Observational Astrophysics
21. Astronomical Polarimetry

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1 Introduction

To understand polarization we have to treat light as a wave phenomenon. As before, apart from giving a few definitions that are useful to have when trying to understand polarimeters, I am not going to describe the physics of light polarization (and I will mostly try to not repeat equations here). For those that feel like they would benefit from some refreshing of the physics, I recommend a few texts for reading below.

Polarization is a property describing the geometrical orientation of the vibrations in a transverse wave. Electromagnetic theory requires that the vibrations of the electric, **E**, and magnetic, **B**, vectors of light are in the plane perpendicular to the direction of propagation (i.e., light is a transverse wave). Mathematically, to treat the light wave we only need to consider the electric vector. The magnetic vector stays perpendicular to it and with the same magnitude as **E**. A predominance of vectors vibrating in one given direction is the indicative of polarized light.

A monochromatic wave where the electric vector simply oscillates in one fixed direction is said to be "linearly polarized". This direction together with the direction of propagation define the "plane of polarization". The solution of the wave equation, however, might require the combination of two such oscillations in perpendicular directions. In the special case where these two waves have equal amplitude and their phases differ by 90 degrees, then the tip of the electric field will describe a circle in the fixed plane that contains the wavefront. In this case, the light is said to be "circularly polarized". In the more general case where the two plane waves do not have the same amplitude and/or have a phase difference of an angle that is not 90 degrees, the tip of the electric field will describe an ellipse instead, and the the light is said to be "elliptically polarized". Linear and circular polarization can be seen as special cases of elliptical polarization. Such polarization can be described by the "polarization ellipse" (Fig. 1). The ellipse can be traced in a clockwise or counter-clockwise sense. These cases can be called "right-handed" (or negative helicity) and "left-handed" (or positive helicity) elliptical polarization, respectively.

More generally, sources of light tend to produce waves of a range of frequencies that are incoherent and of random polarization. The light thus produced is said to be unpolarized. This just means that the polarization is changing quickly and randomly in time. Light might be "partially polarized", if there is more power in one certain polarization state. It is common to describe such cases as a combination of light that is unpolarized with light that is polarized. It is possible to define a

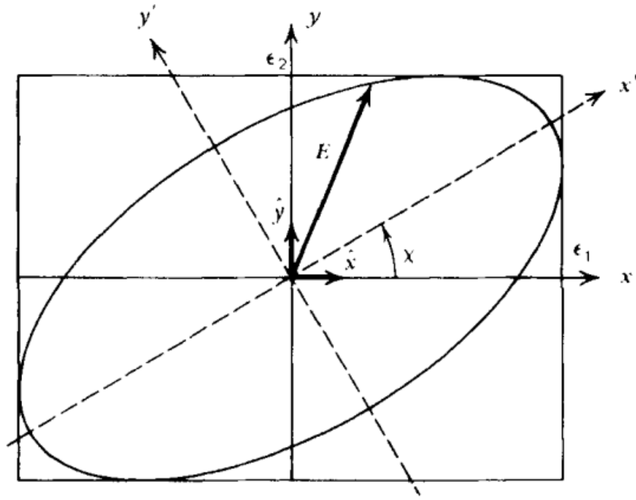

Figure 1: The polarization ellipse. Credit: Figure 2.4 from Rybicki & Lightman (2004).

"degree of polarization", which is a measure of the flux of the polarized radiation with respect to the total flux.

Several phenomena can polarize light including reflection, scattering, double refraction, dichroism, and magnetic fields. Scattering and magnetic fields are perhaps the main processes of interest for astronomical applications (not forgetting also reflection from solar system objects). The polarimetric signatures of scattering by dust allow the study of astrophysical phenomena at scales that can not be directly observed by other means. Getting rid of the unpolarized light from a bright star is also useful to observe faint polarized signal from circumstellar structures or from exoplanets. The degree of polarization is, however, usually small and thus difficult to measure.

