Practical Observational Astronomy

Lecture 3

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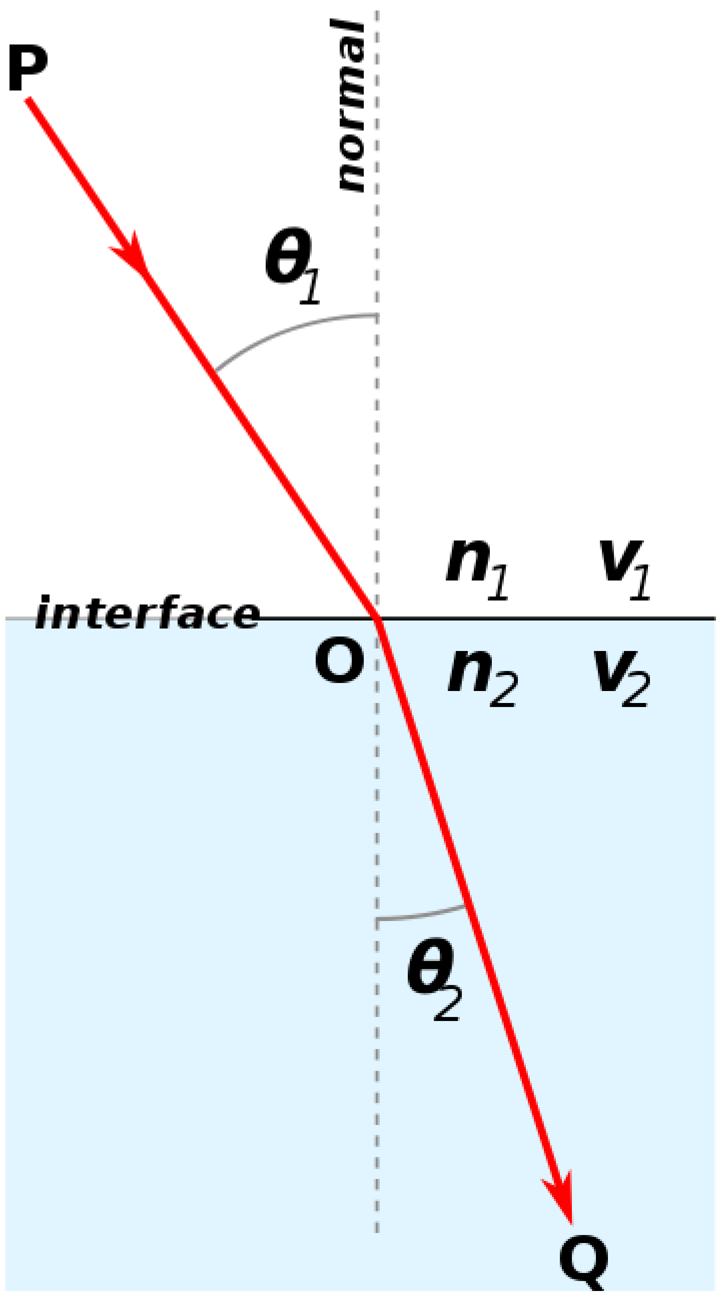
Refraction of light



Snell's law (law of refraction) - a formula used to describe the relationship between the angles of incidence and refraction, when referring to light or other waves passing through a boundary between two different isotropic media, such as water, glass, or air.

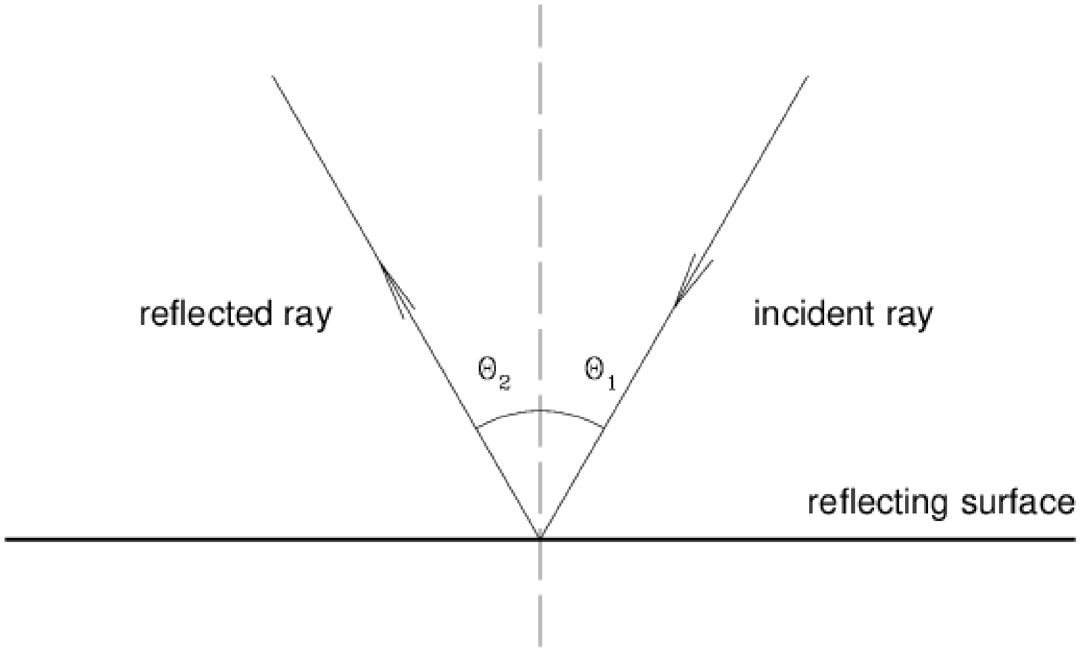
$$\frac{\sin\theta_1}{\sin\theta_2}=\frac{v_1}{v_2}=\frac{\lambda_1}{\lambda_2}=\frac{n_2}{n_1}$$

$$v_1=\frac{c}{n_1}$$

$$v_2=\frac{c}{n_2}$$

n - refractive index of a medium

Reflection of light



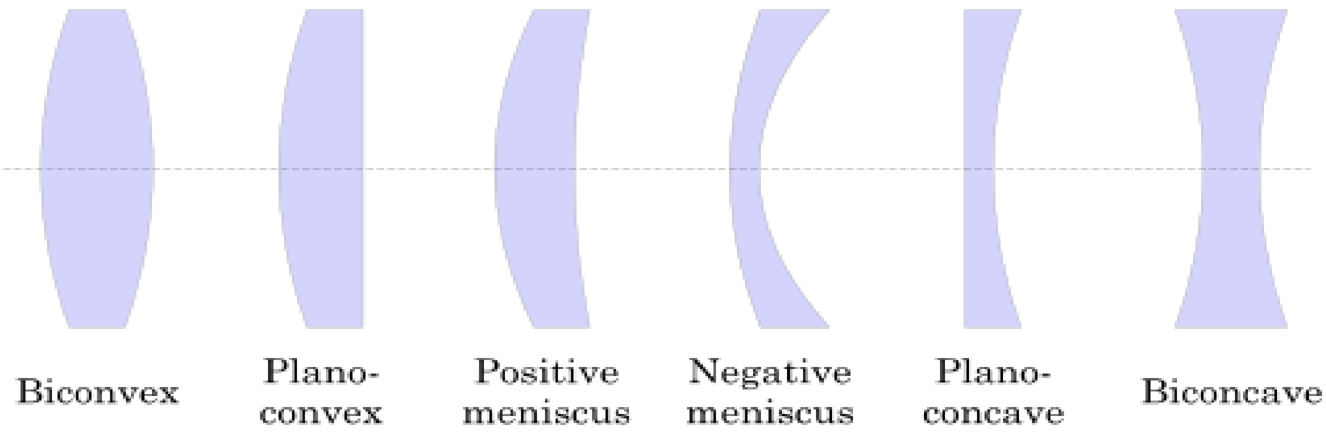
Law of reflection → incidence angle is equal to reflection angle.

$$\theta_1 = \theta_2$$

Lenses

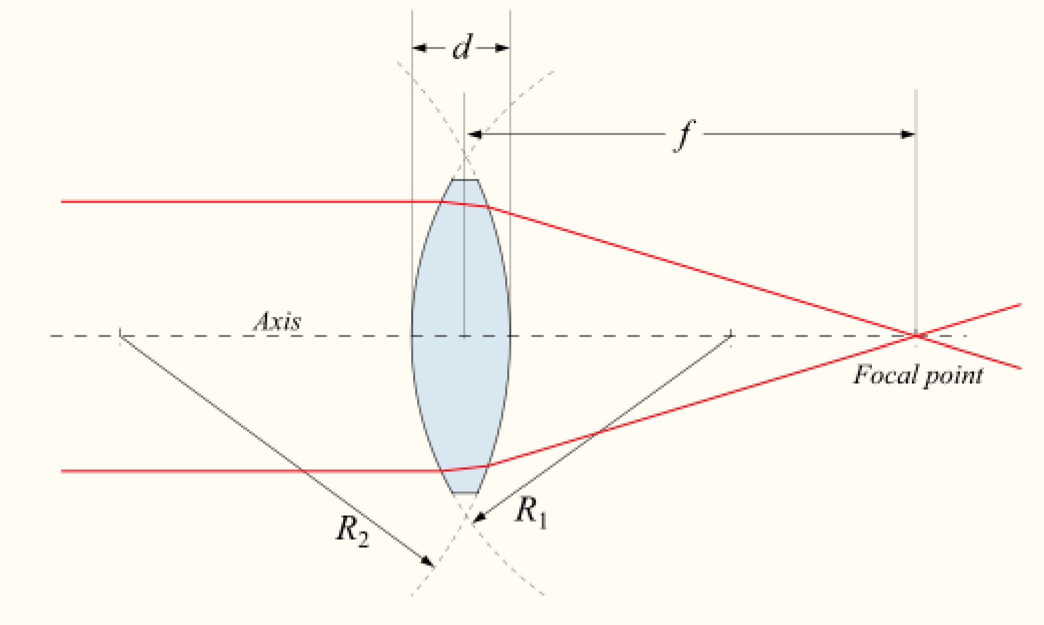
Lens - a transmissive optical device that affects the focus of a light beam through refraction.

Types of lenses:



Lenses

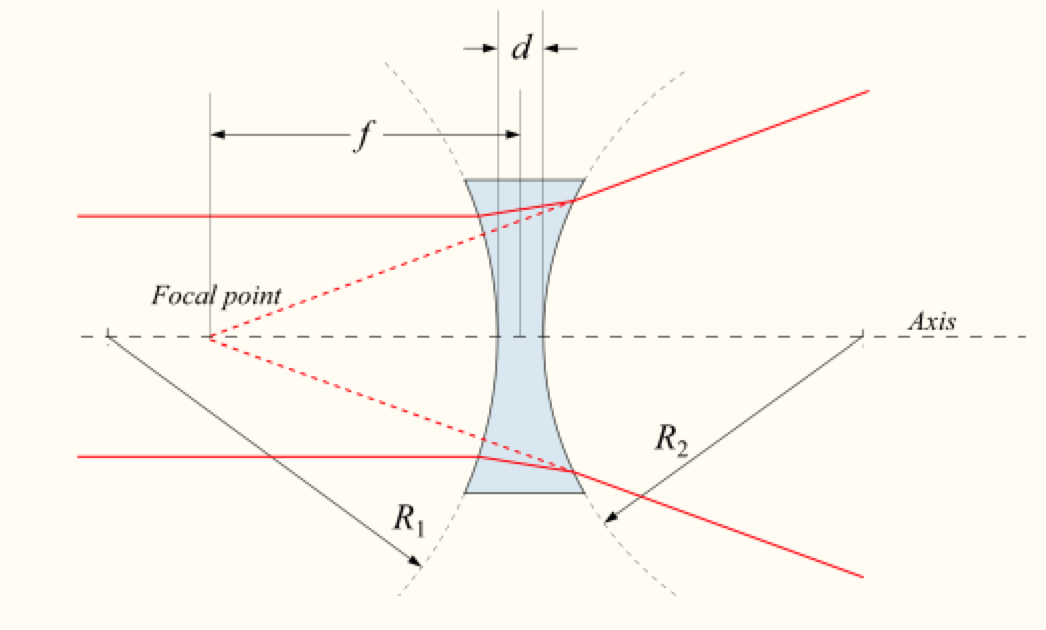
If the lens is biconvex or plano-convex, a collimated beam of light passing through the lens converges to a spot (a focus) behind the lens. In this case, the lens is called a positive or converging lens. The distance from the lens to the spot is the focal length of the lens, which is commonly abbreviated *f* in diagrams and equations.



Positive (converging) lens

Lenses

If the lens is biconcave or plano-concave, a collimated beam of light passing through the lens is diverged (spread); the lens is thus called a negative or diverging lens. The beam, after passing through the lens, appears to emanate from a particular point on the axis in front of the lens. The distance from this point to the lens is also known as the focal length, though it is negative with respect to the focal length of a converging lens.



Negative (diverging) lens

Lensmaker's equation

The focal length of a lens in air can be calculated from the lensmaker's equation:

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right]$$

where
f is the focal length of the lens,
n is the refractive index of the lens material,
R_1 is the radius of curvature (with sign) of the lens surface closest to the light source,
R_2 is the radius of curvature of the lens surface farthest from the light source,
d is the thickness of the lens (the distance along the lens axis between the two surface vertices)

The focal length *f* is positive for converging lenses, and negative for diverging lenses.

For thin lens :

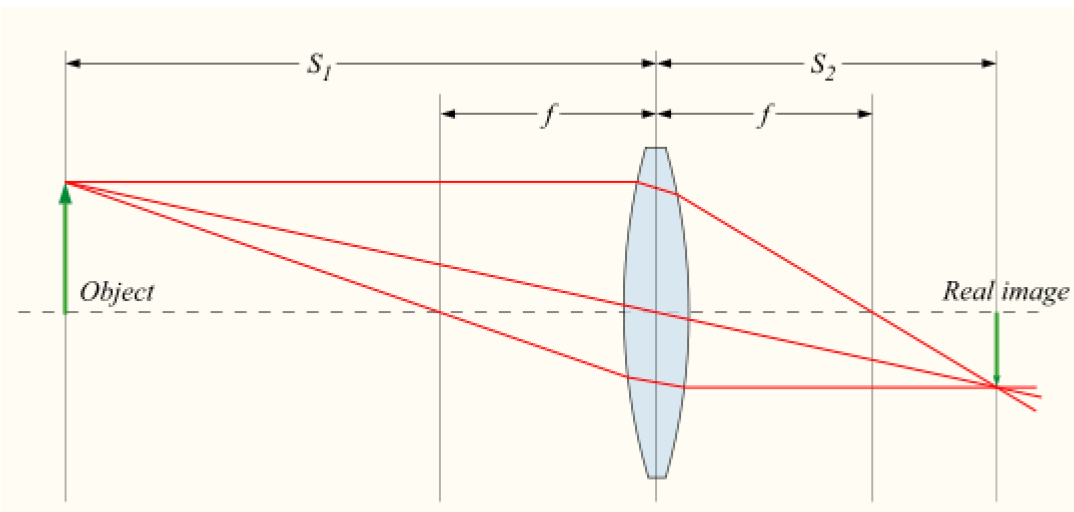
$$\frac{d}{R} \ll 1$$

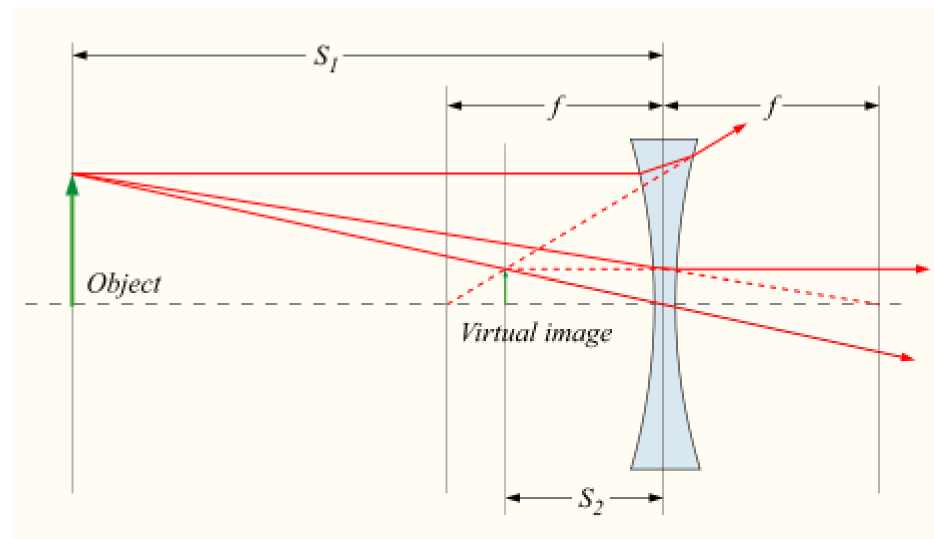
$$\frac{1}{f} \approx (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

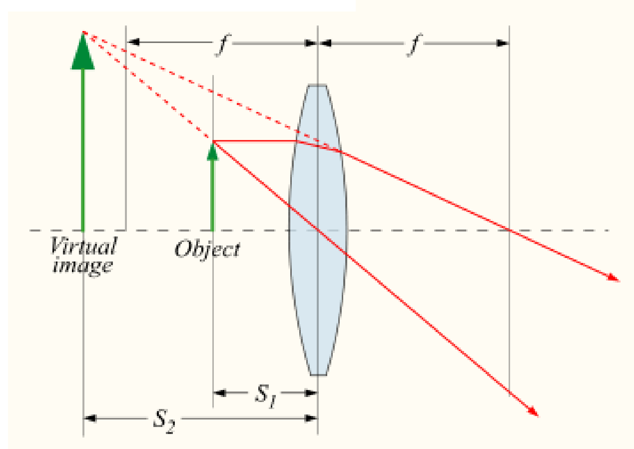
Lensmaker's equation

If the distances from the object to the lens and from the lens to the image are S_1 and S_2 respectively, for a lens of negligible thickness, in air, the distances are related by the **thin lens formula**:

