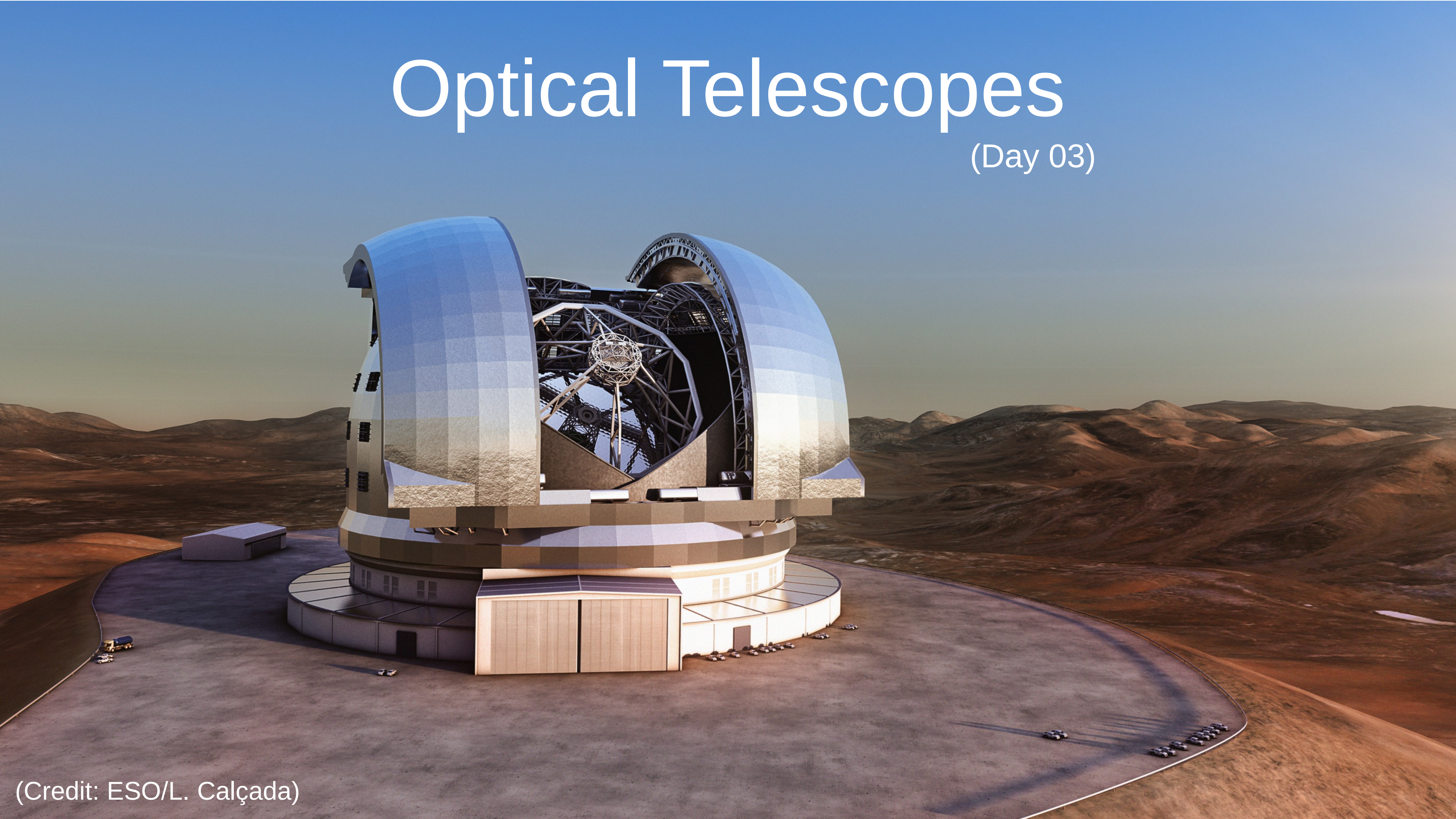
Optical Telescopes

(Day 03)



(Credit: ESO/L. Calçada)

Today

1. Optical telescope
2. Image formation
3. Active and adaptive optics

You go for observations to Paranal.
What happens there?

You go for observations to Paranal. What happens there?

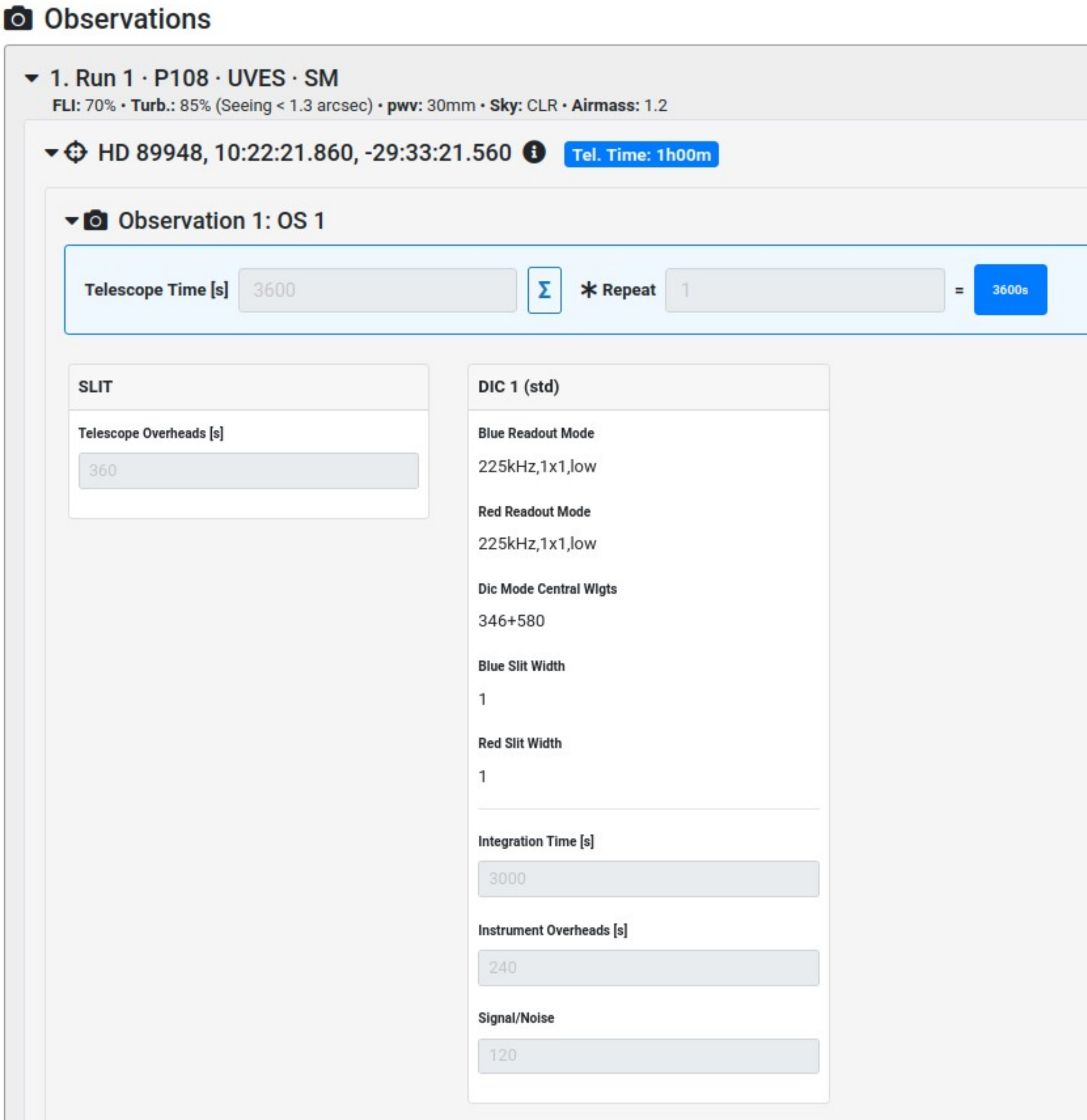
1. During the day, the night astronomer will meet you and discuss your observing program
2. The observations should be prepared in advance of the night

(Credit: ESO)

You should have

1. Objects to be observed during the whole night
2. A plan to observe them at the best possible airmass
3. Calibrations and standards
4. Back up objects, in case of worse sky conditions (or the observatory reverts to Service Mode)

Then you go to the control room by the end of the afternoon

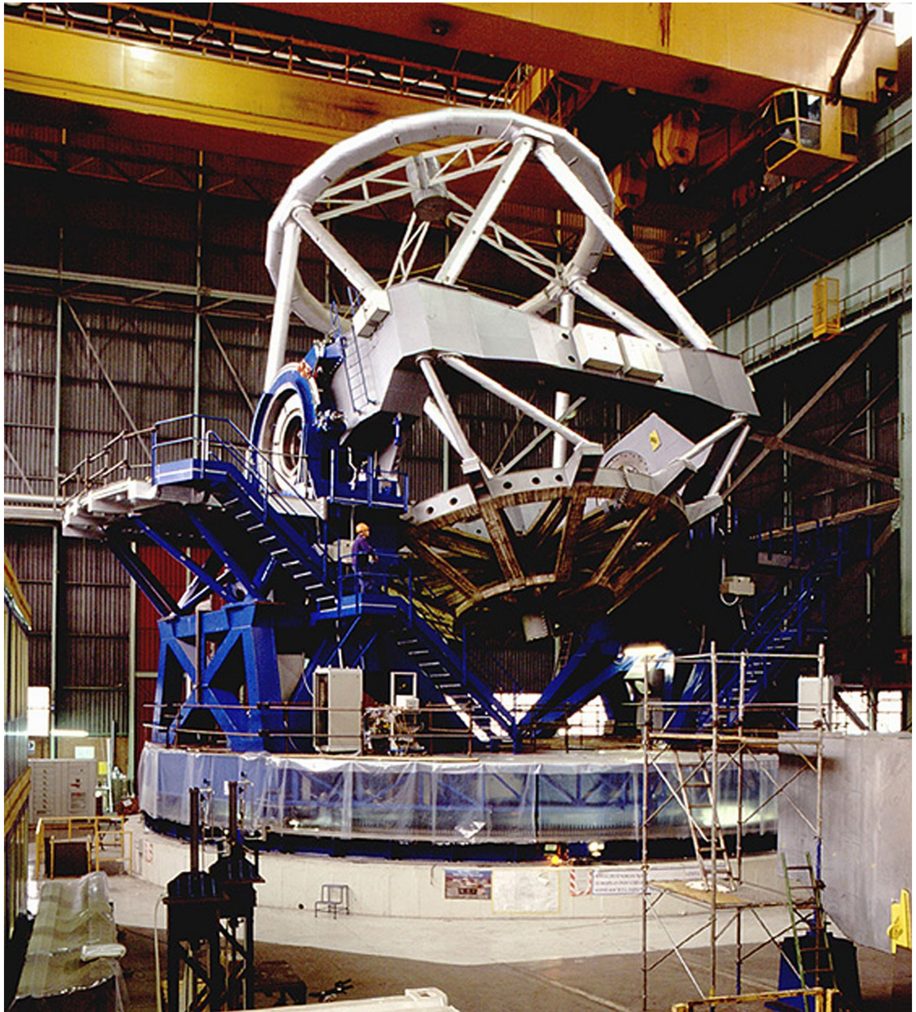


Ready to observe, what do you do?
What does the telescope do?

(Credit: ESO)

The Telescope

1. Points to the target
2. Tracks the target
3. Collects the light
4. Provides angular resolution
5. Feeds the light to the instrument

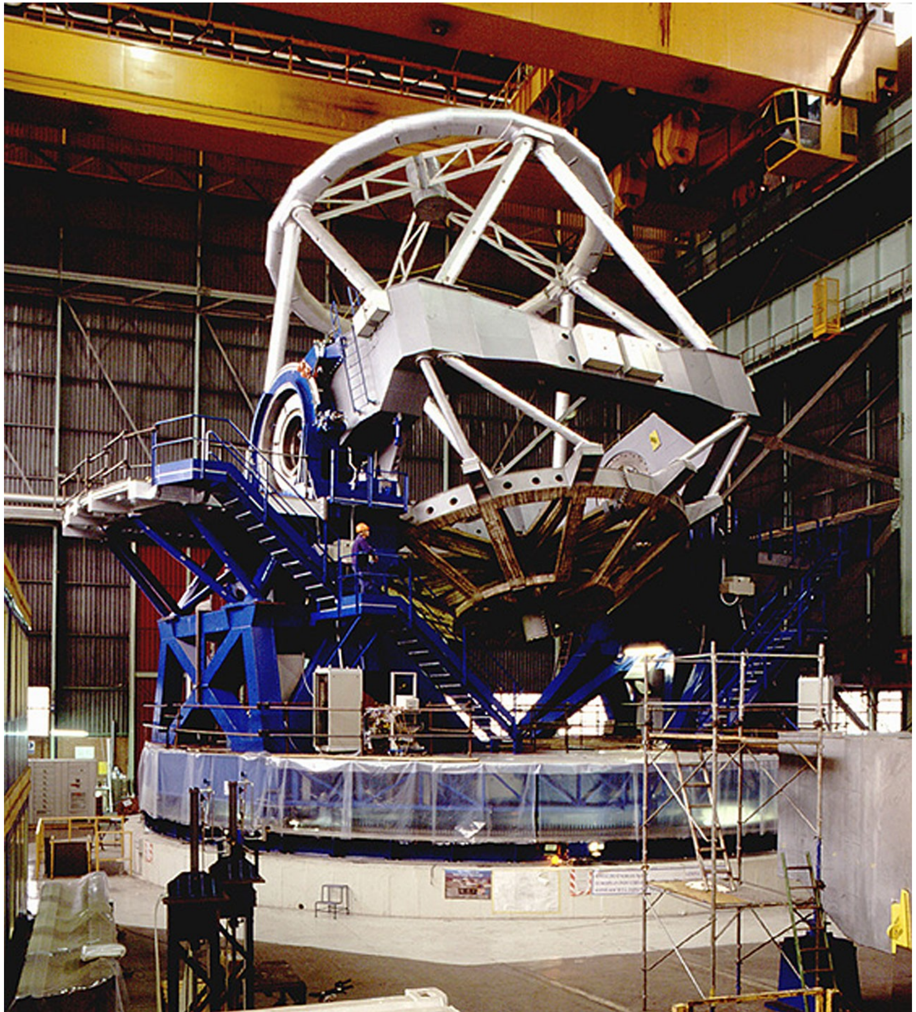


(Credit: ESO)

The Telescope

1. Points to the target
2. Tracks the target
3. Collects the light
4. Provides angular resolution
5. Feeds the light to the instrument

- Software control system
- Electronics
- Mechanical structure
- Optical elements



(Credit: ESO)

Pointing

- Input coordinates (α, δ) and the telescope moves to find the target

Table 1.7. Order of magnitude of pointing corrections

Atmospheric refraction	2′ at 60° zenith distance
Precession	50″ per year
Annual aberration	20″
Velocity aberration (spacecraft only)	5″
Stellar parallax	<1″
Proper motion	< 1″
Diurnal aberration	0.3″
Differential velocity aberration (spacecraft only)	20 mas (LEO)

Note: mas = milliarcsecond; LEO=low-Earth orbit

(Credit: Bely 2003)

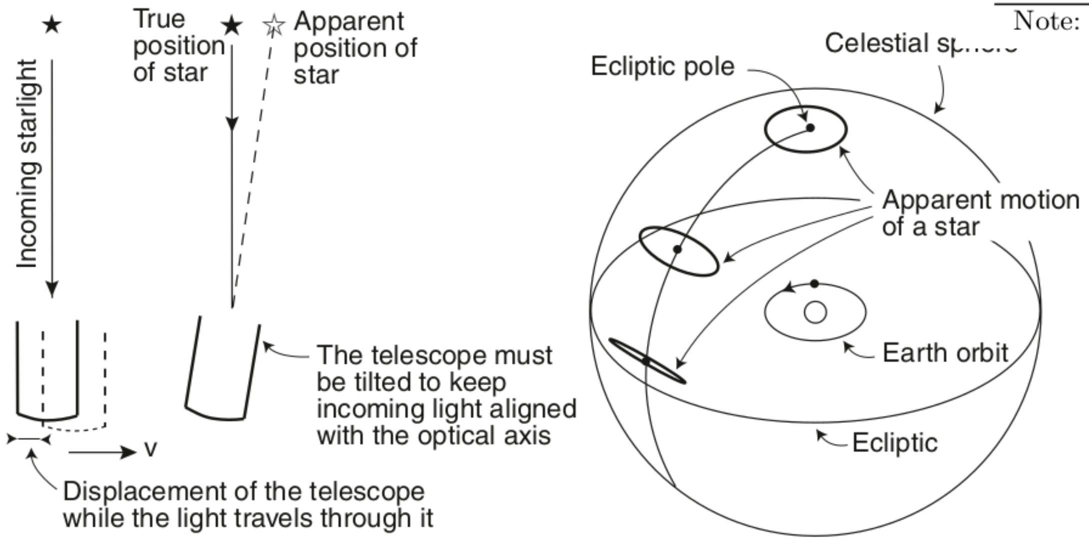


Fig. 1.21. At left, aberration of starlight due to the motion of the observer. At right, velocity aberration due to Earth's rotation around the Sun. The corresponding apparent motion of a star is a circle at the ecliptic poles and an ellipse elsewhere.

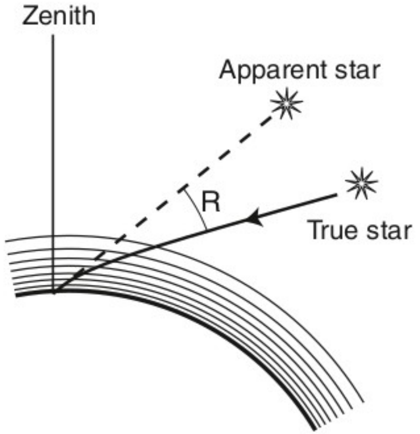
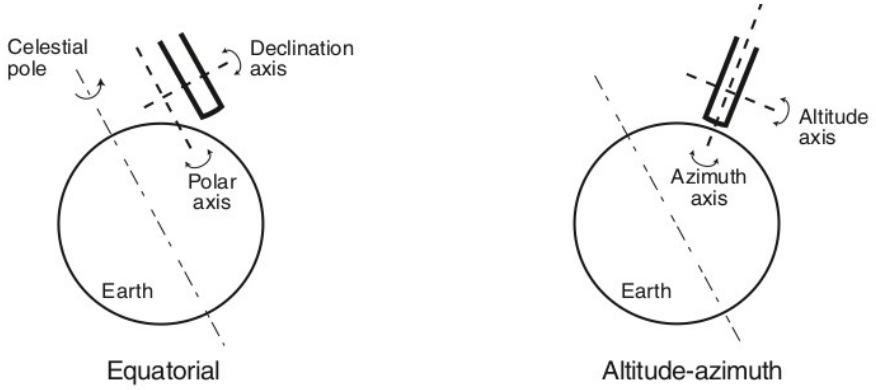


Fig. 1.7. Refraction in the atmosphere

How to check if the correct object was found?

How does the telescope move?



(Credit: Bely 2003)

Fig. 6.36. Principle of the equatorial mount (left) and the alt-az mount (right).

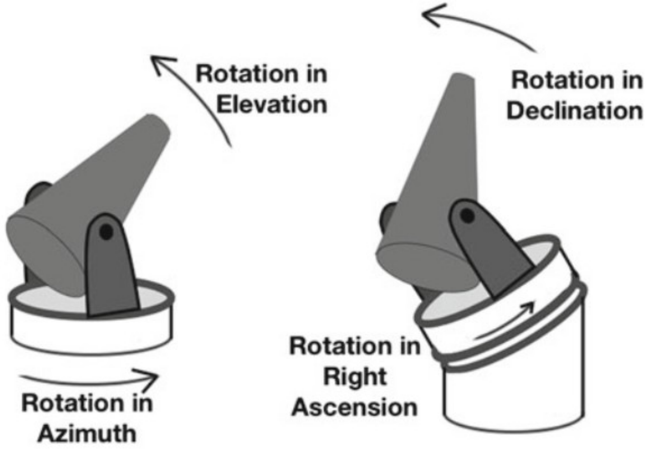


Fig. 1 Alt/Az versus equatorial mounts

(Credit: Conrad 2014)

(Credit: Cheng 2009)

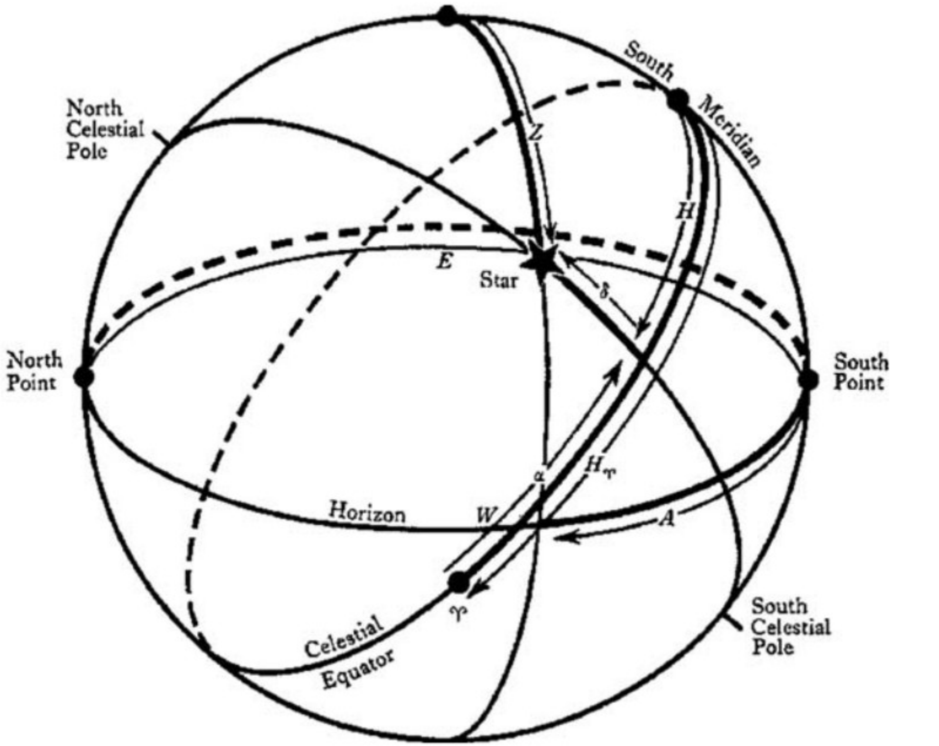


Fig. 3.3. The relationship between alt-azimuth and equatorial coordinate systems.