2 Read this text (and the others if you see the need)

For the summary of the basic polarization concepts, the astronomical context, and the tools used for astronomical polarimetry, please read the text "Astronomical Polarimetry: Polarized Views of Stars and Planets" by Snik & Keller (2013)[1]. The other texts below are for those that need to refresh the concepts or would like to read better explanations than what is provided in the summarized text of Snik & Keller (2013).

If you would like to refresh the basic concepts of polarization and the Stokes parameters, then have a look at Section 2.4 of the book "Radiative Processes in Astrophysics" by Rybicki & Lightman (2004)[2]. And for those that need a bit more, maybe reading sections 2.1 to 2.3 would also be useful.

An introduction to polarization with an account of the main ways of producing polarized light is given in Chapter 24 of "Fundamentals of Optics" by Jenkins & White (1976)[3].

[1] https://link.springer.com/referenceworkentry/10.1007%2F978-94-007-5618-2_4

[2] https://www.google.com/search?q=Rybicki+Lightman+Radiative+Processes+in+Astrophysics

[3] https://archive.org/details/fundamentalsofop00jenk

3 Summary of concepts

- It is convenient to introduce the Stokes parameters (Stokes 1852)[4] to deal with light that is partially polarized. The four parameters are now called I, Q, U, and V, as used by Chandrasekhar (1946, 1947)[5] even though they were called A, B, C, D by Stokes. The four parameters are usually written in the form of a column vector, the "Stokes vector" (**S**). These parameters are combinations of the orthogonal amplitudes of the electric field in the polarization ellipse and the phase difference of the two orthogonal waves. They provide a set of quantities that describe the ellipse size, the azimuth of the major axis, the ellipticity and the sense of the rotation.

- For a elliptically polarized wave, the Stokes parameters are:

$$I = E_{x_0}^2 + E_{y_0}^2 \tag{1}$$
$$Q = E_{x_0}^2 - E_{y_0}^2 \tag{2}$$
$$U = 2\,E_{x_0}\,E_{y_0}\,\cos\delta \tag{3}$$
$$V = 2\,E_{x_0}\,E_{y_0}\,\sin\delta, \tag{4}$$

  where E_{x_0} and E_{y_0} are constants in the wave equations of the projections of the electric field in the x and y axis, and δ is the phase difference.

- For pure elliptical polarization, the four parameters are not independent. For perfectly polarized light: $I^2 = Q^2 + U^2 + V^2$.

- The parameter I is non-negative and represents the total energy flux or intensity of the wave. The parameter V describes the circular polarization. For positive V the wave has right-handed polarization, for negative V it has left-handed polarization, and $V = 0$ is the condition for linear polarization. The parameters Q and U describe the (two-dimensional) state of linear polarization. $Q = U = 0$ is the condition for circular polarization.

- In a beam of unpolarized light, many waves and vibrations are simultaneously present. Their electric fields provide a distribution of orientations and phases. If we could take snapshot pictures of such light crossing a plane that is parallel to the wavefront, with exposure time longer than a few times the reciprocal of the frequency, a series of ellipses would be recorded. For each snapshot, there would be changes in the ellipticity and orientation of the major axis. The Stokes parameters in this case would correspond to time averaged values of these quantities over the exposure time.

- In such cases, with partial randomization of the phases of the waves with time, we have the condition: $I^2 \geq Q^2 + U^2 + V^2$.

- For a completely unpolarized wave, the time averages $\langle E_{x_0}^2 \rangle$ and $\langle E_{y_0}^2 \rangle$ tend to be the same. Moreover, the time average of $\langle E_{x_0}\,E_{y_0}\,\cos\delta \rangle = \langle E_{x_0}\,E_{y_0}\,\sin\delta \rangle = 0$, meaning that for unpolarized light $Q = U = V = 0$. Conversely, when one of these parameters is different from zero, then there is some polarization in the light beam.