$$\frac{1}{S_1}+\frac{1}{S_2}=\frac{1}{f}$$

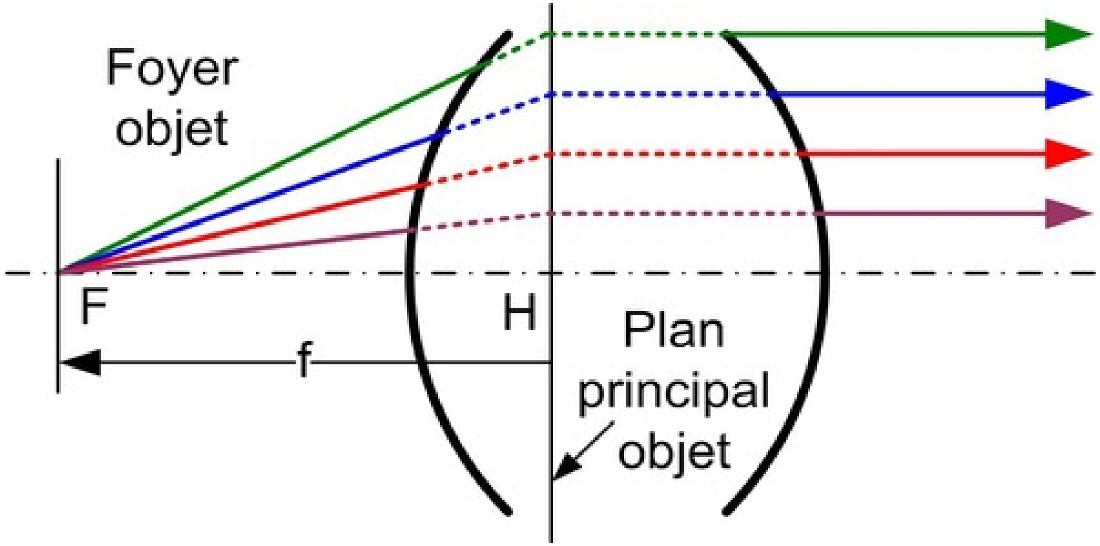


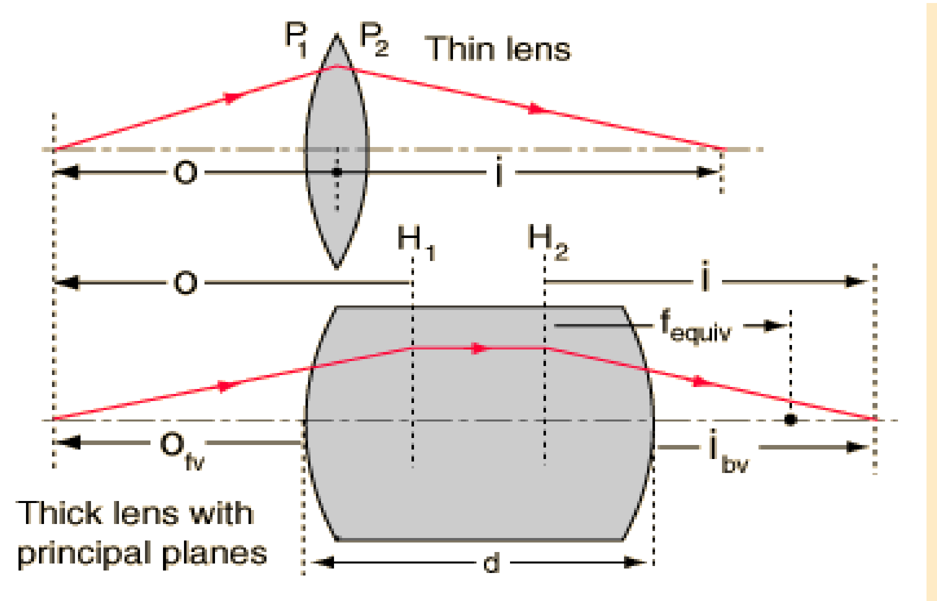




Focal length for system of lenses

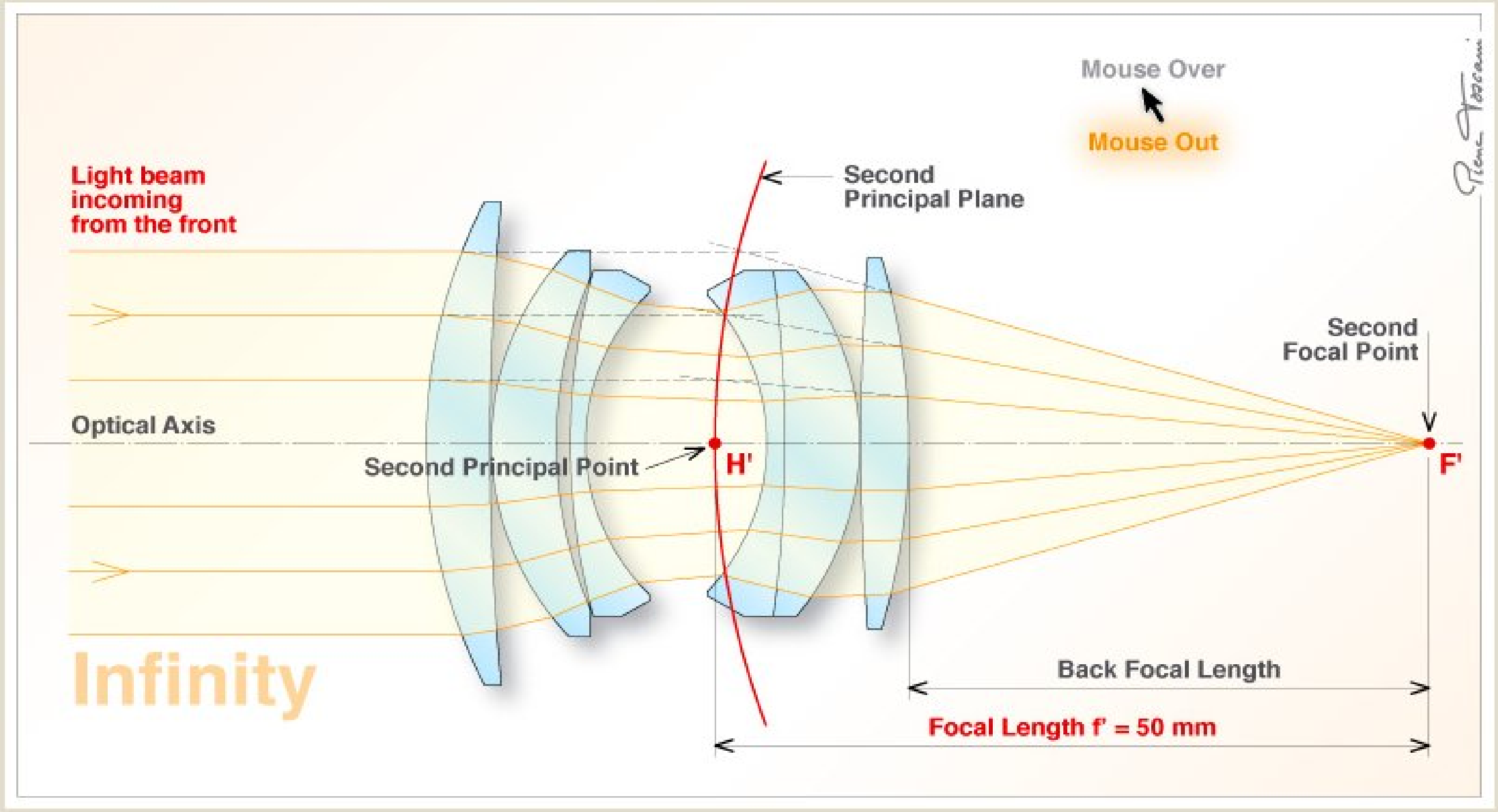
The principal planes are two hypothetical planes in a lens system at which all the refraction can be considered to happen.





Focal length for system of lenses

The focal length for system of lenses is the distance of focal point from main focal plane for infinite rays.

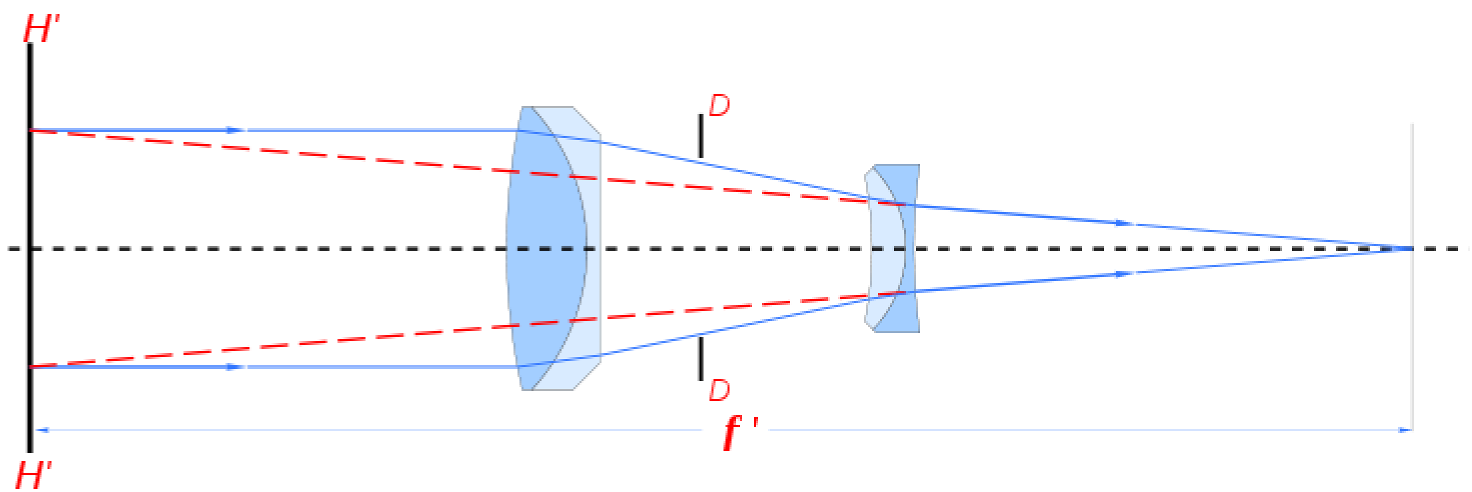


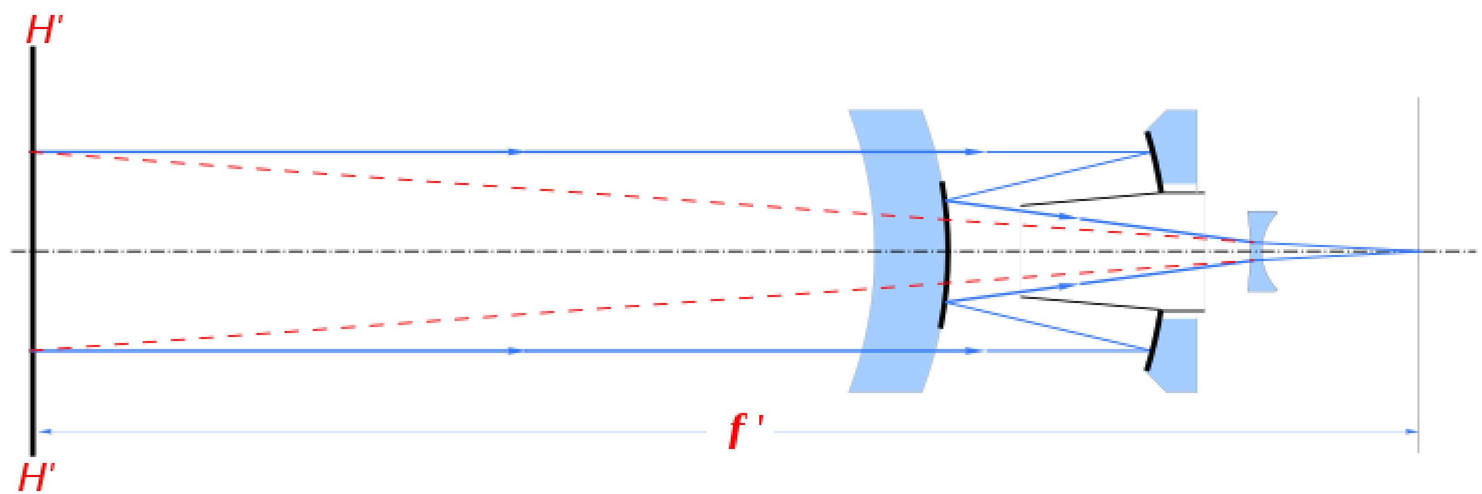
Focal length for system of lenses

The focal length for system of lenses is the distance of focal point from main focal plane for rays falling from infinity.

The main plane can be far away from the lens system → teleobjective.

Barlow (negative) lenses can increase the focal lenght by factor of 2-5.

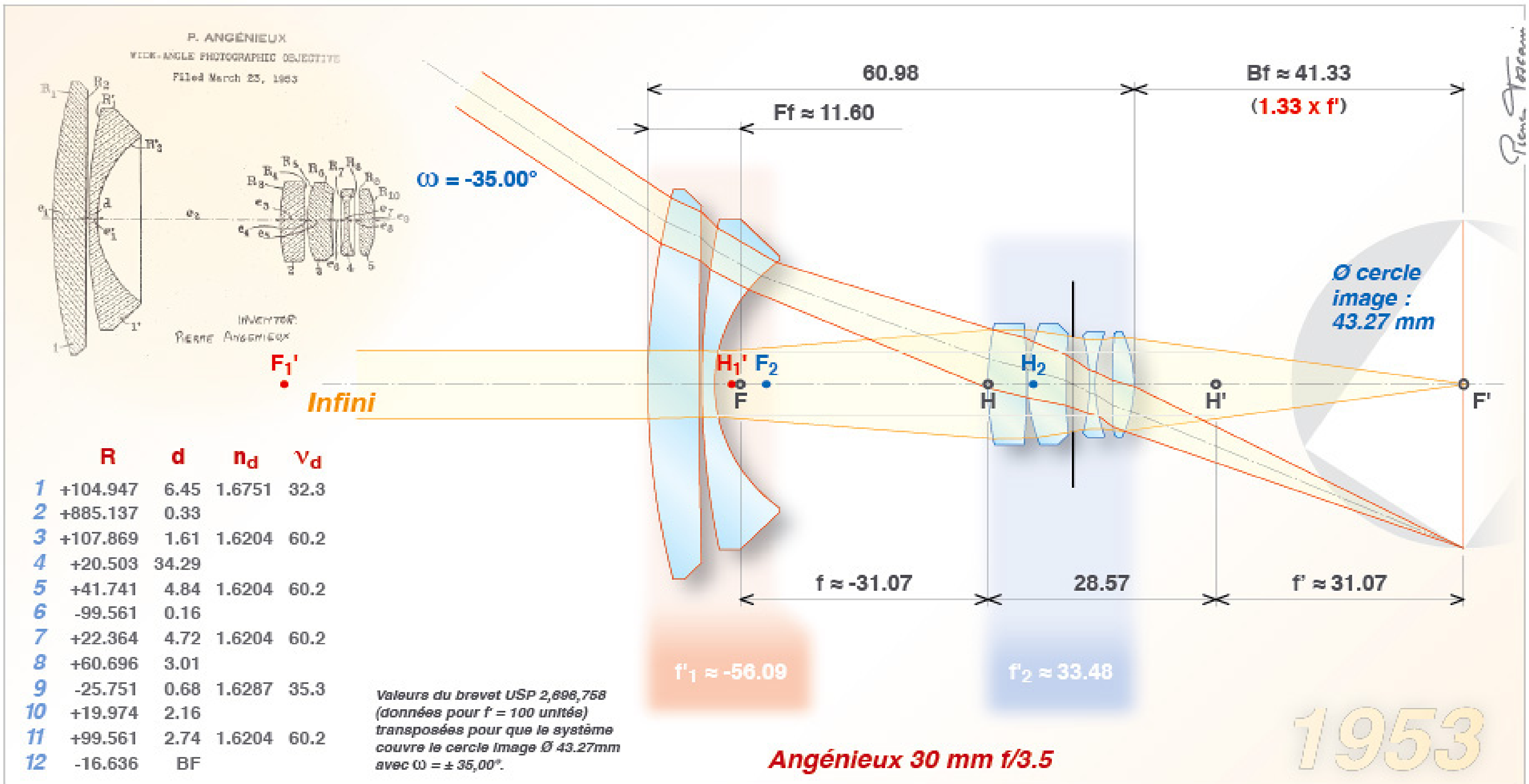




Focal length for system of lenses

The focal length for system of lenses is the distance of focal point from main focal plane for rays falling from infinity.

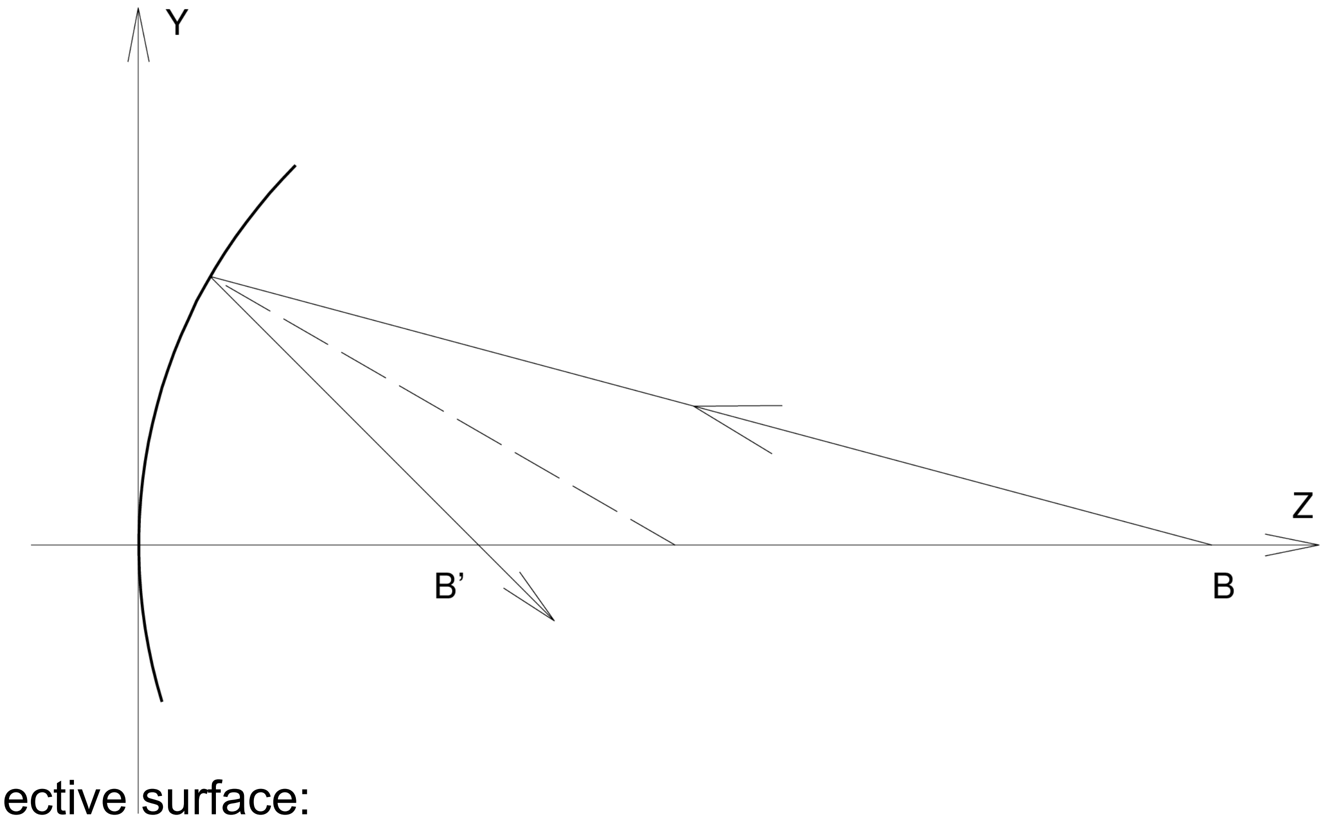
Opposite situation. In wide field lenses the main plane is between the lens and a detector.



	R	d	n_d	ν_d
1	+104.947	6.45	1.6751	32.3
2	+885.137	0.33		
3	+107.869	1.61	1.6204	60.2
4	+20.503	34.29		
5	+41.741	4.84	1.6204	60.2
6	-99.561	0.16		
7	+22.364	4.72	1.6204	60.2
8	+60.696	3.01		
9	-25.751	0.68	1.6287	35.3
10	+19.974	2.16		
11	+99.561	2.74	1.6204	60.2
12	-16.636	BF		

Concave mirror

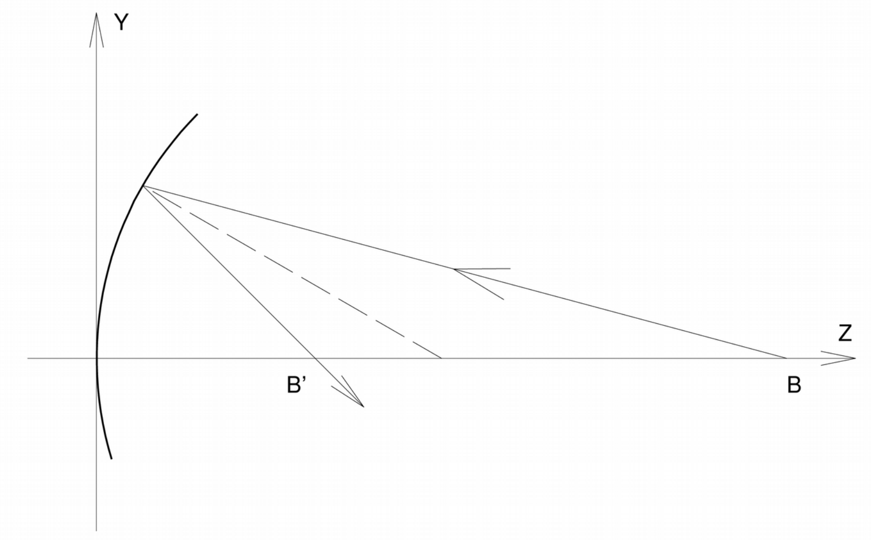
General view



Equation for reflective surface:

$$y^2 - 2Rz + (1 - e^2)z^2 = 0$$
$$K = -e^2$$

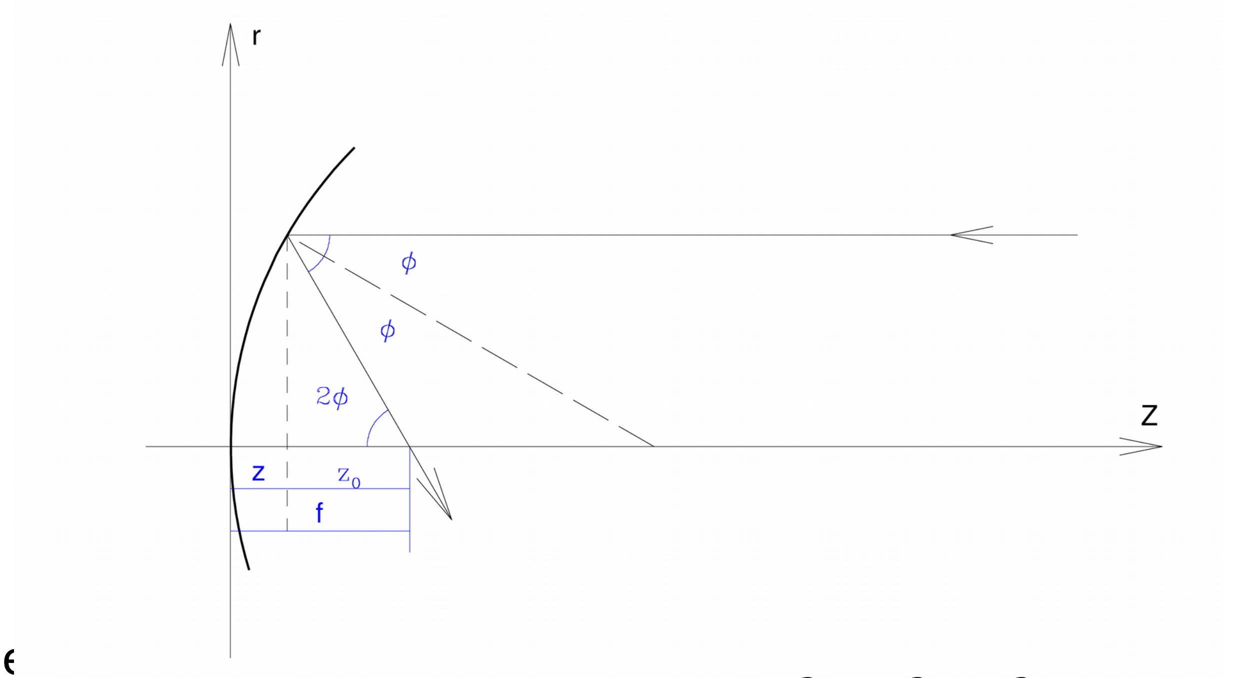
Concave mirror



$$y^2 - 2Rz + (1 - e^2) z^2 = 0 \qquad K = -e^2$$

oblate ellipsoid	e < 0	K > 0
sphere	e = 0	K = 0
prolate ellipsoid	0 < e < 1	-1 < K < 0
paraboloid	e = 1	K = -1
hyperboloid	e > 1	K < -1