Equatorial vs. altitude-azimuth

- The mount supports the tube and rotates it for pointing and tracking
- Equatorial:
  - Once pointed, needs to rotate around the polar axis only
  - And at constant speed
  - No field rotation
  - No blind spot at the zenith

Hale Telescope
(5.1 m)
at Palomar Obs.
(built 1949)

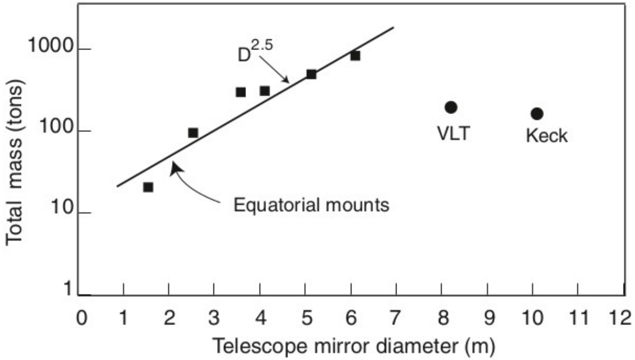
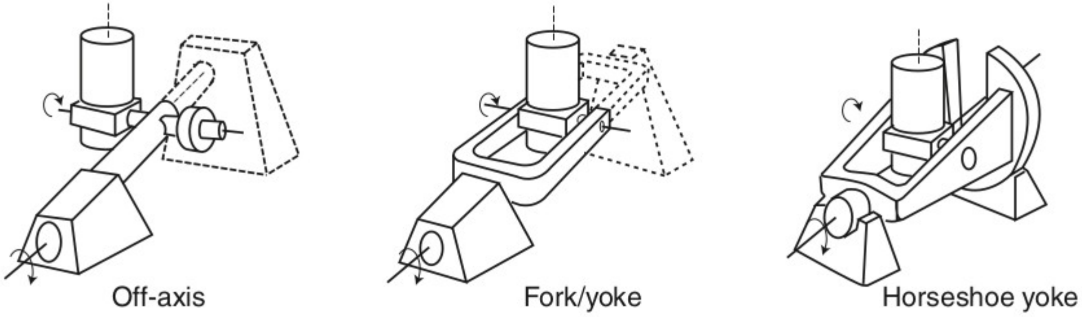


Fig. 6.38. Mass of telescope versus aperture size.



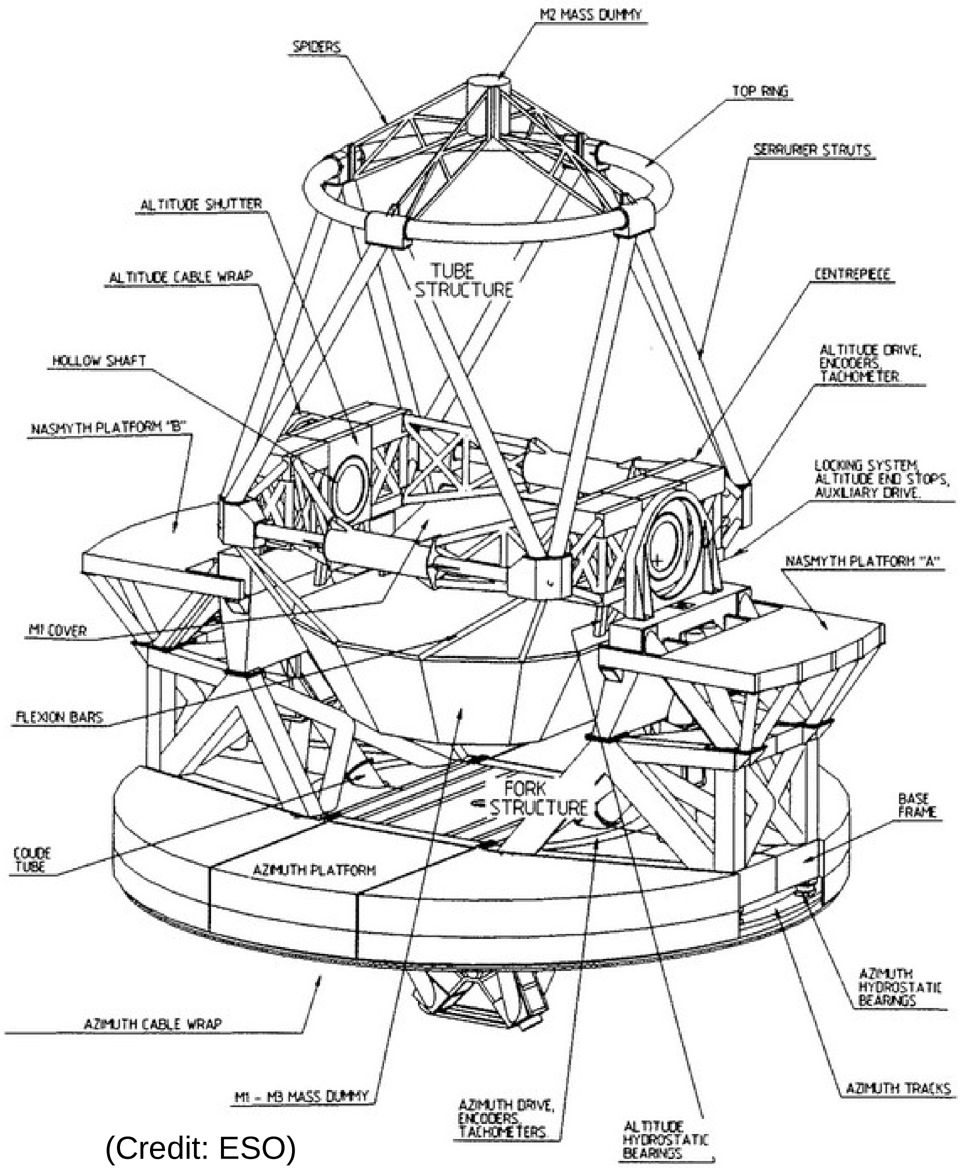
(Credit: Bely 2003)

Equatorial vs. altitude-azimuth

- Alt-azimuth:
  - Vertical (azimuth) and horizontal (altitude)
  - Simpler and stronger
  - Reduction in mass and cost
  - But the field rotates
  - Variable speed in each axis
  - Blind spot near the zenith
  - Tracking software is more complex

$$\tan A = \frac{\sin t}{-\sin\phi\cos t + \cos\phi\tan\delta}$$

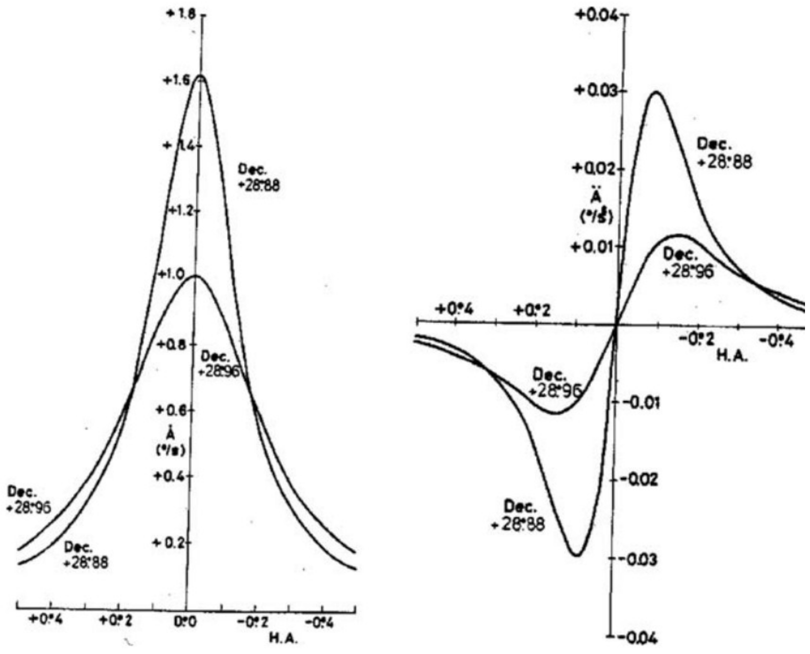
$$\cos Z = \sin\phi\sin\delta + \cos\phi\cos\delta\cos t$$



(Credit: ESO)

Zenith blind spot

- Tracking is limited by the maximum azimuth velocity and acceleration



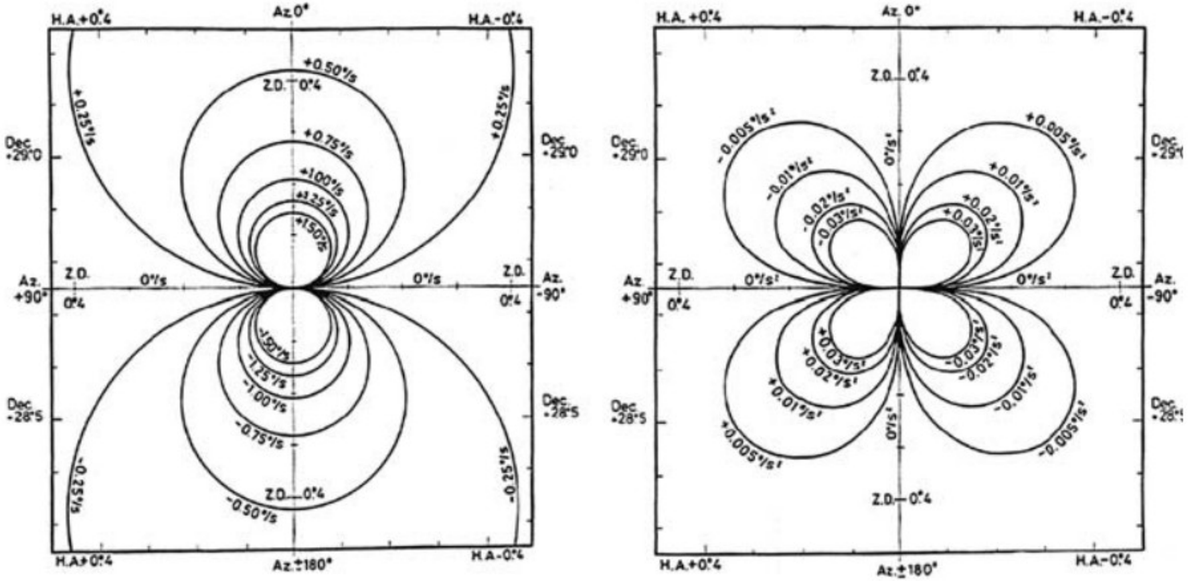


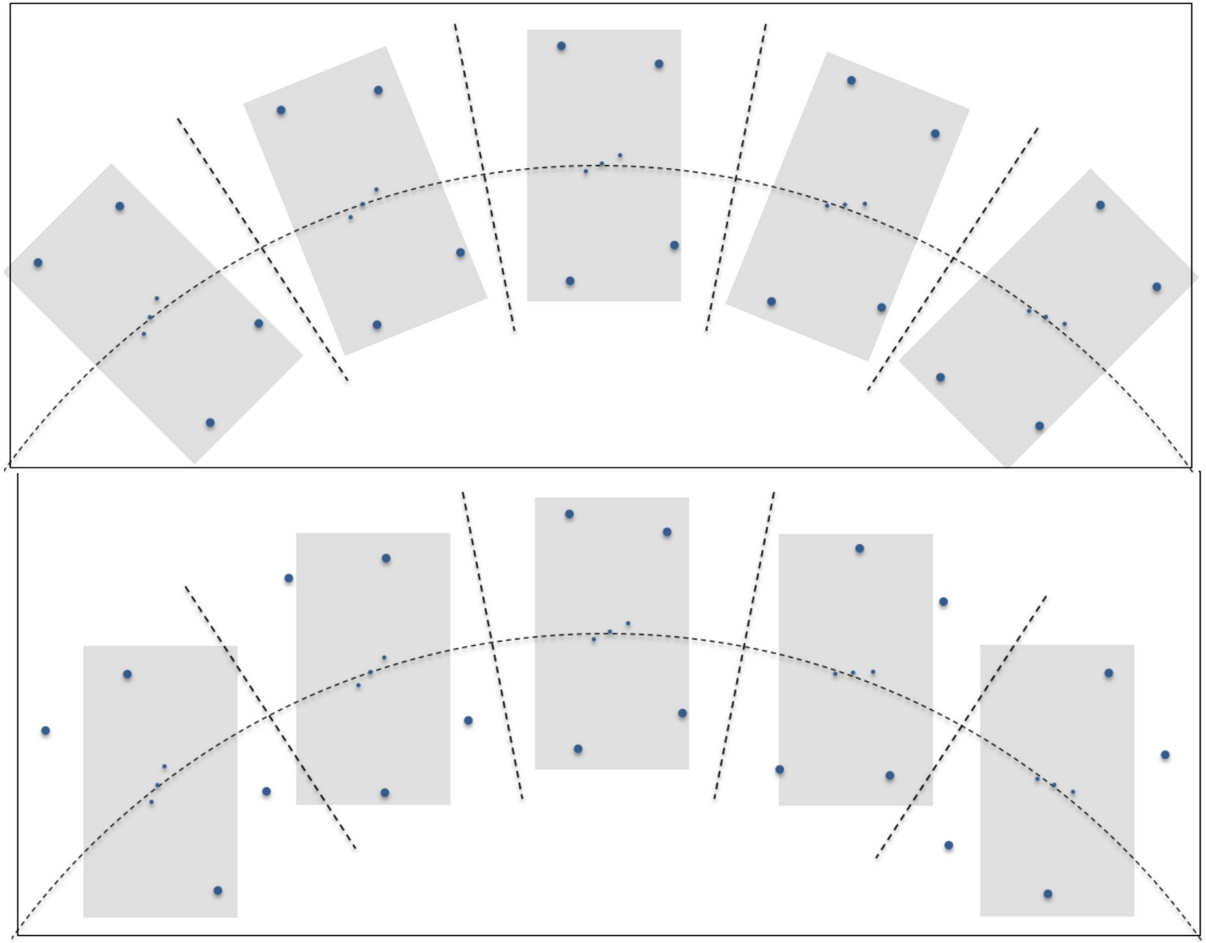
Fig. 3.5. Contours of various maximum azimuth tracking velocities (*left*) and tracking accelerations (*right*) at a latitude of 28.75° (b, *right*) (Watson, 1978).

(Credit: Cheng 2009 – azimuth velocity and acceleration for 0.1° and 0.2° zenith distances)

(Credit: Cheng 2009)

Field rotation

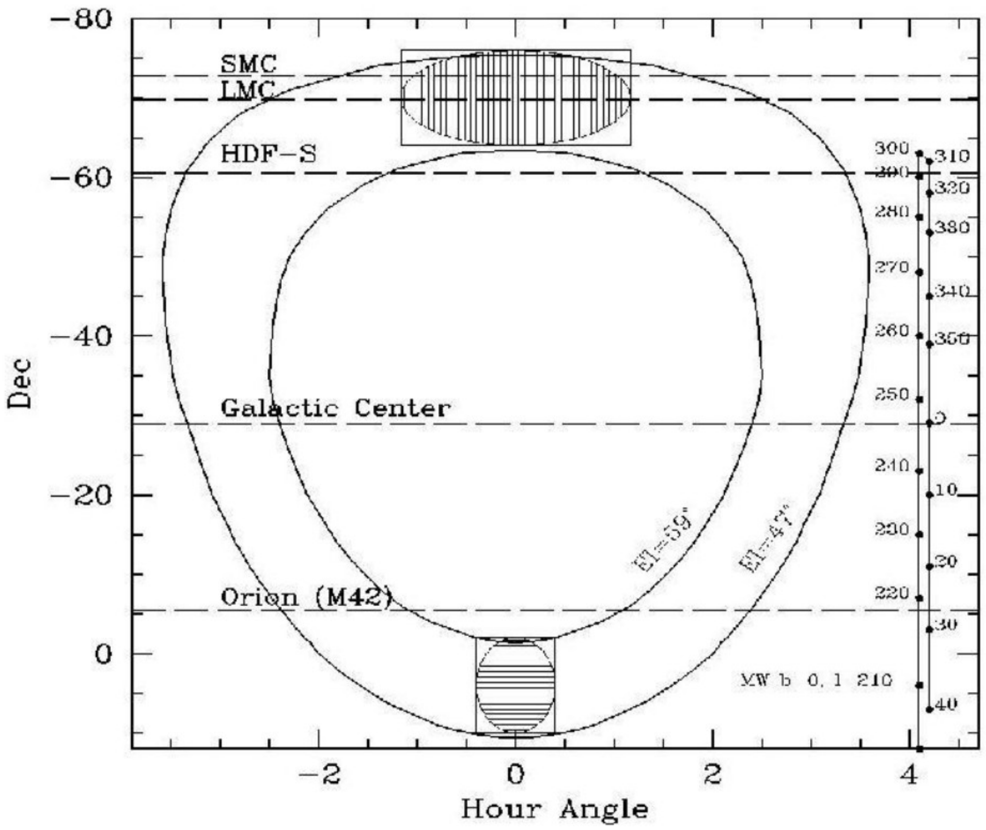
- Parallactic angle (q)
- Angle between target-zenith arc and the pole-target-pole circle
- Position angle of “straight up”
- Refraction disperses light direction to the zenith



(Credit: https://kelly.flanagan.io/)

Fixed altitude mount

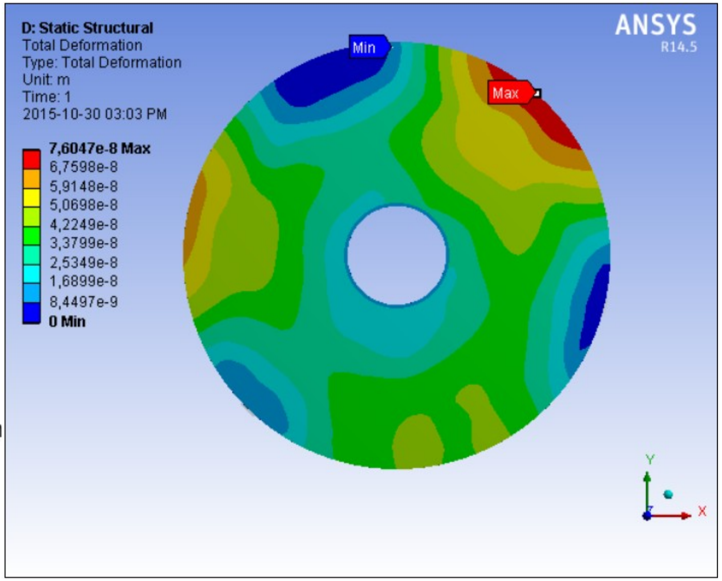
- Fixed altitude
  - One rotation axis
  - Primary mirror does not change direction with respect to gravity (simpler support)
  - Spherical mirror
  - All these make it cheaper
  - Reduced sky coverage
  - Limited exposure time
  - Spherical aberration
  - Small field of view



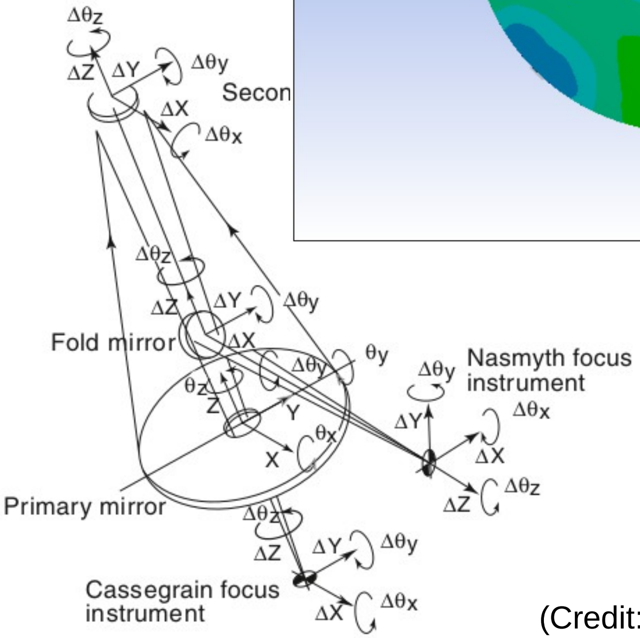
(SALT visibility; Credit: Brink et al. 2008)

Mechanical structure

- Support and maintain the optics aligned during the observations (see Chapter 6 of Bely 2003)
- First defence against gravity, wind gusting and thermal effects
- Mechanical structure is the passive response system
- Active optics provides complementary help to correct small-scale effects
- Thermal properties of the materials, rigidity of the structure, freedom of movement, connections…
- Survive earthquakes and emergency braking



(Credit: Nkosi 2015)



(Credit: Bely 2003)

Questions?