[4] After the Irish physicist and mathematician George Gabriel Stokes. See https://mathshistory.st-andrews.ac.uk/Biographies/Stokes/ for a biography and the original work in https://www.biodiversitylibrary.org/item/19878#page/421/mode/1up

[5] See https://ui.adsabs.harvard.edu/abs/1946ApJ...104..110C/abstract and https://ui.adsabs.harvard.edu/abs/1947ApJ...105..424C/abstract

- If independent beams of light, travelling in the same direction, are co-added, the Stokes parameters of the combined beam is the sum of parameters of the individual beams.

- It is thus conceptually useful to consider a partially polarized beam as the addition of one part that is completely polarized and another that is unpolarized. One can then write the Stokes vectors as:

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} I_p \\ Q_p \\ U_p \\ V_p \end{bmatrix} + \begin{bmatrix} I_u \\ Q_u \\ U_u \\ V_u \end{bmatrix} \tag{5}$$

which, using the conditions for unpolarized light, simplifies to:

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} I_p \\ Q_p \\ U_p \\ V_p \end{bmatrix} + \begin{bmatrix} I_u \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

- And we can define the degree of polarization as:

$$P = \frac{\sqrt{Q_p^2 + U_p^2 + V_p^2}}{I_p + I_u} = \frac{\sqrt{Q_p^2 + U_p^2 + V_p^2}}{I}. \tag{7}$$

Where we see that the degree of polarization is 1 for a completely polarized beam and zero for an unpolarized beam.

- Or one can define, separately, the degree of linear polarization (P_L) and the degree of circular polarization (P_V):

$$P_L = \frac{\sqrt{Q_p^2 + U_p^2}}{I} \tag{8}$$

$$P_V = \frac{V_p}{I} \tag{9}$$

- You might also find the use of fractional or normalized Stokes parameters, i.e. $q = Q/I$, $u = U/I$, and $v = V/I$.

- The 4×4 Müller matrix (M) represents the interaction of a Stokes vector with matter. The effect of n optical elements on the path of a beam can be represented by the (non-commutative) multiplication of Müller matrices:

$$\mathbf{S}_{\text{out}} = M_n M_{n-1} \ldots M_2 M_1 \tag{10}$$

- Polarimeters, i.e. instruments for the measurement of the state of polarization of the light, include optical elements that change the state of polarization of the radiation in a controlled way. Such optical elements can be divided into polarizers (also called analyzers), retarders (also called converters or phase plates), and depolarizers.

- Polarizers are devices for which the emergent beam is linearly polarized in some specific direction, regardless of the polarization state of the incident beam. Converters are devices that convert elliptically polarized light into linearly polarized light (or vice versa), by altering the phase difference between the two orthogonal components of elliptically polarized light. Depolarizers are devices that convert polarized radiation into unpolarized radiation (when averaged over wavelength, time or area).

- Polarizers are usually characterized by their "extinction ratio". This is defined as the transmitted intensity for the light in the polarizing direction divided by the transmitted intensity for the light in the perpendicular direction.

- Basically, a photopolarimeter is an imaging polarimeter obtained by adding some of the polarization-specific optical elements to the design of a photometer/camera. A spectropolarimeter is a spectrograph with such optical elements.

- To measure the polarization, one will actually measure the photon fluxes according to different polarization states. Detectors like CCDs only measure the flux. The various measurements then need to be combined to retrieve the original polarization state of the incident light.

- In practice, the implementation of a polarimetric mode in a given instrument does require some careful consideration during the design. Spurious polarization signals can be created by the optical elements within the instrument or even at the telescope. For example, imperfections in the coating can create polarizing areas on the mirror surface. Mirrors where the incidence of the light is non-normal have to be avoided, because some degree of polarization is introduced (see below). Actually, this is the reason why polarimeters are normally not placed at a Nasmyth or Coudé focus, as the optical path towards these focii include folding mirrors where the beam incidence is non-normal. In truth, the very fact that mirror surfaces are curved means that reflection is not exactly normal, so the state of polarization will be somewhat altered by the time light arrives at the telescope focus (and this has to be measured and dealt with during the data analysis).