Concave mirror for infinite rays



For axisymme

Equation for mirror surface is:

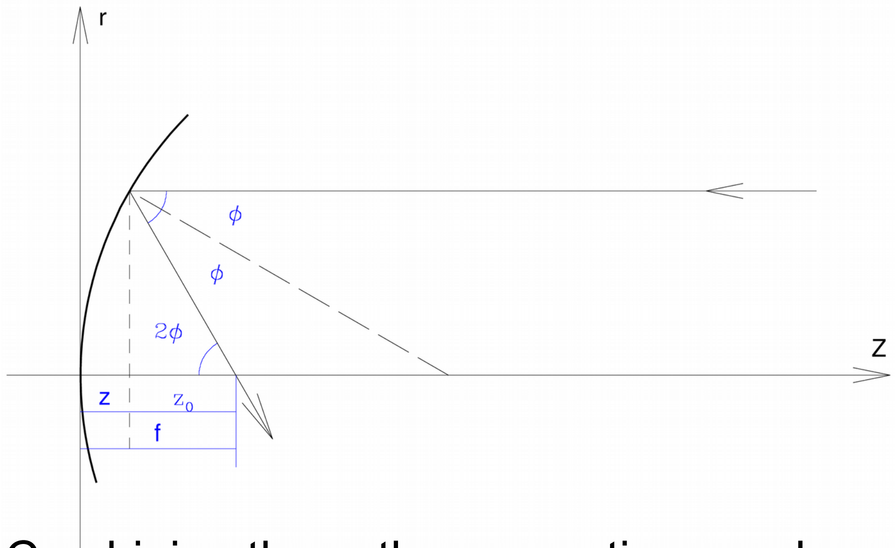
$$r^2 = x^2 + y^2$$
$$r^2 - 2Rz + (1+K)z^2 = 0$$

But: $z_0 = \dfrac{r}{\tan 2\phi} = \dfrac{r(1-\tan^2\phi)}{2\tan\phi}$ and simply: $\tan\phi = \dfrac{dz}{dr}$

Computing the derivative of mirror equation:

$$\frac{dz}{dr} = \frac{r}{R-(1+K)z} = \tan\phi$$

Concave mirror for infinite rays



$$z_0 = \frac{r}{\tan 2\phi} = \frac{r\left(1-\tan^2\phi\right)}{2\tan\phi}$$

$$\frac{dz}{dr} = \frac{r}{R-\left(1+K\right)z} = \tan\phi$$

$$f = z + z_0$$

Combining these three equations we have general equation for focal length:

$$f = \frac{R}{2} + \frac{\left(1-K\right)z}{2} - \frac{r^2}{2\left[R-\left(1+K\right)z\right]}$$

But we can solve the mirror equation for *z* and assume *r/R* as small.

$$z = \frac{R}{1+K}\left[1-\sqrt{1-\frac{r^2}{R^2}\left(1+K\right)}\right] = \frac{r^2}{2R} + \left(1+K\right)\frac{r^4}{8R^3} + \left(1+K\right)^2\frac{r^6}{16R^5} + \ldots$$

Concave mirror for infinite rays

Combination of two equations:

$$f=\frac{R}{2}+\frac{(1-K)z}{2}-\frac{r^2}{2[R-(1+K)z]}$$

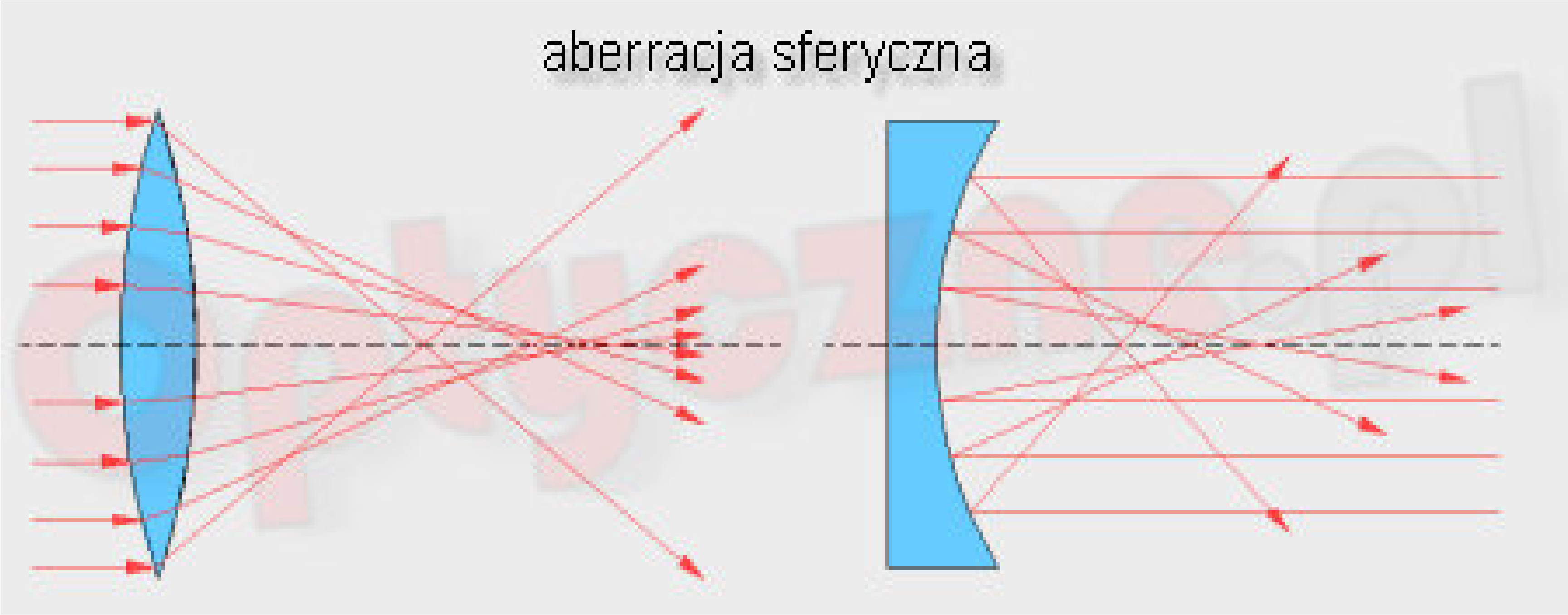
$$z=\frac{R}{1+K}\left[1-\sqrt{1-\frac{r^2}{R^2}(1+K)}\right]=\frac{r^2}{2R}+(1+K)\frac{r^4}{8R^3}+(1+K)^2\frac{r^6}{16R^5}+\dots$$

Gives us focal length of the mirror:

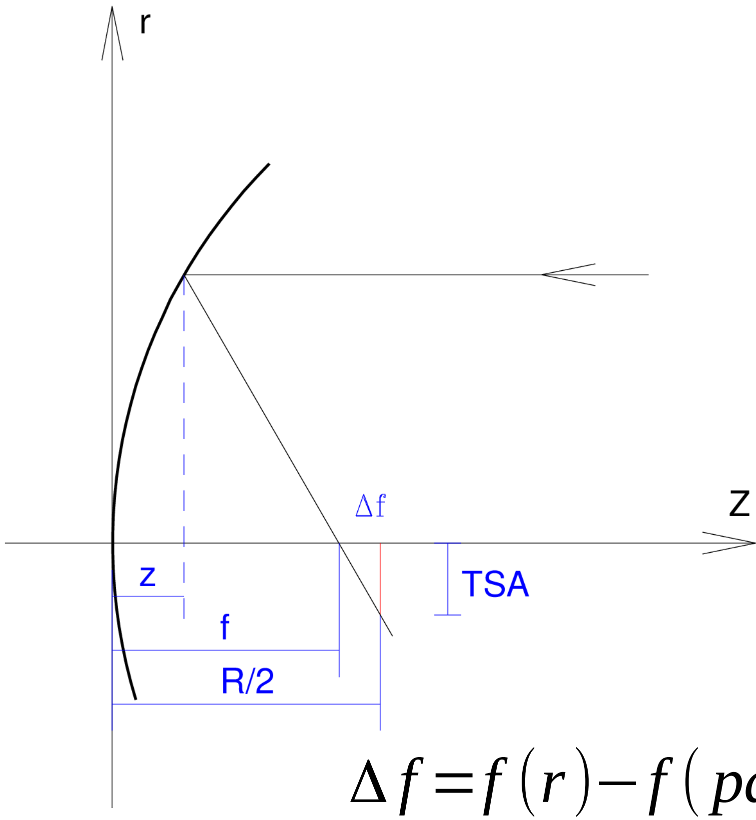
$$f=\frac{R}{2}-\frac{(1+K)r^2}{4R}-\frac{(1+K)(3+K)r^4}{16R^3}-\dots$$

Spherical aberration

Spherical aberration is an optical effect observed in an optical device (lens, mirror, etc.) that occurs due to the increased refraction of light rays when they strike a lens or a reflection of light rays when they strike a mirror near its edge, in comparison with those that strike nearer the centre.



Spherical aberration



Longitudal SA (LSA or Δf) and tranverse SA (TSA)

$$f=\frac{R}{2}-\frac{(1+K)r^2}{4R}-\frac{(1+K)(3+K)r^4}{16R^3}-\dots$$

$$\Delta f=f(r)-f(paraxial)=\frac{-(1+K)r^2}{4R}-\frac{(1+K)(3+K)r^4}{16R^3}-\dots$$

$$\frac{TSA}{LSA}=\frac{r}{f-z}$$

$$TSA=-(1+K)\frac{r^3}{2R^2}-3(1+K)(3+K)\frac{r^5}{9R^4}+\dots$$

Spherical aberration

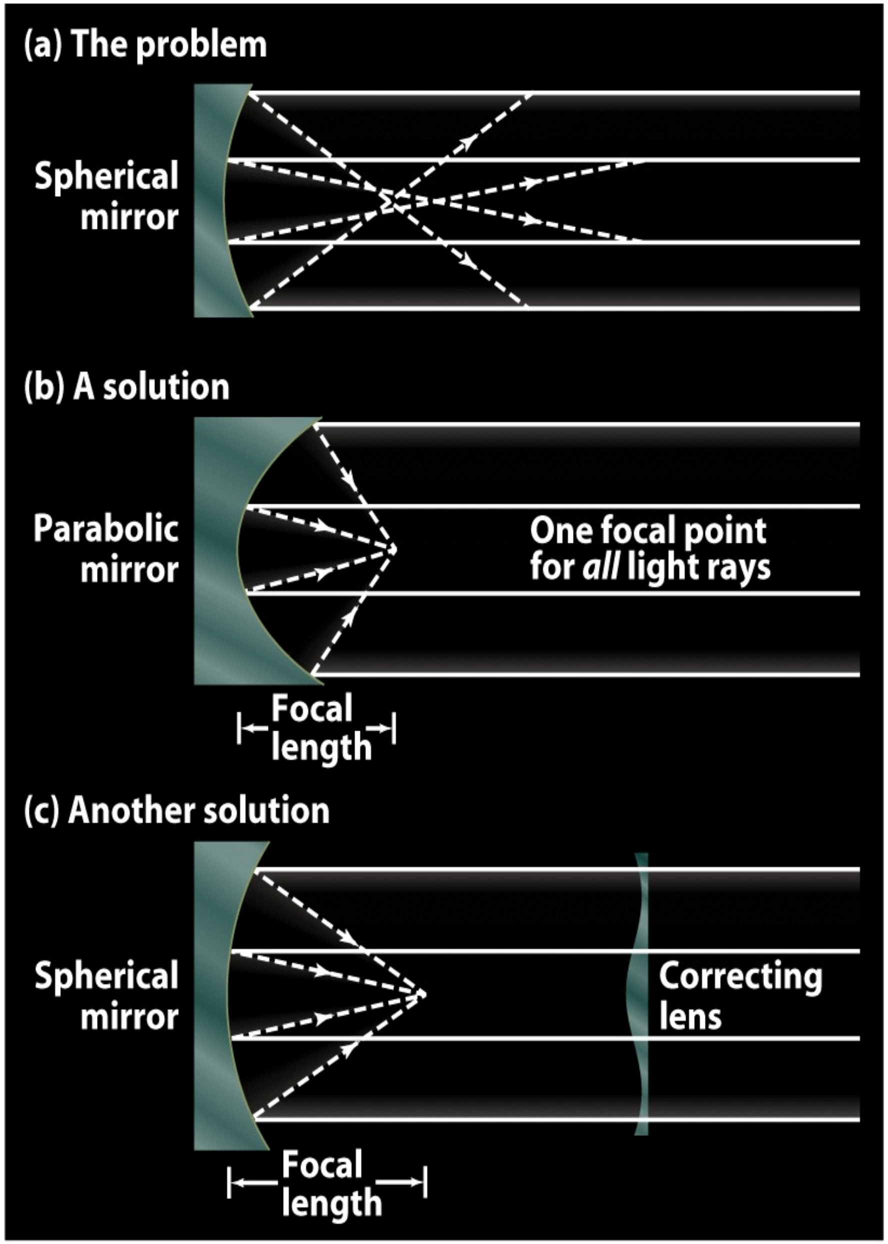
Fighting with spherical aberration.

$$\Delta f = f(r) - f(paraxial) = \frac{-(1+K) r^2}{4R} - \frac{(1+K)(3+K) r^4}{16 R^3} - \ldots$$

$$TSA = -(1+K)\frac{r^3}{2R^2} - 3(1+K)(3+K)\frac{r^5}{9R^4} + \ldots$$

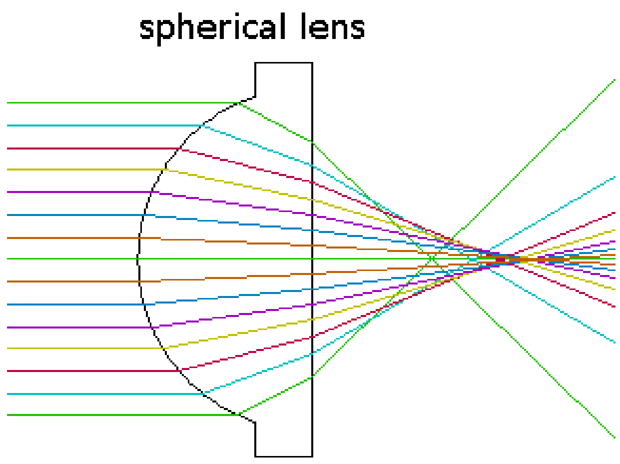
Solution 1 → use parabolic mirrors (*K=-1*)

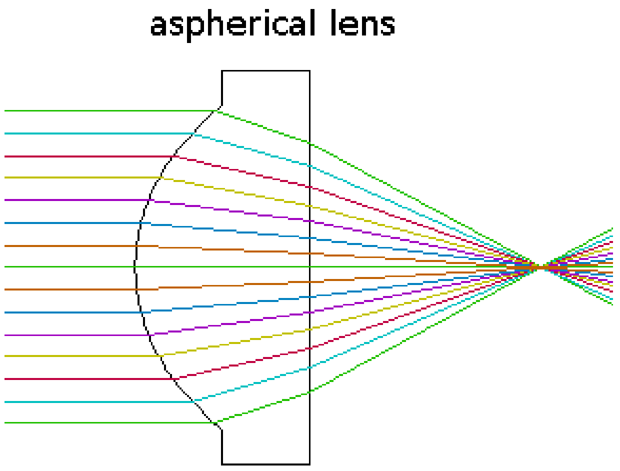
Solution 2 → use correcting lens



Spherical aberration

Fighting with spherical aberration.
For lenses → use aspherical lenses.





Chromatic aberration

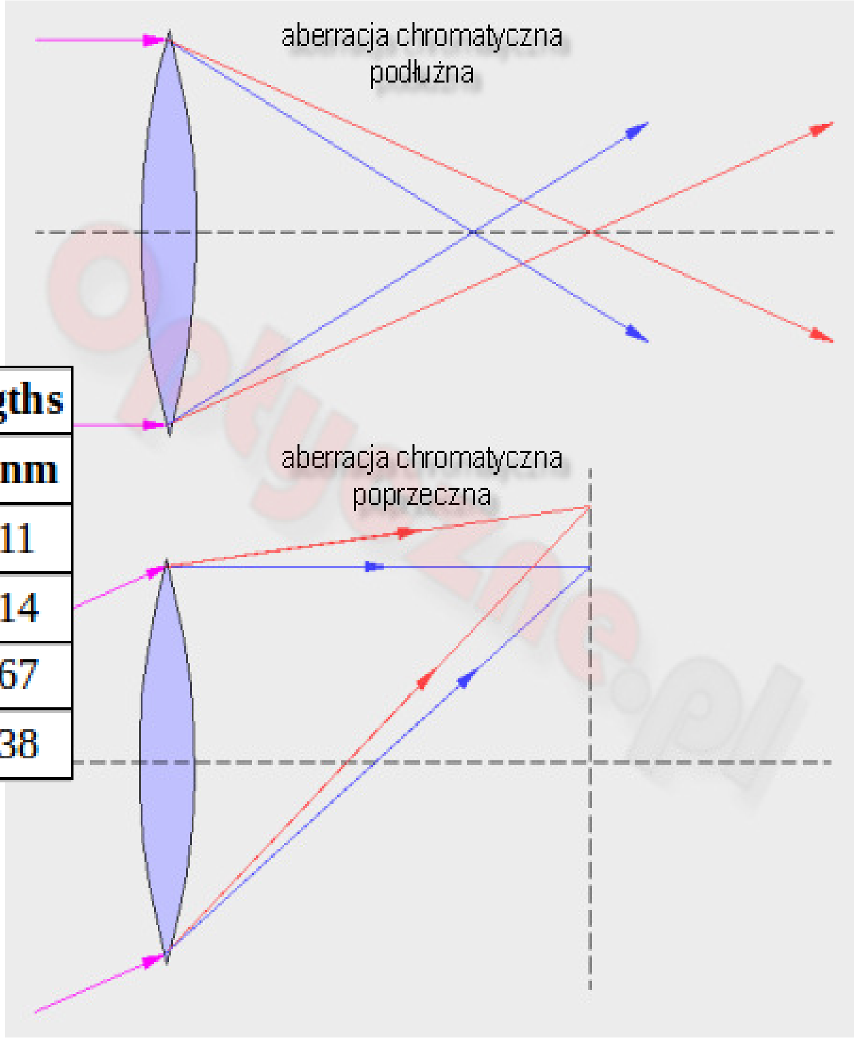
Chromatic aberration - is an effect resulting from dispersion in which there is a failure of a lens to focus all colors to the same convergence point. It occurs because lenses have different refractive indices for different wavelengths of light.

