

# Observational Astrophysics

## 2. Electromagnetic Radiation

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### 1 Introduction

It is fair to say that most of what we know from the Universe comes from interpreting the electromagnetic radiation that we receive from astronomical objects. The radiation received from the object can come from a combination of physical processes, but most of these processes tend to have a dominant energy scale (and thus a dominant wavelength or frequency).

The variation in frequency/wavelength along the electromagnetic spectrum is enormous, of about  $10^{25}$  orders of magnitude from the high-frequency, short-wavelength gamma rays to low-frequency, long-wavelength radio waves (see Figure 1).

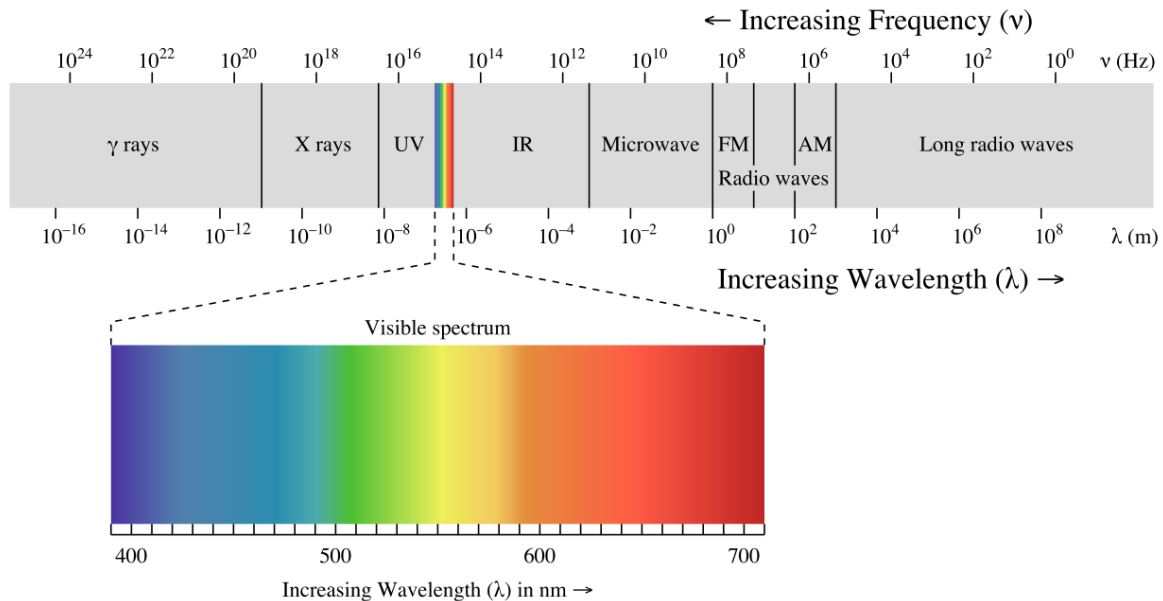


Figure 1: The different bands of the electromagnetic spectrum and their typical frequencies (wavelengths). Credit: Unknown.

Table 1 gives a list of typical values of wavelength, frequency, energy, and associated source temperature (for thermal sources, following Wien's displacement law,  $1/\lambda \propto T$ , see Section 3.1.2 below).

Band	$\lambda$	$\nu$	Energy	Temp. (K)	Typical source
$\gamma$ -ray	$\lesssim 10$ pm	$\gtrsim 30$ EHz	$\gtrsim 100$ keV	$\gtrsim 10^8$	Pulsars, magnetars, AGNs
X-ray	10 pm–10 nm	30 EHz – 30 PHz	100–0.1 keV	$10^8 - 10^5$	Hot cosmic gas and the above
UV	10–300 nm	30 – 1 PHz	100 – 4 eV	$10^6 - 10^4$	Hot stars, white dwarfs
Optical	300–1000nm	1 PHz – 300 THz	4–1 eV	$10^4 - 10^3$	Stellar photospheres
IR	1 $\mu$ m–1mm	300 THz – 300 GHz	1– $10^{-3}$	$10^3 - 10$	Star forming regions
microwave	1mm–1m	300 GHz – 300MHz	$10^{-3}$ – $10^{-6}$ eV	10–0.1	Interstellar medium, CMB
Radio	>1m	<300 MHz	<300MHz	< 10	ISM, molecular clouds

Table 1: Typical values of wavelength, frequency, energy, and temperature in different bands of the electromagnetic spectrum. Yes, sometimes there is overlap as definitions are not clear-cut. Examples of typical astronomical sources of photons in those bands are given in the last column.

Note that in atomic and nuclear physics, the distinction between  $\gamma$ - and X-rays seems to be mostly in terms of origin and not energy (X-rays are produced by electrons and  $\gamma$ -rays by the atomic nuclei). In addition, my impression is that microwave and radio are usually covered within “radioastronomy”. There is also “mm/sub-mm astronomy” (not part of my expertise), which seems to encompass from the far infrared up to some part of the microwave/radio bands.

As you know, there are two complementary ways to describe the properties and behaviour of radiation, either as electromagnetic waves or as massless particles (i.e. photons). Neither way accounts for all aspects of radiation and we change which interpretation to use as needed.

Electromagnetic waves travel through vacuum with the speed of light<sup>1</sup> ( $c$ ). In other medium, its speed  $v$  is given by

$$v = \frac{c}{n}, \quad (1)$$

where  $n$  is the refractive index of that medium. In addition to frequency ( $\nu$  or sometimes  $f$ ) and wavelength ( $\lambda$ ), you might find reference to wavenumber (sometimes written as  $\tilde{k}$ ,  $\tilde{\nu}$ , or  $\eta$ ), angular frequency ( $\omega$ ) and angular wavenumber ( $k$ ) which are related to each other in vacuum as:

$$\nu = \frac{c}{\lambda} = \frac{\omega}{2\pi} = c\tilde{k} = \frac{ck}{2\pi} \quad (2)$$

The frequency of light does not change when it changes the propagating medium. The energy of the photon has to be conserved. However, the wavelength (and wavenumber) changes. When in a different medium,  $c$  in Eq. 2 has to be replaced by the appropriate velocity in that medium.

The energy ( $E$ ) carried by the photon is given by

$$E = h\nu, \quad (3)$$

<sup>1</sup>Its exact value is 299 792 458 m s<sup>-1</sup>, in SI units, as the meter is now defined as the length travelled by light in vacuum during 1/299 792 458 second (in 2019, some of the basic SI units were redefined to be expressed in terms of a few constants of nature with exact values).

where  $h$  is the Planck constant<sup>2</sup>. You will sometimes find energy expressed in electronvolt units (eV), where  $1 \text{ eV} = 1.602176634 \times 10^{-19} \text{ J}$ .

## 2 A bit of the history of the electromagnetic spectrum

If you have an interest in the history of optics and the concepts of light and vision, try to find the book “A History of Optics: from Greek Antiquity to the Nineteenth Century”, By Olivier Darrigol (where there is an obvious focus on the western history of these concepts, Darrigol 2012). Although there were several alternative ways of trying to explain light, colours, rainbows, etc, it was only with the work of Isaac Newton that colours were understood as an intrinsic property of light<sup>3</sup> You can read a transcription of Newton’s notes about colours in the *Newton project* website<sup>4</sup>. And it seems that he was the first one to use the word *spectrum* to refer to the colours observed when light is dispersed by a prism (Newton 1672)<sup>5</sup>.

Frederick William Herschel, German-born British astronomer, reported the discovery of infrared radiation in 1800 (Herschel 1800)<sup>6</sup>. He was experimenting with different filters for solar observation, trying to find the one that would transmit most light but least heat. He used a prism to separate the colors of the sunlight and a cardboard slit to select the color that would illuminate a thermometer. Two other thermometers stayed at the shade of the cardboard, for comparison purposes. He noticed that the temperature increased more when the thermometer was exposed to red light than to violet<sup>7</sup>. Apparently, he eventually decided to measure the temperature just beyond the red, where there was no visible colour and found it to be even hotter than the red colour. His conclusion was that the maximum heat of the sunlight was given by invisible light (the correct interpretation of the findings, regarding the nature of light and heat, took a bit longer. See e.g. Barr 1960)<sup>8</sup>.

Ultraviolet radiation was discovered about one year later, by Johann Wilhelm Ritter, German chemist/physicist (actually born in the region of nowadays Silesia, Poland, but then part of Prussia). He was conducting experiments with silver chloride (AgCl), a white crystal. Upon illumination with violet light, it separates into silver and chlorine, turning to a grey-black colour. In the experiments, he noticed that the invisible light, beyond the violet, was even more efficient in darkening AgCl. His decision to experiment with light on the other end of the visible spectrum was motivated by Herschel’s discovery. His first note about the discovery appeared in a letter to the editor of *Annalen der Physik*, which commented about Herschel’s discovery of infrared light<sup>9</sup>.

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<sup>2</sup>Its exact value in SI units is  $6.62607015 \times 10^{-34} \text{ J Hz}^{-1}$ .

<sup>3</sup>See [this short article](#) by Kate Storey-Fisher published in March 2020 in the *astrobites* website (and follow the links there).

<sup>4</sup><http://www.newtonproject.ox.ac.uk/view/texts/normalized/NATP00004>

<sup>5</sup><https://royalsocietypublishing.org/doi/abs/10.1098/rstl.1671.0072> or <http://www.newtonproject.ox.ac.uk/view/texts/normalized/NATP00006>

<sup>6</sup><https://www.jstor.org/stable/pdf/107056>

<sup>7</sup>How would you explain that? Blue photons have higher energy than red photons...

<sup>8</sup><https://ui.adsabs.harvard.edu/abs/1960AmJPh..28...42B/abstract>

<sup>9</sup>I did not manage to have access to the original letter (*Annalen der Physik*, 1801, 7, 527) which seems to have appeared in a collection of Excerpts from many letters to the editor. Reference to this note is made in Frercks et al. (2009): <https://www.sciencedirect.com/science/article/pii/S003936810900020X>. A full account of the motivation and experiment is given in Ritter’s book *Physisch-chemische Abhandlungen in chronologischer Folge*, Volume 2, pages 81-107, published in 1806.

The “extreme ultraviolet”, below 200 nm, was first detected by Victor Schumann, German physicist, in 1893 (Schumann 1894)<sup>10</sup>. Cornu (1879)<sup>11</sup> suggested that the cutoff in the observed solar spectrum below 300 nm was caused by the atmosphere. Motivated by this, Schumann eventually constructed a spectrograph able to operate under vacuum and started to use special photographic plates (Lyman 1914; Hagmann 2014)<sup>12</sup> in his experiments for observing the spark spectrum of several metals. By gradually improving his methods and materials, he was able to observe the spectrum down to about 120 nm. The solar UV spectrum down to 2100 Å was first recorded by a spectrograph mounted on a V-2 rocket captured by the Americans (Baum et al. 1946)<sup>13</sup>.