- Consider a beam of light incident on a material that can partially reflect and partially transmit the light, as depicted in Fig. 2. The two directions of electric field vibration can be considered to be parallel and perpendicular to the plane of incidence (the plane that contains the surface normal and the incident beam). Brewster's law defines the angle of incidence (called Brewster's angle or polarizing angle) for which only the component perpendicular to the plane of incidence can be reflected:

$$\theta_B = \arctan \frac{n_2}{n_1}, \tag{11}$$

  where n_1 is the refractive index of the medium containing the incident beam and n_2 is the index of the medium containing the refracted beam. In this case, the angle between the reflected and refracted beams is of 90 degrees. The reflected ray is fully polarized, in the direction perpendicular to the plane of incidence. It is easy to see that this needs to be the case, as light waves are strictly transverse (one of the electric field components would need to be vibrating at the same direction of motion of the reflected ray, something that can not happen). The refracted beam will be partially polarized (all the light vibrating parallel to the plane of incidence is transmitted together with part of the light vibrating perpendicular to the plane of incidence).

- One can then imagine an arrangement of several plates of the same material, where the partially polarized light refracted by one plate will then strike the next plate. The process will then repeat, with part of the light vibrating perpendicular to the plane of incidence being reflected and part being transmitted. With several such plates, the part being transmitted can be reduced more and more. The final transmitted beam will then be highly polarized in the direction parallel to the plane of incidence. Such arrangement of plates is one type of polarizer (based on reflection).

- In such contexts you might also hear about s- and p-polarized light. For the s-polarized light, the electric field is perpendicular to plane of incidence. For the p-polarized light, the electric field is parallel to plane of incidence.

- Birefringence is a property of some materials where the refractive index changes according to the orientation of the electric vector of the incident radiation (or in other words, the material has different refractive index for each one of the three-dimensional orientation axis defined by its own structure, $n_x \neq n_y \neq n_z$).

- The simplest case of birefringence is that of uniaxial materials. This means there is one direction inside the material with distinct refractive index ($n_x \neq n_y = n_z$). That direction is called the "optic axis" of the material. One also defines the "fast axis" as the axis with the smallest refractive index, since that is the axis for which the speed of light will be the fastest.

- Let's picture a beam of light entering an uniaxial material and travelling along the optic axis. The vibrations of the electric field are in the directions perpendicular to the optical axis, and thus they are subject to the same refractive index and just ordinary refraction happens. Rotation around the optic axis does not change the optical behaviour. Let's now picture a beam of light entering the same uniaxial material but now travelling along one of the directions that is perpendicular to the optic axis. Now each component of the electric field will be subject to a different refractive index. The original beam splits into two beams with orthogonal linear polarizations (double refraction takes place).

- In the case of double refraction above, when the material is subject to a rotation, the behaviour of one beam does not change (this beam is called the ordinary ray). The other beam, however, will change its direction of travel (this beam is called the extraordinary ray). It is possible to show (Wahlstrom 1960) that the ordinary ray, which is the component that is always perpendicular to the optic axis, is always subject to the same refractive index (let's call it n_o). The extraordinary ray, when is fully parallel to the optic axis is subject to a different refractive index (let's call it n_e). However, upon rotation of the material, the refractive index affecting the extraordinary ray will change, to be something between n_o and n_e.

- The magnitude of the double refraction effect in a material is quantified by the "birefringence", defined as $\Delta n = n_e - n_o$. If the velocity of the component travelling along the optic axis is lower than that of the perpendicular component, the material is said to exhibit "positive birefringence" and is said to be "optically positive". If the opposite, the material exhibits "negative birefringence" and is said to be "optically negative".