Two types:

→ Longitudal CA

→ Latteral CA

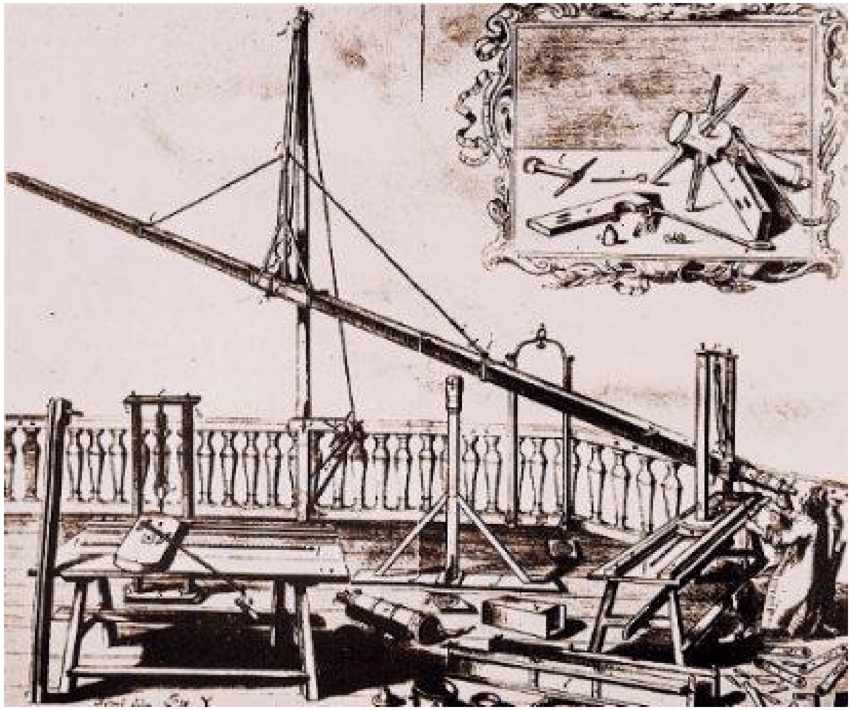
	Refractive index at the specified wavelengths				
Glass type	**361 nm**	**486 nm**	**589 nm**	**656 nm**	**768 nm**
Crown	1.539	1.523	1.517	1.514	1.511
High dispersion crown	1.546	1.527	1.520	1.517	1.514
Light flint	1.614	1.585	1.575	1.571	1.567
Dense flint	1.705	1.664	1.650	1.644	1.638

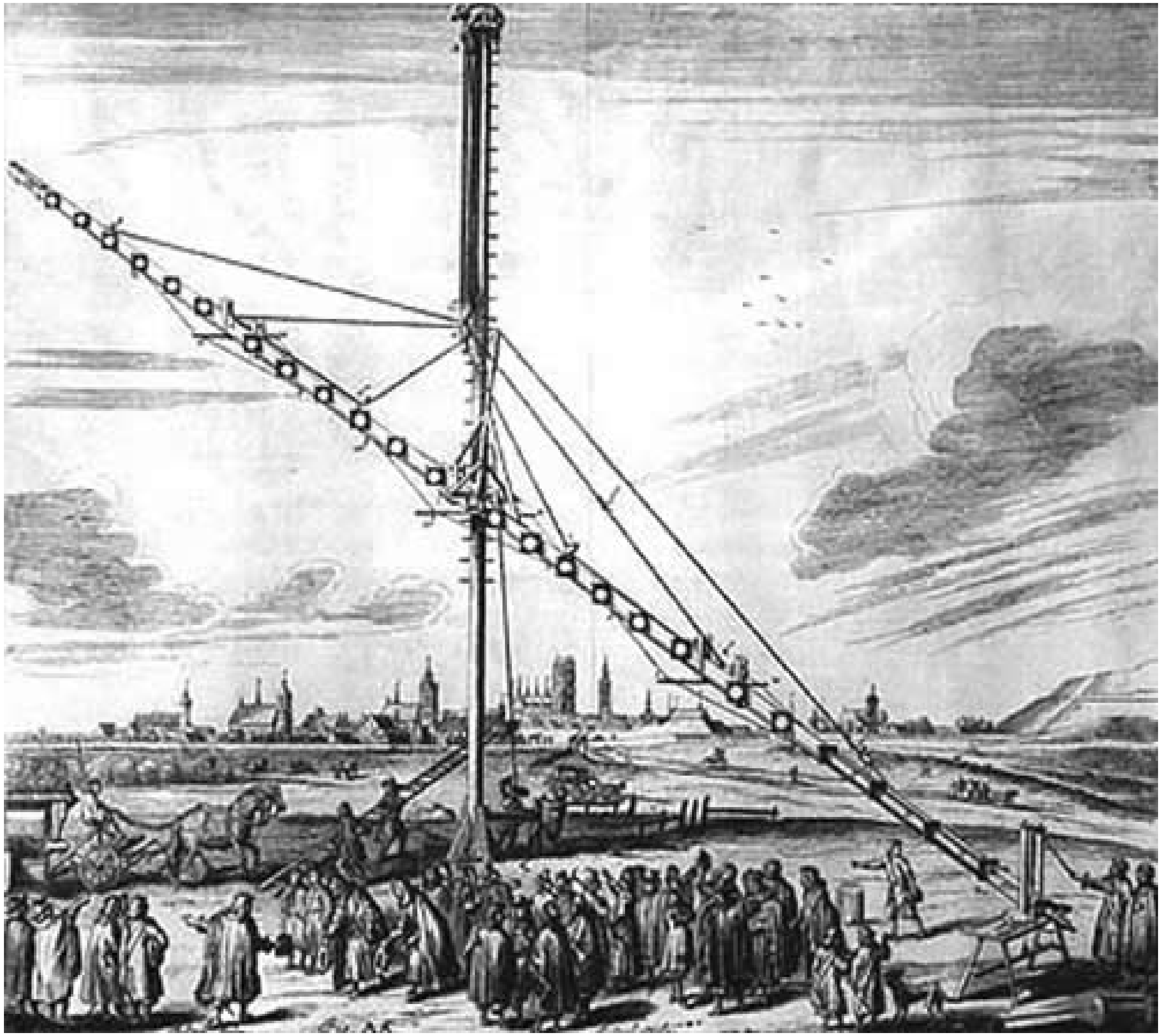


Chromatic aberration

Chromatic aberration is small for slow (high F-ratio) lenses. *F=f/D*

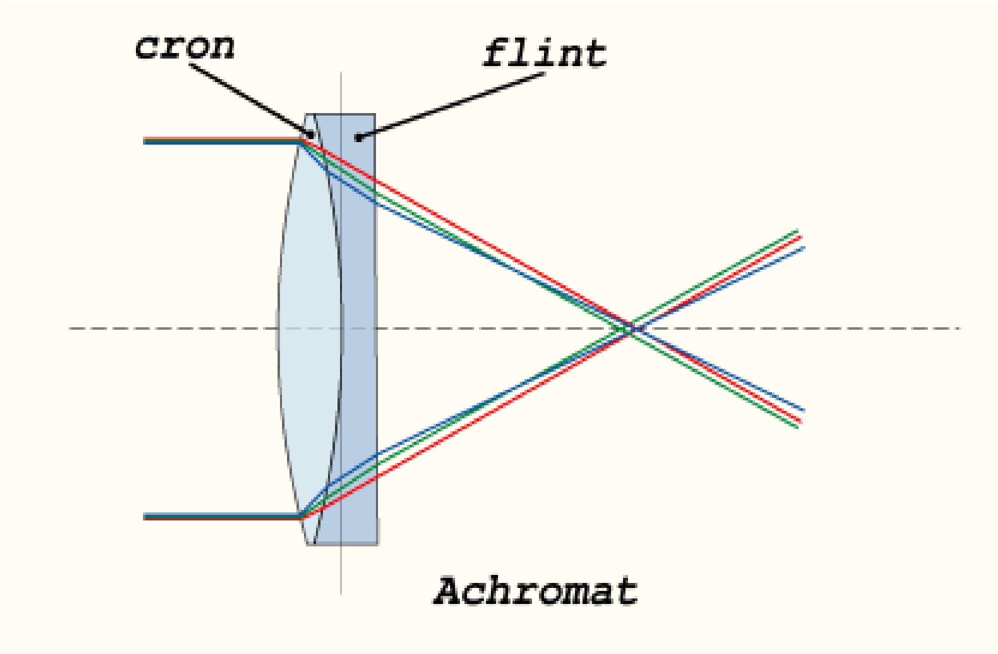
Hevelius telescope with length of 45 meters!

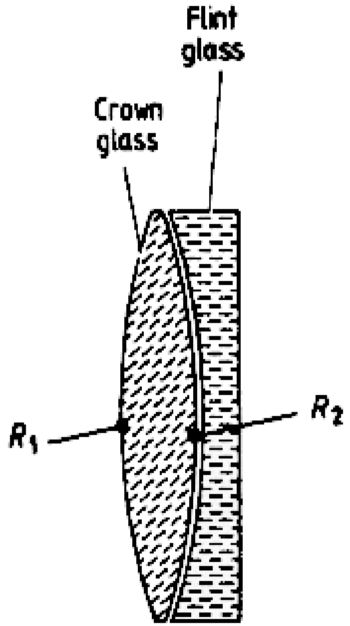




Chromatic aberration

Better solution → achromatic lens consisting of two lenses with different refractive index.





Relation between curvature radii and differences in refractive indices.

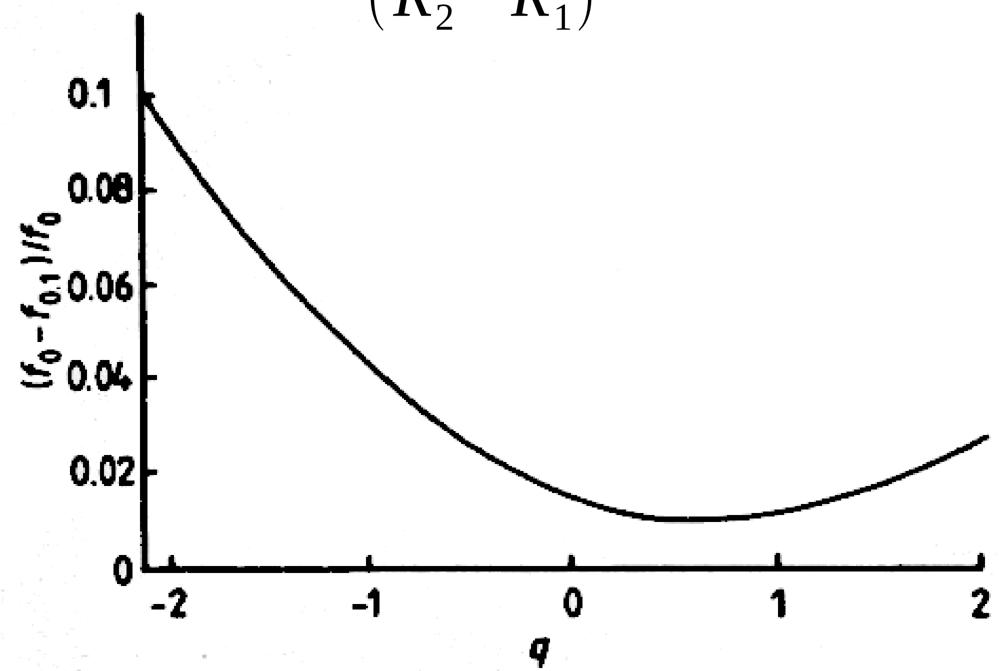
$$\frac{R_1 + R_2}{R_1} \Delta n_C = \Delta n_F$$

Chromatic aberration

Constructing achromat objective lens one can also minimalize spherical aberration.

Let's introduce q parameter:

$$q=\frac{(R_2+R_1)}{(R_2-R_1)}$$



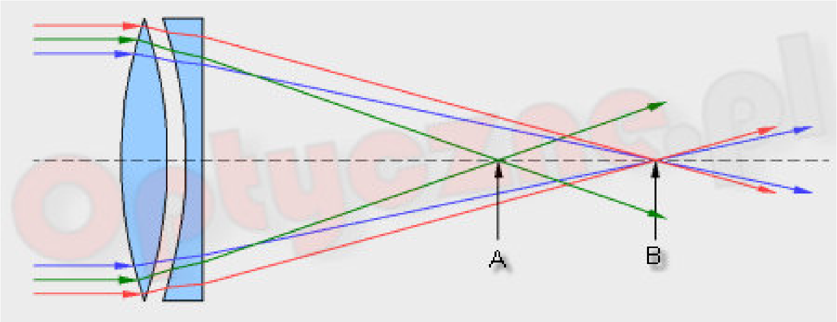
For *q=0.6* spherical aberration has its minimum value.

We have two equations with two unknowns → R_1 and R_2.

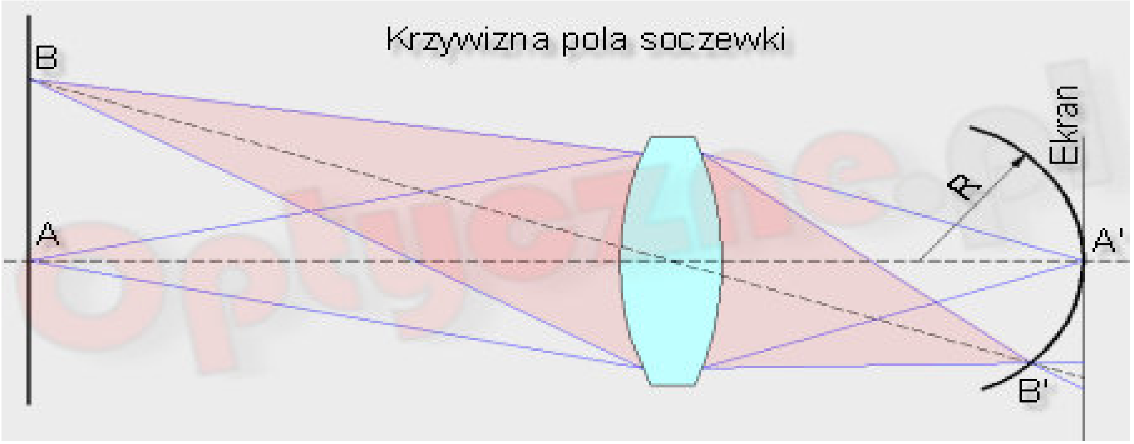
Chromatic aberration

Disadvantages of achromats:

1. secondary spectrum.

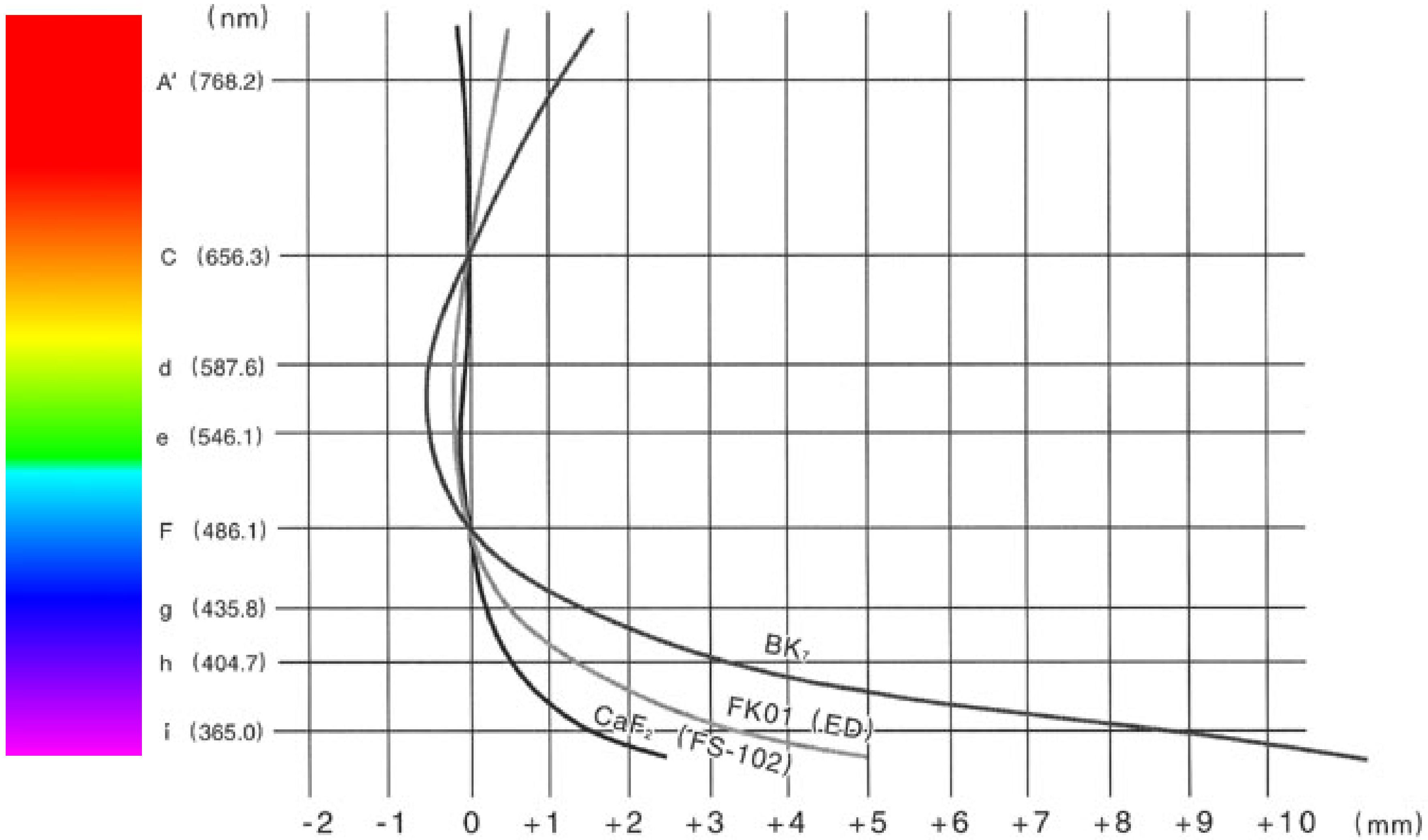


2. significant field curvature (important in times of large photographic plates)



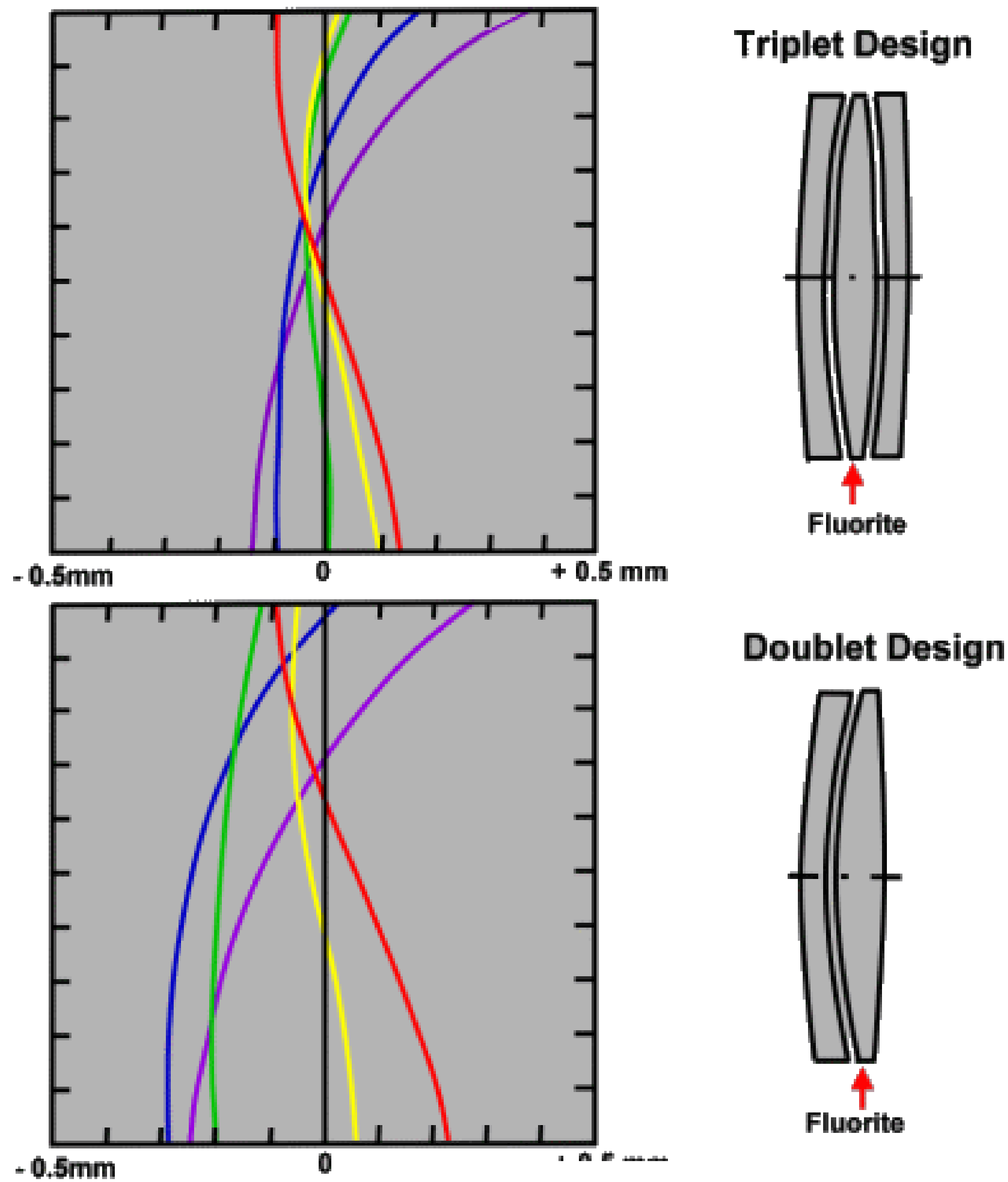
Chromatic aberration

1960-1970s introduction of low dispersion glass (e.g. fluorite)



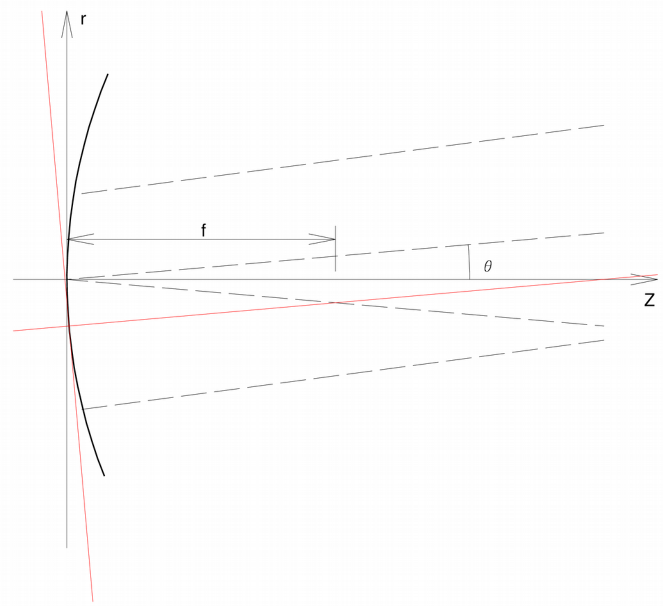
Chromatic aberration

1960-1970s introduction of low dispersion glass (e.g. fluorite) → apochromats



Off-axis aberrations

4,Off-axis aberrations – optical aberrations caused by rays inclined to optical axis.