(Credit: Shutterstock)

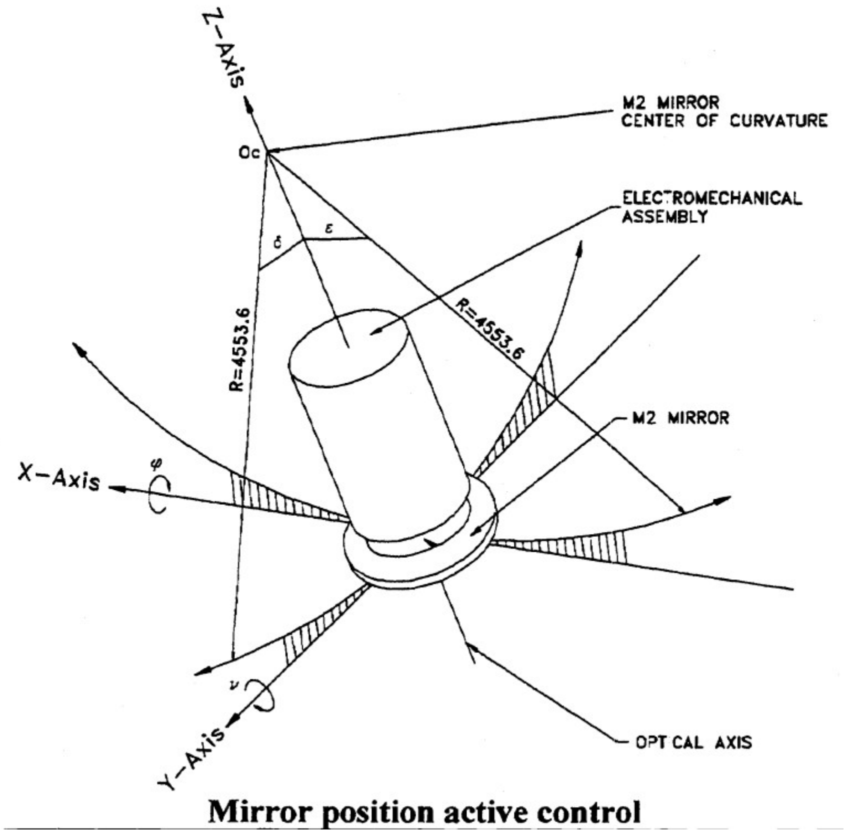
Primary mirror of the VLT

- Single “meniscus” with 8.2 m diameter and a central hole with 1 m diameter
- 17.5 cm thick; weights 23.5 tons
- 14.4 m focal length
- Ritchey–Chrétien telescope (hyperbolic primary and secondary mirrors)
- Zerodur (a lithium-aluminosilicate glass-ceramic produced by Schott AG)
- Low-thermal expansion coefficient, good hardness, non-porous, high-affinity to coating
- M1 Cell: vertical support by 150 actuators, lateral by 64 actuators
- Has to be positioned with precision < 1mm

(Credit: ESO)

Secondary mirror of the VLT

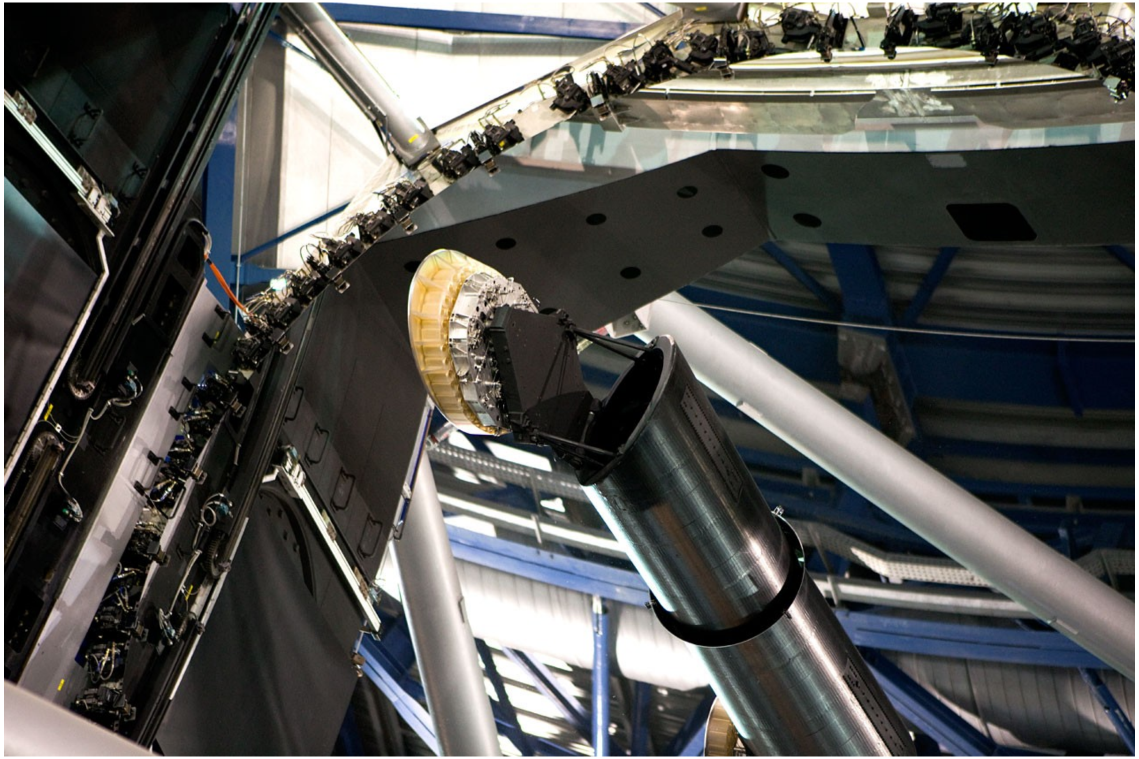
- 1.12 m diameter; 56 Kg
- Made of beryllium (low weight, high stiffness)
- Electroless nickel coating (Ni + P)
- Mirror supported in three points by an electromechanical structure
- Movement in 5 degrees of freedom: focusing, centering, tilt/chopping



(Credit: Cayrel et al. 1996)

Tertiary mirror of the VLT

- Flat mirror (elliptical shape, 890 X 1260 mm^2).
- It can be rotated to feed each of the two Nasmyth foci
- In Cassegrain mode, the M3 is flipped out of the way, parking parallel to the tower
- Equipped with active optics
- Made of Zerodur
- 140mm thick, 170-176 Kg
- 13 axial and 6 lateral support actuators



(Credit: ESO)

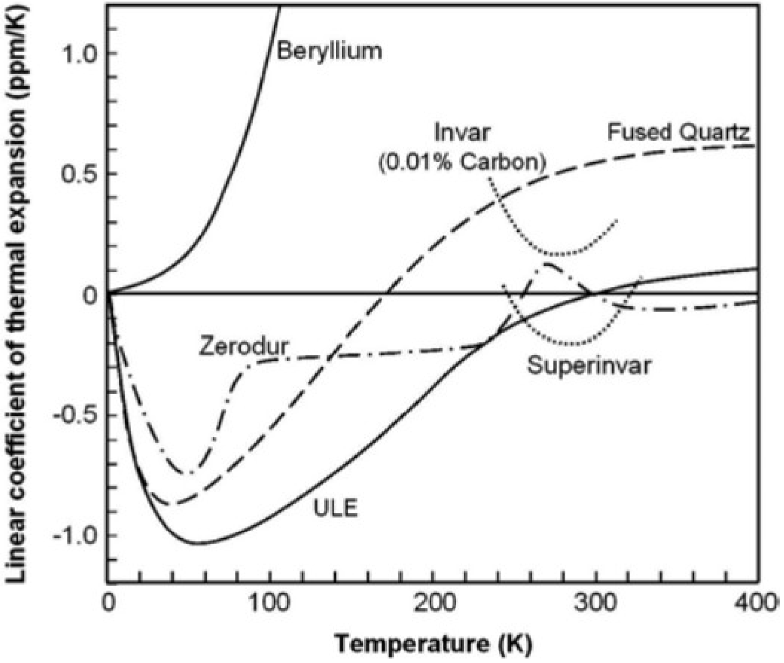
Mirror blank materials

- Stable to maintain optical properties for decades
- Not deform with environment temperature changes
- Produced in large size
- Rigidity and strength to allow handling and mounting
- Possible to achieve fine surface polishing
- High affinity to coating
- Low internal stress to not deform the material

Table 4.6. Properties of commonly used blank materials

Material	Density ρ kg/m^3	Young Mod. E GPa	Poisson ratio ν	Max stress σ_t MPa	CTE 273K α_{273} 10^{-6}/K	CTE 40K α_{40} 10^{-6}/K	Therm. cond. κ W/m K	Specific heat C_p J/kg K
Borosilicate	2200							
ULE	2200							
Zerodur	2500							
SiC (CVD)	3200							
Beryllium	1850							
Aluminum	2700							

Sources: Barnes [29], 1



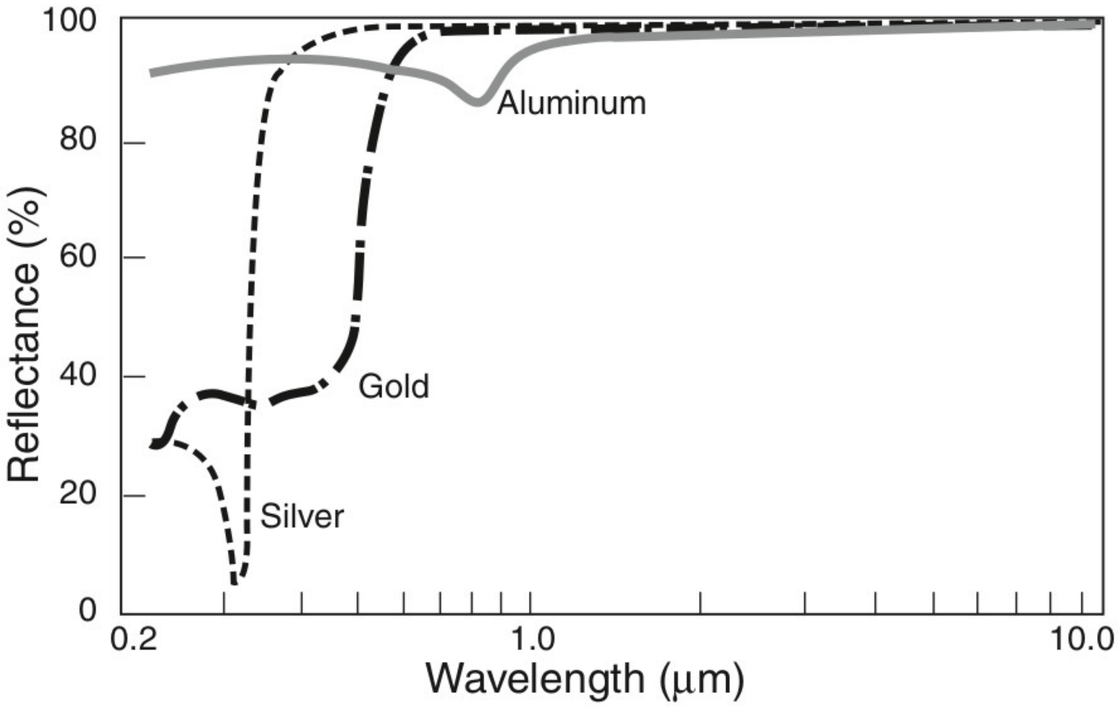
(Credit: Bely 2003)

Coating and cleaning

- Aluminum behaves well from UV to IR
- Silver reflects better than Aluminum >400nm
- Silver has to be protected against oxidation
- Gold used for IR (bare gold is soft and easily damaged)
- Bare aluminum coat thickness 100nm (~5%)

- Coating is done in a vacuum chamber
- Thermal evaporation; electron beam; ion sputtering

- Mirror cleaning is also needed (CO_2 snow cleaning)



Coating

- Primary and tertiary are coated every 18 months
- Reflectivity >90% at 670nm when freshly coated
- Dust is “grey” and reflectivity ~85% after 18 months
- Secondary is inspected and cleaned, but no recoating so far

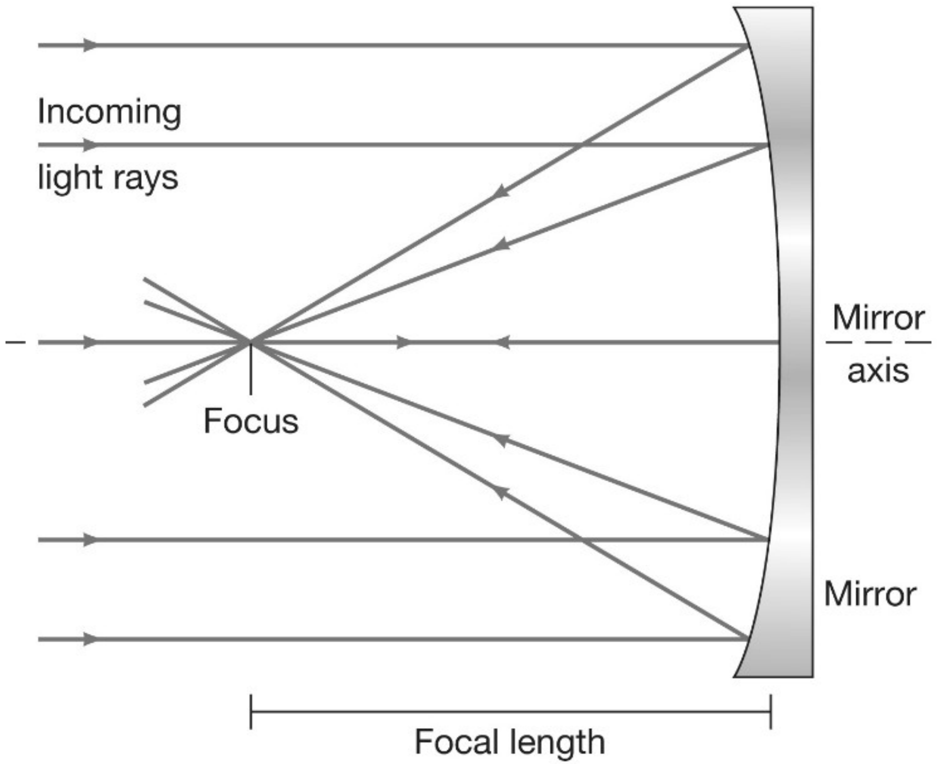
(Credit: ESO)

Mirror cleaning

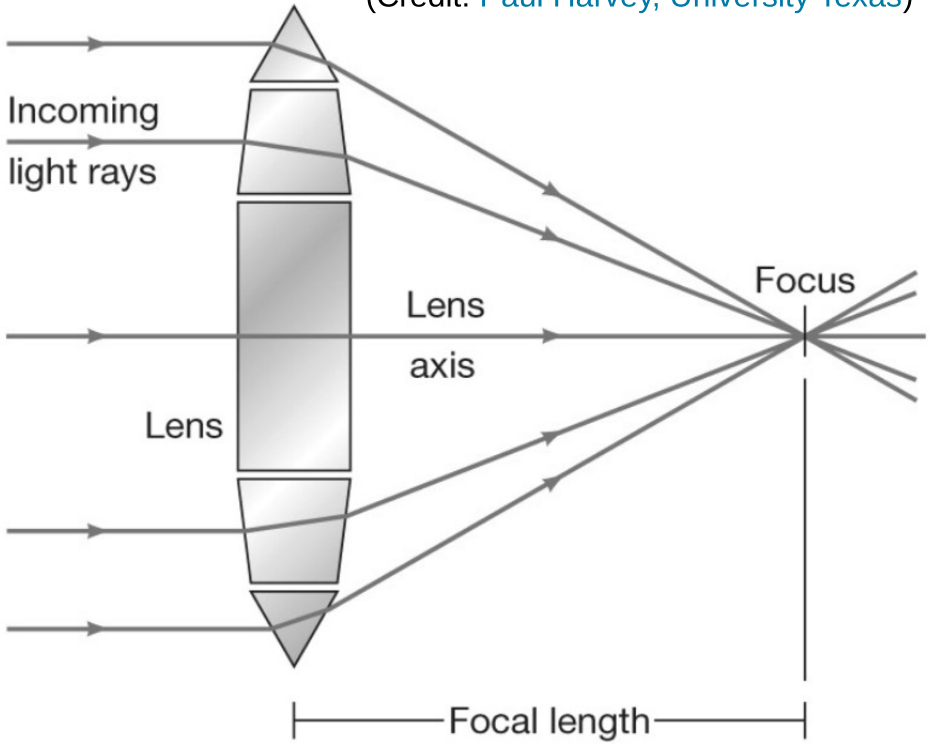
(Credit: ESO)

Mirror or lens

(Credit: Paul Harvey, University Texas)



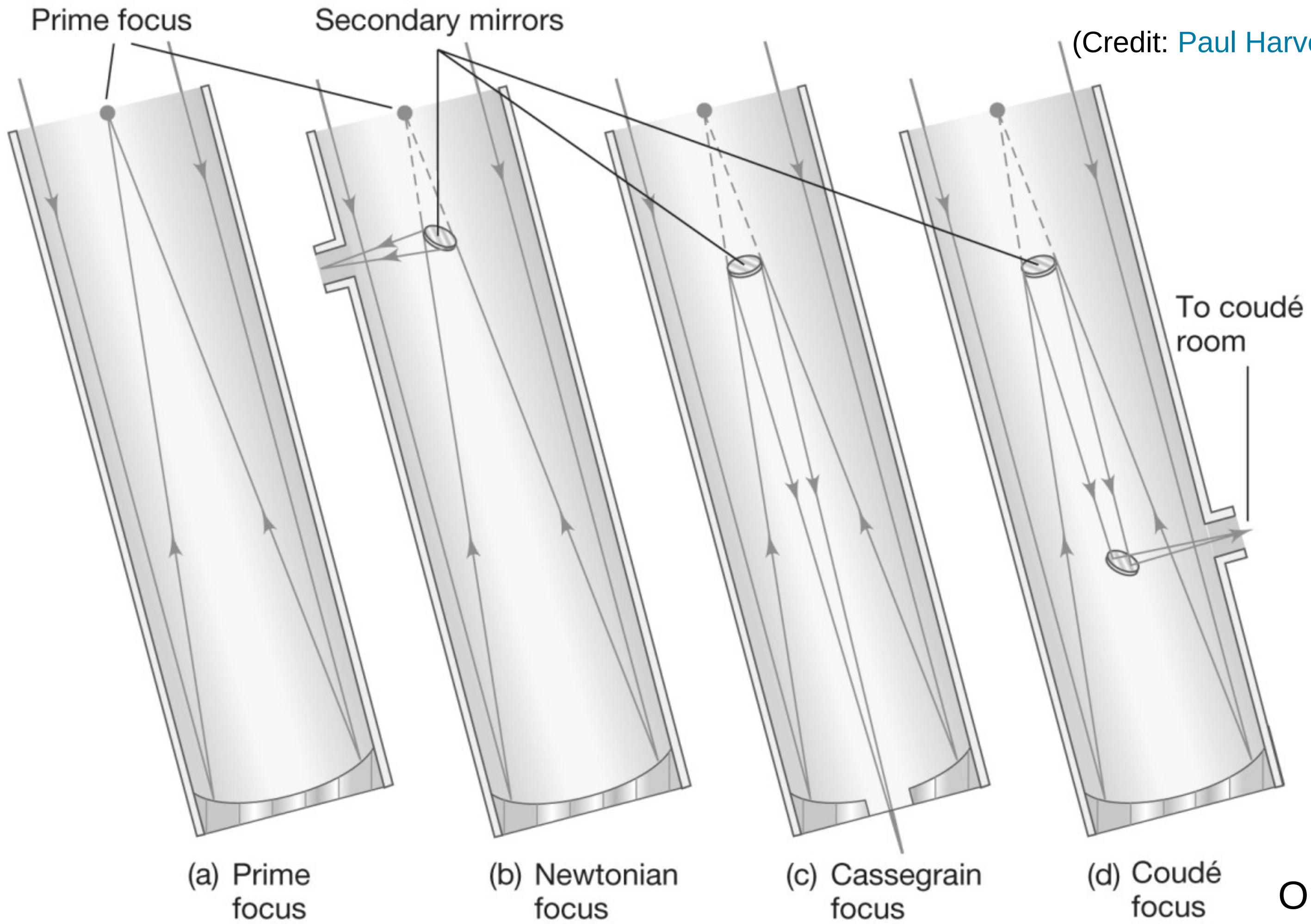
Catoptric system (reflections)



Dioptric system (refraction)

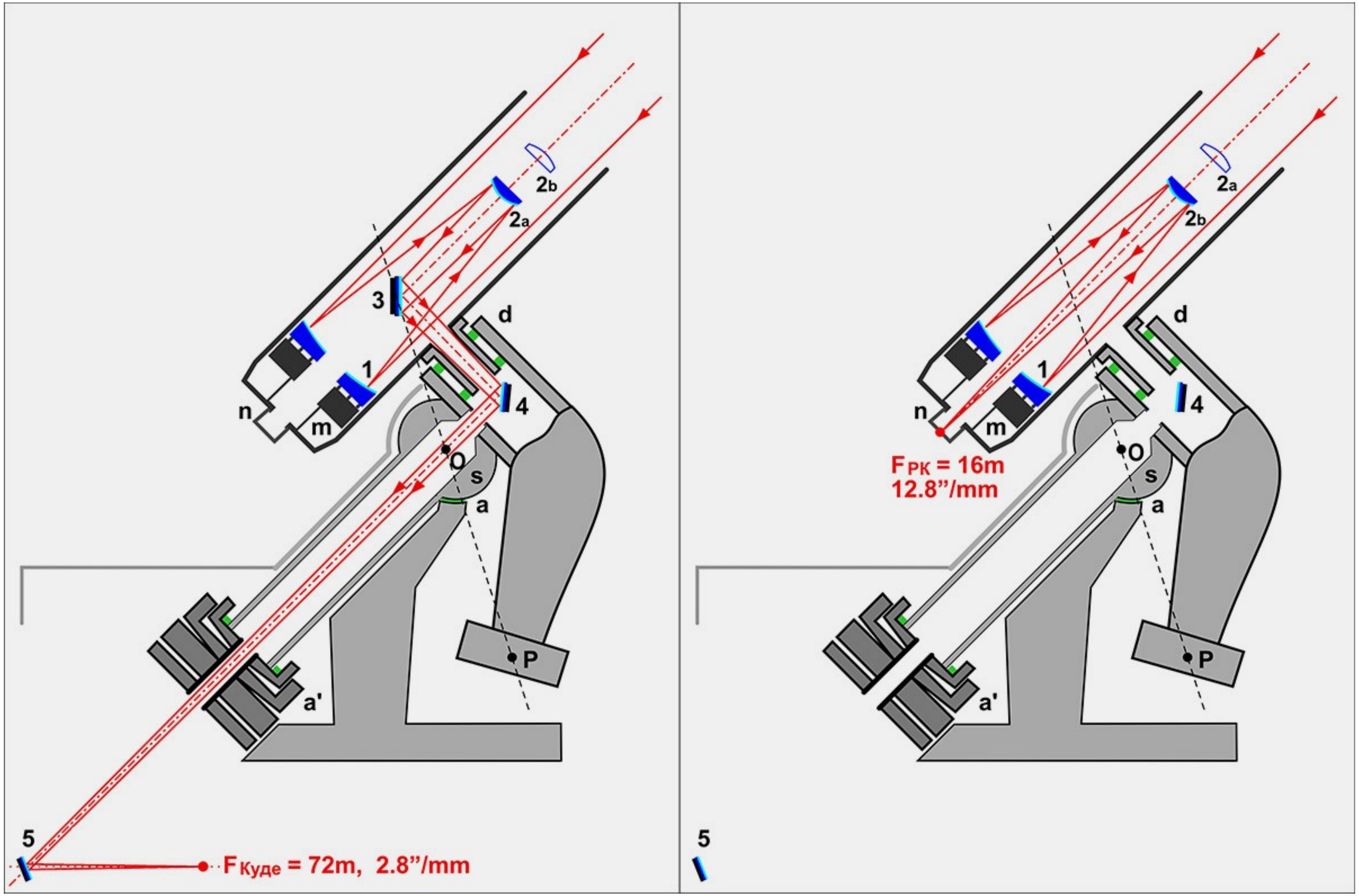
Types of reflectors

(Credit: Paul Harvey, University Texas)