- Materials showing birefringence include calcite (a form of calcium carbonate, $CaCO_3$), quartz and magnesium fluoride (MgF_2). Indeed, most crystals exhibit some natural birefringence. In other crystals and some others substances, such as glass, birefringence can be induced by stress.

- Some crystals, like quartz, are birefringent and also “optically active”. This means that the plane of polarization of the incident beam of radiation will also be rotated as it passes through the material.

- There are two basic forms of polarizers that make use of birefringence. One achieves beam separation simply using the differences in refractive index between the two orthogonal directions of vibration. The second type involves total internal reflection of one of the beams at the interface between different birefringent materials. Such polarizers that work by splitting the beam componets are also called “polarizing beam displacers” or “beam-splitters”.

- Examples of beam-splitters based on birefringence include:

  1. **Nicol prism**: made of two calcite prisms glued together by a substance called “Canada balsam”. The configuration is chosen in a way that the ordinary ray suffers total internal reflection on the interface between the calcite and the Canada balsam. It has some disadvantages and because of them it has been superseded by better polarizers: the entrance and exit faces are inclined with respect to the optical path of the beam, the transmitted beam laterally displaced, and some elliptical polarization is introduced on the emergent beam.
  2. **Rochon prism**: made of two prisms, of the same material, joined together so that the optic axis of the first is parallel to the direction of beam propagation. The optic axis of the second is perpendicular to the direction of beam propagation. The end result is that the ordinary ray is transmitted without deviation while the extraordinary ray is deviated. The extraordinary ray also suffers some spectral dispersion.
  3. **Wollaston prism**: made of two prisms of an uniaxial crystal cemented together in a way that the their optic axis are perpendicular to each other, and perpendicular to the direction of beam propagation. The extraordinary ray in the first prism becomes the ordinary ray in the second (and vice versa) so the end result is that the beams are deviated in opposite directions. Three-element Wollaston prisms are also available.
  4. **Glan-Thompson prism**: made of two calcite prisms that are cemented together with optic axes aligned in the same direction. The ordinary beam will suffer total internal reflection and is then absorbed by black paint on the side of one of the prisms. The extraordinary beam crosses the system straight through.
  5. **Foster prism**: is a variation of the Glan–Thompson prism where the ordinary beam is not absorbed but exits the prism at some angle.
  6. **Glan-Foucault prism**: made of two calcite prisms that are separated by an air gap, where the prisms have optic axes aligned in the same direction which is parallel to the entrance and exit faces of the system.
  7. **Savart plate**: made of two plates of birefringent material with their optic axes orthogonal to each other. Inside the first plate, the ordinary and extraordinary beams split. Inside the second plate, the beams exchange role (the ordinary becomes extraordinary and vice versa). As consequence, they are further displaced from each other but in the orthogonal direction. The result is that the emerging beams are displaced along a diagonal relative to the edges of the entrance face.

- Certain materials (some crystals and polymers) show a property called dichroism. The name comes from a discovery made by Wollaston (1804)[6] that light transmitted through potassium

[6] https://royalsocietypublishing.org/doi/pdf/10.1098/rstl.1804.0019

chloropalladite (K2PdCl4) would show red or green colour, depending on the direction of of travel of the light. The phenomenon is the result of different dependency of the absorption as a function of wavelength for the two directions of vibration of the electric field. Nowadays, material are known that absorb almost 100% of the light vibrating in one direction and largely transmit the light vibrating in the perpendicular direction.

- Dichroism is used in making the so-called sheet polarizers also known by the commercial name of "Polaroid". One type is made by heating and stretching a sheet of polyvynil alcohol (PVA) which is then an iodine solution. The axis showing maximum transmission for linearly polarized light is the one that is perpendicular to the direction of the stretching.