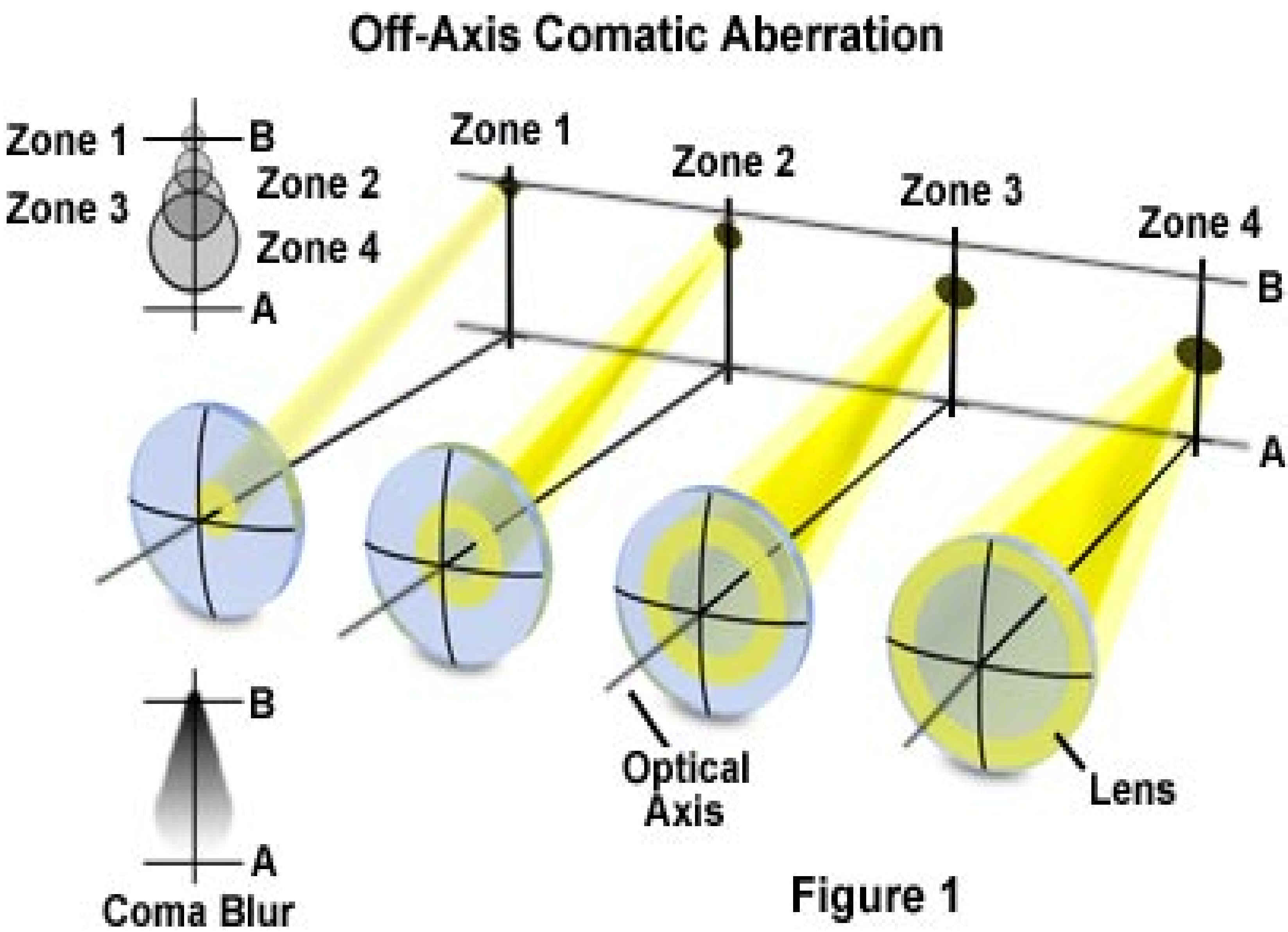
$$AA3 = 3a_1\frac{y^2\theta}{R^2} + 2a_2\frac{y\theta^2}{R} + a_3\theta^3$$

Coma Astigmatism Distorsion

AA3 → 3rd order off-axis aberrations

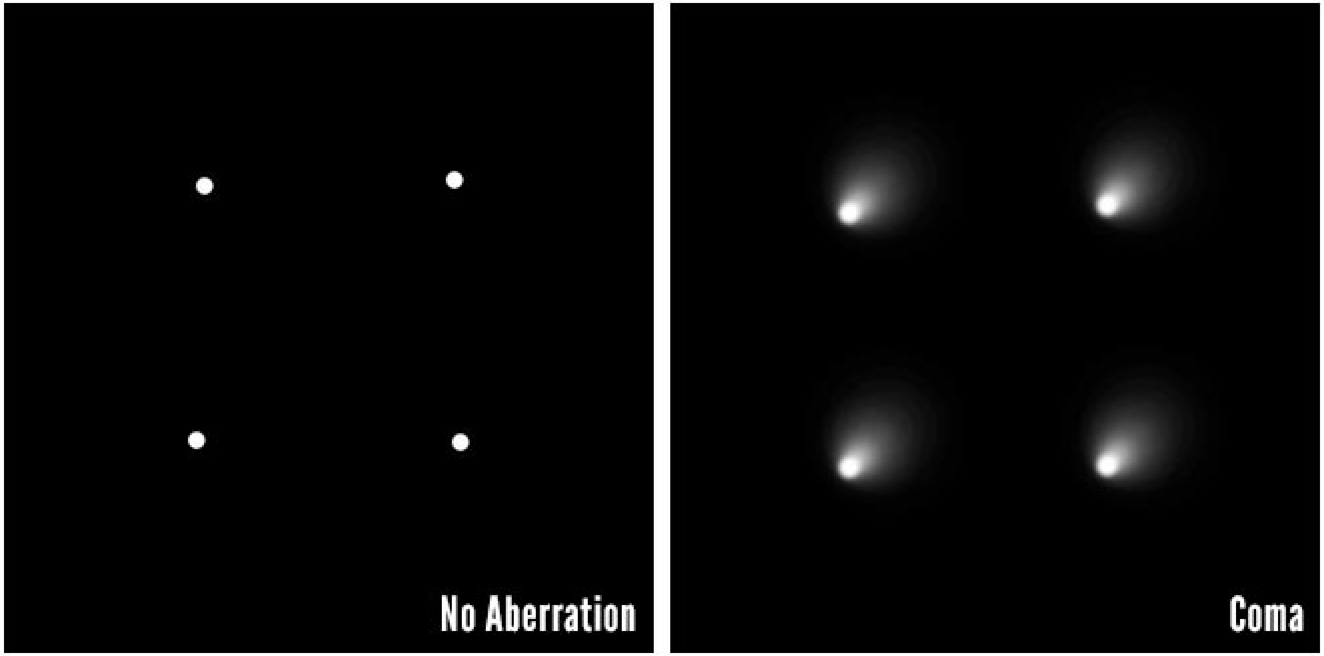
Off-axis aberrations

Coma



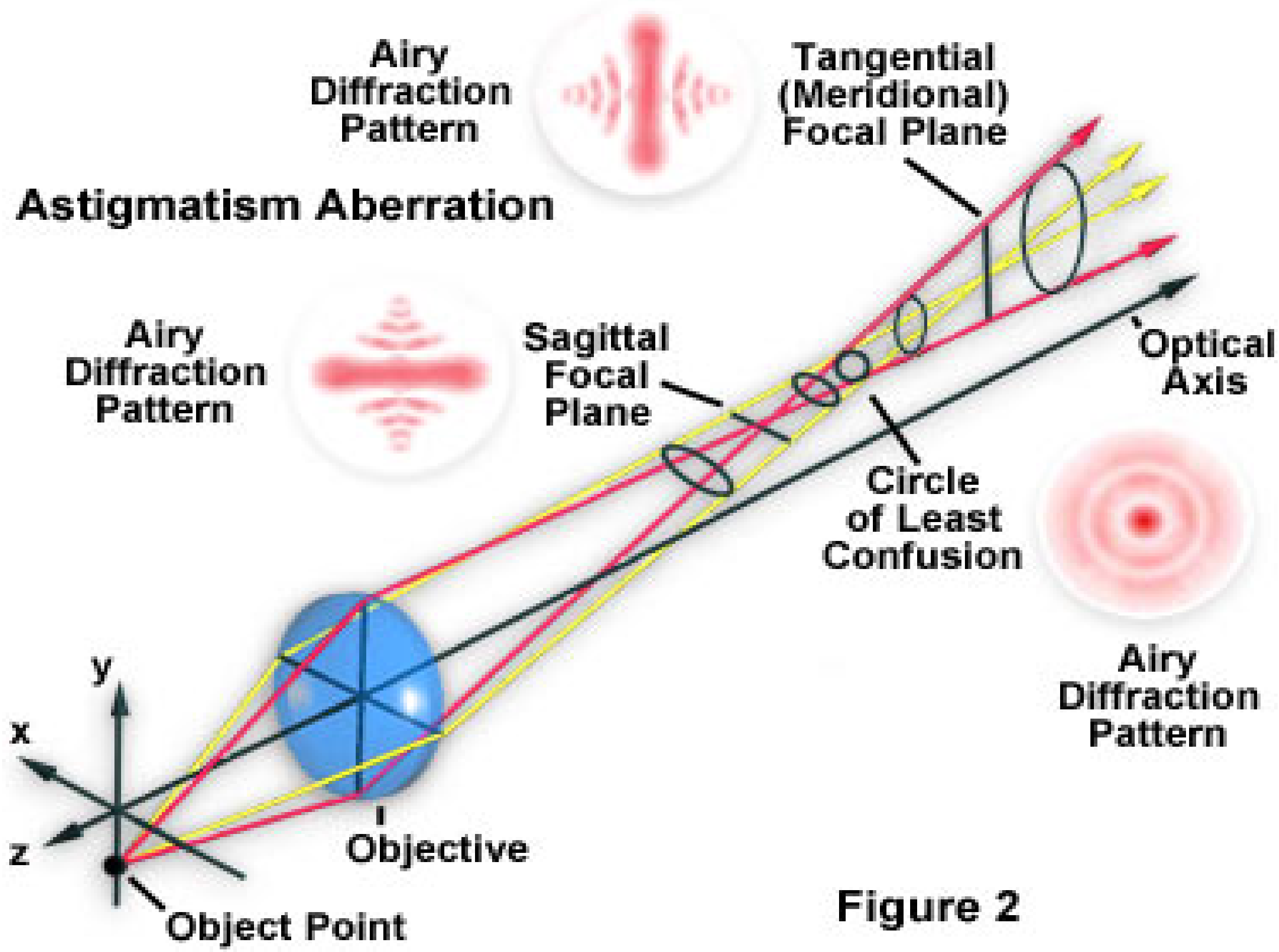
Off-axis aberrations

Coma



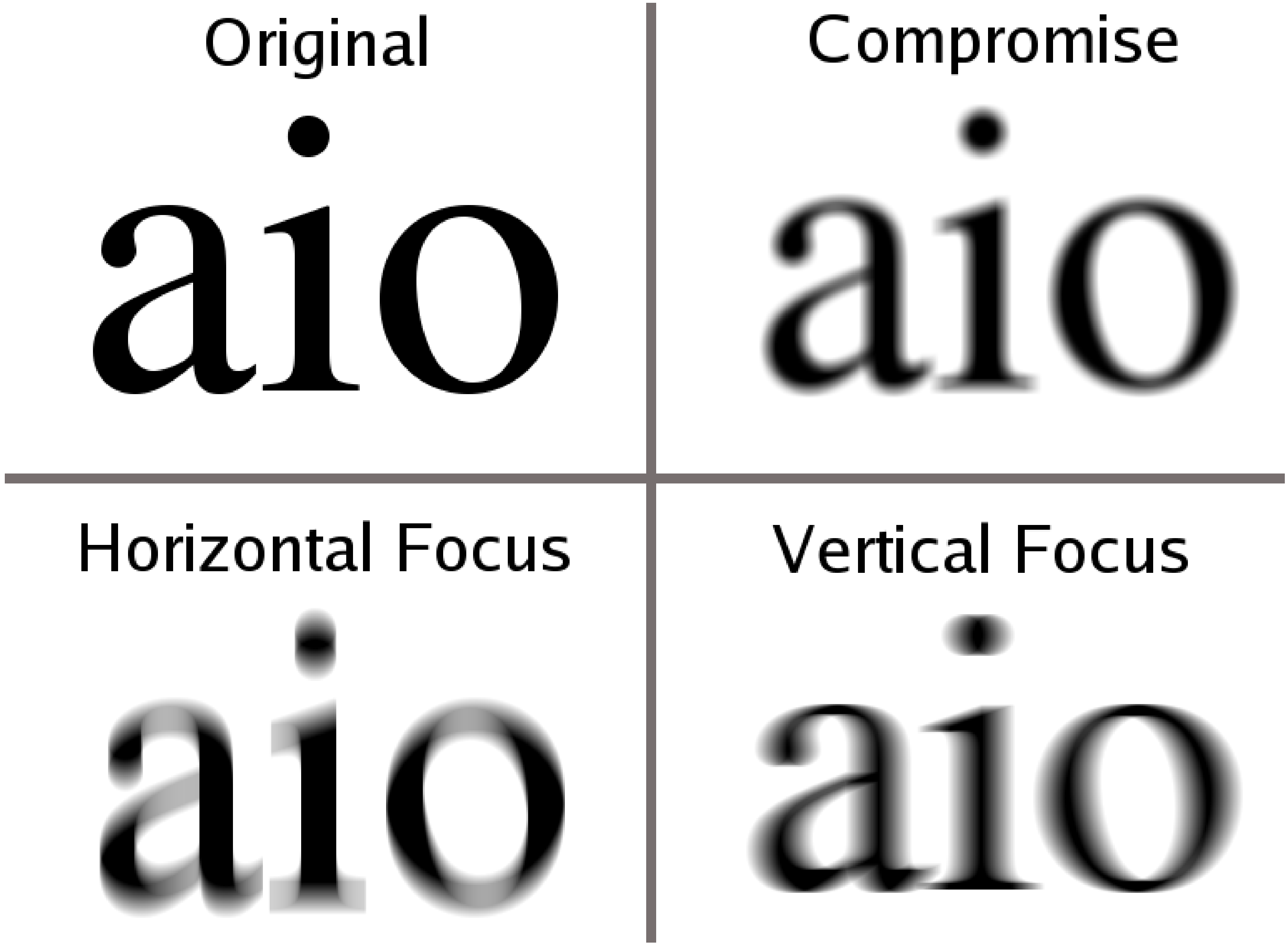
Off-axis aberrations

Astigmatism



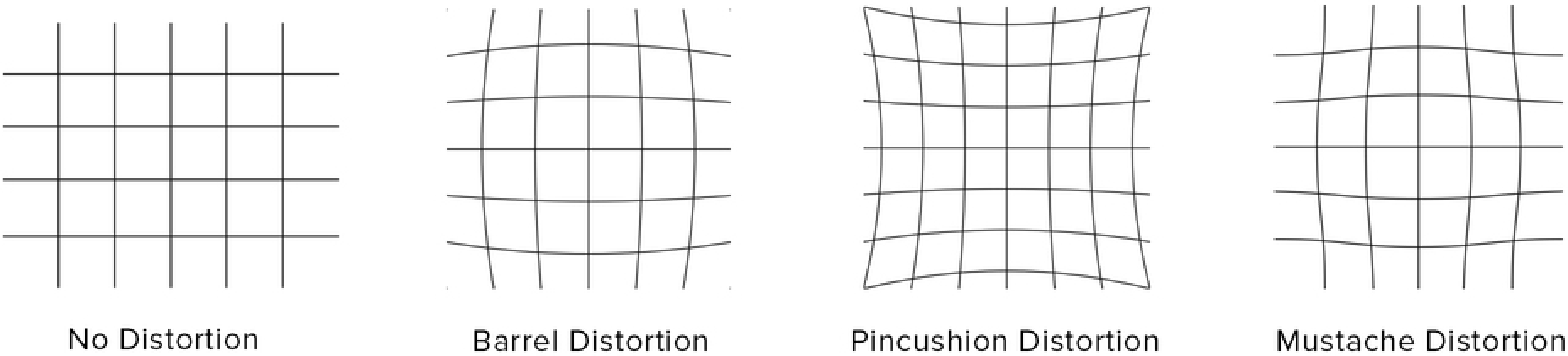
Off-axis aberrations

Astigmatism



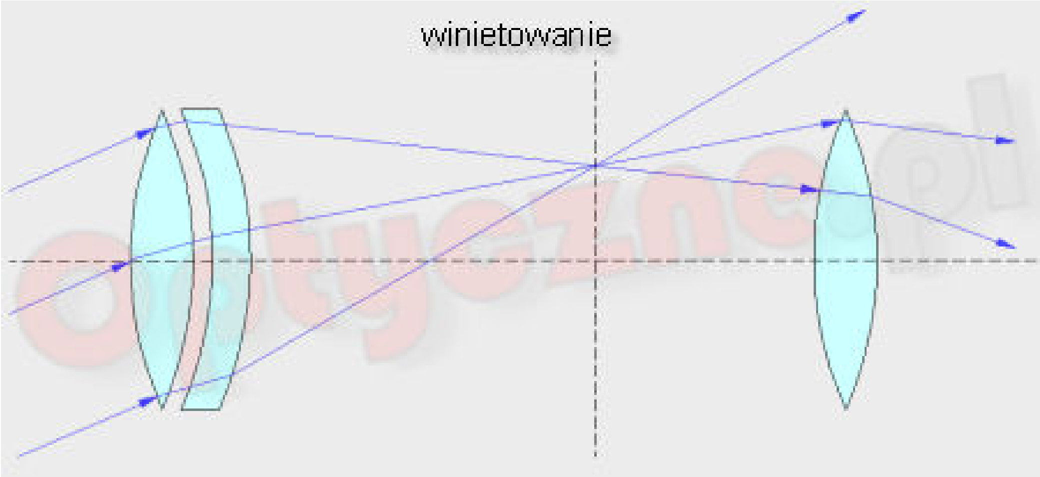
Off-axis aberrations

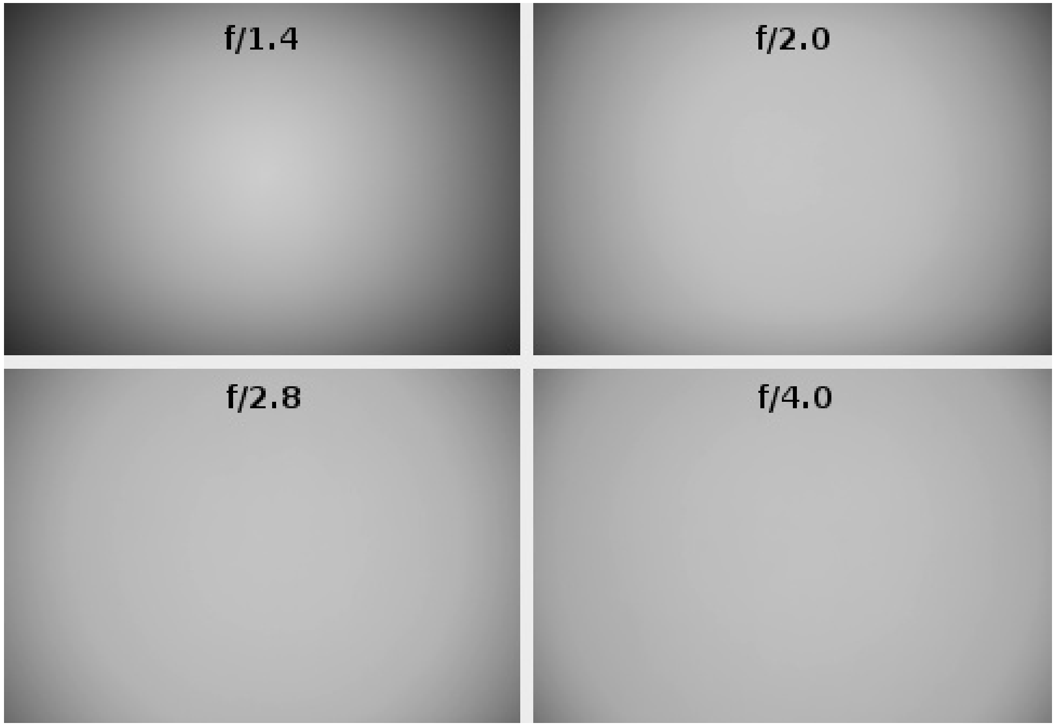
Distortion - deviation from rectilinear projection, a projection in which straight lines in a scene remain straight in an image. It is caused by different magnification of the image with increasing distance from the optical path.



Vignetting

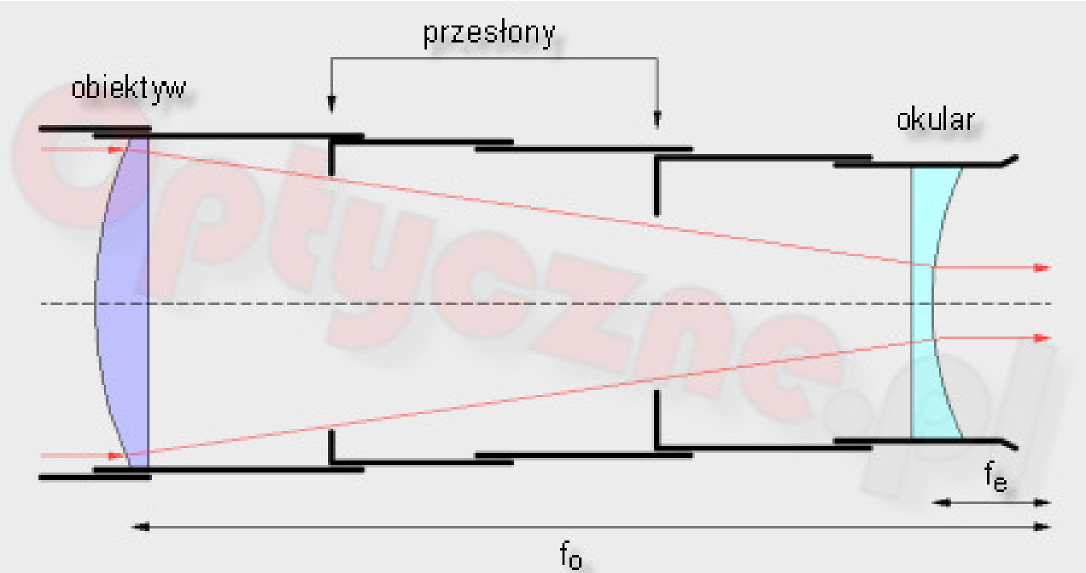
Vignetting - is a reduction of an image's brightness or saturation at the periphery compared to the image center.