X-rays were reported by Wilhelm Rontgen in 1895 (in: *Sitzungsberichte der Würzburger Physik.-Medic.-Gesellschaft.* 1895). He was experimenting with the effects of high voltage on electrical discharge in diluted gases in vacuum tubes. He noticed that, in a dark room, a paper covered with barium platinocyanide, Ba[Pt(CN)<sub>4</sub>], would show fluorescence when close to the tube. When testing if any material could stop the rays causing the fluorescence, he brought his hand on the way and saw the shadow of his own bones on the paper screen. The radiation was referred to as “X” to indicate it was of an unknown type. He refused to patent the invention. The discovery gave him the first Physics Nobel prize in 1901. A translation to English of his report appeared in *Nature*, in January 1896<sup>14</sup>. A note that is present just after this *Nature* report actually mentions that the discovery was not completely novel, as others had reported the same effect before, although perhaps not with as many details as Rontgen. It seems that William Morgan, a Welsh physician/physicist/actuarial, was the first to recognize the invisible radiation we now call X-rays, already in 1785<sup>15</sup>.

Gamma rays were discovered by the French chemist/physicist Paul Villard in 1900 (Villard 1900)<sup>16</sup>. He was experimenting with the magnetic field deflection of “ $\beta$  rays” emitted from radium. In addition to the “ $\beta$  rays”, he noticed a non-refracted beam that travelled on a straight line and was not affected by the magnetic field. The nature of the  $\gamma$  rays was only elucidated later, with the development of quantum theory of radiation. From the doctoral thesis of Marie Curie, it seems that it was Ernest Rutherford who introduced the designation  $\gamma$  ray (Gerward & Rassat 2000)<sup>17</sup>.

James Clerk Maxwell, Scottish scientist, published in 1865 his theory on electromagnetism, bringing together electricity and magnetism and deducing that light is an electromagnetic wave (Maxwell 1865)<sup>18</sup>. It seems that in 1867, Maxwell predicted that there should be light with even longer wavelengths than infrared light (citation missing). A series of experiments conducted by the German physicist Heinrich Rudolf Hertz, between 1886 and 1889, demonstrated the reality of the radio waves (e.g. D’Agostino 2001)<sup>19</sup>. In one of these experiments, in 1886, using a Leyden jar<sup>20</sup>, Hertz observed in a corner of his lab some secondary sparks that could be explained by waves with about 80 MHz of frequency (Schwab & Fischer 1998)<sup>21</sup>.

<sup>10</sup><https://ui.adsabs.harvard.edu/abs/1894PASP....6...66S/abstract>

<sup>11</sup><https://www.jstor.org/stable/113734?refreqid=excelsior%3A61d6e38eab22d7906f1e37d3f1ba7c56>

<sup>12</sup><https://ui.adsabs.harvard.edu/abs/1914ApJ...39...1L/abstract> and <https://ui.adsabs.harvard.edu/abs/2014AnP...526A..11H/abstract>

<sup>13</sup><https://ui.adsabs.harvard.edu/abs/1946PhRv...70..781B/abstract>

<sup>14</sup>[www.nature.com/articles/053274b0.pdf](http://www.nature.com/articles/053274b0.pdf)

<sup>15</sup><https://doi.org/10.1017/S0071368600003001> and <https://archive.org/details/philtrans00580668>

<sup>16</sup>See <https://gallica.bnf.fr/ark:/12148/bpt6k3086n/f1010.item> for the original in French

<sup>17</sup><https://www.sciencedirect.com/science/article/pii/S1296214700010945>

<sup>18</sup><https://royalsocietypublishing.org/doi/10.1098/rstl.1865.0008>

<sup>19</sup><https://www.springer.com/gp/book/9781402002441>, behind a paywall.

<sup>20</sup>[https://en.wikipedia.org/wiki/Leyden\\_jar](https://en.wikipedia.org/wiki/Leyden_jar)

<sup>21</sup><https://ieeexplore.ieee.org/abstract/document/681365>

## 3 Origin of electromagnetic radiation

### 3.1 Black body radiation

The concept of a black body was introduced by the German physicist Gustav Robert Kirchhoff<sup>22</sup> (Kirchhoff 1860)<sup>23</sup>: “...bodies can be imagined which, for infinitely small thicknesses, completely absorb all incident rays, and neither reflect nor transmit any. I shall call such bodies perfectly black, or, more briefly, black bodies.”

#### 3.1.1 Stefan–Boltzmann law

The total “radiant emittance” (another name for the radiant flux per unit area, or what we called total flux) of a black body is given by Stefan-Boltzmann law,

$$F = \sigma T^4, \quad (4)$$

where  $\sigma$  is the Stefan–Boltzmann constant<sup>24</sup>.

Josef Stefan<sup>25</sup> was a mathematician and physicist born in the Austrian Empire but of Slovenian roots. The “ $T^4$ ” law was first published in Stefan (1879)<sup>26</sup>. In there, he also used the law to estimate the temperature of the Sun (finding values between 5307-5838 K). Before Stefan, the best expression known for the radiant emittance was that empirically proposed by Dulong & Petit (1817)<sup>27</sup> in their studies about the “laws of cooling”;  $F = \mu a^\theta$  (where  $\theta$  is the temperature, their proposed value for  $a = 1.0077$ , and  $\mu$  is a constant that depended on the material). It seems that different reasons lead Stefan to suspect this law was not accurate (Dougal 1979; Crepeau 2007, 2008)<sup>28</sup>.

It turns out, however, that Stefan made mistakes in his analysis to arrive at the  $T^4$  law (Dougal 1979). In addition, the data he used, from the Dulong & Petit paper, was not from something that could be considered a (near) perfect black body radiator (Crepeau 2008). Nevertheless, somehow he got the correct relation between  $F$  and  $T$ .

It was Boltzmann who presented a derivation of the law from theoretical grounds, combining ther-

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<sup>22</sup><https://mathshistory.st-andrews.ac.uk/Biographies/Kirchhoff/>

<sup>23</sup><https://onlinelibrary.wiley.com/doi/10.1002/andp.18601850205> for the original in German, or see a translation in <https://www.tandfonline.com/doi/abs/10.1080/14786446008642901>

<sup>24</sup>In SI units it has the exact value of  $5.670374419 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

<sup>25</sup>[https://mathshistory.st-andrews.ac.uk/Biographies/Stefan\\_Josef/](https://mathshistory.st-andrews.ac.uk/Biographies/Stefan_Josef/)

<sup>26</sup><http://www.ing-buero-ebel.de/strahlung/Original/Stefan1879.pdf> – It seems there is no English translation of the paper (Dougal 1979) but a short summary appeared in the Notes section of Nature in May that year (Nat 1879).

<sup>27</sup>See <https://gallica.bnf.fr/ark:/12148/bpt6k6568729f/f231.item> for a scan of the original in French.

<sup>28</sup>See the articles in <https://iopscience.iop.org/article/10.1088/0031-9120/14/4/312/pdf>, <https://www.sciencedirect.com/science/article/abs/pii/S0894177706001361>, and <https://www.afhalifax.ca/magazine/wp-content/sciences/AMA2009/stefan/2008crepeau.pdf>

modynamics and Maxwell's electromagnetic equations (Boltzmann 1884)<sup>29</sup> to show that Stefan's expression was indeed correct. Ludwig Eduard Boltzmann<sup>30</sup> was an Austrian physicist. He obtained his PhD in the Institute of Physics of the University of Vienna and had Stefan as his advisor.

### 3.1.2 Wien's displacement law

Wien's displacement law, not to be confused with Wien's approximation (Section 3.1.4 below), is a relation that says that the wavelength corresponding to the peak of emission of a black-body is inversely proportional to the temperature,

$$\lambda = \frac{b}{T}, \quad (5)$$

where  $b$  is a constant (with exact value  $2.897771955 \times 10^{-3} \text{ m K}$ )<sup>31</sup>. The law was first published in Wien (1893)<sup>32</sup>, derived using thermodynamics. The value of the constant was first determined by Lummer & Pringsheim (1899a,b)<sup>33</sup>.

Wilhelm Wien<sup>34</sup> was a German physicist. For his work in radiation, Wien received the Nobel Prize in Physics of 1911. It seems that, at the time, the thermodynamics developed by Boltzmann was still not well accepted. Wien's PhD advisor, Hermann von Helmholtz, was against using the method (Crepeau 2009)<sup>35</sup>.

### 3.1.3 Planck's law

Understanding the black body radiation had a fundamental part in the early development of quantum theory. It was the German theoretical physicist Max Karl Ernst Ludwig Planck<sup>36</sup> that, trying to reconcile the electromagnetic theory of radiation with experimental data, proposed the law that could explain the black body radiation (Planck 1900b,a, 1901)<sup>37</sup>. In the paper of 1901, Planck makes the assumption that energy had to be proportional to frequency ( $E = h\nu$ )<sup>38</sup>.

<sup>29</sup><https://onlinelibrary.wiley.com/doi/10.1002/andp.18842580616>

<sup>30</sup><https://mathshistory.st-andrews.ac.uk/Biographies/Boltzmann/>

<sup>31</sup><https://physics.nist.gov/cgi-bin/cuu/Value?bwien>

<sup>32</sup><https://www.biodiversitylibrary.org/item/93363#page/75/mode/1up>

<sup>33</sup>See <https://archive.org/details/verhandlungende63unkngoog/page/n34/mode/2up> and <https://archive.org/details/verhandlungende63unkngoog/page/n225/mode/2up>

<sup>34</sup><https://mathshistory.st-andrews.ac.uk/Biographies/Wien/>

<sup>35</sup><https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.721.2955&rep=rep1&type=pdf>

<sup>36</sup><https://mathshistory.st-andrews.ac.uk/Biographies/Planck/>

<sup>37</sup>See the originals in German here: <https://archive.org/details/verhandlungende01goog/page/n212/mode/2up> and here <https://onlinelibrary.wiley.com/doi/10.1002/andp.19013090310> or the English translations here: <http://materias.df.uba.ar/f4ba2015c1/files/2012/08/papers-planck.pdf> and here [http://www.ffn.ub.es/luisnavarro/nuevo\\_maletin/Planck%20\(1901\),%20Energy%20distribution.pdf](http://www.ffn.ub.es/luisnavarro/nuevo_maletin/Planck%20(1901),%20Energy%20distribution.pdf)

<sup>38</sup>Nevertheless, he did not think at first that this really meant energy was discontinuous (i.e., existed in discrete quanta). It seems that he only came to accept that later, sometime between 1906-1910, after further work by Ehrenfest, Einstein and others (Kuhn 1987, see Chapter VIII of the book in Google-book preview [here](#)). Max Planck was awarded the Nobel Prize in Physics of 1918 for the discovery of the energy quanta.