- Wire-grid polarizers are made building a pattern of parallel conducting wires with with spacing between them that is smaller than the wavelength. The polarization of the transmitted light is linear and perpendicular to the wires.

- Retarders (or converters), are optical elements that delay the phase of one polarization state with respect to the orthogonal state. This will result in a change of the type of polarization and/or its orientation. Retarders that use birefringence to achieve the change in phase are known as "phase plates" or "wave plates".

- Wave plates are made of a plane-parallel plate of some birefringent material aligned in a way that the optic axis is parallel to the faces of the plate. The beam that hits the face normally will split into two components that will propagate with different velocities (because of the different refractive indices of the birefringent material). The optical paths are then different and a phase delay is introduced. Common material for such plates include mica and crystalline quartz. The required phase delay (Δ) at a reference wavelength is obtained by controlling the thickness (d) of the plate:

$$\Delta = \frac{2\pi d}{\lambda}(n_e - n_o) \tag{12}$$

- Although plates for arbitrary phase delays can be manufactured, most frequently one encounters the so-called "quarter-wave plates" (which introduce a delay of 90 degrees, converting linear into circular polarization, or vice versa) or "half-wave plates" (which introduce a delay of 180 degrees, changing the orientation of linear polarization without affecting circular polarization).

- Like in most materials, the refractive indices in birefringent materials are a function of wavelength, and thus some chromatic dispersion is expected. A phase plate designed that works as a quarter-wave plate at 800 nm will be close to a half-wave plate for light with 400 nm. Similarly to how one can build achromatic lenses by combining two or more lenses, two or more wave plates can be combined to correct for chromatic effects.

- Of course, if the delay of a quarter wave plate is $n \times 360 + 90$ degrees, the geometrical effect is the same (and the same reasoning applies to other types of wave plates). However, such plates will be thicker and the chromatic dispersion will be stronger. The thin plates that achieve the phase delay with $n = 0$ are called "true zero-order wave plates".

- Achromatic wave plates that are made of two plates of different material are also called "bi-crystalline retarder" (Clarke 1967)[7]. With two plates, what one achieves is essentially a

[7] https://ui.adsabs.harvard.edu/abs/1967AcOpt..14..343C/abstract

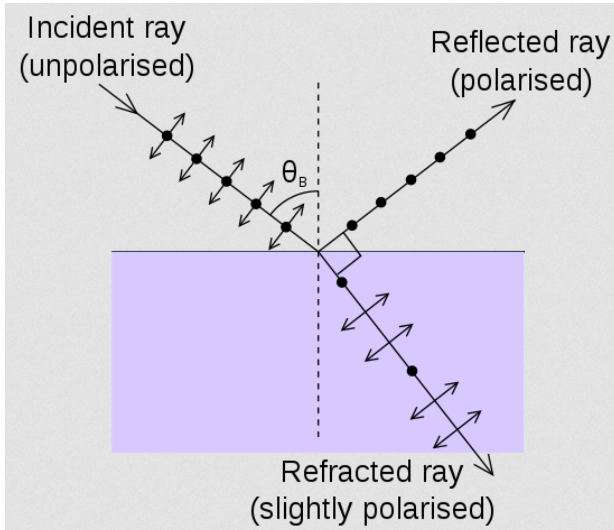

Figure 2: Illustration of the Brewster's angle and the polarization of the reflected light beam.

wave plate that works in the same way at two wavelengths with a small spectral variation. Alternatively, three or five plates of the same material with optic axes in different orientation can be used. These are also called "Pancharatnam retarders" (Pancharatnam 1955)[8]. The so-called "superachromatic" wave plates combine both designs (Serkowski 1974)[9].

- In polarimeters, wave plates are placed in front the other optical components that separate the light in different spectral regions, because they introduce some polarization in the light.

- Reflection losses happen when light reaches the entrance and exit faces of the plates. Because the refractive indices are different for each orthogonal direction of the electric field, the losses will be different and will introduce some spurious polarization in the signal. This effect can be reduced (but not eliminated completely) with anti-reflection coatings applied to the surfaces of the plates.