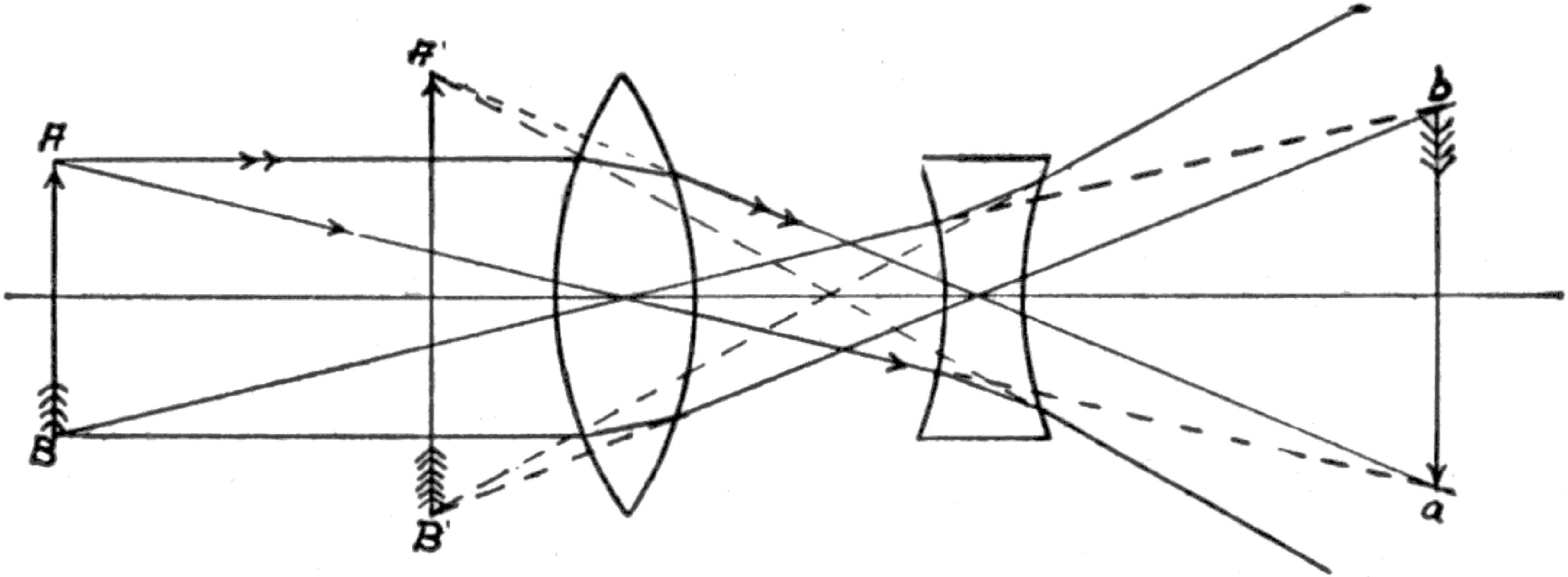




Types of telescopes - refractors

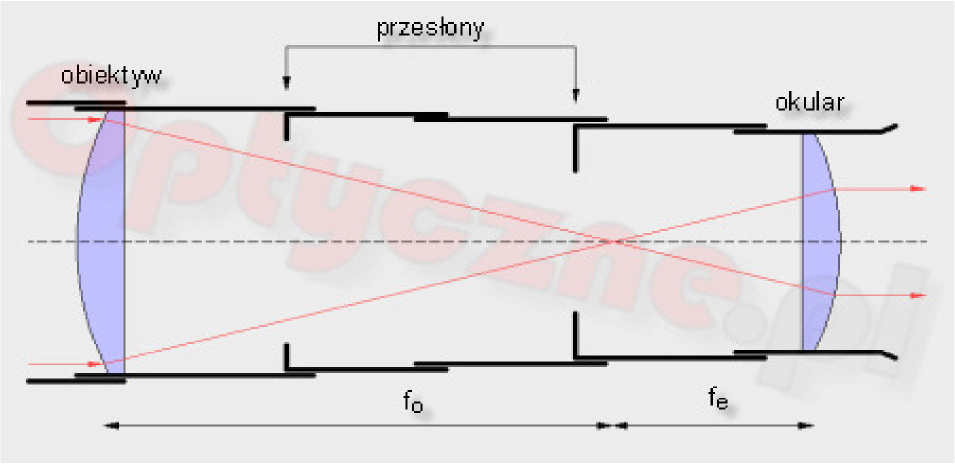
Galilean telescope (1609) → positive lens as objective + negative lens as eyepiece → results in a non-inverted and upright image

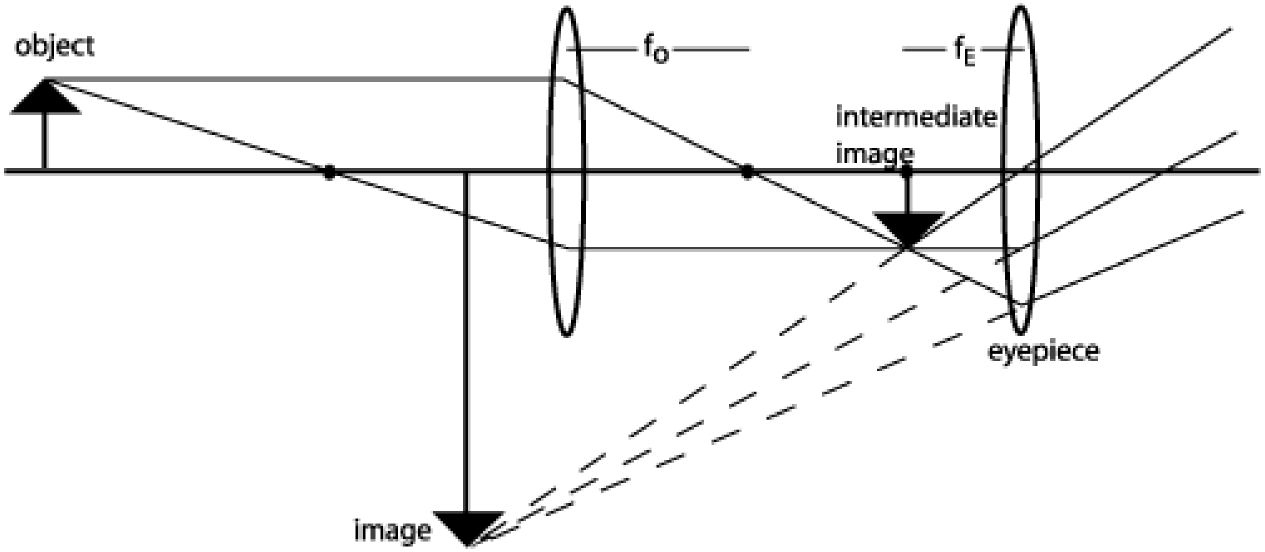




Types of telescopes - refractors

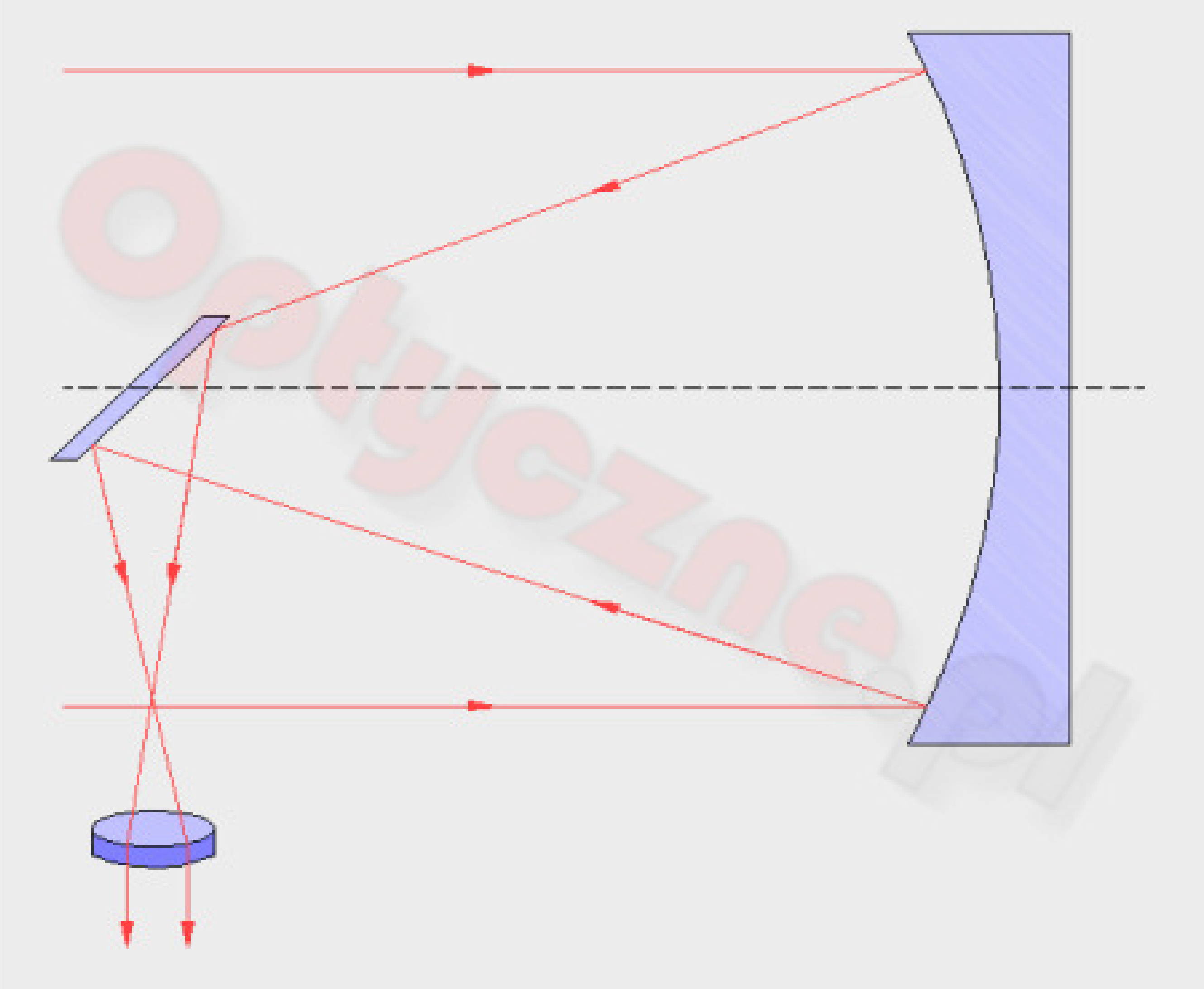
Keplerian telescope (1611) → positive lens as objective + positive lens as eyepiece. The advantage of this arrangement is that the rays of light emerging from the eyepiece are converging. This allows for a much wider field of view and greater eye relief, but the image for the viewer is inverted.





Types of telescopes - reflectors

Newtonian telescope (1668) → concave mirror + flat mirror + eyepiece.



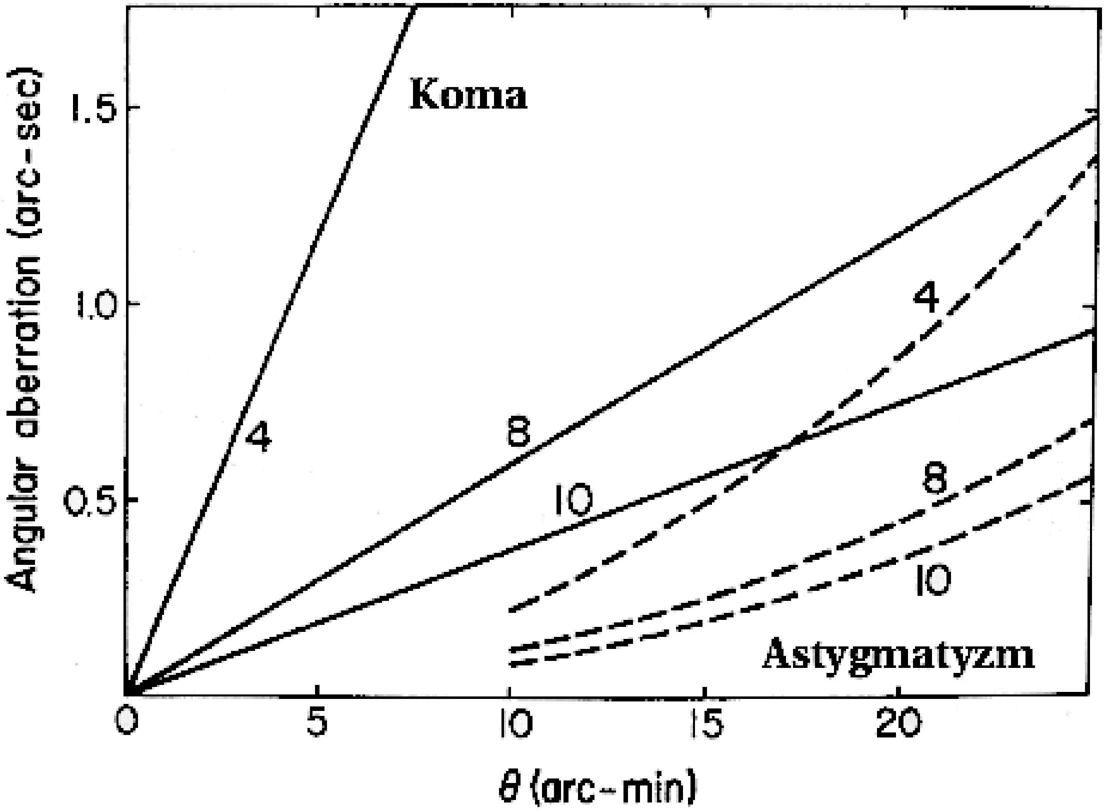
Types of telescopes - reflectors

In parabolic mirror the coma is main source of blur especially for fast newtonians...

Limiting Field Radius
for Good[a] Images
Paraboloid Telescope

F	Θ [arcmin]
4	1.42
8	5.59
10	8.89

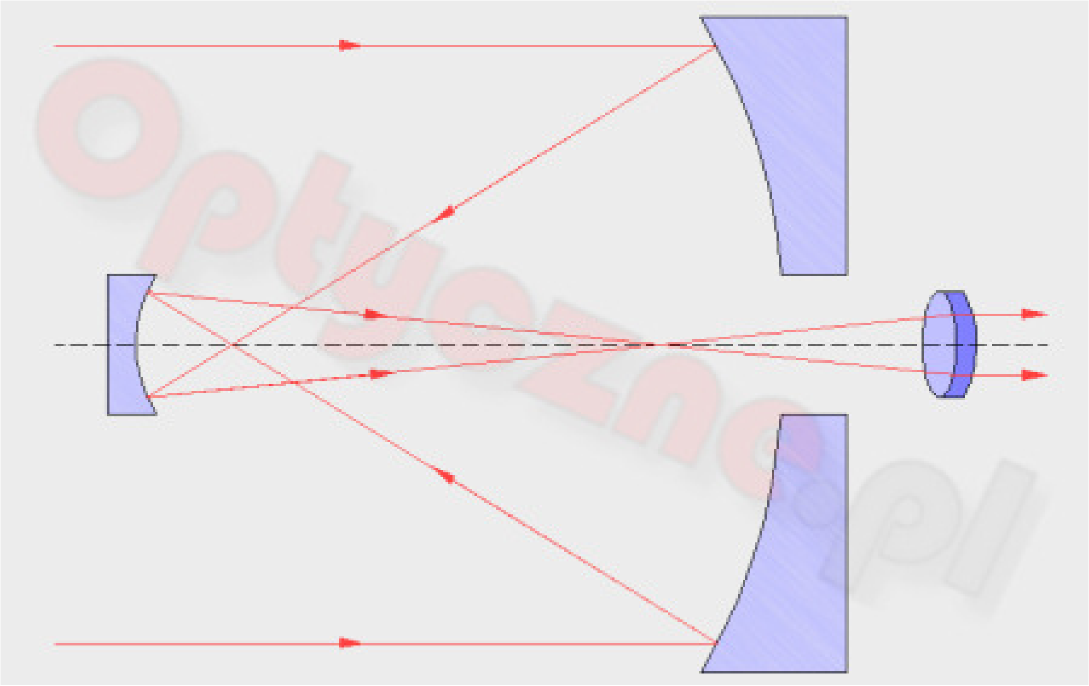
[a]Good defined as tangential coma that measures 1 arcsec.



Types of telescopes - reflectors

Gregory's telescope (1663) → concave mirror + ellipsoidal mirror + eyepiece.

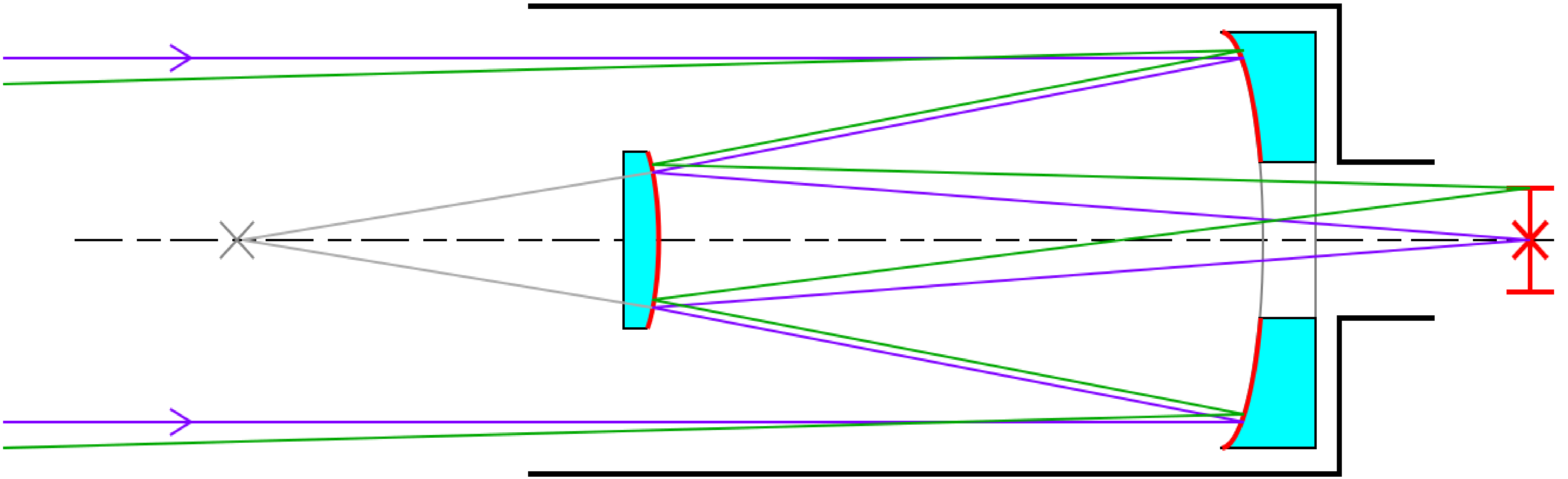
This design renders an upright image, making it useful for terrestrial observations. It also works as a telephoto in that the tube is much shorter than the system's actual focal length.



Types of telescopes - reflectors

Cassegrain telescope (1672) → parabolic primary mirror + hiperbolid mirror + eyepiece.

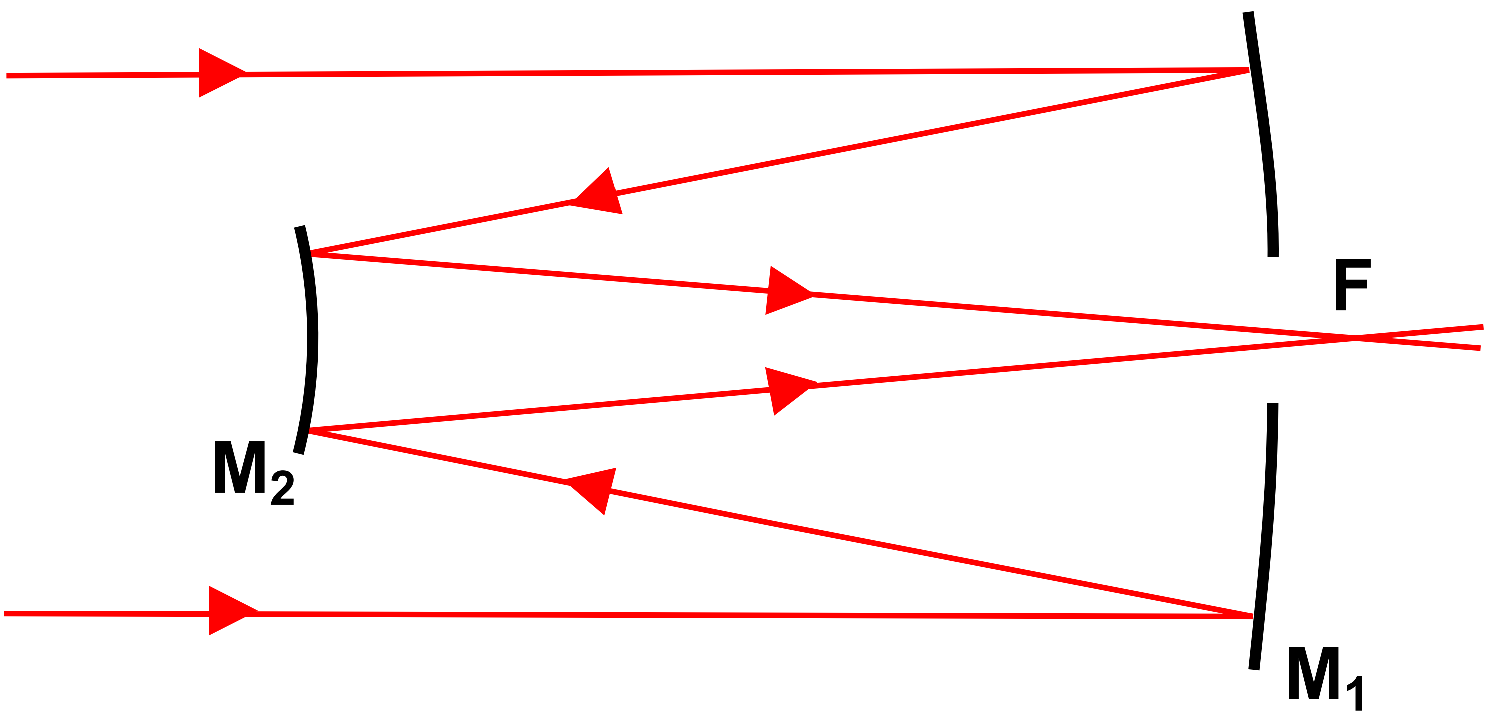
This design puts the focal point at a convenient location behind the primary mirror and the convex secondary adds a telephoto effect creating a much longer focal length in a mechanically short system.