For the radiation emitted by a black body in thermodynamic equilibrium, the spectral radiance (specific intensity,  $I_\nu$  or  $I_\lambda$ ) depends only on frequency (wavelength) and on the temperature of the black body and is given by Planck's law:

$$I_\nu = B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1} \quad (6)$$

or

$$I_\lambda = B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1} \quad (7)$$

where  $c$  is the speed of light,  $h$  is the Planck constant, and  $k$  (sometimes written as  $k_B$ ) is the Boltzmann constant<sup>39</sup>.

Figure 2 displays the Planck law curves for three different temperatures both in terms of frequencies and of wavelengths. It is interesting that depending if you write the equation using  $\nu$  or  $\lambda$ , the region where we find the peak of emission changes (right?).

### 3.1.4 Wien's approximation

Wien's approximation (or Wien's law, not to be confused with Wien's displacement law in Section 3.1.2 above) can be used to describe the specific intensity of the black body in short wavelengths (or high frequencies) but diverges from the truth for the far infrared; it is valid for the case where  $h\nu \gg kT$ :

$$I_\nu = \frac{2h\nu^3}{c^2} \exp\left(-\frac{h\nu}{kT}\right) \quad (8)$$

Wien's law actually precedes Planck's law, but of course in a version without the introduction of the Planck constant. It was derived based on thermodynamic arguments (Wien 1896)<sup>40</sup>. Out of curiosity, the expression derived by Wien was this one:

$$\phi_\lambda = \frac{C}{\lambda^5} \exp\left(-\frac{c}{\lambda\theta}\right), \quad (9)$$

where  $C$  and  $c$  are unknown constants and  $\theta$  is the temperature. The German physicist Friedrich Paschen, more or less at the same time, arrived at the same expression but from the experimental side. Paschen's expression is actually cited in Wien's paper as a private communication. Paschen's work appears in published form in the same journal of Wien's paper, one issue earlier (Paschen

<sup>39</sup>Which has an exact value in SI units of  $1.380649 \times 10^{-23}$  J K<sup>-1</sup>.

<sup>40</sup><https://onlinelibrary.wiley.com/doi/10.1002/andp.18962940803> in German, with an English translation in Wien (1897) in <https://www.equipes.lps.u-psud.fr/Montambaux/histoire-physique/Wien-1897.pdf>

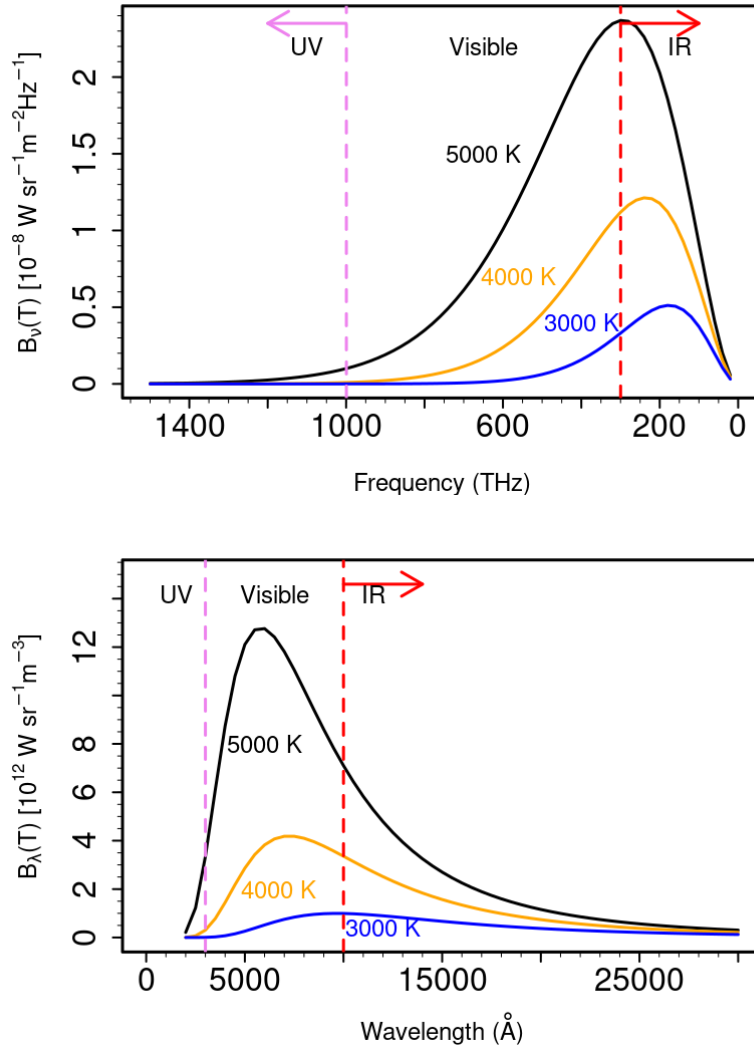


Figure 2: Planck law in frequencies (top) and wavelengths (bottom).

1896)<sup>41</sup>. The difference is that Paschen had one additional unknown constant in his expression; Wien's  $\lambda^5$  was  $\lambda^\alpha$  for Paschen. Although we now call equation 9 as the Wien's law, it seems that for a while in those times it was actually called Paschen-Wien law (Duncan & Janssen 2019)<sup>42</sup>.

### 3.1.5 Rayleigh-Jeans law

Rayleigh-Jeans law can be used to describe the case for long wavelengths (or low frequencies, i.e. radio). It is valid when  $h\nu \ll kT$  and then we can make use of the expansion  $e^x = 1 + x$ , so that equation 6 becomes:

<sup>41</sup><https://onlinelibrary.wiley.com/doi/abs/10.1002/andp.18962940703>

<sup>42</sup>Page 47 in the Google Book preview [https://scholar.google.com/scholar?hl=en&as\\_sdt=0%2C5&q=%22Constructing+Quantum+Mechanics%22+Duncan&btnG=](https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=%22Constructing+Quantum+Mechanics%22+Duncan&btnG=)



$$I_\nu = \frac{2\nu^2}{c^2} kT, \quad (10)$$

or using wavelength:

$$I_\lambda = \frac{2c}{\lambda^4} kT \quad (11)$$

Interestingly, the specific intensity equation in this form does not contain the Planck constant.

The first suggestion of the expression with  $\lambda^4$  like in Eq. 11 appeared in a short two-pages note by Rayleigh (1900)<sup>43</sup>. The English physicist Lord Rayleigh<sup>44</sup>, 3rd Baron Rayleigh, was actually called John William Strutt (Rayleigh was part of his title only).

Rayleigh was raising some objections to Wien’s law (Eq. 9), in particular the implication that energy would no longer increase with temperature if  $\lambda\theta$  is much larger than the constant  $c$ , something that could already be tested as it would happen within capabilities of experiments of that time ( $\lambda \sim 60 \mu\text{m}$  and 1000 K). His argument was actually to substitute the factor  $\lambda^5$  by  $\frac{T}{\lambda^4}$ . This was better developed in Rayleigh (1905)<sup>45</sup> but with an error in the constant by a factor of eight that was pointed out by Jeans (1905b)<sup>46</sup>.

James Hopwood Jeans<sup>47</sup> was an English mathematician, physicist, and astronomer. During 1905, Jeans and Rayleigh conducted a discussion on the topic in several letters to Nature and continued to study the issue in other publications (I tried to follow, but it was not easy to find everything, see Kuhn 1987)<sup>48</sup>. Jeans was able to derive the relation with  $\lambda^4$  in many independent ways (see his book, Jeans 1914)<sup>49</sup>. He also showed that with the Rayleigh-Jeans law, he could recover Stefan’s law for the total radiation and Wien’s displacement law (Jeans 1905a)<sup>50</sup>. Nevertheless, the Rayleigh-Jeans law can not apply in nature for all wavelengths. For small wavelengths, the energy emitted by the black body diverges. This is the problem known as the “ultraviolet catastrophe”, one of the strong motivations for the quantum theory (see discussion in Jeans’ book, Jeans 1914).

If the Rayleigh-Jeans law is applicable, as is the usual case in radio-astronomy, one can define the “brightness temperature”,  $T_b$ . This is the temperature of a blackbody having the same brightness of your source at a given frequency. So that inverting Equation 12 for the temperature we have

$$T_b = \frac{c^2}{2\nu^2 k} I_\nu. \quad (12)$$

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<sup>43</sup>A complete collection of Rayleigh’s papers is available in the internet. This note can be [found here](#).

<sup>44</sup><https://mathshistory.st-andrews.ac.uk/Biographies/Rayleigh/>

<sup>45</sup><https://www.nature.com/articles/072054c0>

<sup>46</sup>Jean’s paper can be [found here](#). The error is acknowledged by Rayleigh in the version of the paper included in the series of books with his complete works.

<sup>47</sup><https://mathshistory.st-andrews.ac.uk/Biographies/Jeans/>

<sup>48</sup>See [page 144](#) of the preview of the book.

<sup>49</sup><https://archive.org/details/cu31924012330407/mode/2up>

<sup>50</sup><https://royalsocietypublishing.org/doi/pdf/10.1098/rspa.1905.0060>

### 3.1.6 (Approximate) Black bodies in nature

According to Shaviv (2009)<sup>51</sup>, it was Ritter (not the same of the UV radiation above) who first used the  $T^4$  law to relate the energy emission of two stars, finding that the stellar luminosity can be written as product of the surface area and the fourth power of the temperature,  $L = 4\pi R^2 \sigma T^4$ , (Ritter 1881)<sup>52</sup>. August Ritter, German scientist, was a professor of mechanics at the Aachen Technische Hochschule. He was a pioneer in the theory of stellar structure (Meo 2007)<sup>53</sup>. The stellar temperature that we get using Stefan-Boltzmann law for the stellar flux is what we call the “effective temperature” ( $T_{\text{eff}}$ ) of the star.