- In general, the orientation of the wave plate can be changed by rotation. So the intensity arriving at the detector will be a function of both the retardance (the phase delay) and the position angle of the retarder (see e.g. Keller 2002). Measurements at 8 angles are needed to determine all 4 Stokes parameters.

- **Fresnel rhombs**: are devices that achieve phase delay by total internal reflection. It is hard to achieve 90 degrees phase change with one single reflection, but possible to get 45 degrees. Quarter- and half-wave Fresnel rhombs achieve their phase delays with 2 and 4 total internal reflections (see Fig. 3). An achromatic retarder is obtained by combining two Fresnel rhombs.

- Retarders with variable retardance can also be made. Liquid crystals can change retardance with changes in the applied voltage. The so-called "photo-elastic modulator" rely on applying stress in one axis to make glass become birefringent.

[8] https://www.ias.ac.in/public/Volumes/seca/041/04/0137-0144.pdf
[9] https://ui.adsabs.harvard.edu/abs/1974MExP...12..361S/abstract

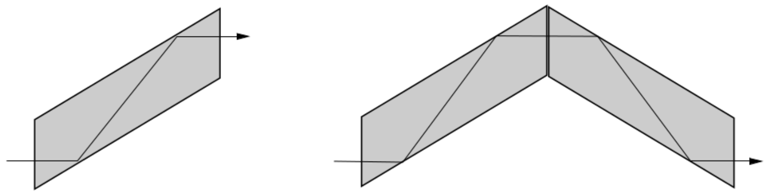

Figure 8. Traditional arrangements for quarter-wave (left) and half-wave (right) Fresnel rhombs.

Figure 3: Arrangement for quarter- and half-wave Fresnel rhombs. Credit: Figure 8 of Keller (2002).

- The common design of a polarimeter will include a part called the "modulator" which decides which polarization state is analyzed by the polarizer/analyzer (placed downstream in the system). The type of modulation is what decides how the Stokes parameters are going to be retrieved. With spatial modulation, the incident beam is split into two or more beams, and their intensities can all be measured. With temporal modulation, the intensity measurements in distinct polarization states are obtained sequentially (as is the case with a rotating wave plate).
- Good polarimeter will also include optics for the polarization calibration. The idea is to use such system to temporarily inject light with known polarization states into the polarimetric instrument, in front of the polarization modulator.
- If one also needs to calibrate the polarization properties of the telescope, the observation of polarimetric standard stars might be needed. Such standards can be polarized or unpolarized, with a state that is known with good accuracy. Polarized standards are also useful to provide the reference axis of the instrument attached to the telescope and to calibrate the modulation efficiency.

4 Additional reading

If you still feel that you need to read more on the mathematical concepts (e.g., Stokes parameters, Muller matrices, etc), then read Chapters 2, 3, and 4 of the book "Stellar Polarimetry" by Clarke (2010). Chapter 1 might also be interesting, as it gives a historical account of polarization and its astronomical use.

For a summary of the applications of polarimetry in many areas of astronomy see Hough (2007)[10].

Some more details about instrumentation for spectropolarimetry can be found in Keller (2002)[11]. In particular, for the data reduction challenges see Section 4 of this same text.

If that is your field, and you want a deep theoretical treatment of spectropolarimetry (with details of

[10] https://ui.adsabs.harvard.edu/abs/2007JQSRT.106..122H/abstract

[11] The text is part of notes from a Winter School on Astrophysical spectropolarimetry, see https://home.strw.leidenuniv.nl/~keller/Teaching/iac2000.pdf

the atomic physics, quantum mechanics, quantum electrodynamics, and radiative transfer involved in the problem) then the book "Polarization in Spectral Lines" by Landi Degl'Innocenti & Landolfi (2004)[12] might be what you are looking for.

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