Types of telescopes - reflectors

Ritchey-Chrétien telescope (1910) → hiperbolic primary mirror + hiperbolic secondary mirror + eyepiece.

It is free of coma and spherical aberration at a flat focal plane, making it well suited for wide field and photographic observations. This design is very common in large professional research telescopes, including the Hubble Space Telescope, Keck Telescopes and VLT telescope.

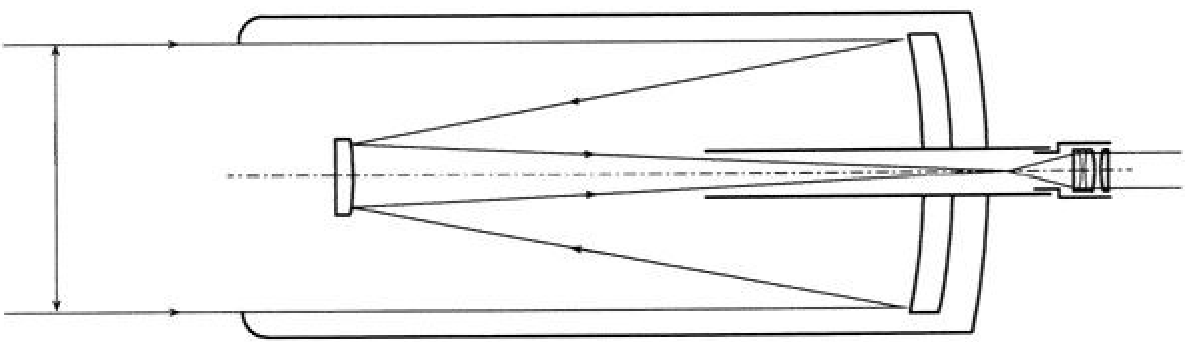


Ritchey-Chrétien

Types of telescopes - reflectors

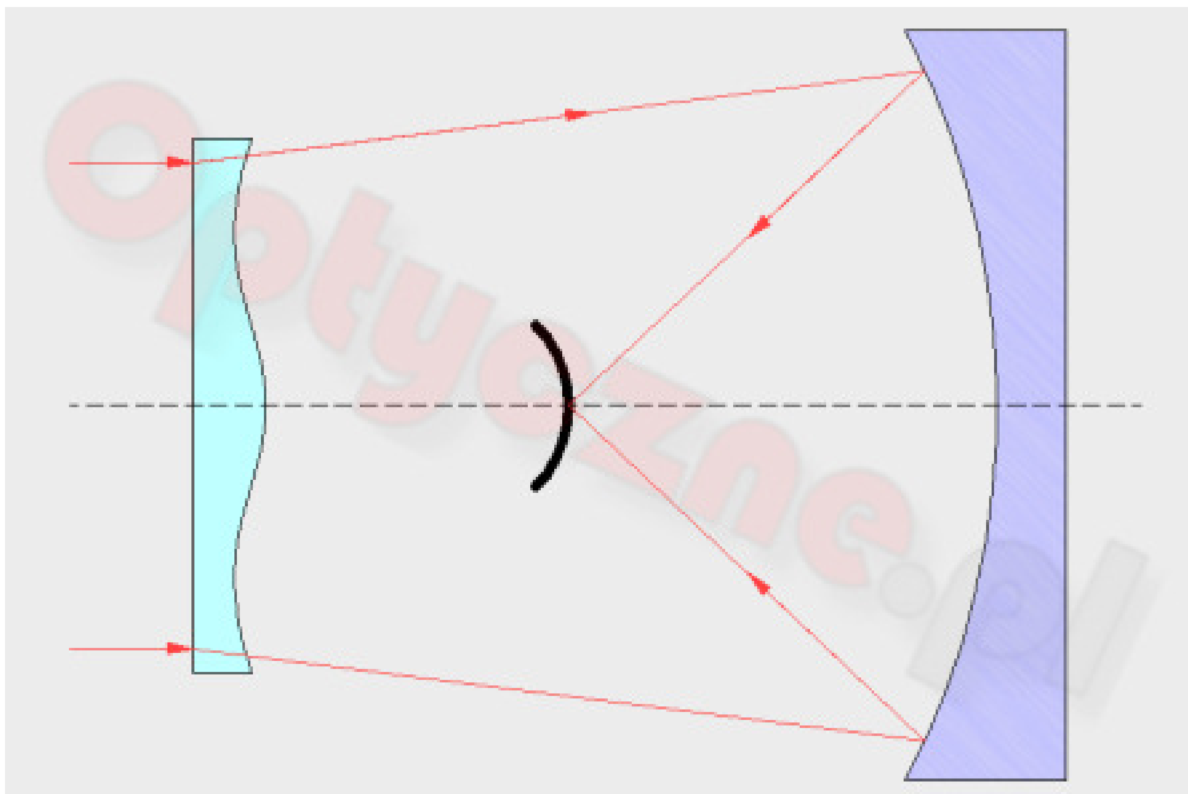
Dall-Kirkham telescope (1928) → elliptical primary mirror + spherical mirror + eyepiece.

This system is easier to polish than a classic Cassegrain or Ritchey-Chretien system, but the off-axis coma is significantly worse, so the image degrades quickly off-axis.



Types of telescopes – catadioptic constructions

Schmidt telescope (1930) → spherical primary mirror + corector lens
Large off-axis aberration free field of view.
Significant field curvature.

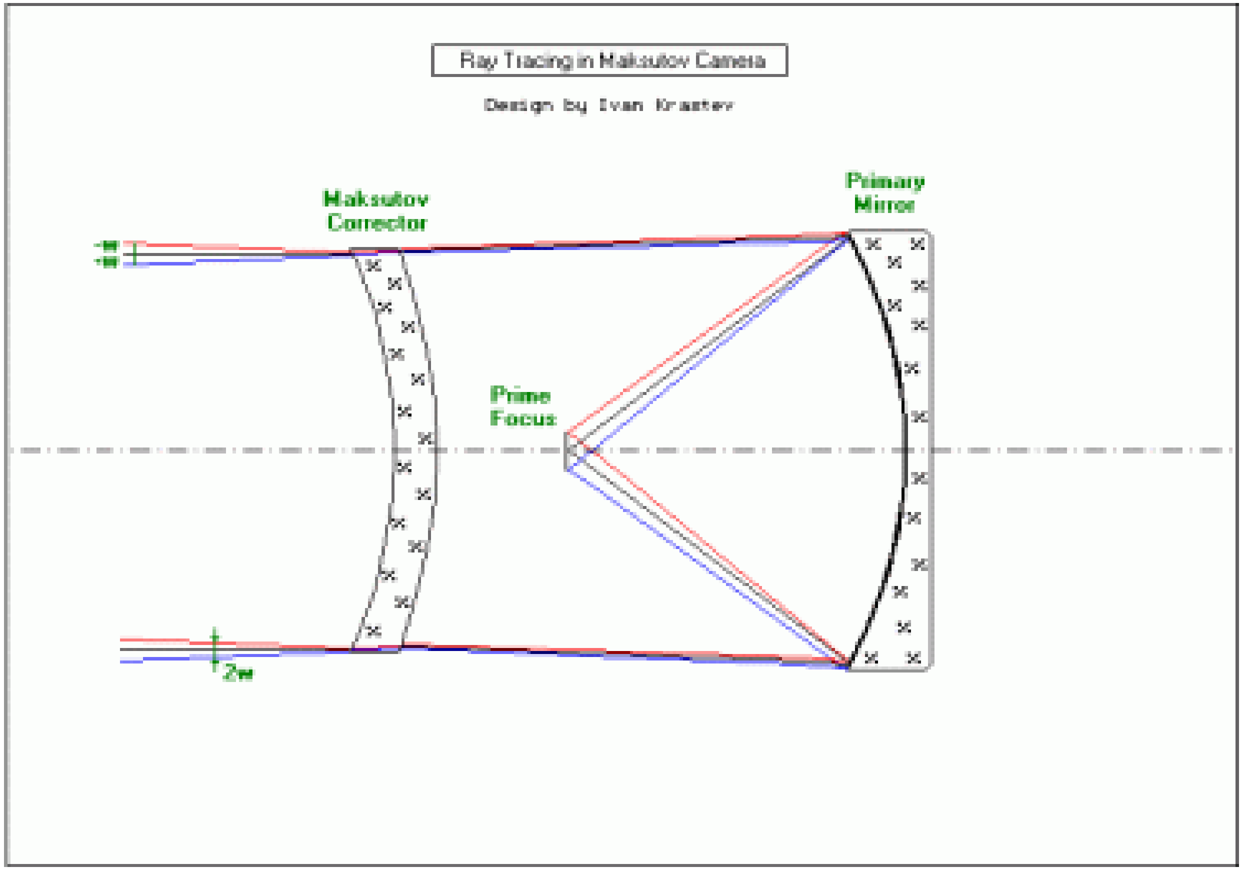


Types of telescopes – catadioptic constructions

Maksutov telescope (1941) → spherical primary mirror + negative meniscus lens

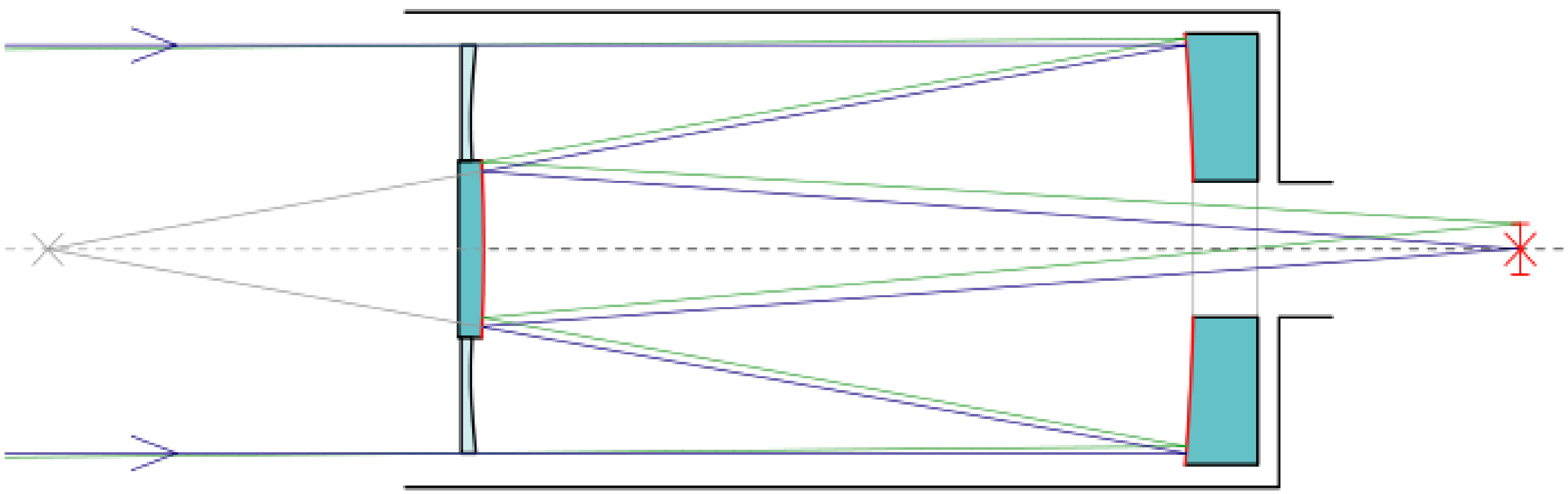
Large off-axis aberration free field of view.

Significant field curvature.



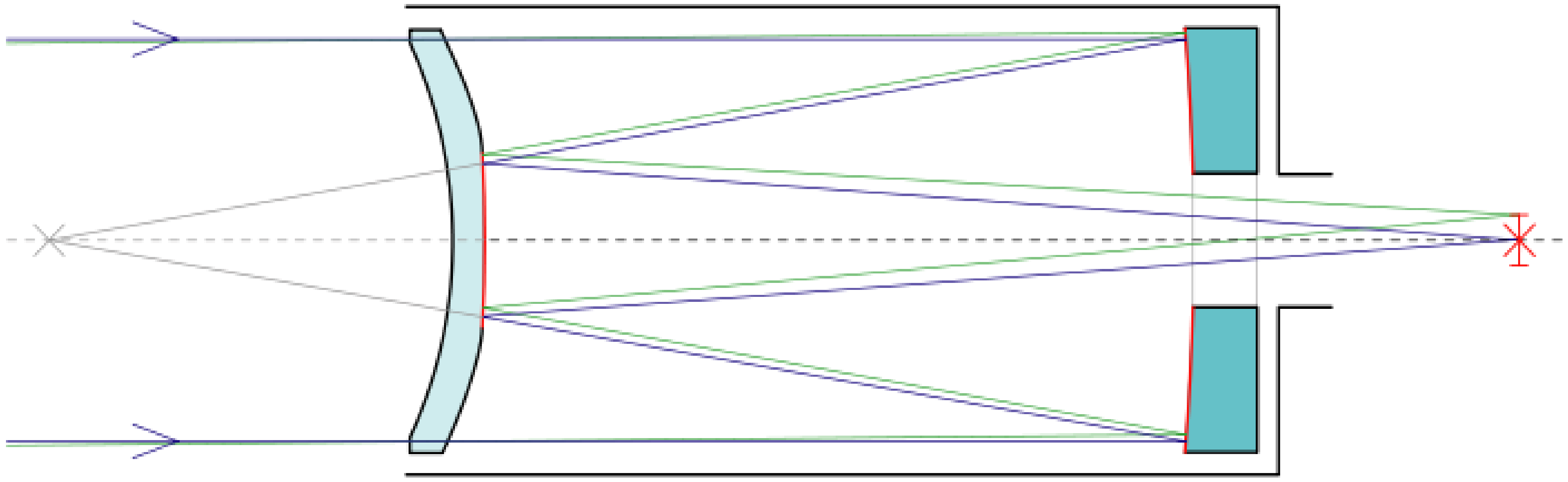
Types of telescopes – catadioptic constructions

Schmidt-Cassegrain telescope (1946) →
parabolic primary mirror + corrector lens + hyperbolic secondary.



Types of telescopes – catadioptic constructions

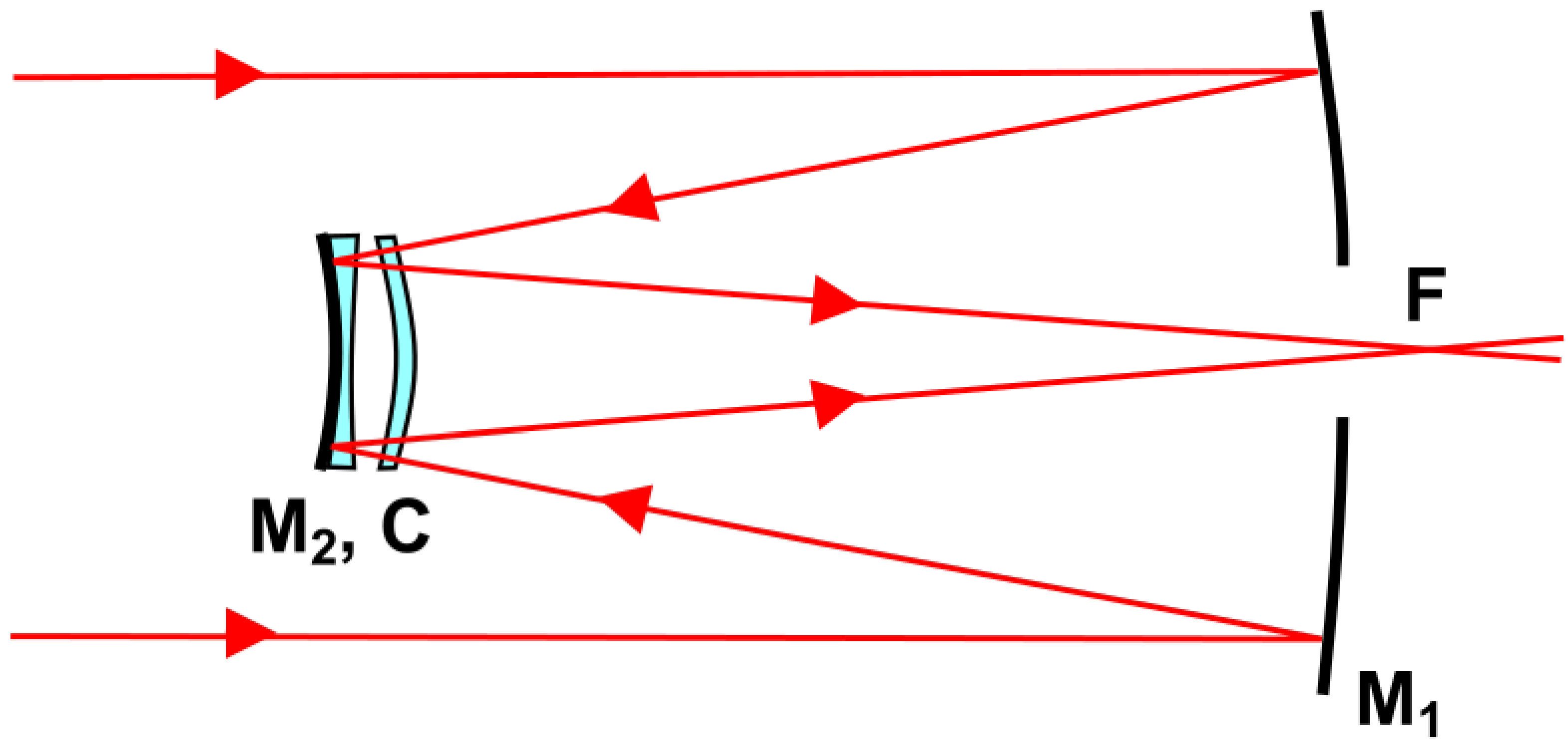
Maksutov-Cassegrain telescope →
parabolic primary mirror + meniscus negative lens + hyperbolic secondary.



Types of telescopes – catadioptic constructions

Klevtsov-Cassegrain telescope →
parabolic primary mirror + Mangin secondary mirror with additional lens corrector.

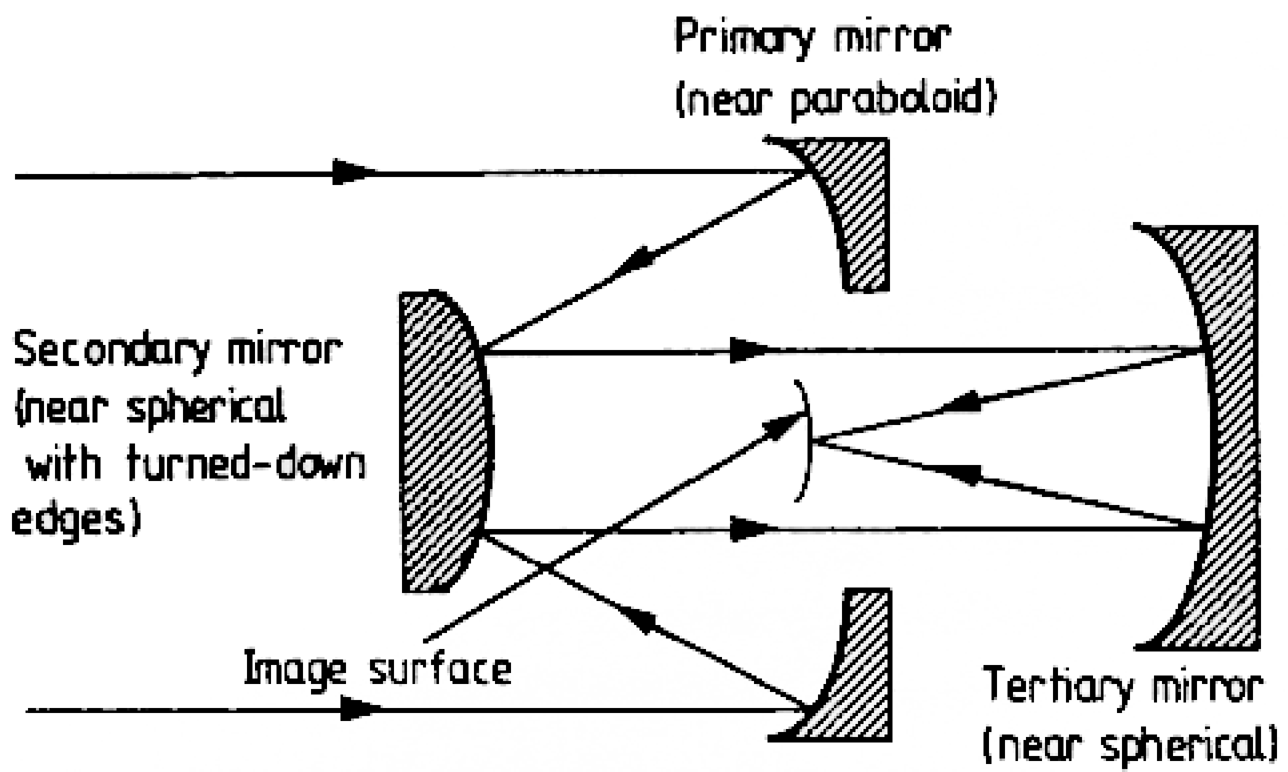
Mangin mirror is a meniscus lens with one air-to-glass surface covered with reflective layer.