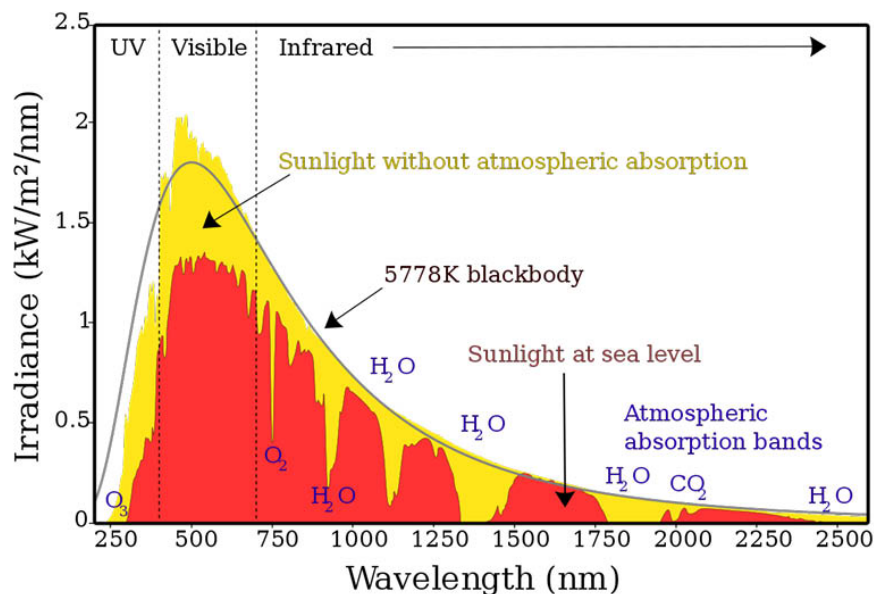


Figure 3: Solar irradiance compared to a black body curve (credit: Landro & Amudsen 2019).

Of course, stars do not emit radiation as black bodies, however their emission does behave approximately like that, enough for the concept to be useful. Figure 3 compares the solar irradiance (above and below the atmosphere) with a black body curve of 5778 K. Strong deviation is seen towards the blue/UV regime, where blanketing caused by absorption lines and continuum opacity sources is strong. Further comparisons for stars in a range of temperatures is shown in Figure 4.

A source that behaves very much like a black body is the Cosmological Background or Cosmic Microwave Background (CMB). The CMB radiation originated when the Universe cooled down enough to allow electrons and protons to form hydrogen atoms, reducing the scattering by free electrons and making the Universe more transparent.

<sup>51</sup>See a [preview of the book here](#).

<sup>52</sup>Between 1878-1893, August Ritter published a study in 18 parts entitled “Researches on the height of the atmosphere and the constitution of gaseous celestial bodies”. According to Shaviv (2009), it is in part 12 that the  $T^4$  is used to find the expression for luminosity. My German is probably failing me, but I see Stefan’s law only in Ritter (1883) which is part 16. The only English translation of Ritter’s papers seems to be exactly of part 16, coming out in ApJ thanks to an initiative of the editor (Ritter 1898), see <https://ui.adsabs.harvard.edu/abs/1898ApJ...8..293R/abstract>. I have not really identified an expression for the luminosity in either part. But I admit also that besides finding the papers, I did not spend much time reading it all in detail yet.

<sup>53</sup>[https://link.springer.com/referenceworkentry/10.1007%2F978-1-4419-9917-7\\_1174](https://link.springer.com/referenceworkentry/10.1007%2F978-1-4419-9917-7_1174)

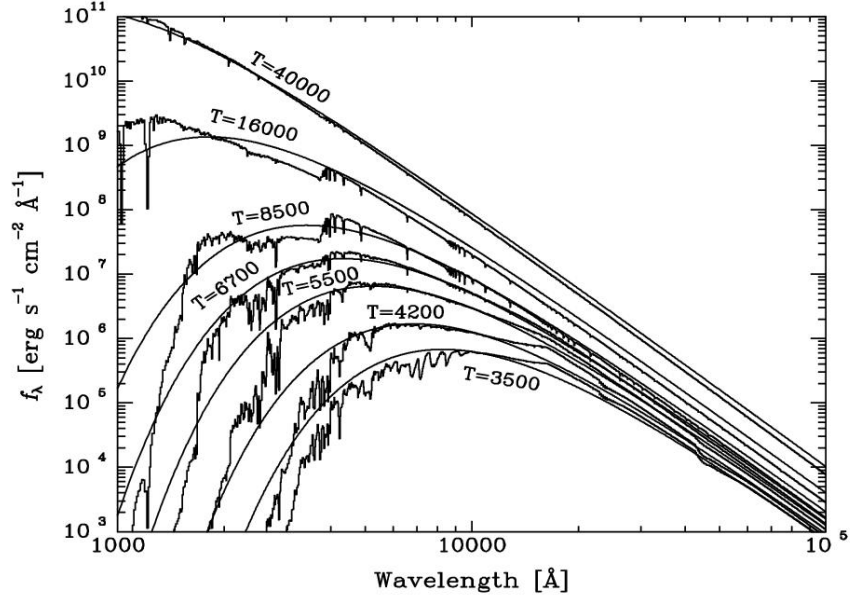


Figure 4: Comparison of stellar spectra to blackbody curves. Credit: Fig. 2.1 of Maoz (2016) obtained from <http://wise-obs.tau.ac.il/~dani/figures.htm>.

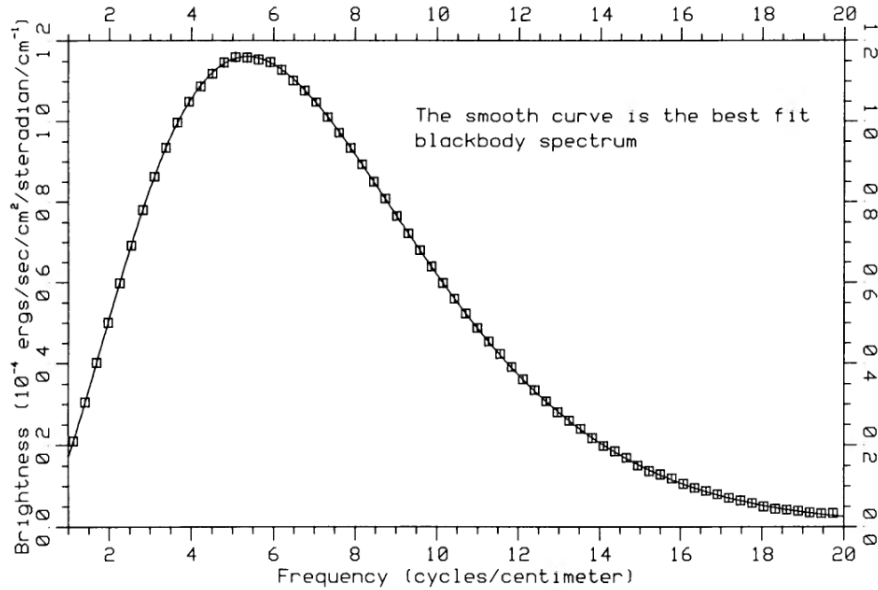


Figure 5: Best fitting black body curve to the preliminary COBE data. Credit: Fig. 2 of Mather et al. (1990), see <https://ui.adsabs.harvard.edu/abs/1990ApJ...354L..37M/abstract>.

The temperature of the CMB was first predicted by Alpher & Herman (to be of 5 K, 1948)<sup>54</sup> in connection to work by Gamow (1948)<sup>55</sup>. The CMB was first discovered by Penzias & Wilson (1965)<sup>56</sup> and then interpreted by Dicke et al. (1965)<sup>57</sup>. The first space mission whose aim was to study the CMB was the Cosmic Background Explorer (COBE, see Mather 1994, and references therein)<sup>58</sup>, launched by NASA in 1989. The CMB spectrum measured by COBE follows closely a black body with temperature of 2.726 K (Mather et al. 1994)<sup>59</sup>, see Figure 5. The COBE mission also confirmed the CMB small level anisotropies (Smoot et al. 1992)<sup>60</sup>.

Black body radiators are also used as standards for calibration of certain instruments and detectors. The FIRAS instrument onboard of COBE, for example, used a temperature-controlled blackbody as reference source to obtain high-precision when measuring the CMB spectrum (see e.g. Boggess et al. 1992)<sup>61</sup>. Technically, black body sources are cavities kept at uniform temperature with a small opening. Typical applications require that the black body cavity emission be known an uncertainty of 0.01% or less (see e.g. Hartmann et al. 2009)<sup>62</sup>.

### 3.2 Thermal Bremsstrahlung

Bremsstrahlung (German for “braking radiation”) is the radiation emitted by a charged particle decelerated in the electric field of another charged particle. Full treatment of the problem requires quantum mechanics, and we are definitely not going to cover that here. Relativity is also needed for dealing with relativistic particles. A classical treatment can be used in certain regimes where the quantum results can be applied as corrections (the Gaunt factor) to the classical results. For details see Chapter 5 of Rybicki & Lightman (1991)<sup>63</sup>, Sections 1.29 and 1.30 of Lang (1980)<sup>64</sup>, and/or Blumenthal & Gould (1970)<sup>65</sup>.

Bremsstrahlung is important in many astrophysical plasmas. It is the main process responsible for the cooling of the intracluster medium, the plasma that permeates a galaxy cluster, which produces X-ray emission (Cavaliere & Fusco-Femiano 1976)<sup>66</sup>. It is also an important process to understand the spectral distortions of the CMB (Chluba & Sunyaev 2012)<sup>67</sup>. Bremsstrahlung also contributes to the continuum radio emission from HII regions (e.g. Subrahmanyan et al. 2001)<sup>68</sup> and planetary

<sup>54</sup><https://ui.adsabs.harvard.edu/abs/1948Natur.162..774A/abstract>

<sup>55</sup><https://ui.adsabs.harvard.edu/abs/1948Natur.162..680G/abstract>

<sup>56</sup><https://ui.adsabs.harvard.edu/abs/1965ApJ...142..419P/abstract> and measured to be about 3 K. Penzias and Wilson were awarded the 1978 Nobel Prize in Physics.

<sup>57</sup>Who were in the process of building an instrument to try to detect the CMB but were scooped, <https://ui.adsabs.harvard.edu/abs/1965ApJ...142..414D/abstract>

<sup>58</sup><https://www.sciencedirect.com/science/article/pii/S1350449594900914>

<sup>59</sup><https://ui.adsabs.harvard.edu/abs/1994ApJ...420..439M/abstract>

<sup>60</sup><https://ui.adsabs.harvard.edu/abs/1992ApJ...396L...1S/abstract>. This and the previous COBE results gave John Mather and George Smoot the Nobel Prize in Physics of 2006.