Or Nasmyth

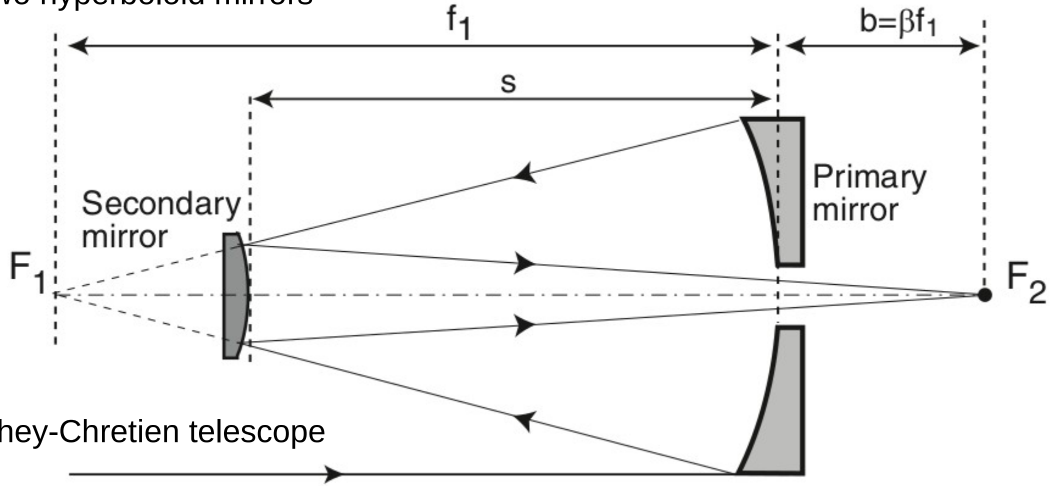
Coude focus



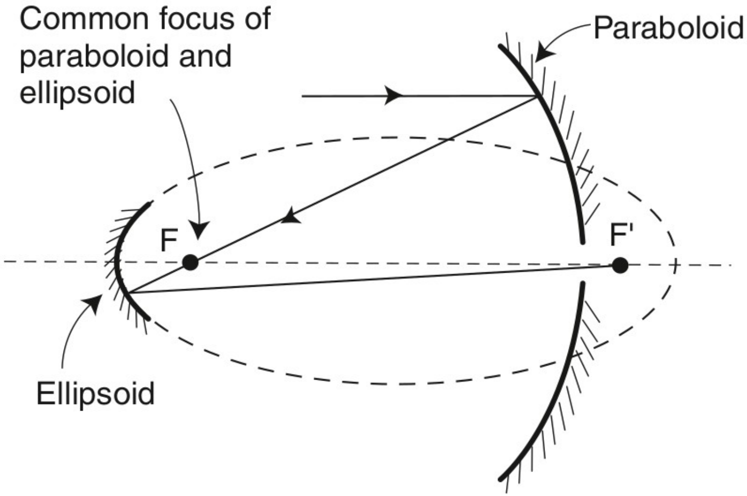
(Credit: NAO-Rozhen, Bulgaria)

Variations of Cassegrain

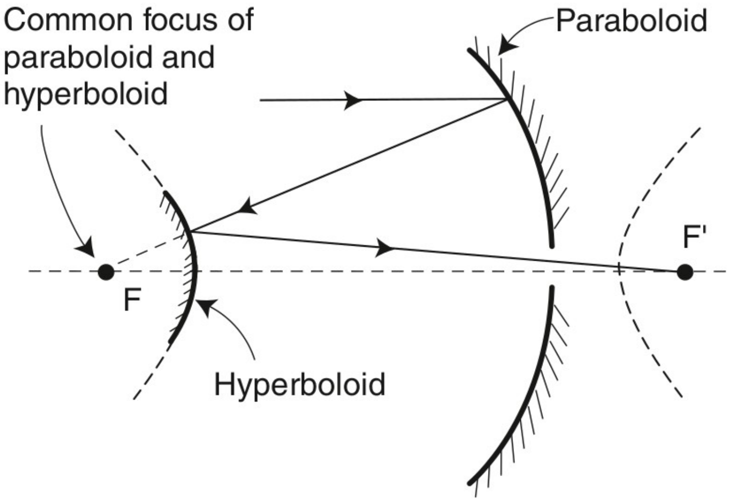
Two hyperboloid mirrors



Ritchey-Chretien telescope



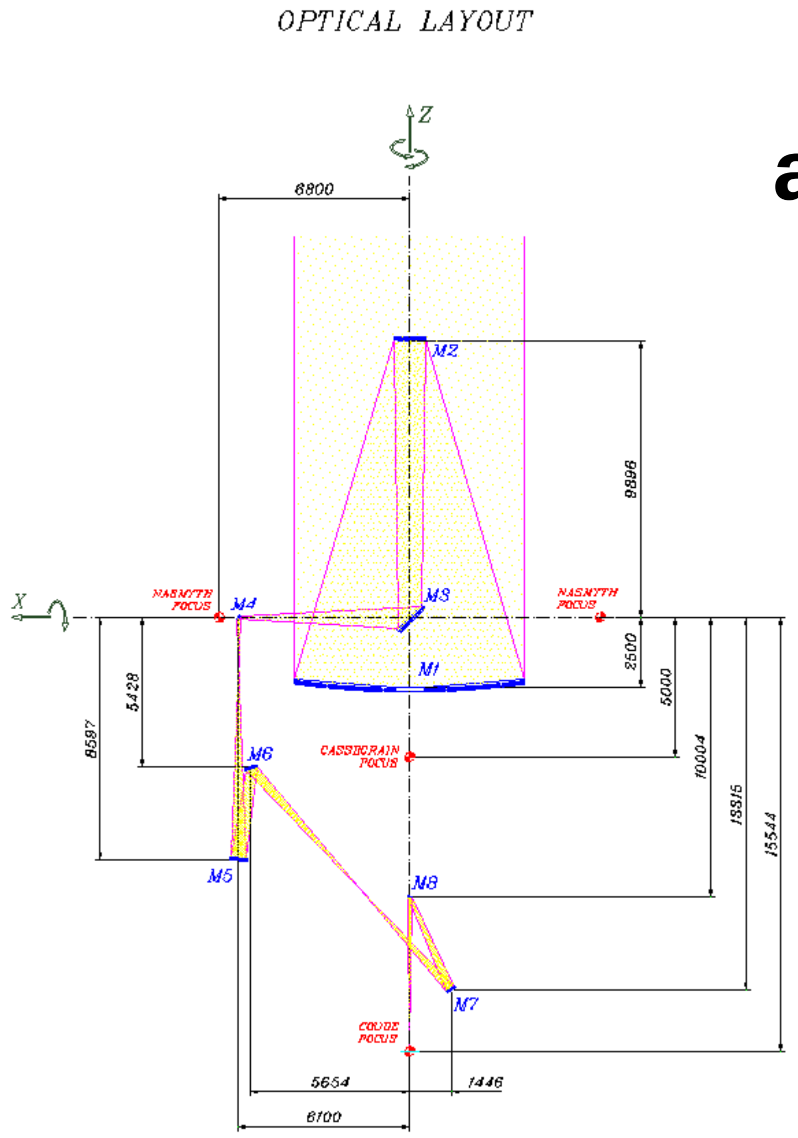
Gregorian telescope

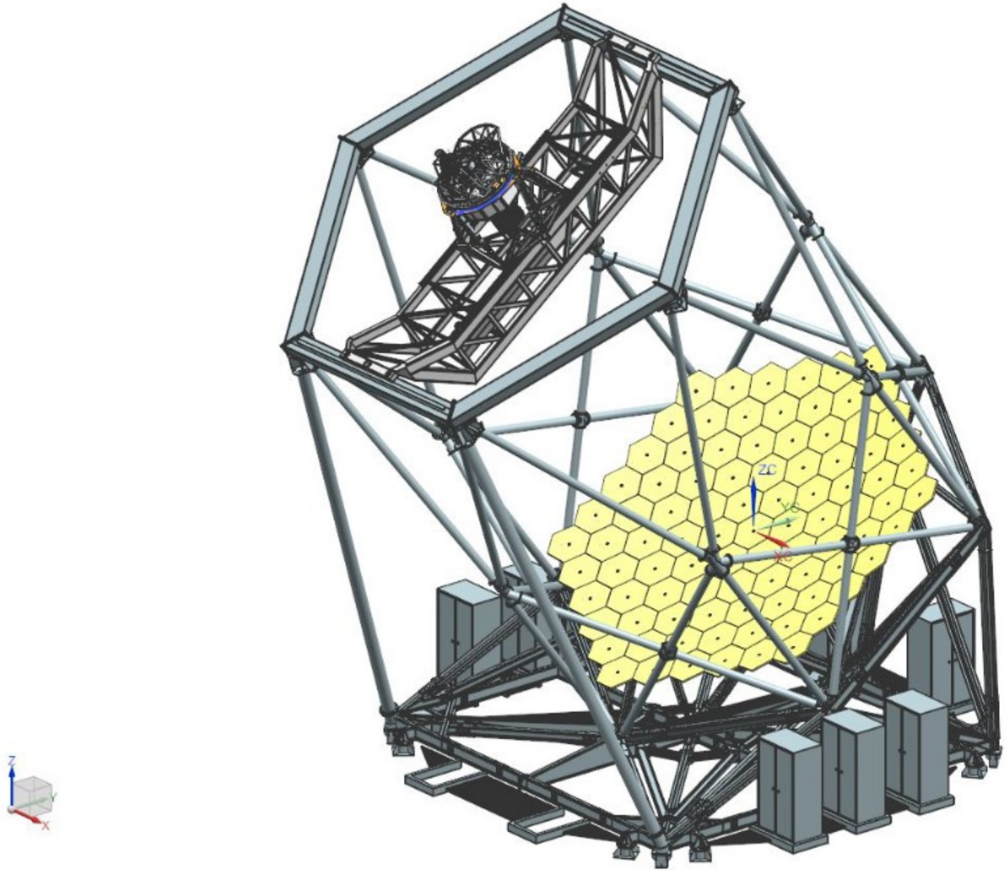


Cassegrain telescope

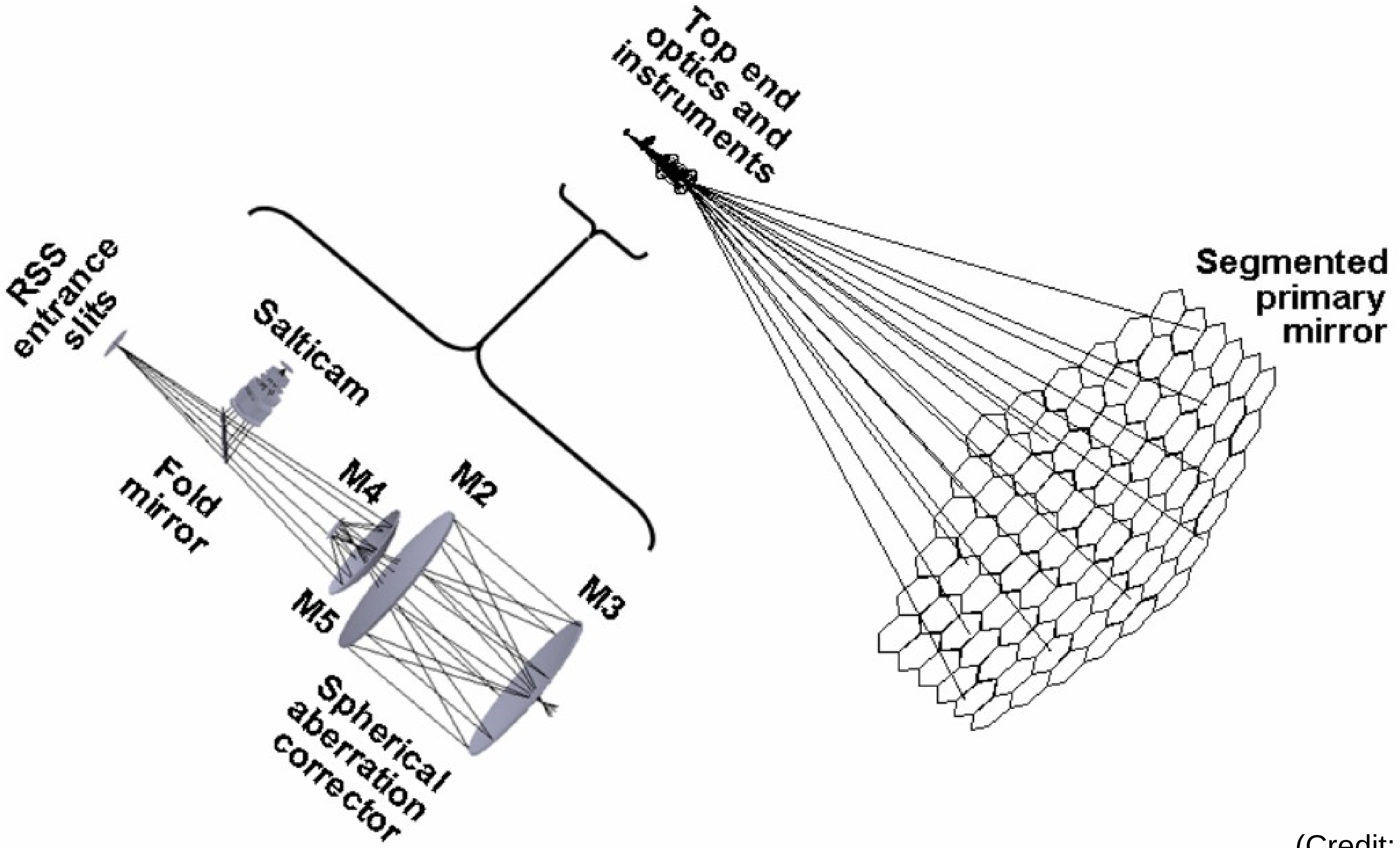
(Credit: Bely 2003)

VLT
and SALT


OPTICAL LAYOUT
Z
6800
M2
9896
X
NASMYTH FOCUS
M4
M3
NASMYTH FOCUS
2500
M1
5000
5428
8597
M6
CASSEGRAIN FOCUS
10004
13315
15544
M5
M8
M7
COUDE FOCUS
5654
1446
6100



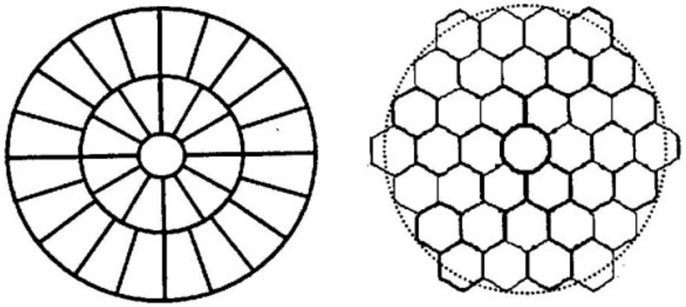
SALT Layout



(Credit: O’Donoghue 2008)

Segmented mirrors

- 8m is the limit for monolithic mirror (because of transport and facilities for melting, assembly, polishing)
- Segmented mirrors: surface is not continuous:
  - Carefully made as part of the same shape
  - Maintained in place (in spite of gravity, wind, thermal effects)
- Hexagon geometry preferred. Optimal size 1-2m.



(Credit: Bely 2003)

Fig. 4.34. The two main segmented mirror geometries: petal (left) and hexagonal (right).

(Credit: ESO)

Questions?

(Credit: Shutterstock)

Stops and pupils

- **Aperture stop:** the element that determined the amount of light reaching the image
- Mostly, the primary mirror (can be the secondary in IR observations)
- **Field stop:** the element that determines the angular size of the field imaged by the system
- The edges of the detector or a diaphragm in an image plane ahead of the detector
- **Vignetting:** An intermediate component is not big enough to accept oblique rays entering the aperture

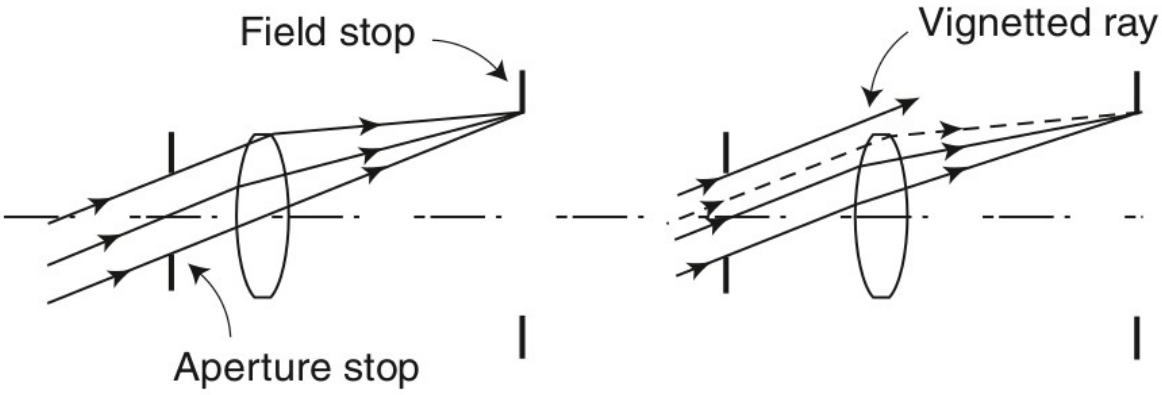
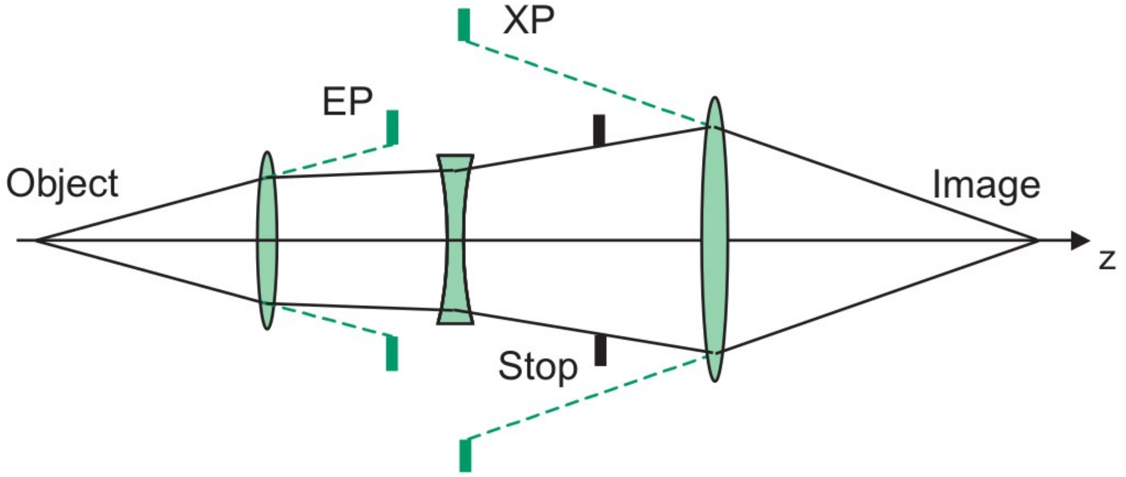


Fig. 4.3. Aperture and field stops (left) and vignetting (right).

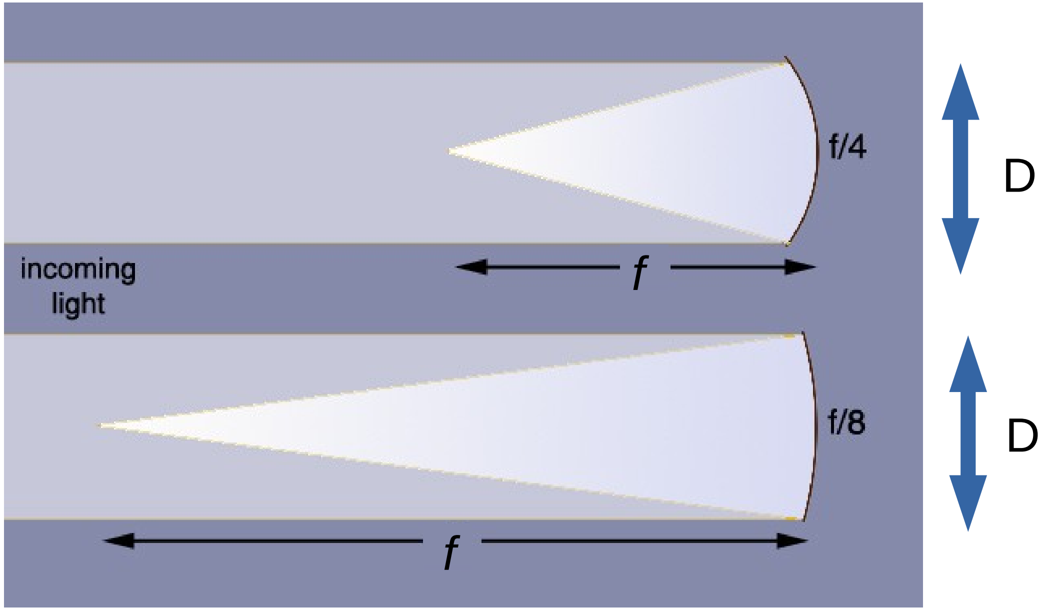
Stops and pupils

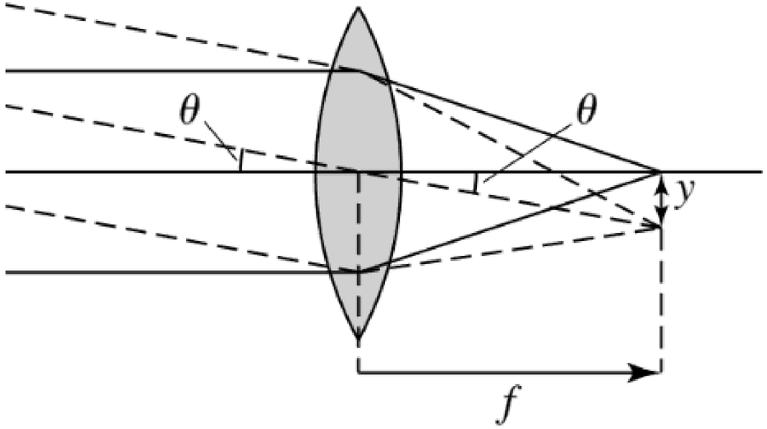
- The pupils define the cones of light entering and exiting the optical system from any object point.
- **Entrance pupil:** image of the aperture stop formed by the optical system preceding it.
- Most telescopes: entrance pupil = aperture stop
- **Exit pupil:** image of the aperture stop formed by the system following it.
- Rays from the boundary of the aperture stop reach the final image as if coming from the boundary of the exit pupil



(Credit: Greivenkamp 2004)

f-number, plate scale



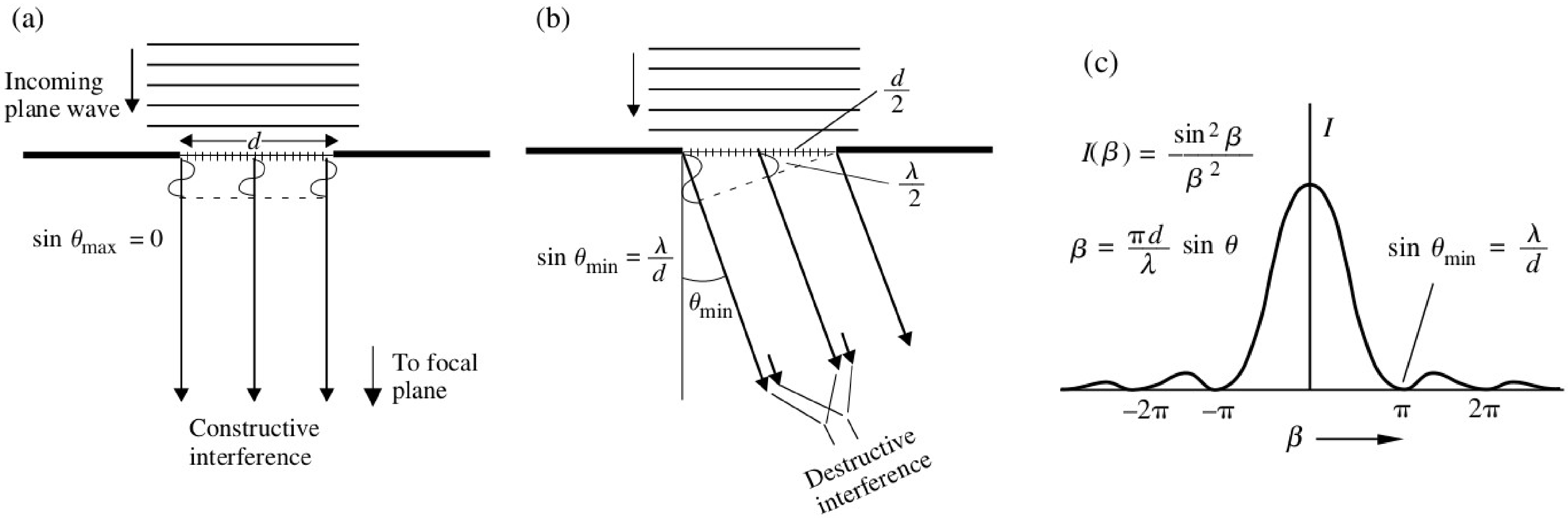


- f = focal length
- D = aperture diameter

- f-ratio = F/D

- y = f tan(θ)
- y = f θ (for small θ)
- dθ/dy = 1/f
- dθ/dy = p = 206265”/f
  (p = plate scale)

