**, 55

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