<sup>61</sup><https://ui.adsabs.harvard.edu/abs/1992ApJ...397..420B/abstract>

<sup>62</sup><https://www.sciencedirect.com/science/article/pii/S1079404209042064>

<sup>63</sup>See in Google Scholar [https://scholar.google.com/scholar?hl=en&as\\_sdt=0%2C5&q=rybicki+lightman&btnG=&oq=Rybicki](https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=rybicki+lightman&btnG=&oq=Rybicki)

<sup>64</sup><https://www.springer.com/gp/book/9783662216422>

<sup>65</sup><https://ui.adsabs.harvard.edu/abs/1970RvMP...42..237B/abstract>

<sup>66</sup><https://ui.adsabs.harvard.edu/abs/1976A%26A...49..137C/abstract>

<sup>67</sup><https://ui.adsabs.harvard.edu/abs/2012MNRAS.419.1294C/abstract>

<sup>68</sup><https://ui.adsabs.harvard.edu/abs/2001AJ...121..399S/abstract>

nebulae.

Most commonly needed is the bremsstrahlung radiation of an electron deflected by an ion. This radiation is sometimes also called free-free radiation, as the electrons are not bound to the atomic nucleus neither before nor after the interaction. Thermal bremsstrahlung is the case where the velocity of the particles is thermal (i.e., it follows Maxwell–Boltzmann distribution). The “volume emissivity” of thermal bremsstrahlung from a plasma (i.e., the power per unit volume per unit solid angle per unit frequency interval) is:

$$\varepsilon_\nu = \frac{8}{3} \left( \frac{2\pi}{3} \right)^{1/2} n_\nu \frac{Z^2 e^6}{m^2 c^3} \left( \frac{m}{kT} \right)^{1/2} N_i N_e g(\nu T) \exp(-h\nu/kT), \quad (13)$$

where  $n_\nu$  is the refractive index,  $Z$  is the ion charge (e.g. 1 for a plasma of protons),  $e$  and  $m$  are the electron charge and mass,  $N_e$  and  $N_i$  are the volume densities of electrons and ions, and  $g(\nu T)$  is the Gaunt factor. Sometimes emissivity will be called the emission coefficient, and sometimes you will encounter the “mass emission coefficient”,  $j_\nu = \varepsilon_\nu/\rho$ , where  $\rho$  is the density in mass per unit volume (so  $j_\nu$  has units of power per unit mass per unit solid angle per unit frequency interval).

The intensity emitted along a certain line of sight,  $L$ , is then

$$I_\nu = \int_0^L \varepsilon_\nu dx \propto g(\nu T) L T^{-1/2} \exp(-h\nu/kT) \quad (14)$$

For the radio continuum emission we have that  $h\nu \ll kT$ . Free-free radiation is also absorbed at radio wavelengths. So to understand the radio continuum of a nebulae, one also needs an expression for the absorption coefficient ( $\kappa_\nu$ ). The source function,  $S_\nu$ , is defined by  $S_\nu = \varepsilon_\nu/\kappa_\nu$ , and in thermal equilibrium  $S_\nu$  is given by Planck’s law (Eq. 6). Since we are talking about the Rayleigh-Jeans limit ( $h\nu \ll kT$ ), the spectral flux density (integrated over solid angle) can be expressed in terms of the brightness temperature:

$$F_\nu = \oint I_\nu d\omega = \frac{2\nu^2 k}{c^2} \oint T_b(\nu) d\omega \quad (15)$$

The optical depth,  $\tau_\nu$ , is defined as

$$\tau_\nu = \int_0^L \kappa_\nu dx. \quad (16)$$

Using the definitions above it is possible to obtain an expression for the optical depth where  $\tau_\nu \propto \nu^{-2.1}$  (see Lang 1980; Wilson et al. 2013). Furthermore,

$$T_b(\nu) \rightarrow T_e \tau_\nu \quad (\text{in the optically thin case}), \quad (17)$$

$$T_b(\nu) \rightarrow T_e \quad (\text{in the optically thick case}) \quad (18)$$

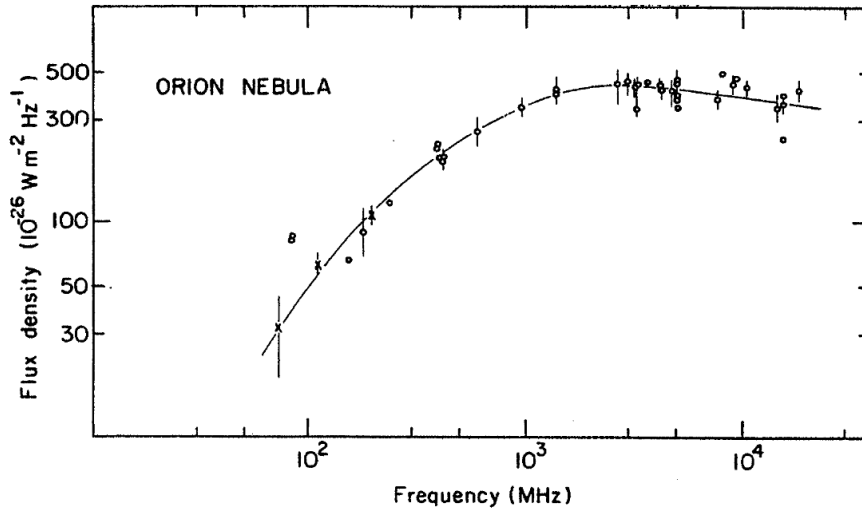


Figure 6: The radio spectrum of the Orion nebula. The optically thick case applies around  $\sim 10^2$  MHz and the optically thin case around  $\sim 10^4$  MHz. Credit: Fig. 2 of Terzian & Parrish (1970), see <https://ui.adsabs.harvard.edu/abs/1970ApL.....5..261T/abstract>.

which, using Eq. 15, results that the bremsstrahlung radio continuum flux is proportional to  $\nu^{-0.1}$  in the optically thin case and to  $\nu^{-2.1}$  in the optically thick case (see Fig. 6).

In addition, one can define the “turn-over frequency” ( $\nu_T$ ) where the optical depth is equal to one:

$$\nu_T^{2.1} \approx 8.24 \times 10^{-2} N_e^2 L T_e^{-1.35} \quad (19)$$

(in GHz for  $L$  given in pc). The brightness temperature and the intensity are shown with respect to the turn over frequency in Fig. 7. Note however that there were a series of assumptions to get to the expressions above, including that the nebula is homogenous<sup>69</sup>. For clumpy nebulae, the equations have to be modified.

And just for historical completeness, the Gaunt factor is named after the physicist John Arthur Gaunt, born in China of English parents. There seems to be little information about him, as he was scientifically active just for a few years. He died in Hong Kong in 1944 as a prisoner of war (Wilson 1945)<sup>70</sup>. It seems his supervisor in Cambridge was Ralph Fowler, the same supervisor of Dirac and Chandrasekhar (Jeffreys 1990)<sup>71</sup>. Oppenheimer (1929)<sup>72</sup> presented a theory of the bremsstrahlung radiation of electrons in the electric field of a positive ion. Gaunt was working on the same problem and found a mistake made by Oppenheimer. He published his own work on the subject in Gaunt (1930b,a)<sup>73</sup>. The theory builds on previous work by Kramers (1923), and because of that sometimes there is instead reference to the Kramers-Gaunt factor. Hendrik

<sup>69</sup>If interested, see also the lecture notes on HII regions by Richard Pogge: <http://www.astronomy.ohio-state.edu/~pogge/Ast871/Notes/Ionized.pdf>

<sup>70</sup><https://ui.adsabs.harvard.edu/abs/1945Natur.155...41W/abstract>

<sup>71</sup><https://www.jstor.org/stable/531586>

<sup>72</sup><https://ui.adsabs.harvard.edu/abs/1929ZPhy...55..7250/abstract>

<sup>73</sup><https://royalsocietypublishing.org/doi/pdf/10.1098/rsta.1930.0005>

**Fig. 10.5** Thermal radiation of a gas cloud. (a) Spectral distribution of the intensity.  $I_0$  is the intensity, the electron temperature is  $T_e$ , and  $\tau$  is the optical depth,  $\nu_0$  is the turn over frequency, where  $\tau = 1$ . (b) Spectral distribution of the brightness temperature

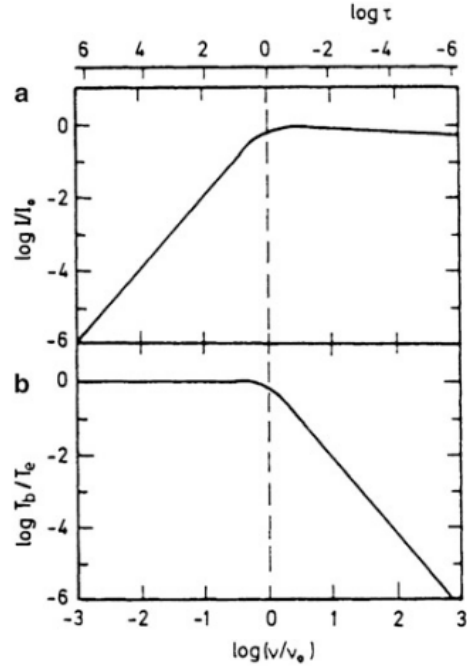


Figure 7: The brightness temperature and the intensity are shown with respect to the turn over frequency. Credit: Fig. 10.5 of Wilson et al. (2013).

Anthony Kramers is better known than Gaunt. He was a Dutch physicist that had Niels Bohr as his advisor<sup>74</sup>. It seems it was Chandrasekhar, in his 1939 book on stellar structure that introduced the name “Gaunt factor” (page 262, Chandrasekhar 1939).

### 3.3 Synchrotron radiation

Synchrotron radiation is emitted by charged particles suffering acceleration by a magnetic field, when the velocity of the particles is relativistic. In astrophysical sources, synchrotron radiation can contribute to the flux in radio, optical, and/or X-rays. In the case of non-relativistic particles, the emission is called cyclotron radiation (see Sec. 3.5.2 below). Magnetic fields are everywhere, although typically they are weak, but there are strongly magnetized objects out there. The interstellar medium has a typical field with strength of the order of  $10^{-6}$  gauss (G)<sup>75</sup>, a stellar atmosphere of  $\sim 1$  G, a supermassive black hole of  $\sim 10^4$  G, a neutron star of  $10^{12}$  G.

Synchrotron radiation is also known as magneto bremsstrahlung radiation as it is the analogous case of bremsstrahlung when the deflection of the particle is caused by a magnetic instead of an electric field. For the mathematical details see Chapter 6 of Rybicki & Lightman (1991), Sections 1.25 to 1.28 of Lang (1980), Sections 10.7 to 10.13 of Wilson et al. (2013), and/or Blumenthal & Gould (1970). Here I will just quickly list some of the synchrotron radiation characteristics without

<sup>74</sup><https://mathshistory.st-andrews.ac.uk/Biographies/Kramers/>

<sup>75</sup>Named after the German mathematician and physicist Carl Friedrich Gauss. In the SI, the magnetic field is given in Tesla (T), named after the Serbian inventor and engineer Nikola Tesla.  $1 \text{ G} = 10^{-4}$ .