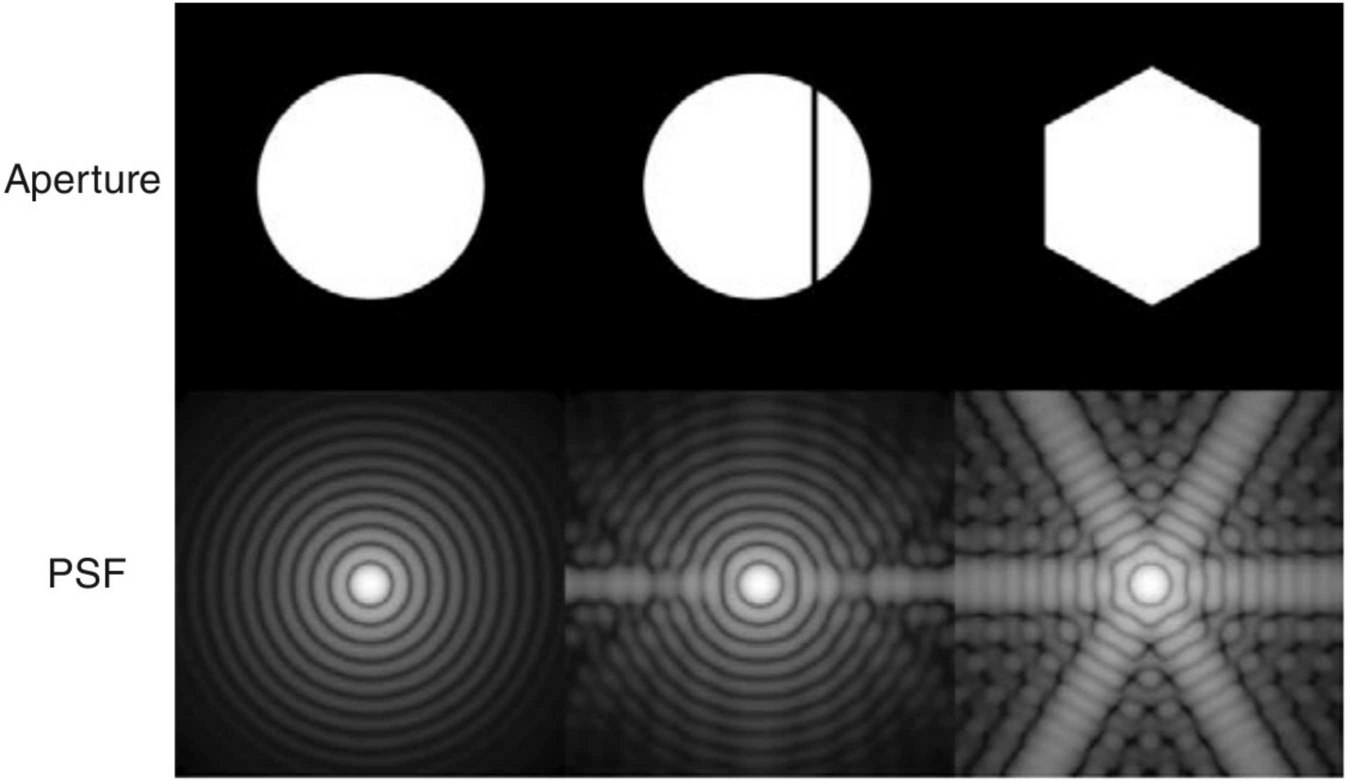
Diffraction



(Credit: Bradt 2004)

Diffraction

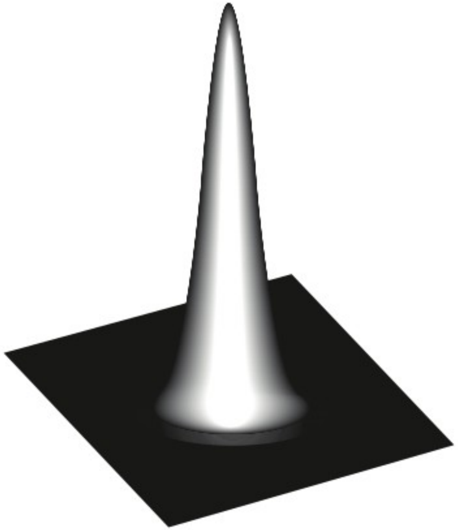
- An optical system will not image a point source as a true point
- Caused at the edges of the aperture and any other obstacle

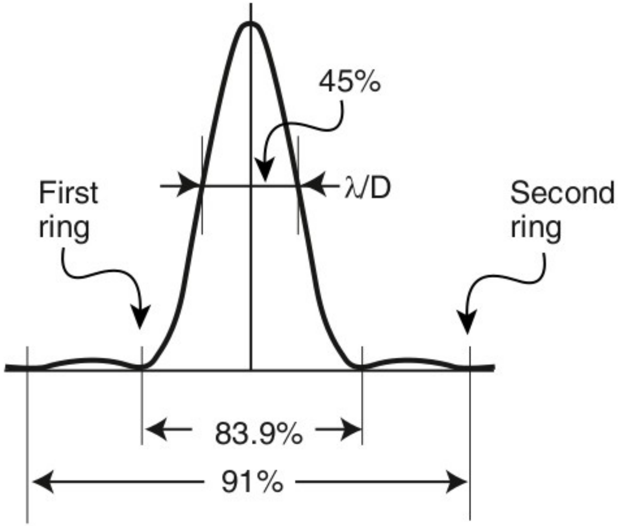


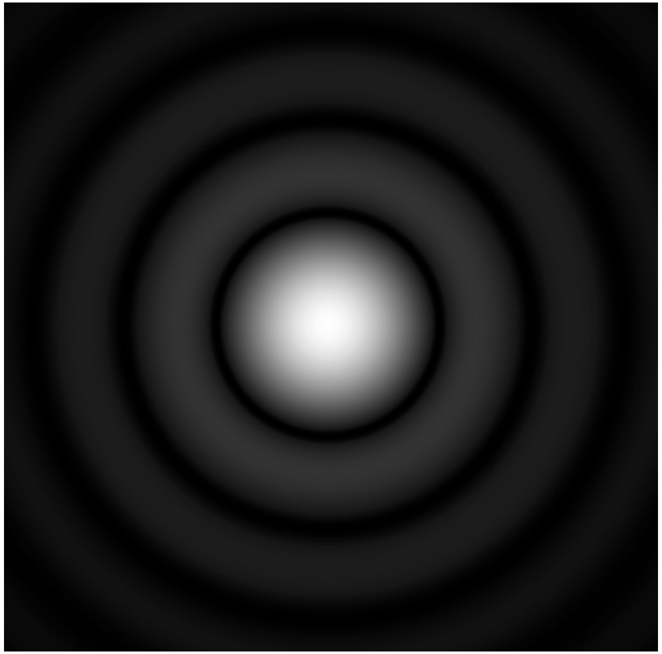
(Credit: Bely 2003)

Image formation

- **Point Spread Function:** distribution of light intensity in the image of a point source
- Extended objects can be considered a collection of point sources







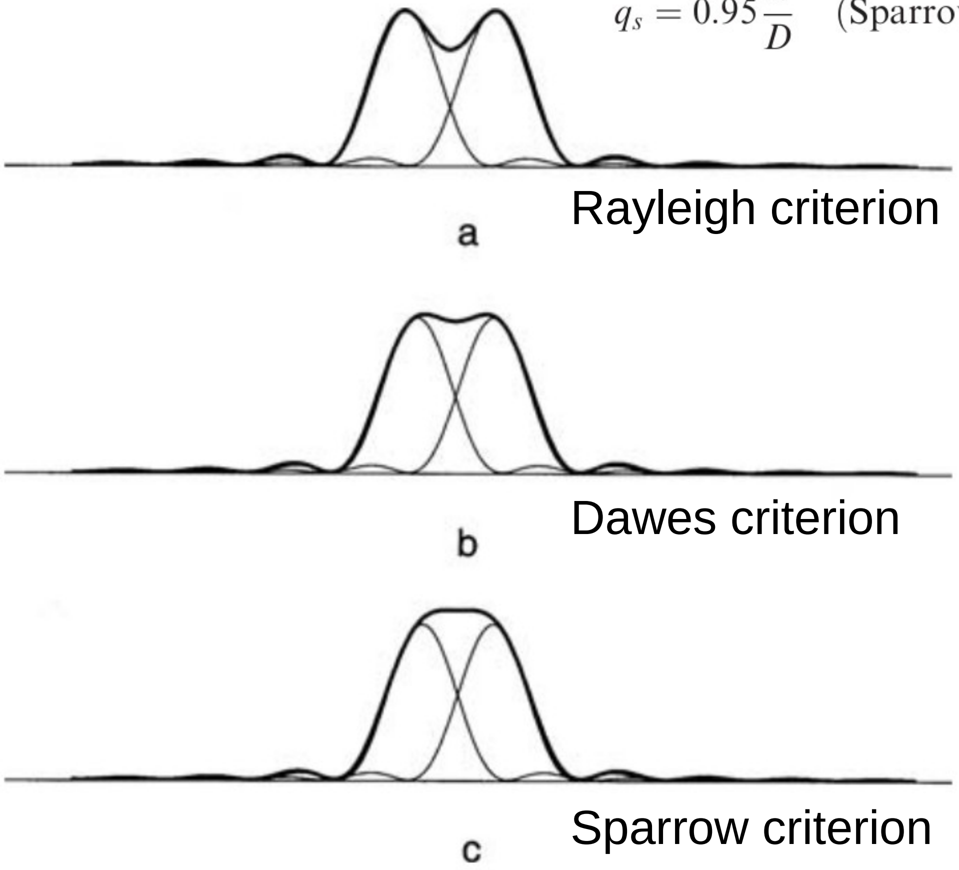
Angular resolution

$$q_r = 1.22\frac{\lambda}{D} \text{ (Rayleigh)}$$

$$q_d = 1.02\frac{\lambda}{D} \quad \text{(Dawes)}$$

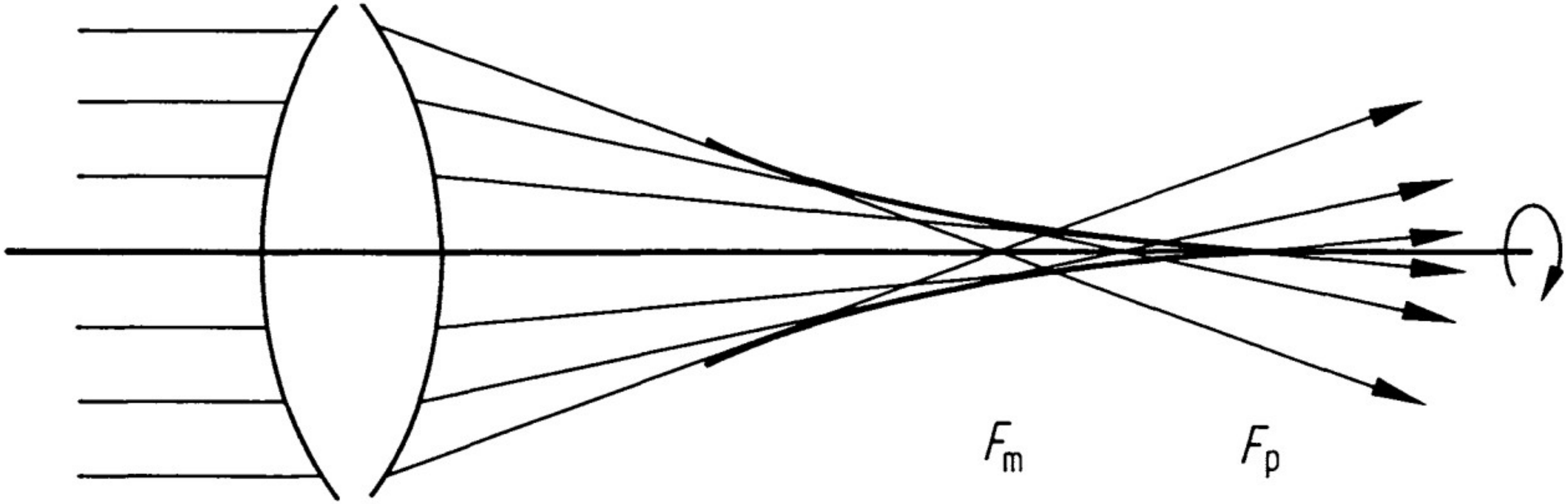
$$q_s = 0.95\frac{\lambda}{D} \quad \text{(Sparrow)}$$

- No system is perfect and the image is not given only by diffraction
- **Strehl-ratio:** ratio between the normalized peak intensity of the actual PSF to that of the perfect image
- Diffraction-limited: Strehl ratio > 0.8
- **Angular resolution:** ability of an optical system to distinguish details in the image
- Or smallest angle between two point sources for which separate images are produced
- Rayleigh criterion: the central peak of one falls upon the first minimum of the other



Aberrations

- **Spherical aberration**
- Each annular zone has its own focus
- Reduced by decreasing the size of the aperture or long focal length
- Or with non-spherical surfaces



Aberrations

- **Coma**
- Rays from an off-axis source do not converge to the same focal plane
- Creates a blur that resembles a comet
- Dominant aberration in Cassegrain systems (with paraboloid primary)
- Increases with distance from the optical axis
- Mirrors with small f-ratio are more affected

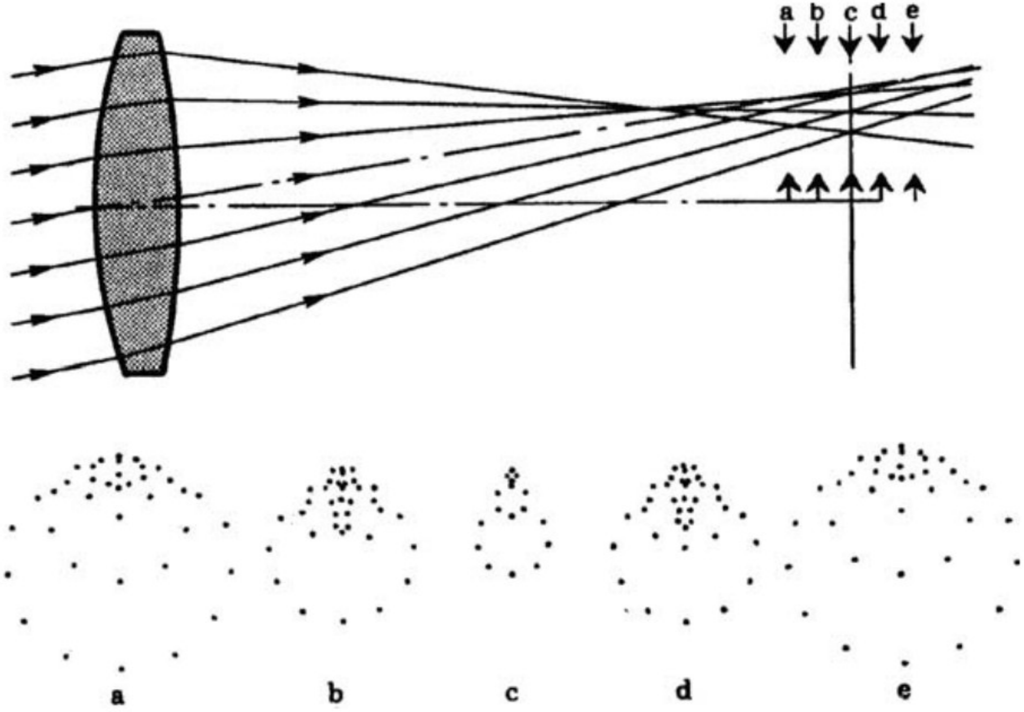
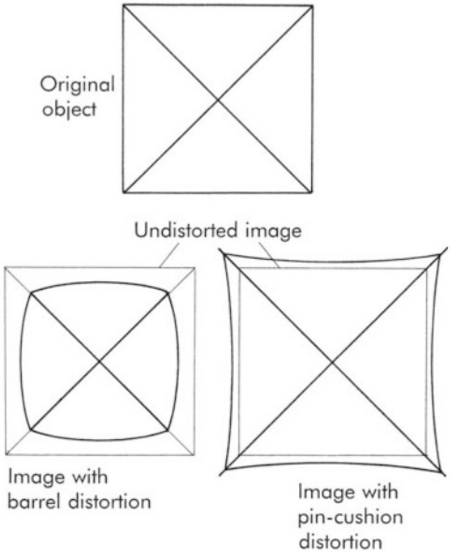
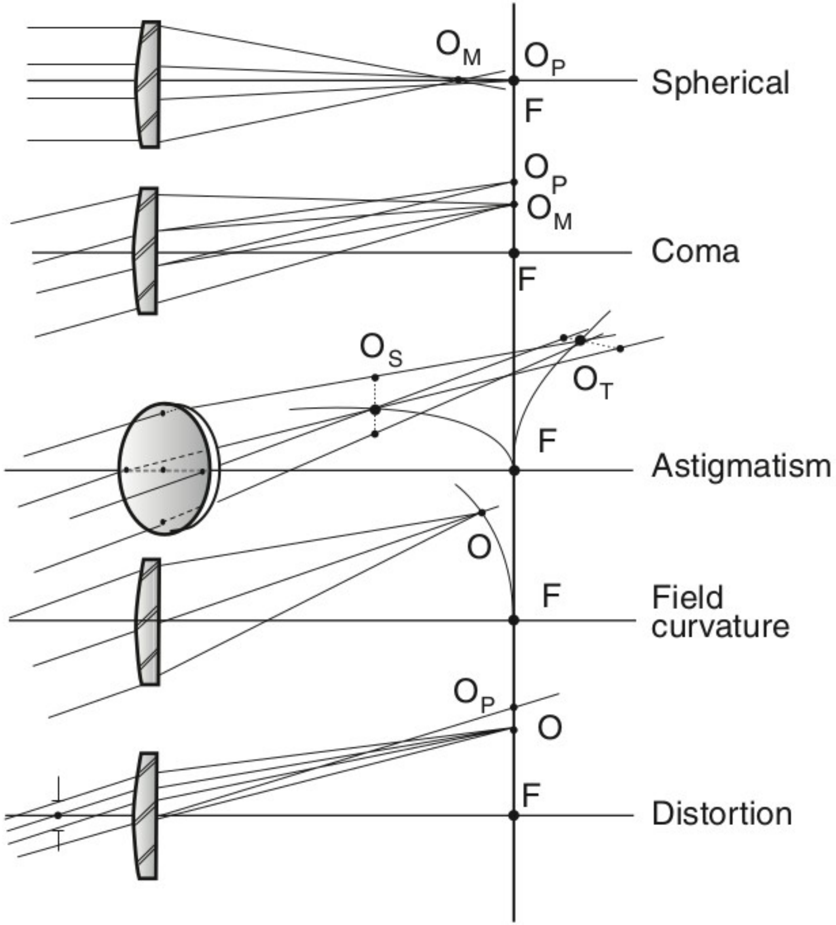


Fig. 1.30. The image of coma along the axis.

Aberrations

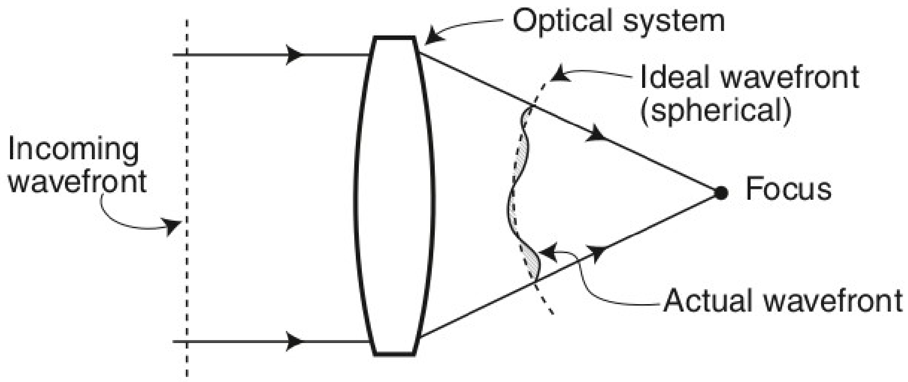
- **Astigmatism**
- Focus of the horizontal plane is different from the one of the vertical plane
- Affects the VLT
- **Field curvature**
- **Distortion**

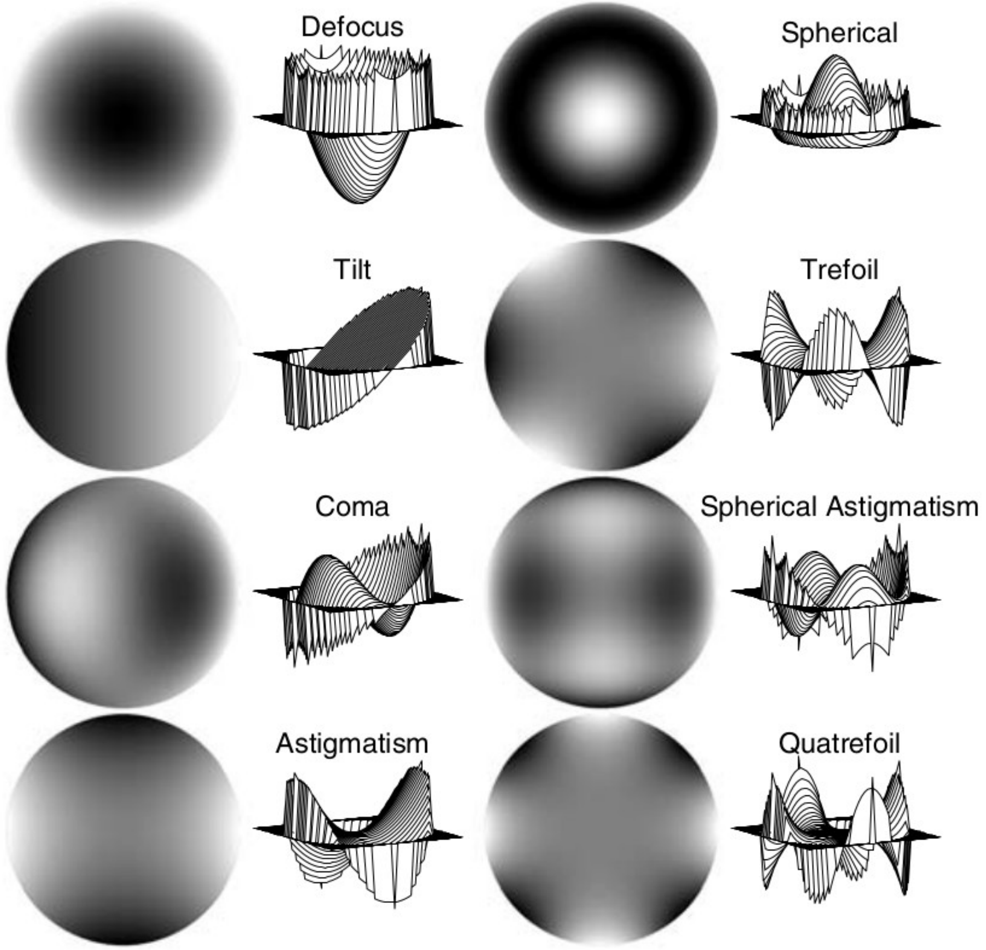




Aberrations

- **Aberrations** can also be described by wavefront errors
- Written as a linear combination of Zernike polynomials





Questions?