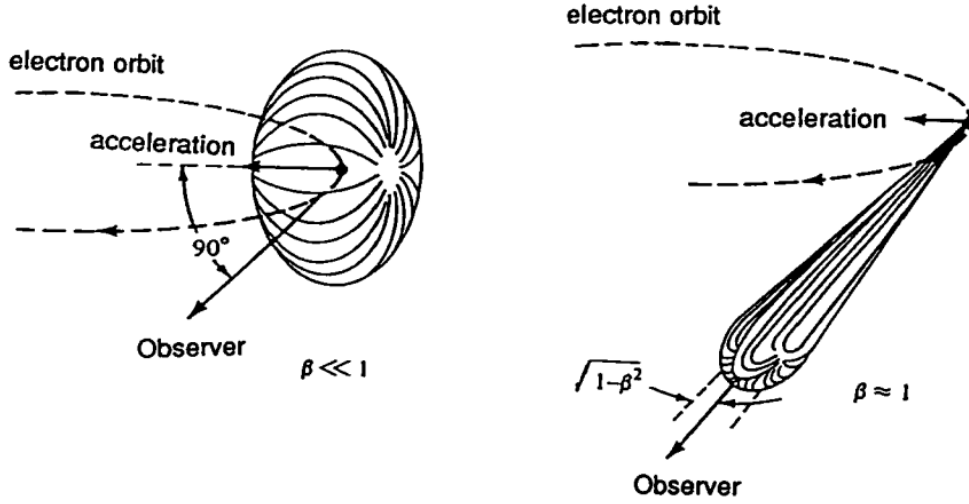


FIG. 1. Radial distribution of the emitted radiation from an accelerating electron as a function of the electron velocity  $\beta = v/c$ .

Figure 8: Distribution of the emitted radiation from an accelerating electron. Credit: Fig. 1 of Johnson (1997).

deriving any of the results (which anyway is not the goal of these lectures). As we are talking about relativistic particles, it is good to remember that:  $\gamma = (1 - \frac{v^2}{c^2})^{-1/2}$  and if we define  $\beta = v/c$  then we can also write  $\gamma = (1 - \beta^2)^{-1/2}$ .

A magnetic field ( $\mathbf{B}$ ) does no work on the particle, so the electron keeps the magnitude of its velocity ( $\mathbf{v}$ ) constant (i.e.,  $|\mathbf{v}| = \text{constant}$ ). Furthermore, it can be shown that there is no acceleration in the direction along the field and that the component of the velocity in this direction remains constant ( $\mathbf{v}_{\parallel}$ ). Together, this means that also  $|\mathbf{v}_{\perp}| = \text{constant}$ . The acceleration in the direction perpendicular to the field only changes the direction of the velocity, resulting in circular motion. The combination of uniform motion (along the field) and circular motion (perpendicular to the field) gives a helical path to the electron, where it gyrates with a constant pitch angle ( $\alpha$ , the angle between its velocity and the field). The angular frequency of gyration is given by:

$$\omega_B = \frac{eB}{\gamma mc}, \quad (20)$$

where  $e$  is the charge and  $m$  the mass of the electron.

As the electron is accelerated in its orbit (in a direction perpendicular to the field), it radiates. With the electron moving at relativistic velocities, the radiation it emits is concentrated into a narrow cone that points in the same the direction of its instantaneous motion (Fig. 8). Since the radiation is emitted in a cone, the observer will receive it in pulses of short time duration (just when the cone is oriented towards the observer). The interval of time ( $\Delta t$ ) in which the observer sees the pulse is  $\Delta t \sim (\gamma^3 \omega_B)^{-1}$ .



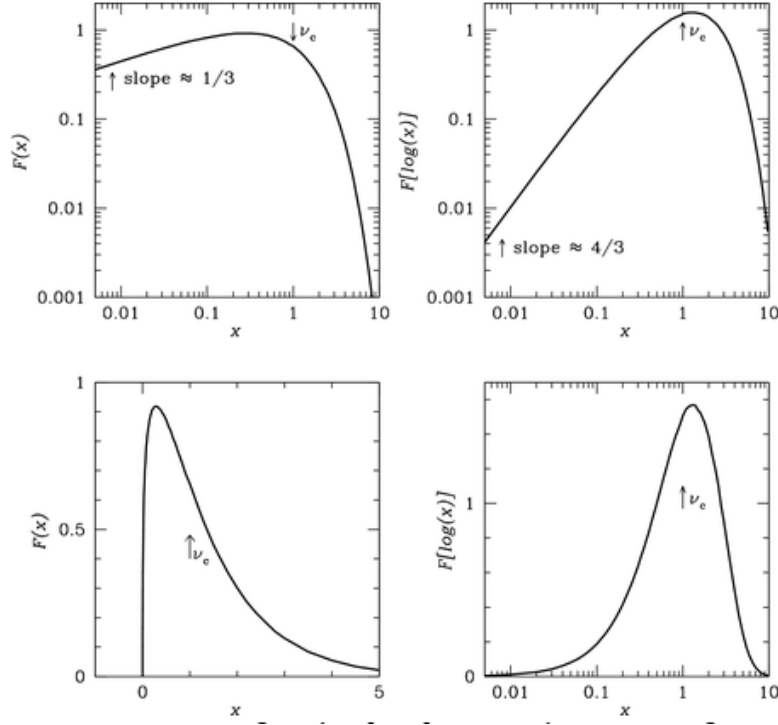


Figure 9: Four different ways to plot the synchrotron spectrum distribution,  $F(x)$ , of a single electron in units of the critical frequency ( $x = \nu/\nu_c$ ). Credit: Fig. 5.6 of <https://www.cv.nrao.edu/~sransom/web/Ch5.html>.

It can be shown that the spectrum radiated by a single electron is fairly broad and that the radiated power is increased because of the relativistic effects. “In principle, the radiation is not a continuum but has a series of spikes in the frequency domain (with  $\Delta\nu < 10^{-3}$  Hz). In practice, however, any small fluctuation in the electron energy, magnetic field strength, or pitch angle would cause shifts that are larger in frequency than  $\Delta\nu$ ”<sup>76</sup>, therefore the synchrotron spectrum of even a single electron would be essentially continuous. A critical frequency, that divides the power spectrum in two equal parts, can be defined as:

$$\nu_c = \frac{3}{4\pi} \gamma^3 \omega_B \sin \alpha. \quad (21)$$

The spectrum distribution for the emission of a single electron in terms of the critical frequency is shown in Fig. 9.

For astrophysical use, one needs to know the synchrotron emission of an ensemble of electrons. In many cases, the energy distribution of relativistic electrons can be written with a power-law density distribution:

$$N(E) dE = kE^{-p} dE, \quad (22)$$

<sup>76</sup>Quoting from <https://www.cv.nrao.edu/~sransom/web/Ch5.html>.

where  $p$  is called the spectral index. The total power radiated by such a distribution is obtained by the integral of  $N(E) dE$  times the expression for the power radiated by a single electron integrated over all energies. To solve the problem, it is usually assumed that each electron of the distribution radiates only at its critical frequency. One then obtains an expression for the total power emitted that is  $P \propto \nu^{-s}$ , where

$$s = \frac{p-1}{2}. \quad (23)$$

So the spectral index of the radiation is related to the spectral index of the electron distribution.

The radiation from a single charge is elliptically polarized. However, for distributions of particles that vary smoothly with pitch angle, the time-averaged polarization is linear. The degree of polarization is very high,  $> 0.5$ .

And for the historical overview:

The first major step in describing what we now call synchrotron radiation was the 327 pages essay by Schott (1912)<sup>77</sup> on “Electromagnetic radiation and the mechanical reactions arising from it”. After that, it seems that further development came only with the work of Iwanenko & Pomeranchuk (1944)<sup>78</sup> on energy losses on betatrons, a type of particle accelerator. Additional theoretical work was motivated in the USA with the development of high energy accelerators (Blewett 1946; Schwinger 1949)<sup>79</sup>. The radiation was then observed in the General Electric synchrotron accelerator and reported by Elder et al. (1947)<sup>80</sup>. The radiation was named after the machine. The machine name appeared in a paper by McMillan (1945)<sup>81</sup>, where he proposed a new type of cyclotron machine that, to work with relativistic electrons, would need a synchronous adjustment to provide the necessary accelerating force. See Blewett (1998)<sup>82</sup> for further details on the early history of synchrotron radiation.

On the astrophysical side, it was Unsöld (1949)<sup>83</sup> who first concluded that the radio emission of the Sun was made of two parts, a steady part due to free-free (thermal) transitions in the solar corona and chromosphere and a variable part that had to be non-thermal. The first suggestion that cosmic radio sources were emitting synchrotron radiation was made by Alfvén & Herlofson (1950)<sup>84</sup>. This suggestion followed new observational discoveries about the radio emission of Galactic sources (see

<sup>77</sup><https://archive.org/details/cu31924011397084>

<sup>78</sup><https://journals.aps.org/pr/abstract/10.1103/PhysRev.65.343>; this is actually a short summary in English of their original paper in Russian that appeared in a Soviet journal. The reference is sometimes given as *Akademia Nauk Doklady*, 1944, 44, 315, but I could not track it down yet.

<sup>79</sup>See <https://journals.aps.org/pr/abstract/10.1103/PhysRev.69.87> and <https://journals.aps.org/pr/abstract/10.1103/PhysRev.75.1912>.

<sup>80</sup><https://journals.aps.org/pr/abstract/10.1103/PhysRev.71.829.5>

<sup>81</sup><https://journals.aps.org/pr/abstract/10.1103/PhysRev.68.143>. Edwin Mattison McMillan was awarded the Nobel Prize in chemistry of 1951, not for inventing the synchrotron machine, but for his work on transuranium elements. He was the first to synthesize Neptunium, an element with  $Z = 93$ .

<sup>82</sup><https://onlinelibrary.wiley.com/doi/abs/10.1107/S0909049597043306>

<sup>83</sup><https://ui.adsabs.harvard.edu/abs/1949ZA....26..176U/abstract>

<sup>84</sup><https://journals.aps.org/pr/abstract/10.1103/PhysRev.78.616>. The Swedish engineer and physicist Hannes Olof Gösta Alfvén was awarded the Nobel Prize in physics of 1970 for his work in magneto-hydrodynamics.

e.g. Smith 1950, and references therein)<sup>85</sup>. Kiepenheuer (1950)<sup>86</sup> suggested that the general Galactic radio emission could be caused by cosmic rays emitting synchrotron in the interstellar medium. In Russia, Vitaly Ginzburg<sup>87</sup> made further contributions regarding the cosmic-ray idea (published in Russian). Iosif Shklovsky<sup>88</sup> suggested that the radiation from the Crab nebula could be explained as radio and optical synchrotron radiation (also published in Russian, but see Ginzburg 1990; Ekers 2014)<sup>89</sup>. In 1954, Mikheil Vashakidze<sup>90</sup> (Georgian astronomer) and Viktor Dombrovsky<sup>91</sup> (Russian astronomer) independently reported the first observations of polarisation in the Crab nebula at optical wavelengths (also in Russian, but see the Introduction of Westfold 1959)<sup>92</sup>. The detection of polarisation was the final evidence needed to prove the synchrotron hypothesis. The additional discovery by Baade (1956) that the jet of M87 also emits polarised light, showed that the synchrotron mechanism was also at work in extragalactic sources. See also Ginzburg & Syrovatskii (1965)<sup>93</sup> for an early review that includes a lot of the math.

### 3.4 Inverse Compton scattering

This is not exactly a process of radiation emission, but it is an important process that changes the characteristics of the radiation that we receive from certain astrophysical sources. Compton scattering is named after the American physicist Arthur Holly Compton<sup>94</sup>. The process involves the scattering of a photon by a charged particle. If there is a decrease in the photon energy, then this is the Compton effect (or scattering). If there is an increase in the photon energy, then this is called the inverse Compton scattering.

When the free electron is basically at rest, i.e. it has little or no kinetic energy, and the photons have small energy ( $h\nu \ll mc^2$ ) then the scattering of radiation can be treated classically considering the radiation as an electromagnetic wave. This is the Thomson scattering<sup>95</sup>. The cross section for the Thomson scattering is independent of wavelength and given by:

$$\sigma_{\text{T}} = \frac{8\pi}{3} r_0^2, \quad (24)$$

<sup>85</sup><https://ui.adsabs.harvard.edu/abs/1950Natur.165..422S/abstract>

<sup>86</sup><https://journals.aps.org/pr/abstract/10.1103/PhysRev.79.738>

<sup>87</sup>Ginzburg was awarded the Nobel prize of physics in 2003 for his work on the theory of superconductors and superfluids. See his biographical account in <https://www.nobelprize.org/prizes/physics/2003/ginzburg/biographical/>.

<sup>88</sup>See [https://link.springer.com/referenceworkentry/10.1007%2F978-1-4419-9917-7\\_1276](https://link.springer.com/referenceworkentry/10.1007%2F978-1-4419-9917-7_1276)

<sup>89</sup><https://ui.adsabs.harvard.edu/abs/1990ARA%26A..28...1G/abstract> and <https://www.sciencedirect.com/science/article/pii/S092765051300087X>

<sup>90</sup><https://observatory.iliauni.edu.ge/mikheil-vashakidze-2/?lang=en>

<sup>91</sup>[https://link.springer.com/referenceworkentry/10.1007/978-1-4419-9917-7\\_371](https://link.springer.com/referenceworkentry/10.1007/978-1-4419-9917-7_371)

<sup>92</sup><https://ui.adsabs.harvard.edu/abs/1959ApJ...130..241W/abstract>

<sup>93</sup><https://ui.adsabs.harvard.edu/abs/1965ARA%26A...3..297G/abstract>

<sup>94</sup>Who received the Nobel Prize in physics of 1927 for his discovery of the Compton scattering. See Compton (1923) in <https://ui.adsabs.harvard.edu/abs/1923PhRv...21..483C/abstract>.

<sup>95</sup>Named after the British physicist Joseph John Thomson, the one credited as the discoverer of the electron, see Thomson (1897) in <http://www.damtp.cam.ac.uk/user/tong/pp/jj1.pdf> but see also Smith (1997) in [https://www.cs.princeton.edu/courses/archive/fall05/frs119/papers/smith97\\_thomson.pfg.pdf](https://www.cs.princeton.edu/courses/archive/fall05/frs119/papers/smith97_thomson.pfg.pdf) for the historical context and information about others doing similar work at the time. Johnson was awarded the Nobel Prize in Physics of 1906 for his work on the conduction of electricity by gases.

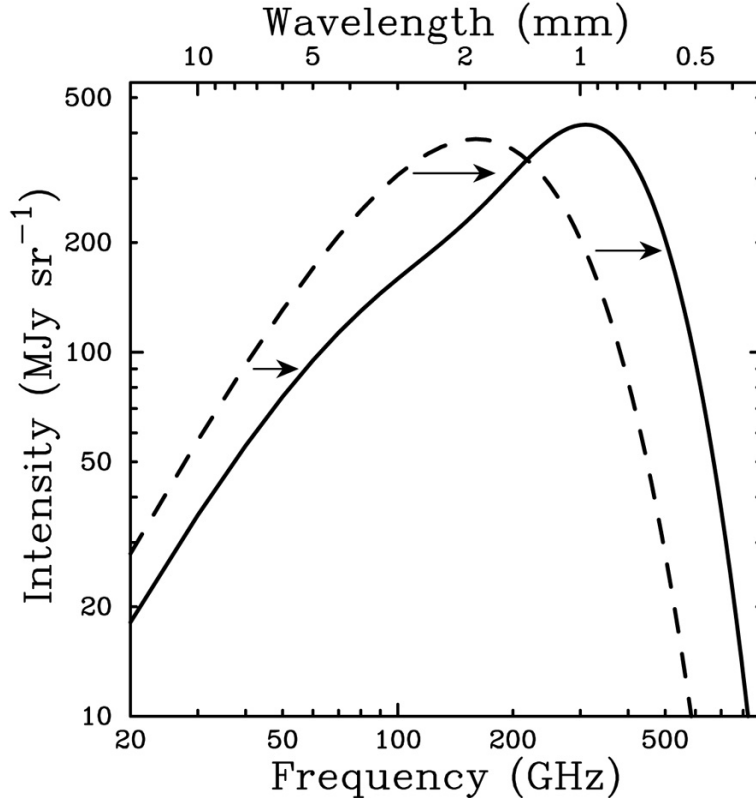


Figure 10: The CMB spectrum compared to a version distorted by the Sunyaev-Zeldovich effect. The distortion is exaggerated for a fictional cluster that is 1000 times more massive than a typical massive galaxy cluster. Credit: Fig. 1 of Carlstrom et al. (2002) in <https://ui.adsabs.harvard.edu/abs/2002ARA%26A..40..643C/abstract>.

where  $r_0$  is the classical electron radius ( $2.8179403227 \times 10^{-15}$  m). Thomson scattering is the case for photons in UV, visible, infrared and radio.

For energetic photons, in the gamma and x-ray regimes, the momentum of the incident photon becomes significant. Then the scattering process needs to be treated as a relativistic particle-particle collision. This is the case of the Compton scattering. The ratio between the photon energy before ( $E_{in}$ ) and after ( $E_{out}$ ) the collision is given by:

$$\frac{E_{in}}{E_{out}} = 1 + \frac{E_{in}}{mc^2}(1 - \cos \theta), \quad (25)$$

where  $\theta$  is the scattering angle (the angle between the direction of motion of the emerging photon with respect to the direction of motion of the incident photon.)

In astrophysical cases, it can happen that hot thermal electrons are scattered against low energy (e.g. radio) photons. In such cases, the photons are scattered with higher energies at the expense of the electron energy. This is the inverse Compton scattering. The process increases the energy of the photon by a factor of the order of  $\gamma$ . For further details, you are referred to Chapter 7 of Rybicki & Lightman (1991), Sections 1.38 and 4.5.1.8 of Lang (1980), Section 10.14 of Wilson et al.

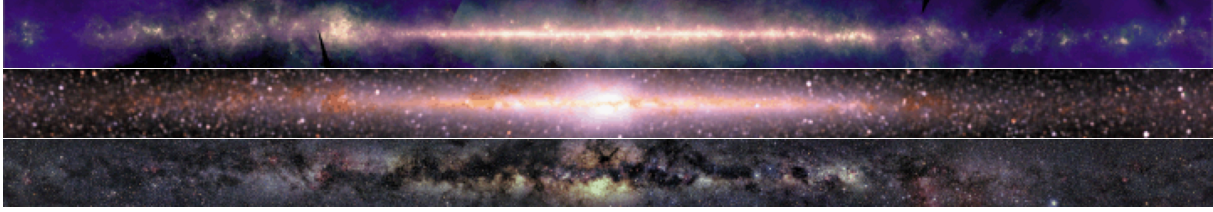


Figure 11: The Milky Way seen in different bands. In the visible (bottom) we notice the dark regions affected by dust extinction. In the near-IR (middle) we have an almost unobscured view of stellar emissino. In the mid- to far-IR (top) we see dust emission. Credit: NASA – Jay Friedlander in consultation with Seth Digel. Taken from [https://asd.gsfc.nasa.gov/archive/mmw/mmw\\_sci.html](https://asd.gsfc.nasa.gov/archive/mmw/mmw_sci.html).

(2013), and/or Blumenthal & Gould (1970).

Photons of the CMB suffer inverse Compton scattering when moving through the hot gas in intra-cluster medium. The electrons of the gas loose energy and the photons become more energetic; this is the so-called Sunyaev–Zeldovich effect(Sunyaev & Zeldovich 1972)<sup>96</sup>, see Fig. 10.

Inverse Compton scattering is also important in the environment around an accreting black holes. Thermal photons from the accretion disc are scattered by the energetic particles that surround the black hole (see e.g. Wilkins & Gallo 2015, and references therein)<sup>97</sup>.

### 3.5 Other sources of electromagnetic radiation

For completeness, here I shortly mention a few other sources of electromagnetic radiation and point out a few additional references for those interested in more details.

#### 3.5.1 Dust emission

Galaxies are not made only of stars and their stellar systems (planets, moons, comets, disks, planetesimals, ...). There is also interstellar matter spread inhomogeneously in the space between stars. In the interstellar medium (ISM) we can find gas (atoms, molecules, ions, and electrons) and dust (small solid particles). The ISM, and dust in particular, affects starlight causing extinction (i.e., making things look fainter), reddening (i.e., making the color of things look redder), and polarization. The effects of absorption are stronger in UV and visible wavelengths. The ISM material can also create absorption lines in stellar spectra and emit radiation over the continuum or in specific lines. See Fig. 11 for an illustration of the effects of dust in different bands.