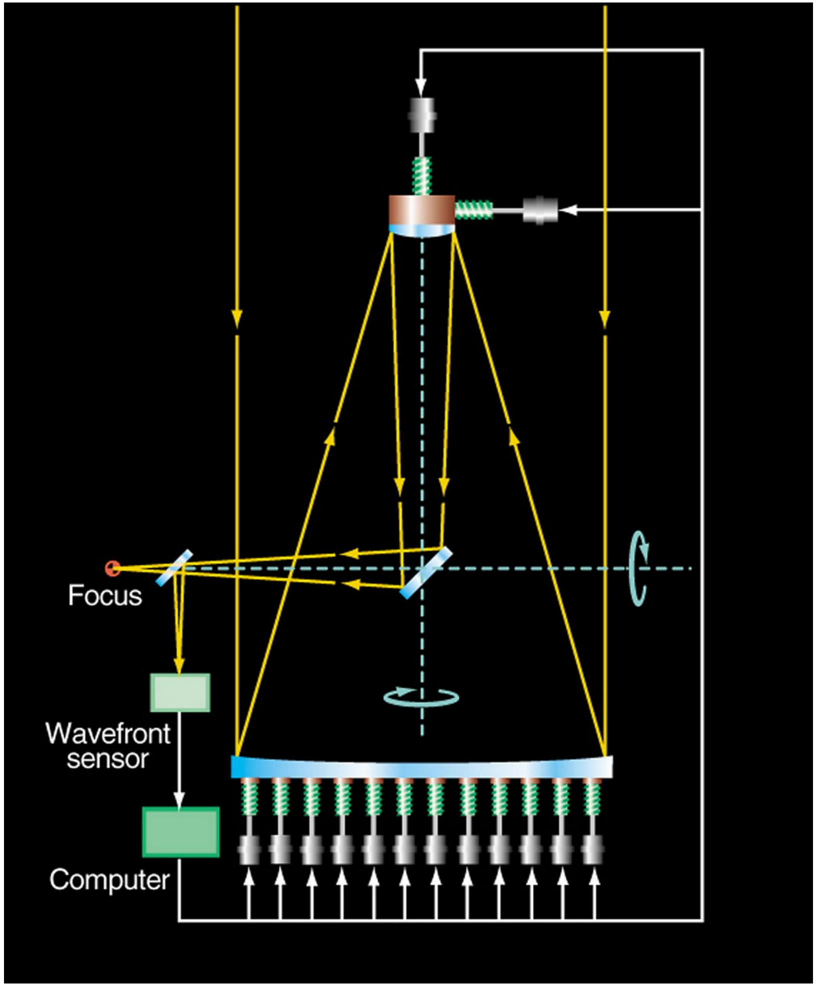
(Credit: Shutterstock)

Active optics

- Actuators correct the shape of the primary mirror and move the secondary
- Correct deformations from thermal effects, wind buffeting, telescope inclination

VLT case (all 4 UTs):

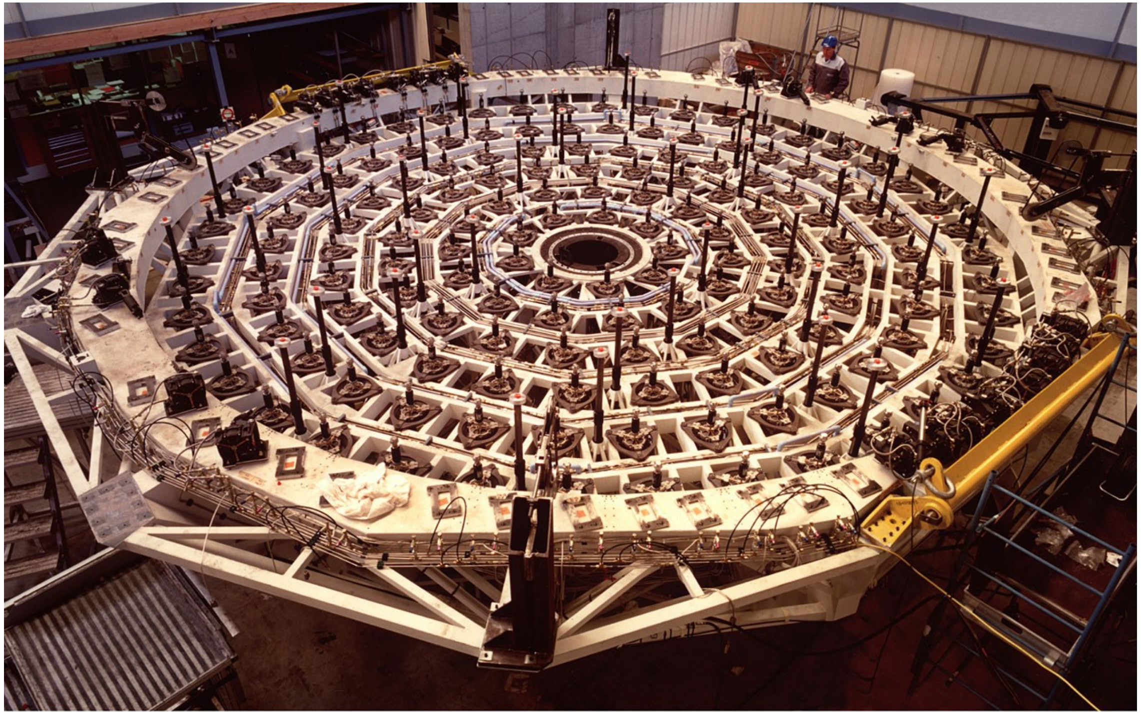
- Wavefront sensor (adapter/rotator): observes a reference star to determine wavefront aberrations through image analysis
- 30s integration used to analyse deviations from best quality
- Decomposes the deviation into single optical contributions (defocus, astigmatism, coma etc…)
- Calculates the force correction for each of 150 actuators to achieve the optimal quality



Active optics

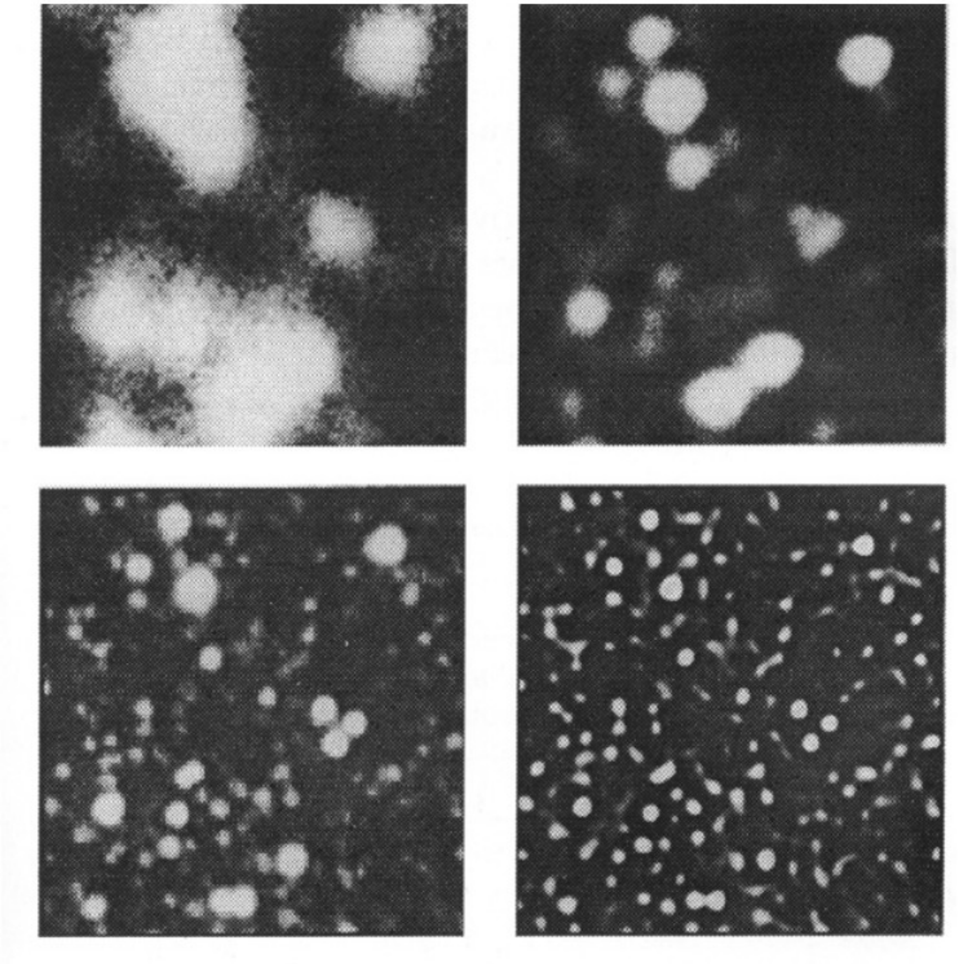
- Two other modes:
  - Fast correction (1s): needs brighter reference and decreases aberrations that can be corrected
  - Open loop: ignores the analysis; predict forces based on tube inclination and temperature

- Active optics is essential to coalign and cophase an array of mirror segments



Active optics

- Field in the globular cluster Omega Centauri.
  - Top left: ESO 1m telescope (2” seeing)
  - Top right: ESO 3.6m telescope (1” seeing)
  - Bottom left: NTT with active optics (0.33” FWHM)
  - Bottom right: same image further processed

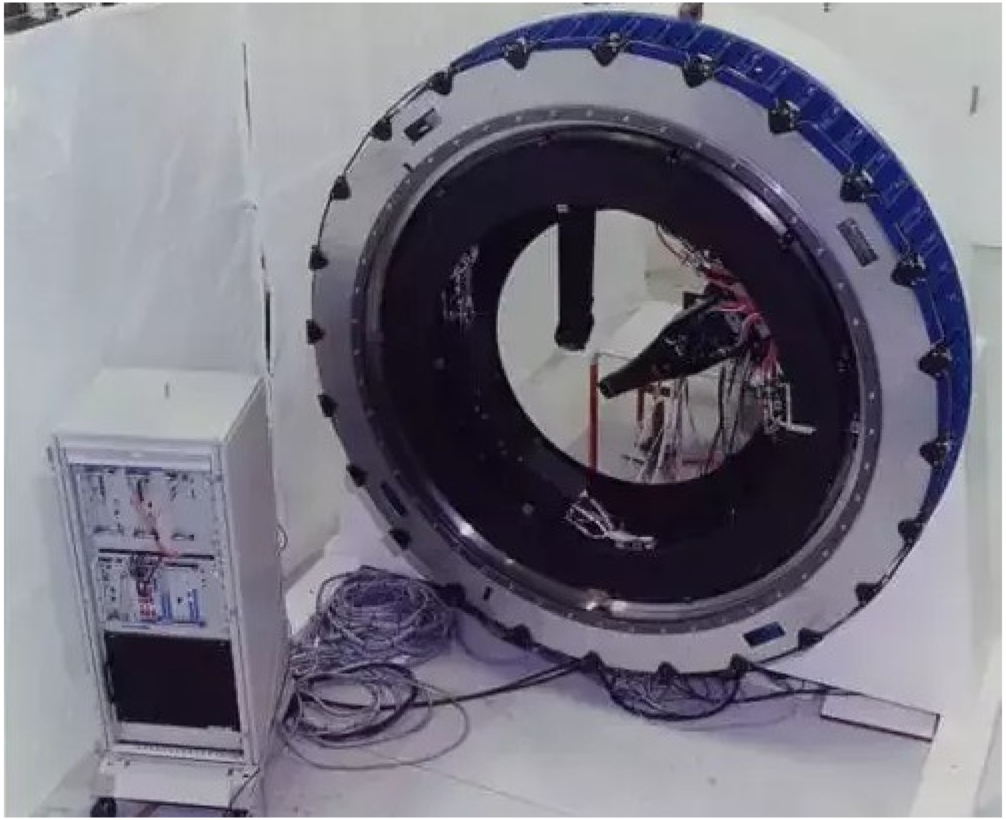


(Wilson 2003)

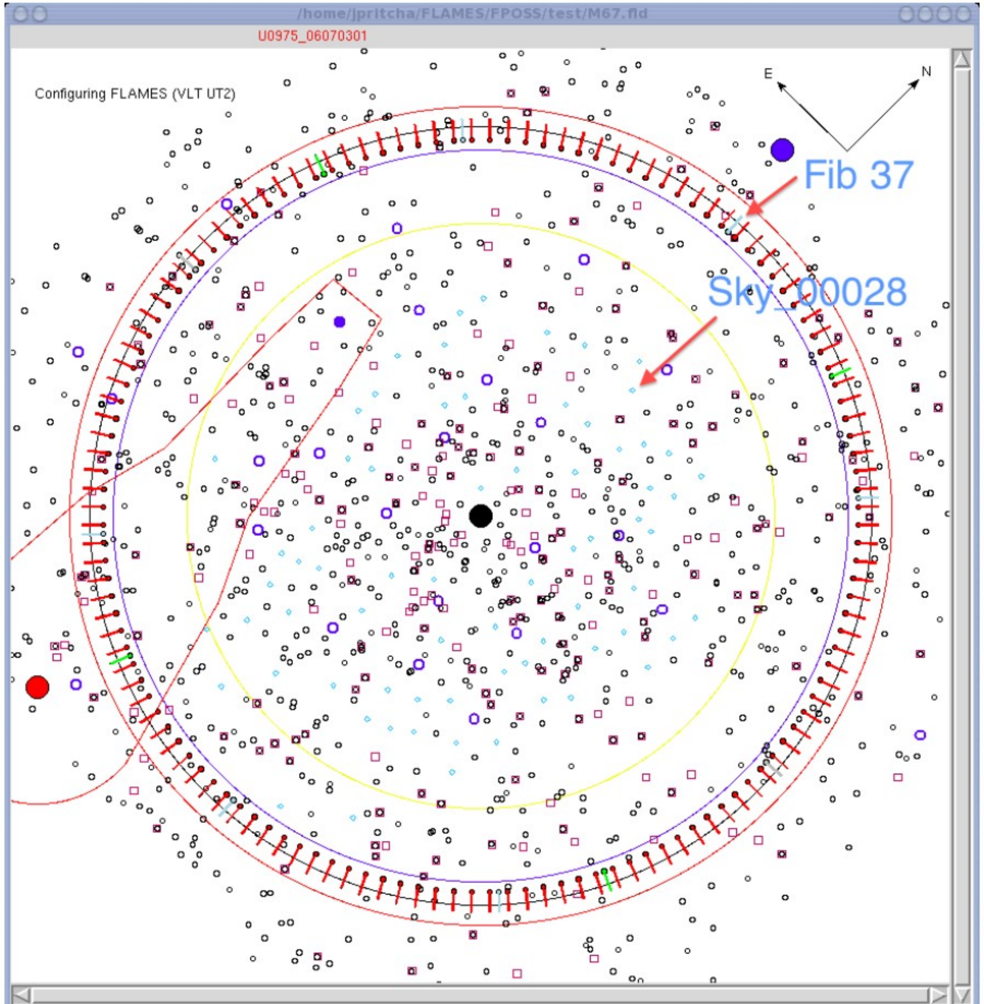
How to check if the correct object was found?

Acquisition/guiding

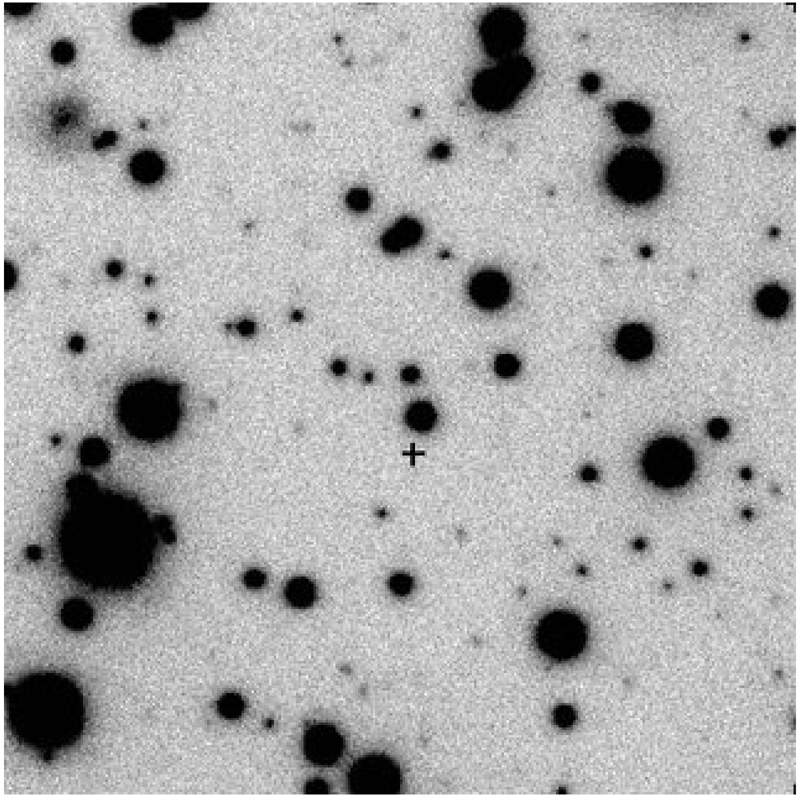
- The adapter/rotator connects the telescope and instrument
- It has a sensor arm
- **Acquisition:** view the central part of the telescope field to provide visual identification of the object to be observed.
- **Guiding:** star close to the object being observed provides a reference position
- Outside the field being observed
- The same star is used to analyse deviations from best quality for the active optics

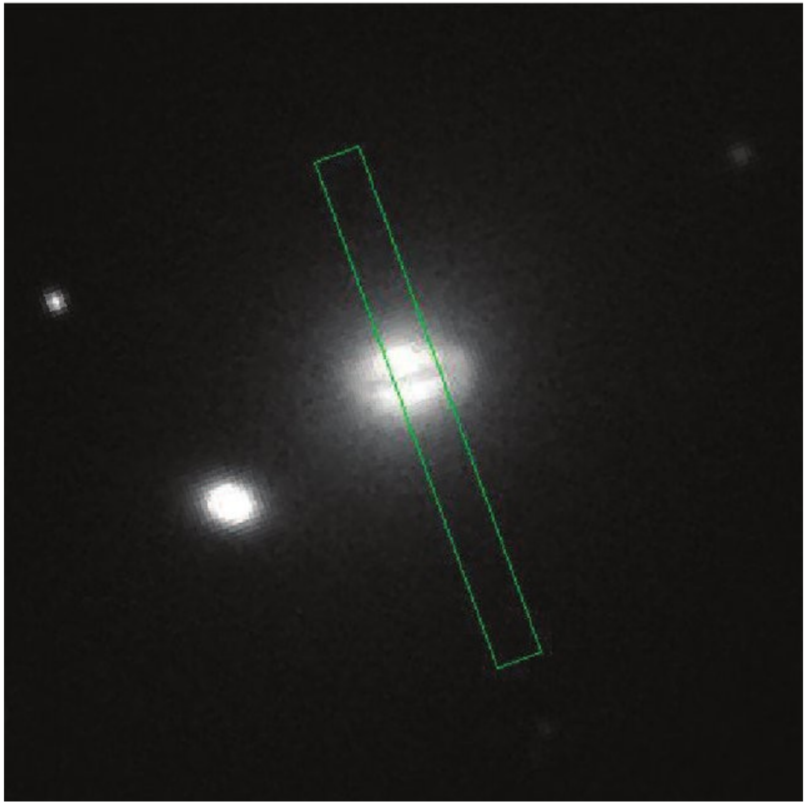


Acquisition/guiding



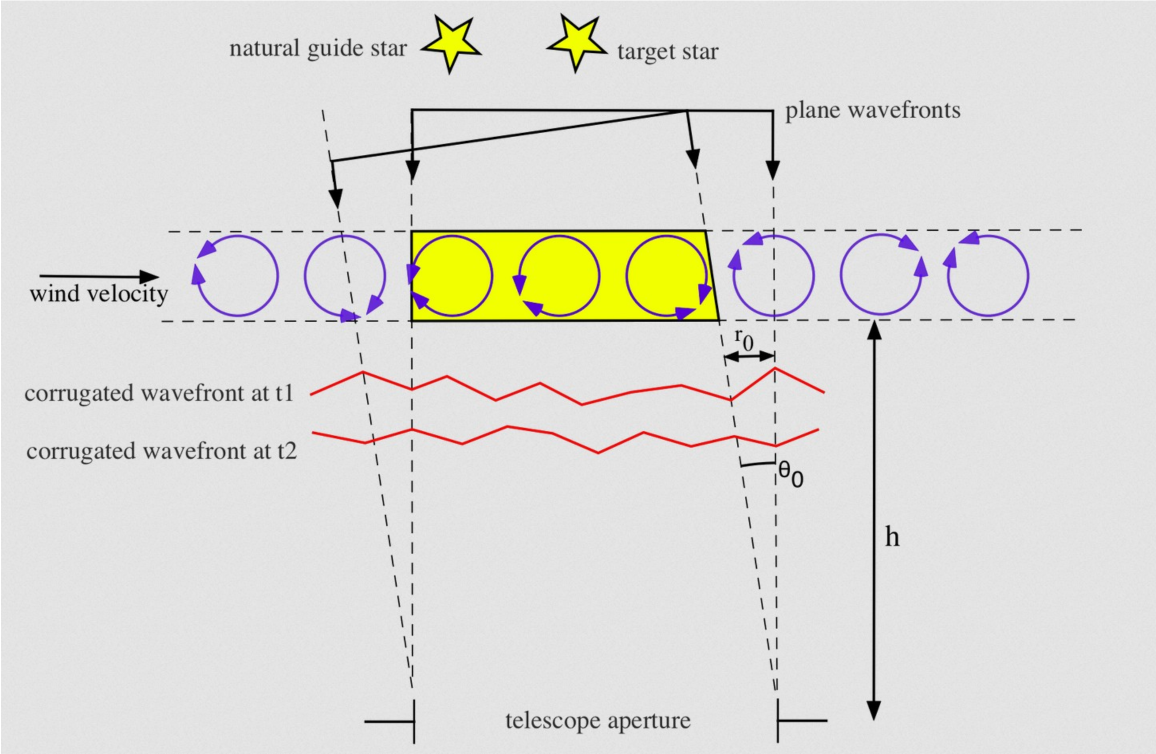
Acquisition/guiding





Adaptive optics (AO)