We will talk more about reddening when discussing photometry. Here, I just want to mention some quick facts about dust emission and point out a few texts where you can find more information if interested. Astrophysical sources of dust include the winds from evolved giant stars, massive stars

<sup>96</sup>See <https://ui.adsabs.harvard.edu/abs/1972CoASP...4...173S/abstract>.

<sup>97</sup><https://ui.adsabs.harvard.edu/abs/2015MNRAS.448...703W/abstract>

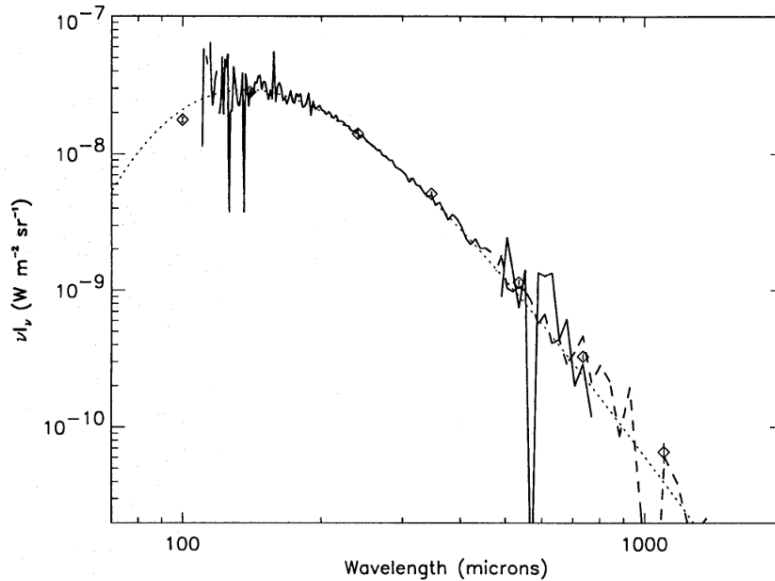


Figure 12: Far-infrared spectrum of dust obtained with instruments on board the COBE satellite. The emission is fitted with a Planck curve of 17.5 K. Credit: Figure 2 of Boulanger et al. (1996), see <https://ui.adsabs.harvard.edu/abs/1996A%26A...312..256B/abstract>.

undergoing mass loss, novae, and probably supernovae (maybe also AGNs, but that seems to be controversial).

Dust grains absorb stellar light, getting heated up to a temperature of 15-20 K or above (depending on the environment). The heat is later mostly radiated in the mid- or far-IR wavelengths (but can cover a broader range, from the near IR to the millimeter regime). Dust seems to be composed mainly of spherical graphite, silicate grains, and polycyclic aromatic hydrocarbon (PAH) molecules. The thermal emission of dust in the IR is shown in Fig. 12.

For more information about dust and the interstellar medium, have a look at Ferrière (2001)<sup>98</sup>, Draine (2003)<sup>99</sup>, Draine (2004)<sup>100</sup>, and/or Steinacker et al. (2013)<sup>101</sup>. These are perhaps not the most recent of references, but they should serve as a good starting point.

### 3.5.2 Cyclotron radiation

Cyclotron radiation is that emitted by a charged particle with a non-relativistic velocity moving through a magnetic field. The name of the radiation also comes from the accelerator used to create the fast moving particles<sup>102</sup>. As discussed for the case of synchrotron radiation (Sec. 3.3), the

<sup>98</sup><https://ui.adsabs.harvard.edu/abs/2001RvMP...73.1031F/abstract>

<sup>99</sup><https://ui.adsabs.harvard.edu/abs/2003ARA%26A...41..241D/abstract>

<sup>100</sup><https://ui.adsabs.harvard.edu/abs/2004tcu...conf..213D/abstract>

<sup>101</sup><https://ui.adsabs.harvard.edu/abs/2013ARA%26A...51...63S/abstract>

<sup>102</sup>The cyclotron was invented by Ernest Orlando Lawrence, who received the Nobel Prize in physics of 1939 for this invention. His student, Stanley Livingston did a lot of the work in building the equipment, see Livingston (1975) in <https://accelconf.web.cern.ch/c75/papers/j-01.pdf>. It seems that a Hungarian inventor, Leo Szilard

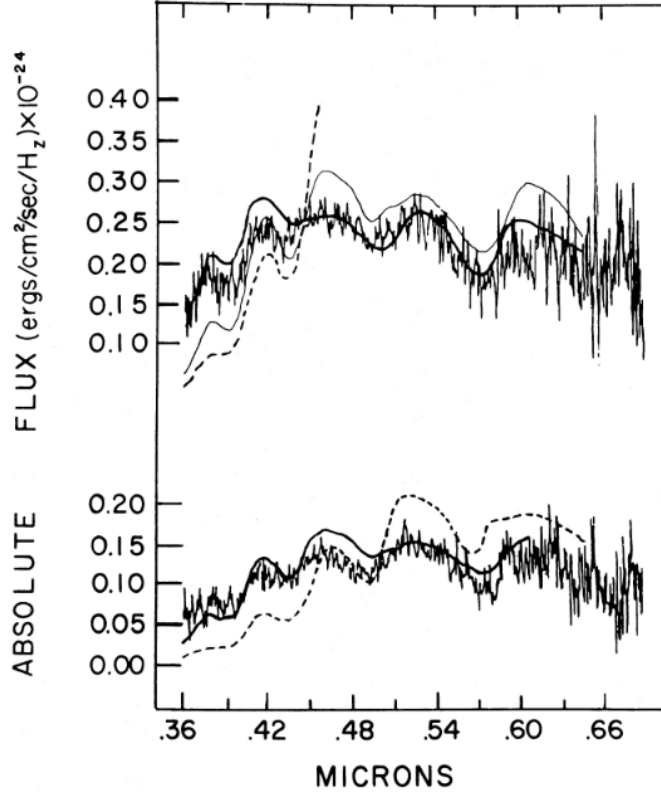


Figure 13: Cyclotron emission lines observed in the spectrum of the accreting magnetic white dwarf VV Pupis. The lines indicate different models for comparison. Credit: Figure 8 of Barrett & Channugam (1985), see <https://ui.adsabs.harvard.edu/abs/1985ApJ...298..743B/abstract>.

charge will spiral around the magnetic field. In this case with an angular frequency of gyration given by:

$$\omega_B = \frac{eB}{mc}, \quad (26)$$

The spectrum emitted is not continuous, but concentrated on this frequency and its harmonics. This type of emission is difficult to observe from astrophysical sources. Cyclotron lines have been detected at least in the spectra of accreting magnetic white dwarfs (see Fig 13) and X-ray accreting pulsars. It seems however that the lines are usually detected not in emission but in absorption (the so-called cyclotron resonant scattering features, see e.g. Schwarm et al. 2017)<sup>103</sup>. X-ray accreting pulsars are systems containing a magnetised neutron star. For more details of the detections see Kreykenbohm et al. (2005)<sup>104</sup>, Staubert et al. (2019)<sup>105</sup>, and references therein.

patented the same idea years before, but never constructed the machine, see Telegdi (2000) in <https://physicstoday.scitation.org/doi/10.1063/1.1325189>.

<sup>103</sup><https://ui.adsabs.harvard.edu/abs/2017A%26A...601A..99S/abstract>

<sup>104</sup><https://ui.adsabs.harvard.edu/abs/2005A%26A...433L..45K/abstract>

<sup>105</sup><https://ui.adsabs.harvard.edu/abs/2019A%26A...622A..61S/abstract>

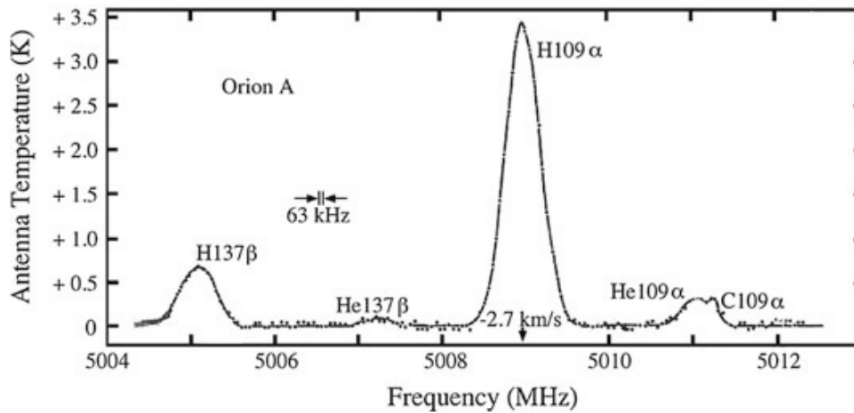


Figure 14: Recombination lines of H, He, and C in the radio spectrum of the Orion nebula. Credit: Figure 11.4 of Lang (2013), see <https://link.springer.com/book/10.1007/978-3-642-35963-7>.

### 3.5.3 Line emission

Planetary nebulae and H II regions, made of gas ionized by hot stellar sources, have spectra that are very rich in emission lines. Bright emission lines are relatively easy to detect in comparison to the weaker continuum level. The spectra of ionized gas nebulae can be measured also in other galaxies.

In the UV, visible and IR bands, we can see collisionally excited emission lines from metal ions and recombination lines of Hydrogen and Helium. There are also recombination lines from highly excited states of H, He, and metals that can be detected in radio (see Fig. 14). Recombination lines appear when an electron is captured by the ion in an excited state, which is followed by a quick cascade of transitions into the ground state, each transition emitting photons of different energy. As first shown by the American astronomer Ira Bowen<sup>106</sup>, many of these emission lines are “forbidden lines” (Bowen 1927, 1928)<sup>107</sup>, in the sense that the upper states are metastable with long lifetimes ( $10^{-2}$ – $10^2$ s).

For the physics behind recombination lines, see [Chapter 4 of the book](#) on gaseous nebulae and active galactic nuclei by Osterbrock & Ferland (2006). For more details and references on nebular spectroscopy see Peimbert et al. (2017)<sup>108</sup>. And for a review about emission lines in the spectra of galaxies, see Kewley et al. (2019)<sup>109</sup>.

<sup>106</sup>See [https://link.springer.com/referenceworkentry/10.1007/978-0-387-30400-7\\_191](https://link.springer.com/referenceworkentry/10.1007/978-0-387-30400-7_191)

<sup>107</sup><https://ui.adsabs.harvard.edu/abs/1927Natur.120..473B/abstract> and <https://ui.adsabs.harvard.edu/abs/1928ApJ....67....1B/abstract>

<sup>108</sup><https://ui.adsabs.harvard.edu/abs/2017PASP...129h2001P/abstract>

<sup>109</sup><https://ui.adsabs.harvard.edu/abs/2019ARA%26A...57..511K/abstract>



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