- Corrects the distortions caused by atmospheric turbulence
- Measures the distortions of another point source in the isoplanatic angle
- Applies the opposite effect to correct the wavefront
- Requires a nearby bright star (natural guide star, NGS). Not always available (R = 9 -11 mag)
- Artificial laser stars are the alternative
- Adaptive optics: to achieve diffraction limit
- **VLT: Laser Guide possible with the Adaptive Optics Facility @ UT4**
- **A few instruments have AO modules (Interferometry, CRIRES, SPHERE)**



Adaptive optics

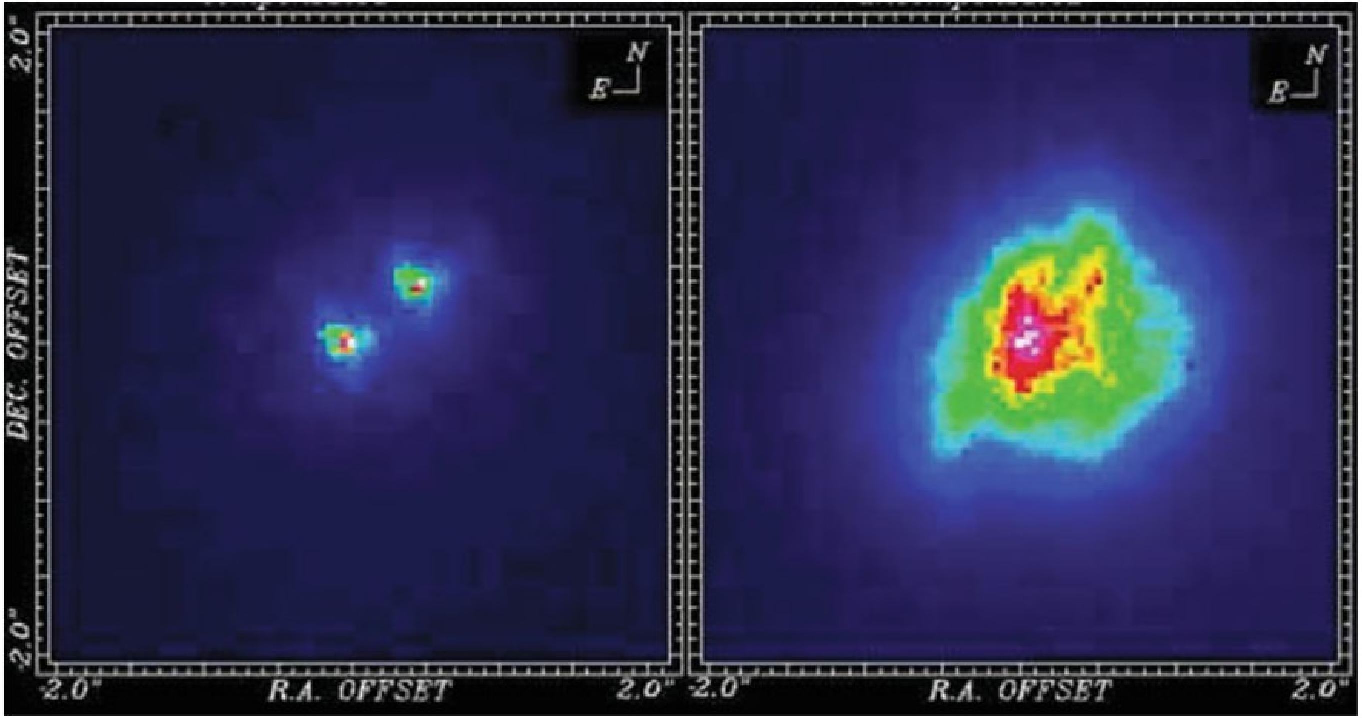
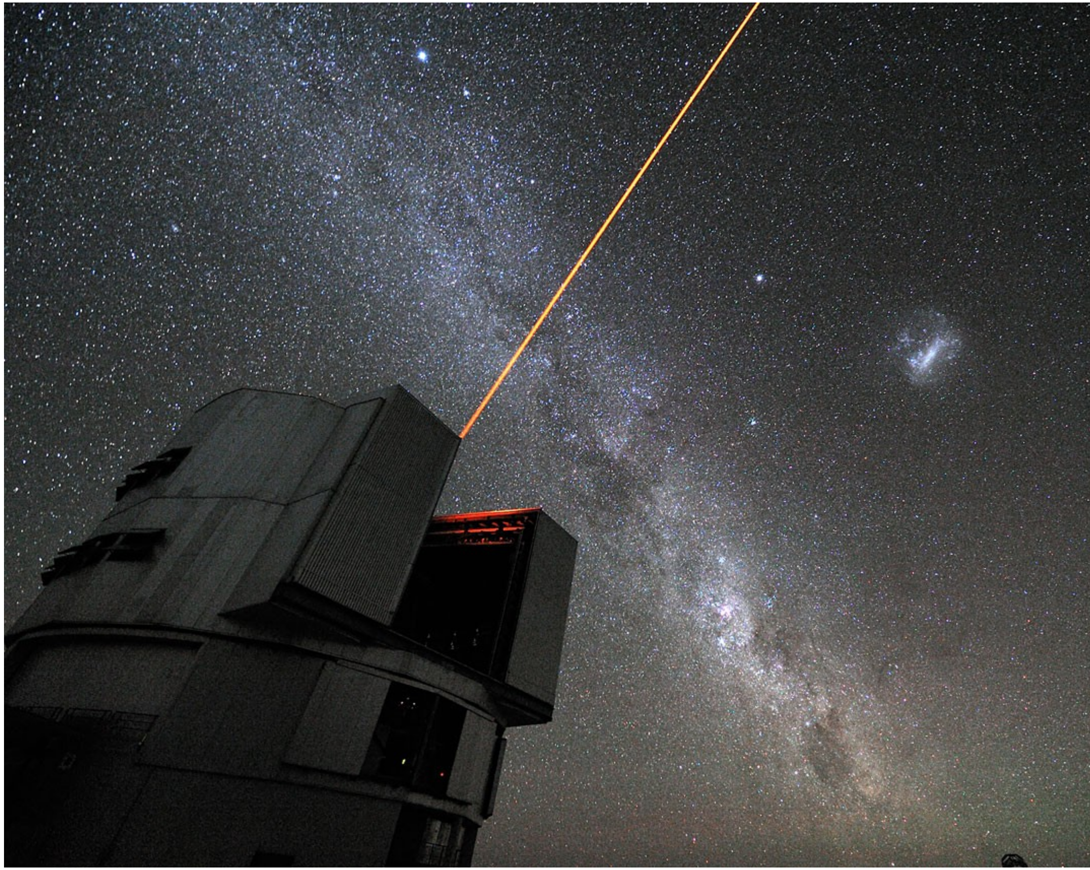


Fig. 6.19 Birth of adaptive optics. Image of a binary system (magnitude $m_V = 13.1$, separation $0.59''$) using a 3.6 m telescope at wavelength 1.65 μm (European Southern Observatory, La Silla, Chile). *Right*: Image affected by atmospheric turbulence (seeing $1.7''$). *Left*: Corrected image, exposure time 10 s, final resolution $0.12''$ (FWHM), 1 pixel = 50 mas. Adaptive system Come-On Plus, 1993. (Picture by Beuzit J.L., Paris Observatory and ESO)

Laser guide stars

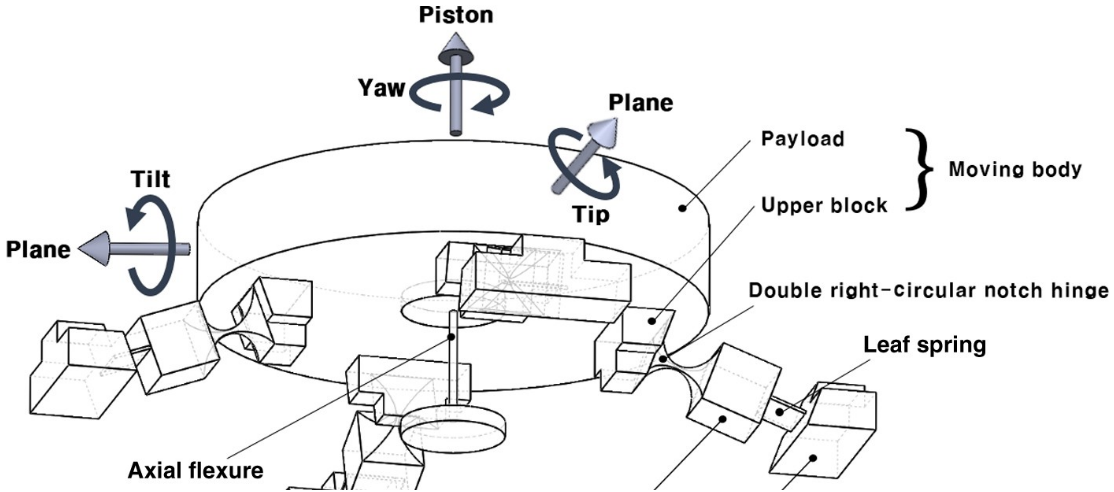
- Two options of LGS:
  - **Sodium laser**
- ~80 km altitude, there is a Na layer regularly refreshed by meteoritic particles
- Beam with 589.3nm excites Na, spontaneous emission provides an almost point source
- Needs a powerful laser
  - **Rayleigh scattering**
- Laser beam focused to 10-20 km
- Backscattered photons are observed
- Still below some of the turbulence

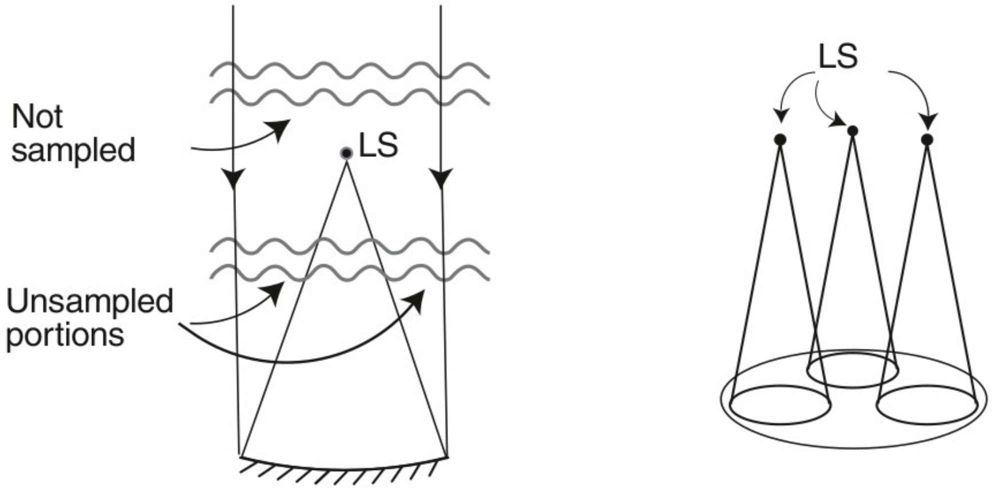


Limitations of LGSs

- LGS does not provide tip-tilt correction
- i.e. 2D offset tilt of the image (the laser crosses the atmosphere twice, and the movement is then cancelled)
- NGS still needed for tip-tilt

  (corrected with fast movement of the secondary)
- LGS samples a light cone, not the full regi affecting the wavefront
- Multiple LGS can help to sample the complete region





Adaptive Optic System: MACAO

- CRIRES: infrared spectrograph with AO (slits 0.2” or 0.4”)
- **MACAO: Multi-Application Curvature Adaptive Optics system**
- AO fiber that redirects light to MACAO and afterwards concentrates it on the entrance slit
- The target can be used as NGS: some light reflected by the slit to the AO system
- Ideally AO star R~11mag
- Brighter to R~0.2mag possible with neutral density filter
- Good performance down to R~14mag
- Not possible R > 15 mag
- 4 MACAO systems for the VLTI (each Coude)

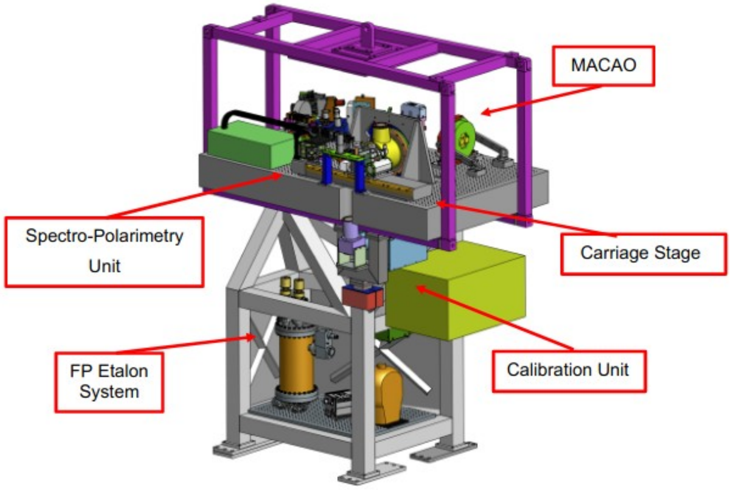


Figure 8: The upgraded CRIRES warm part assembly with etalon system, calibration slide, AO system and de-rotator mechanism

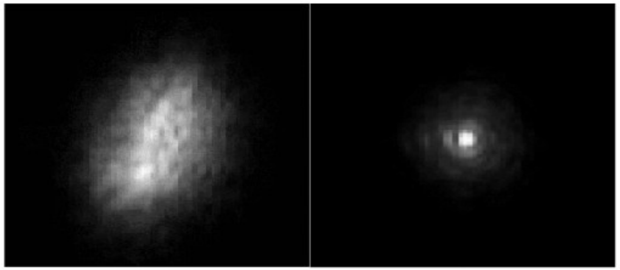
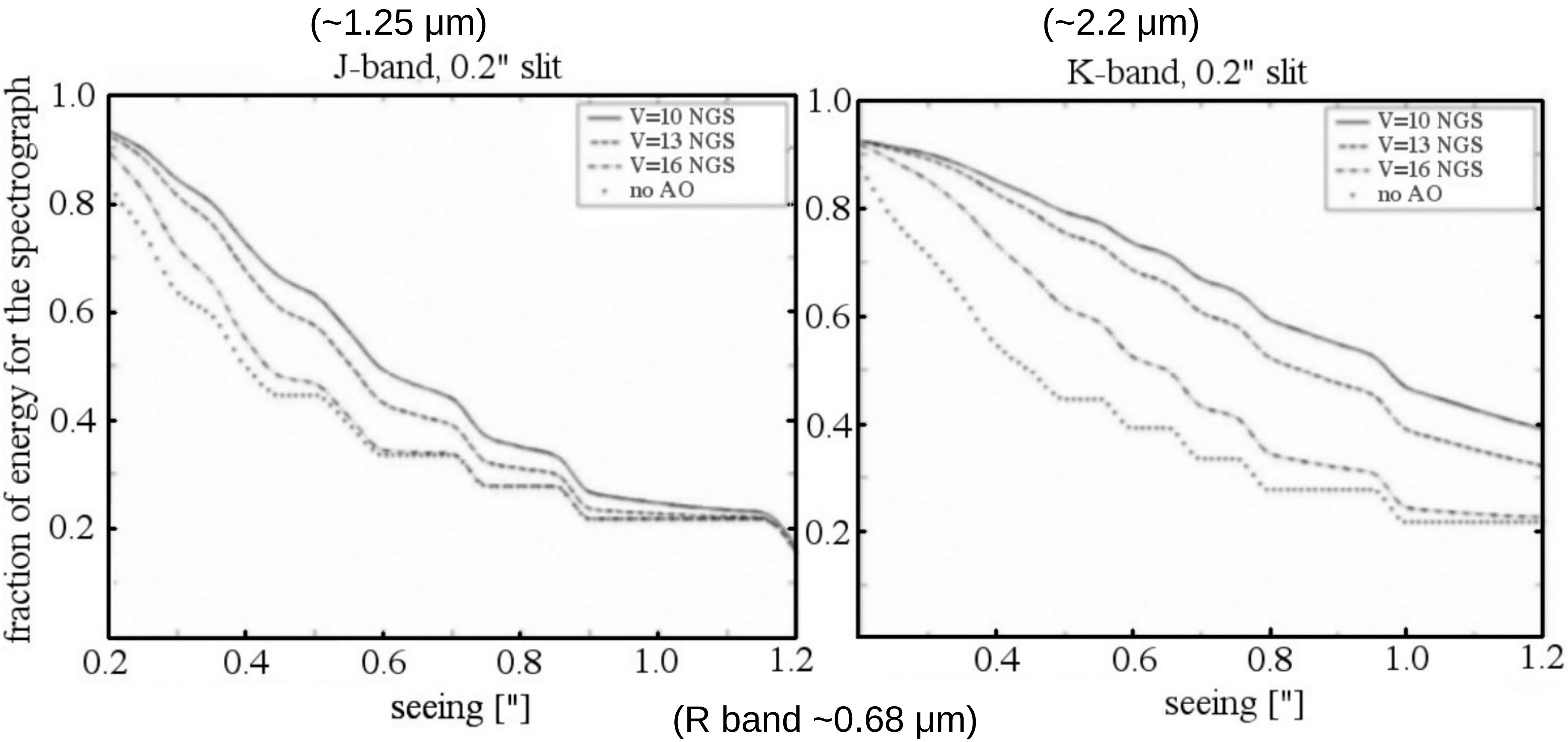


Figure 9: PSF without (left) and with (right) MACAO correction. Images have been taken in lab using a turbulence generator.

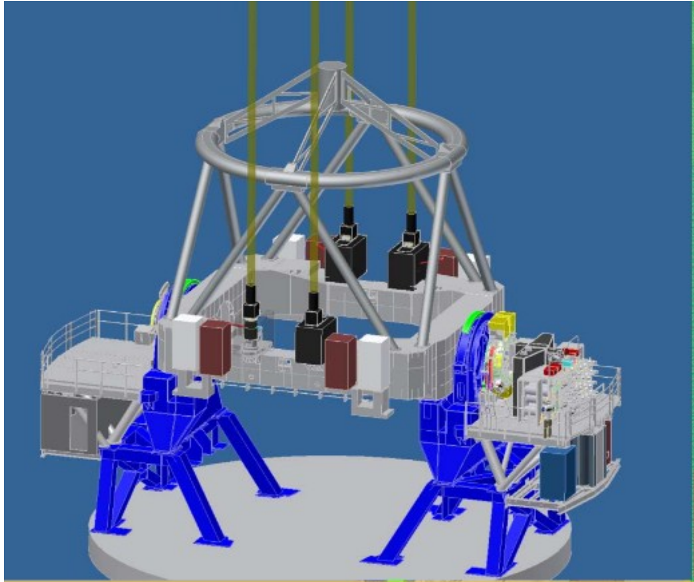
Adaptive Optic System: MACAO



4 Laser Guide Star Facility (4LGSF)

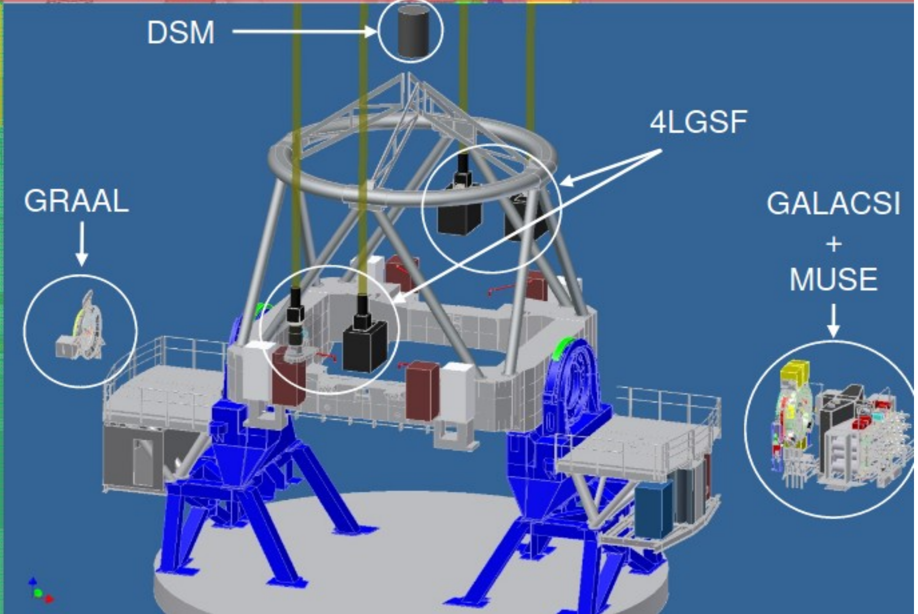
- Only at the UT4
- 4 Na LGS
- Can be used with MUSE (integral field spectrograph) and HAWK-I (near infrared wide field imager)
- Each instrument has its AO system
  1) GALACSI for MUSE
  2) GRAAL for HAWK-I
- The corrections are fed to the DSM (deformable secondary mirror)

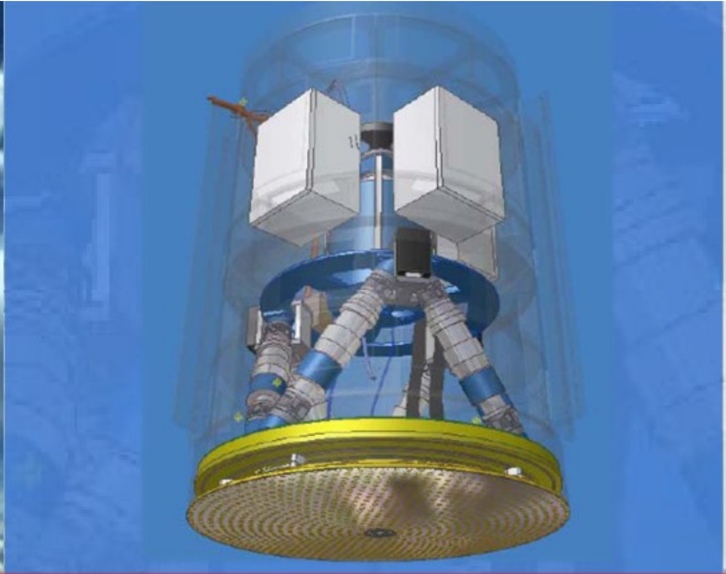
AOF:	ADAPTIVE OPTICS FACILITY:
DSM:	DEFORMABLE SECONDARY MIRROR
4 LGSF:	4 LASER GUIDE STAR FACILITY
GALACSI:	Ground Atmospheric Layer Adaptive Corrector for Spectroscopic Imaging
MUSE:	Multi-Unit Spectrocscopic Explorer
GRAAL:	GRound layer Adaptive optics Assisted by Lasers
HAWK-I:	High Acuity, Wide field K-band Imaging
ASSIST:	Adaptive Secondary Simulator and Instrument Testbed
SPARTA:	Standard Platform for Adaptive optics Real Time Applications



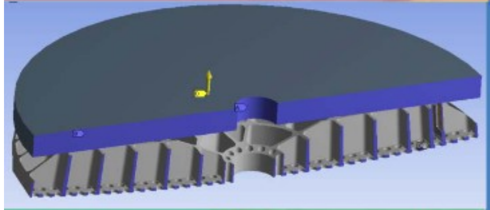
AOF

OVERVIEW



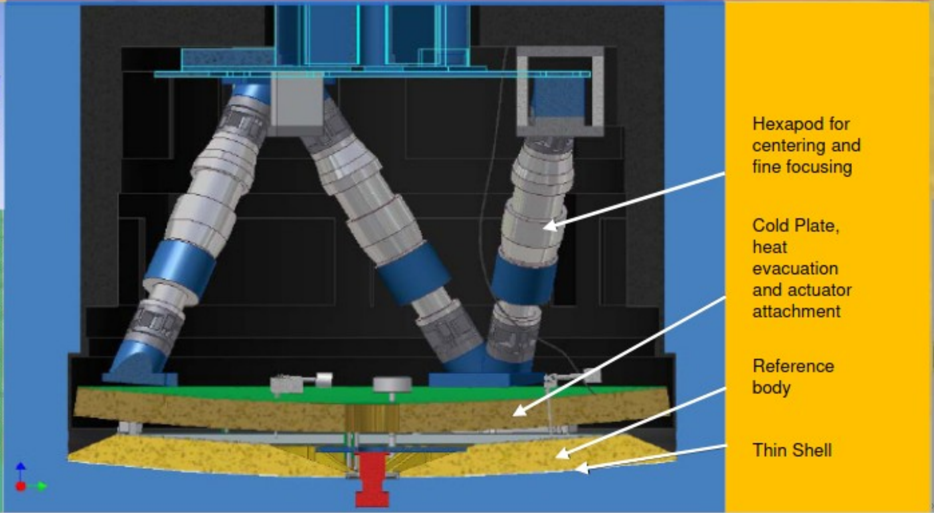


THE DSM: THE CORE OF AOF

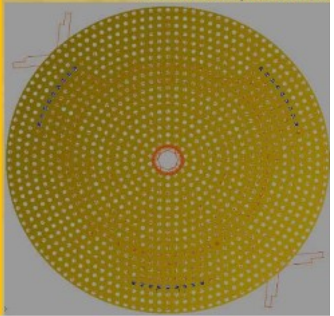


The cold-plate plus the back-plate.

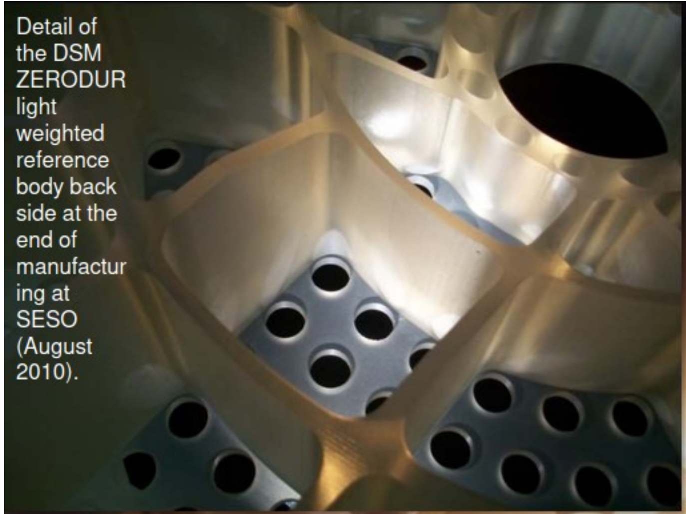
M2 unit is composed by an hexapod, a cold plate, a backplate and a thin shell mirror. The mirror positioning is obtained with an hexapod. The actuators are attached to a cold plate connected to a reference body, on which the thin shell is leaning when not operative.



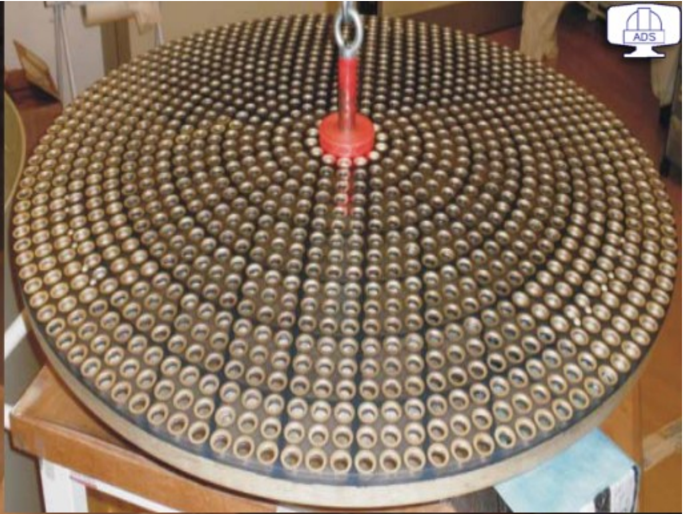
The thin shell mirror has a diameter of 1120 mm, 2 mm thickness, about 9 kg weight. 1170 voice-coil actuators are acting on magnets glued on the back face of the shell (below: actuators pattern).



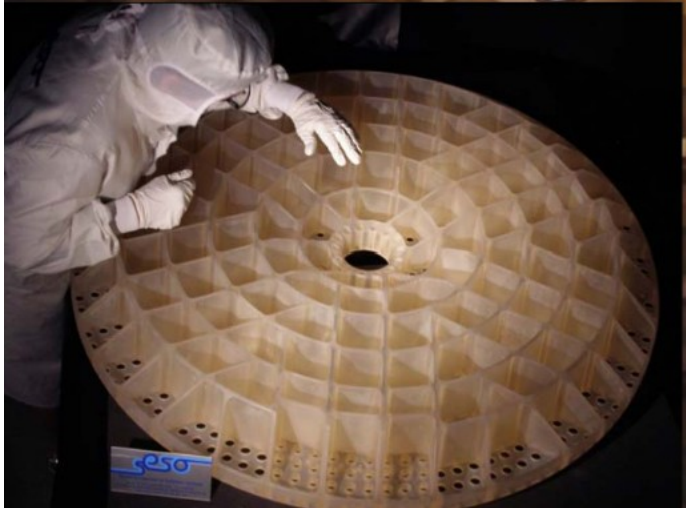
REFERENCE BODY

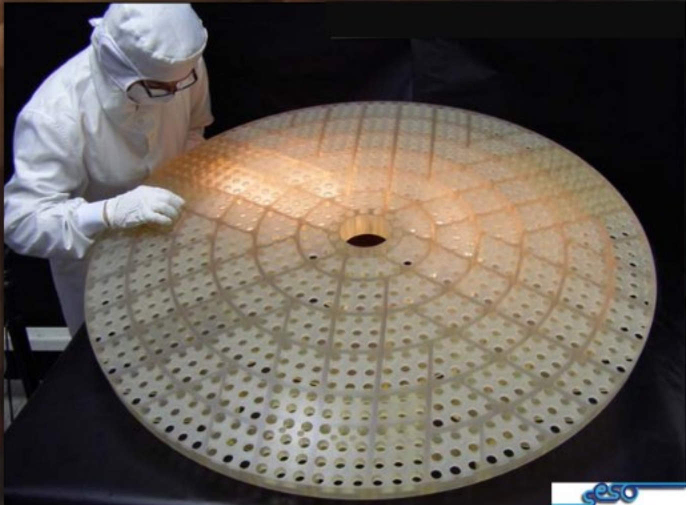


Detail of the DSM ZERODUR light weighted reference body back side at the end of manufacturing at SESO (August 2010).



The reference body at ADS, after silver coating and capacitive sensors etching (March 2011). The total weight of the component is 47 kg.

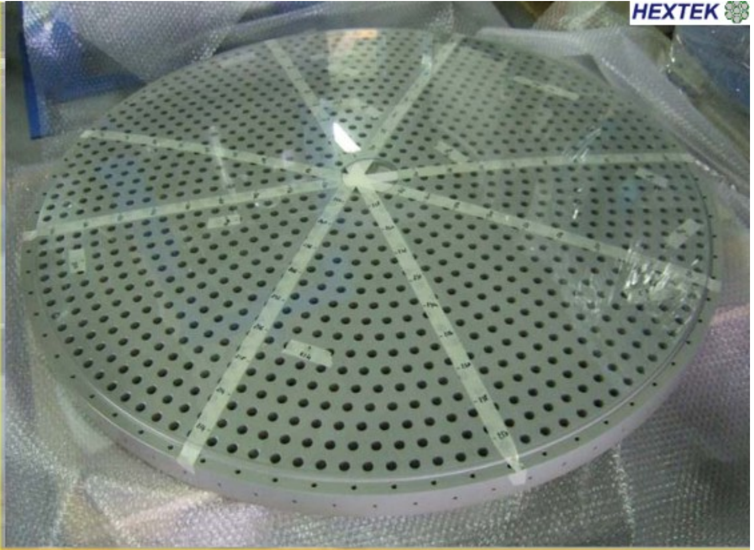




Final inspection of the reference body at SESO. The unit is ready for the metal coating deposition on the internal sides of the holes, for the capacitive sensor.

The M2 Zerodur Test Matrix produced by REOSC for the VLT, inside the barrel (mount) manufactured for the M2 Test matrix by AOF. The test matrix is being inserted into the barrel with a custom designed and fabricated handling tool (April 2007).

Hextec slumped engineering shell, to be used for electromechal testing of the M2-DSM unit.



DSM THIN SHELL

Hextec slumped engineering shell — HEXTEK

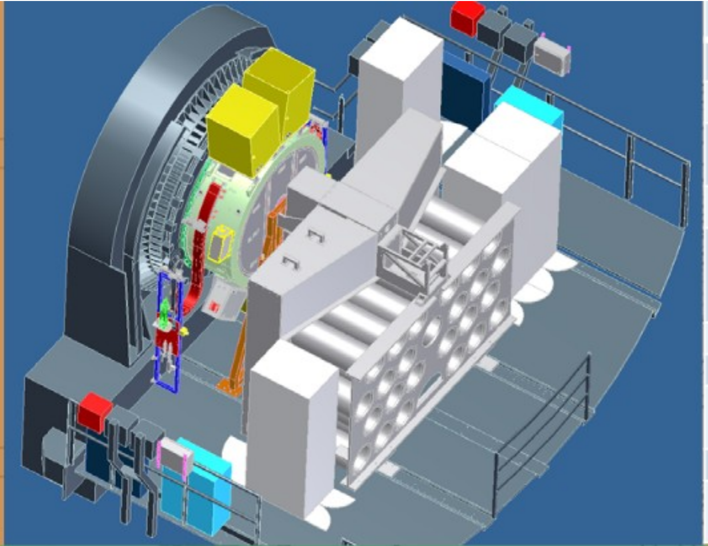
Schott blank for SAGEM (first science shell) — Sagem Défense Sécurité

Left: first thin science shell during the final phase of the convex side polishing at SAGEM (inspection, February 2010). The Zerodur blank (Schott) grinding started in December 2009. In February 2011 the thickness was about 2.8 mm.

Right: The thin shell in February 2011 at SAGEM. In March 2011 the goal thickness of 2 mm has been reached: it's the first time in Europe for a curved shell.

GALACSI goal is to concentrate the energy of a Point Spread Function (PSF) over a large FoV (1') for a visible-light integral field spectrograph (MUSE: Multi Unit Spectrographic Explorer), a second generation instrument for VLT.

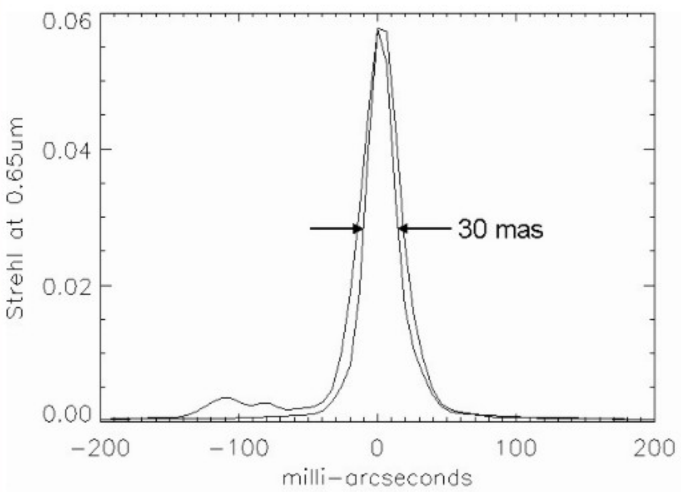
Here: 3-D model of GALACSI (left) on the Nasmyth platform with MUSE (right).

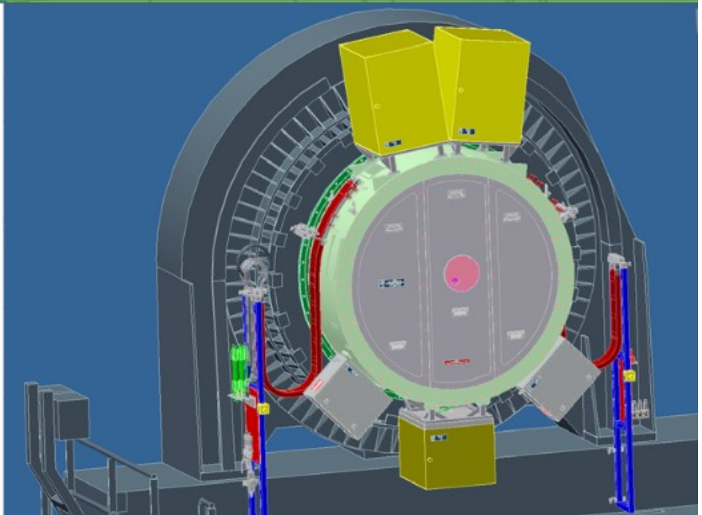


Field of view	1' WFM (7.5" NFM)
Instrument	Muse (VIS 3D-spectrograph)
Modes	GLAO, LTAO
Performance GLAO	×2 in ensquared energy (central pixel), 95% sky coverage
Performance LTAO	Strehl Ratio >5% @0.65μm
WFS	4 LGS L3-CCD (1 e$^-$ Read out Noise) 1 TT L3-CCD 1 TT IR
Loop frequency	= 1 kHz
SPARTA	HW=GRAAL
4LGSF	4 stars ∅2'/∅20" LTAO drives LGS power
ASSIST	Full FoV
Status	FDR passed

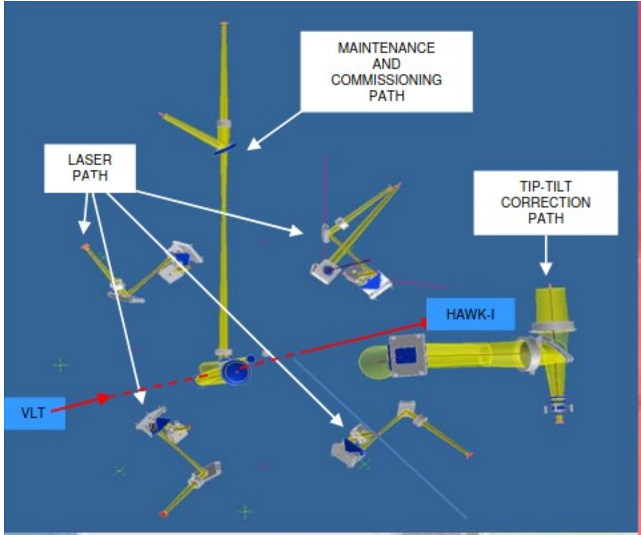
GALACSI: AO MODULE FOR MUSE

GALACSI will operate in two modes: Wide Field Mode (WFM, seeing reducer over 1' FoV at 750 nm) and Narrow Field Mode (NFM, SR ≅ 6% at 650 nm, in 7.5" FoV). Here is shown a typical simulation of the PSF as expected with GALACSI in NFM.



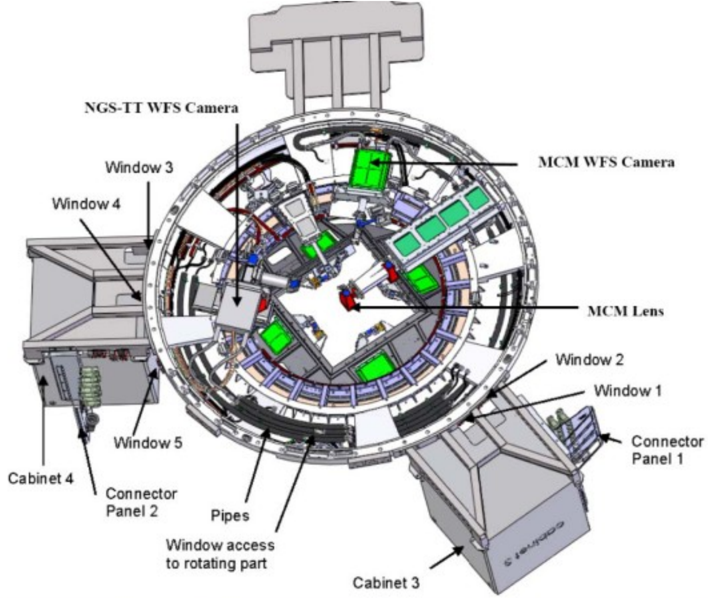


GRAAL DESIGN AND PERFORMANCES



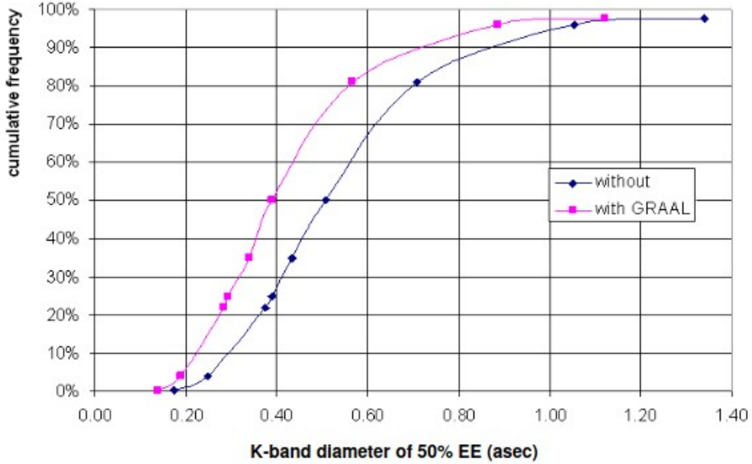
The complexity of the optical design relies in the tight arrangement of GRAAL optics in the space available between the telescope adapter and Hawk-I, requiring some optics to be located in a complex 3-D geometry. SESO started the manufacturing of the optics in June '09.

GRAAL main assembly descriptive view. The main assembly has been designed as a plug-and-play unit: no modification of any Hawk-I internal part is necessary during the installation. The design concept has been developed at ESO.



Field of view	7.5' (10" MCM)
instrument	Hawk-I (IR imager)
modes	GLAO, SCAO
Performance GLAO	x1.7 (central pixel), 95% sky coverage
Performance SCAO	(80% in K-band)
WFS	4 LGS L3-CCD (1 e^- Read out Noise) 1 TT L3-CCD 1 NGS L3-CCD
Loop frequency	≥ 700 Hz
SPARTA	HW=GALACSI
4LGSF	4 stars Ø12'/-
ASSIST	Limited FoV
Status	Detailed design, sub-contracted main assembly

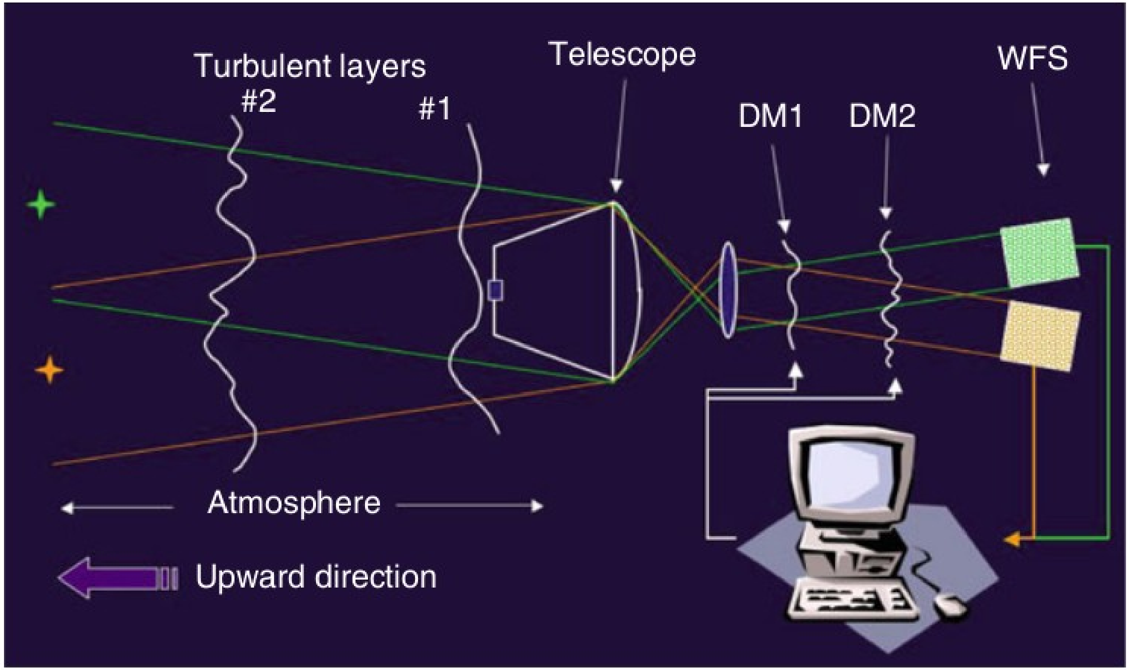
GRAAL is expected to provide about a factor 2 of improvement in the occurrence of good images (<0.4"). Seeing reducer at 60% occurrence is a factor 0.8 (in K-band). Improvement is expected for all seeing conditions




Adaptive optics
No Adaptive optics

AO Flavors

- **MCAO: Multi-layer adaptive optics**
  - Multiple mirrors to correct turbulence from different layers
- **GLAO: Ground-layer adaptive optics**
  - Designed to correct only close to ground turbulence
- **MOAO: Multi-object adaptive optics**
  - Multiple correctors, with small number of actuators, associated to each object
- **EXAO: Extrema adaptive optics**
  - AO corrects the core and first fringe of the diffraction pattern
  - Residuals accumulate in long exposures
  - Increased # of actuators and corrections



Questions?

(Credit: Shutterstock)

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