Klevtsov-Cassegrain

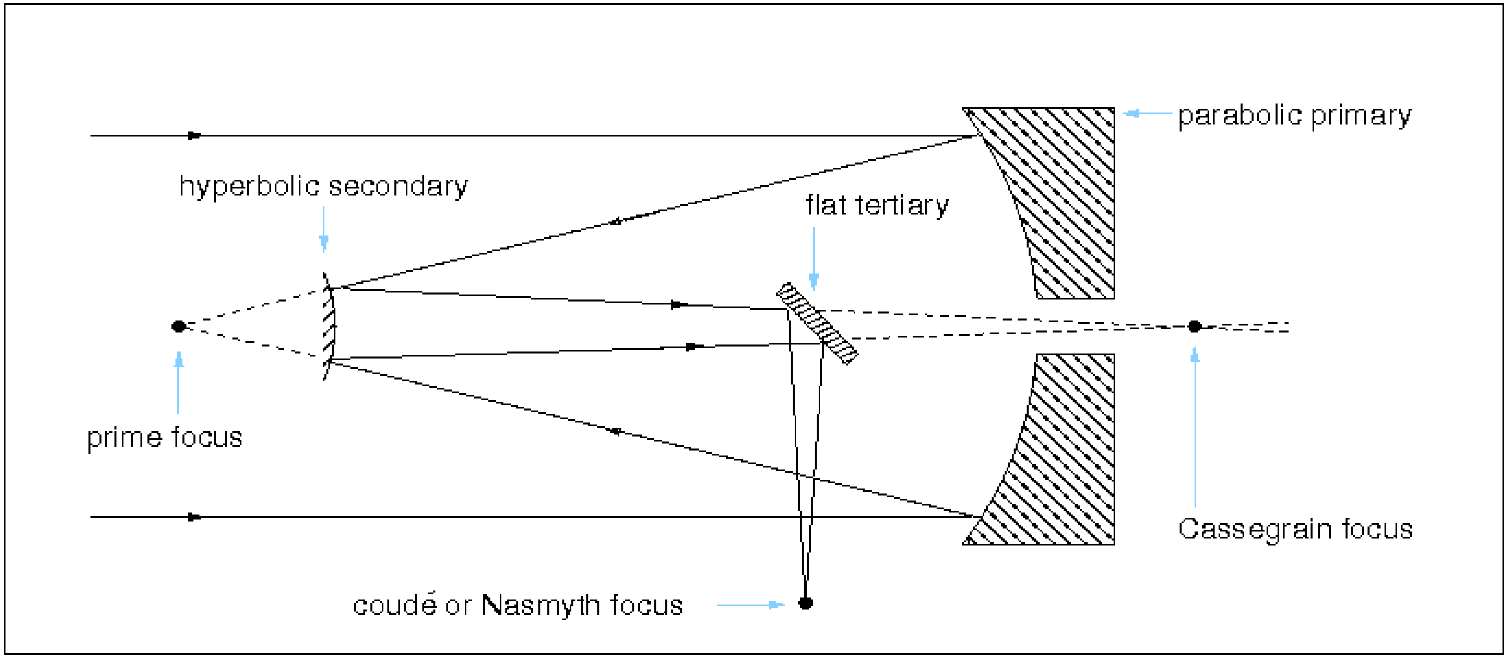
Types of telescopes – more complicated systems...

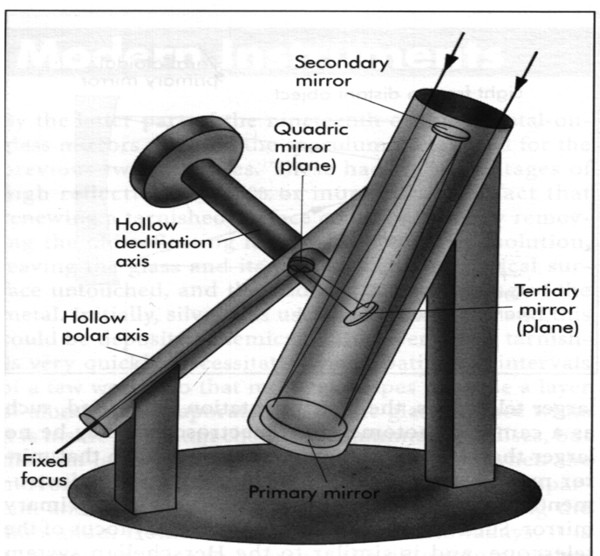
Willstrop telescope →
large field of view free of aberrations.
0.5-m telescope in Cambridge.

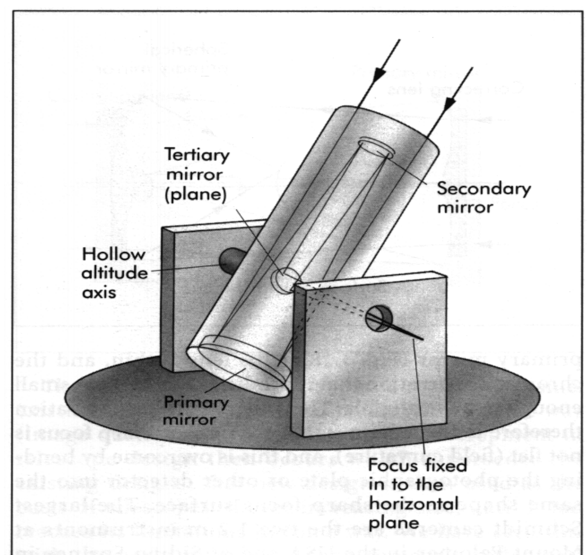


Types of telescopes – more complicated systems...

Coude or Nasmyth telescope/focus

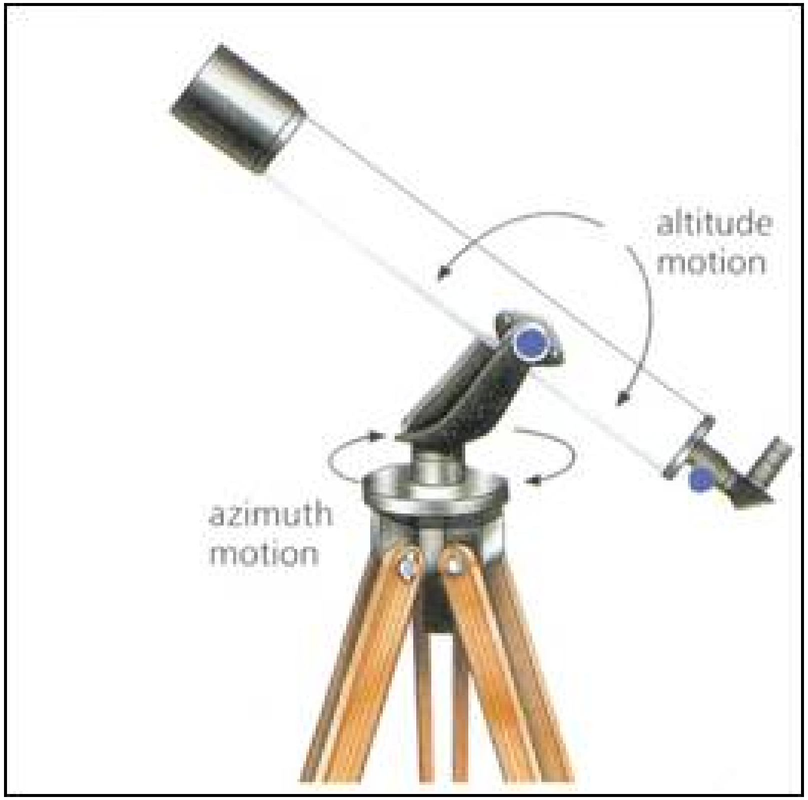


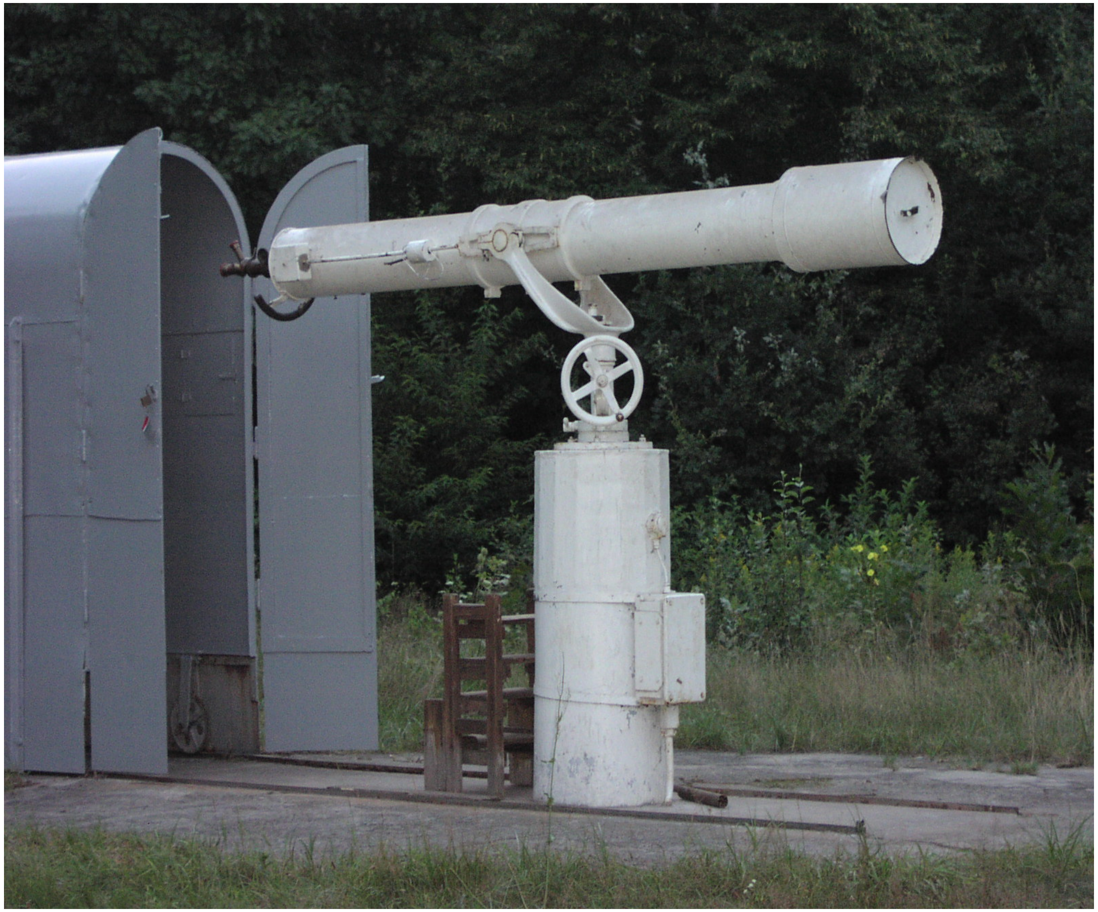




Telescope mounts

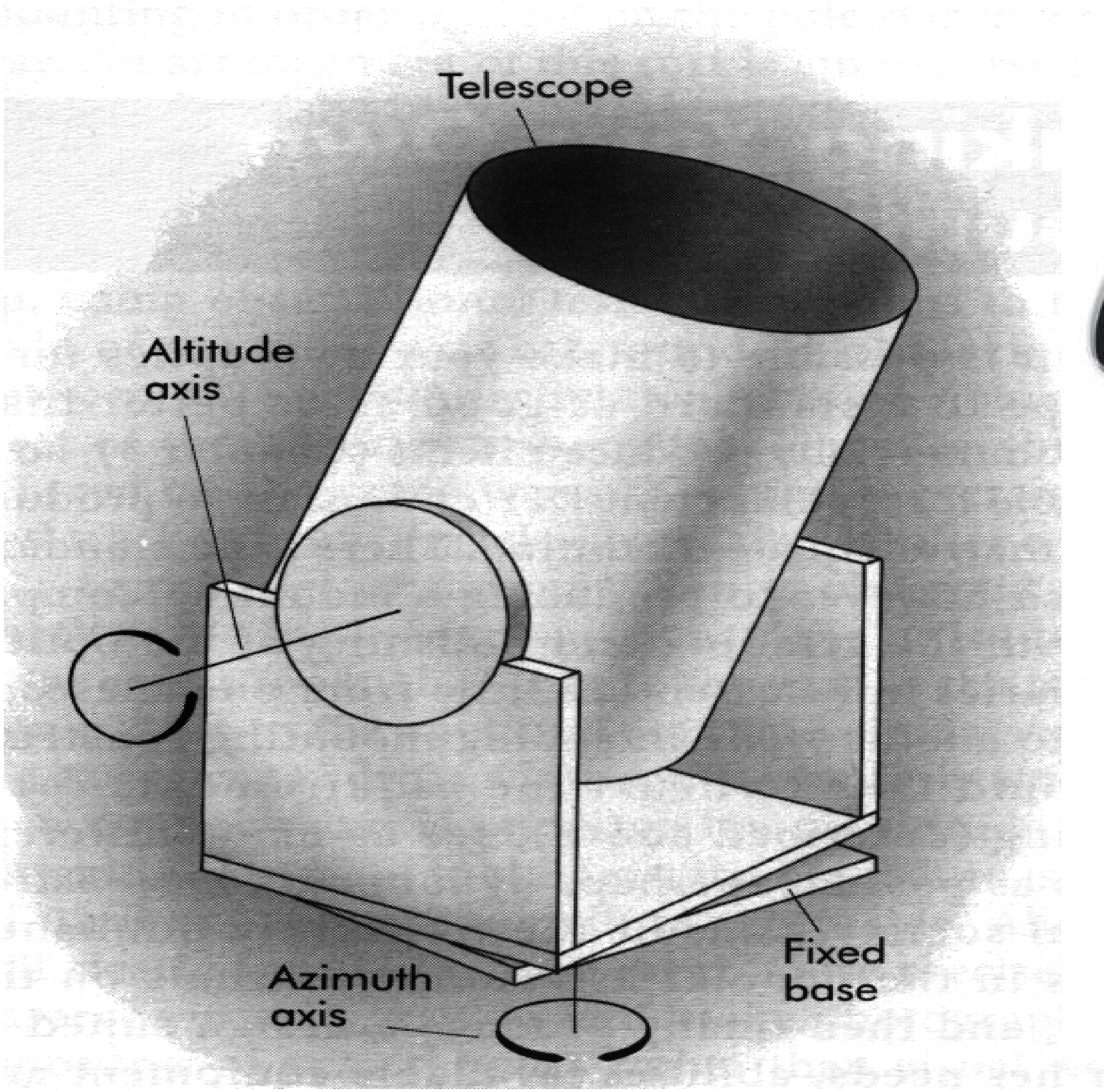
Horizontal mount





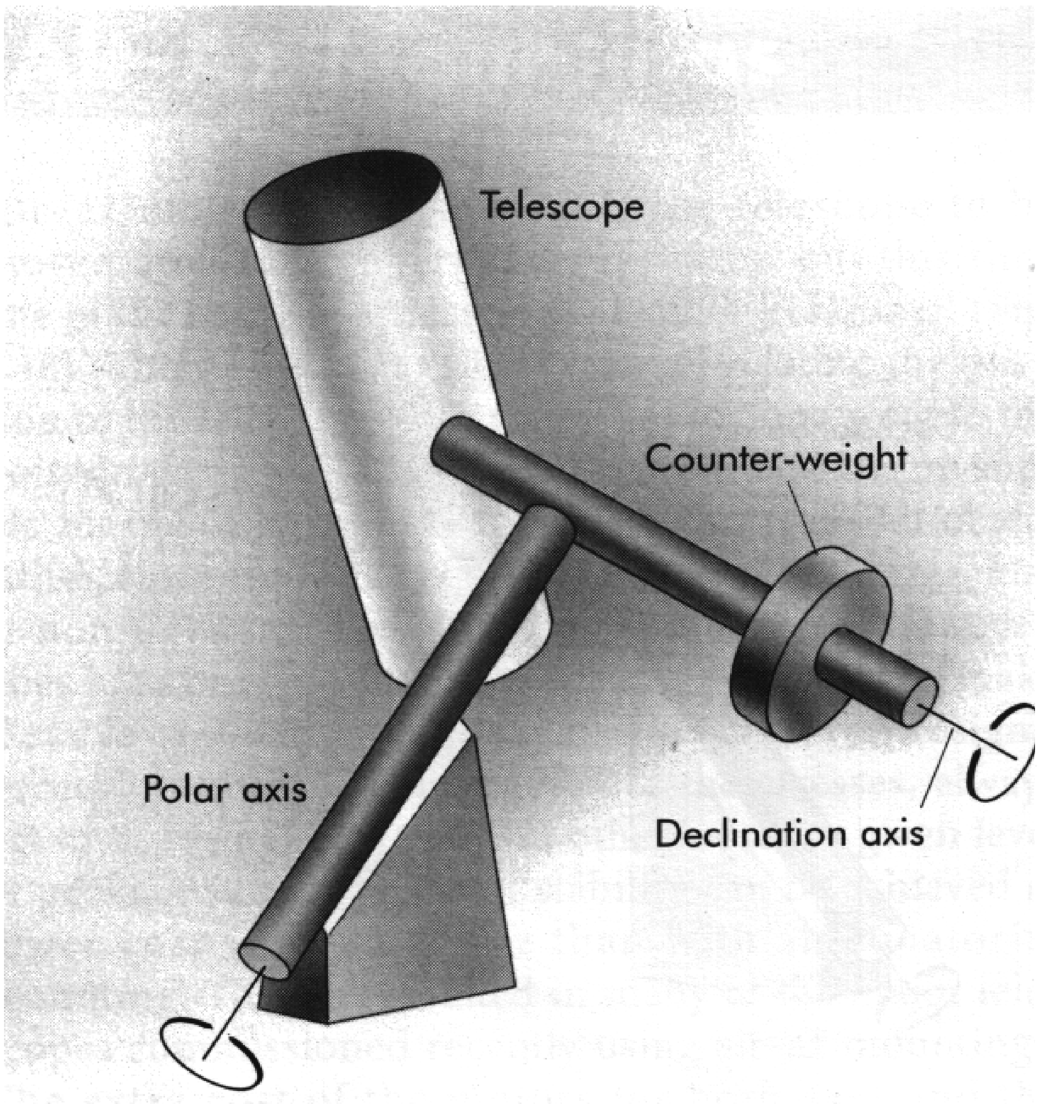
Telescope mounts

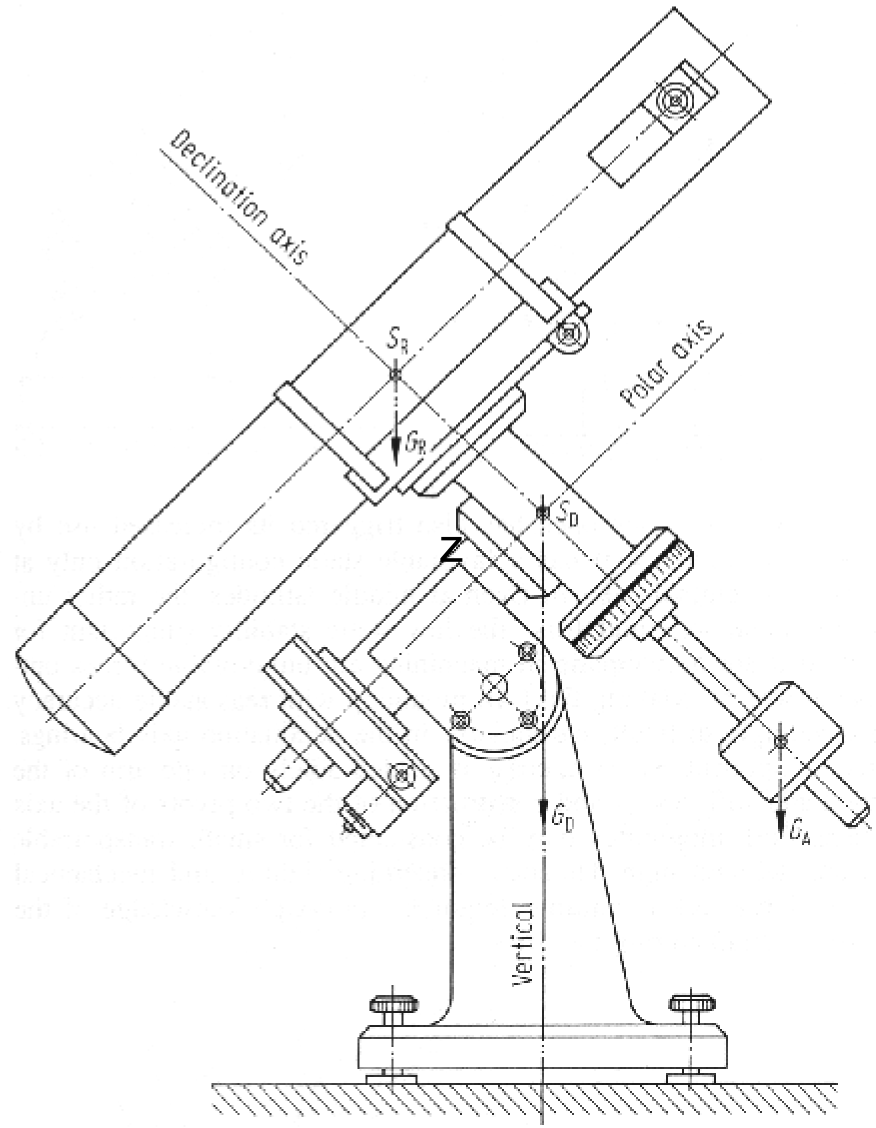
Horizontal Dobsonian mount



Telescope mounts

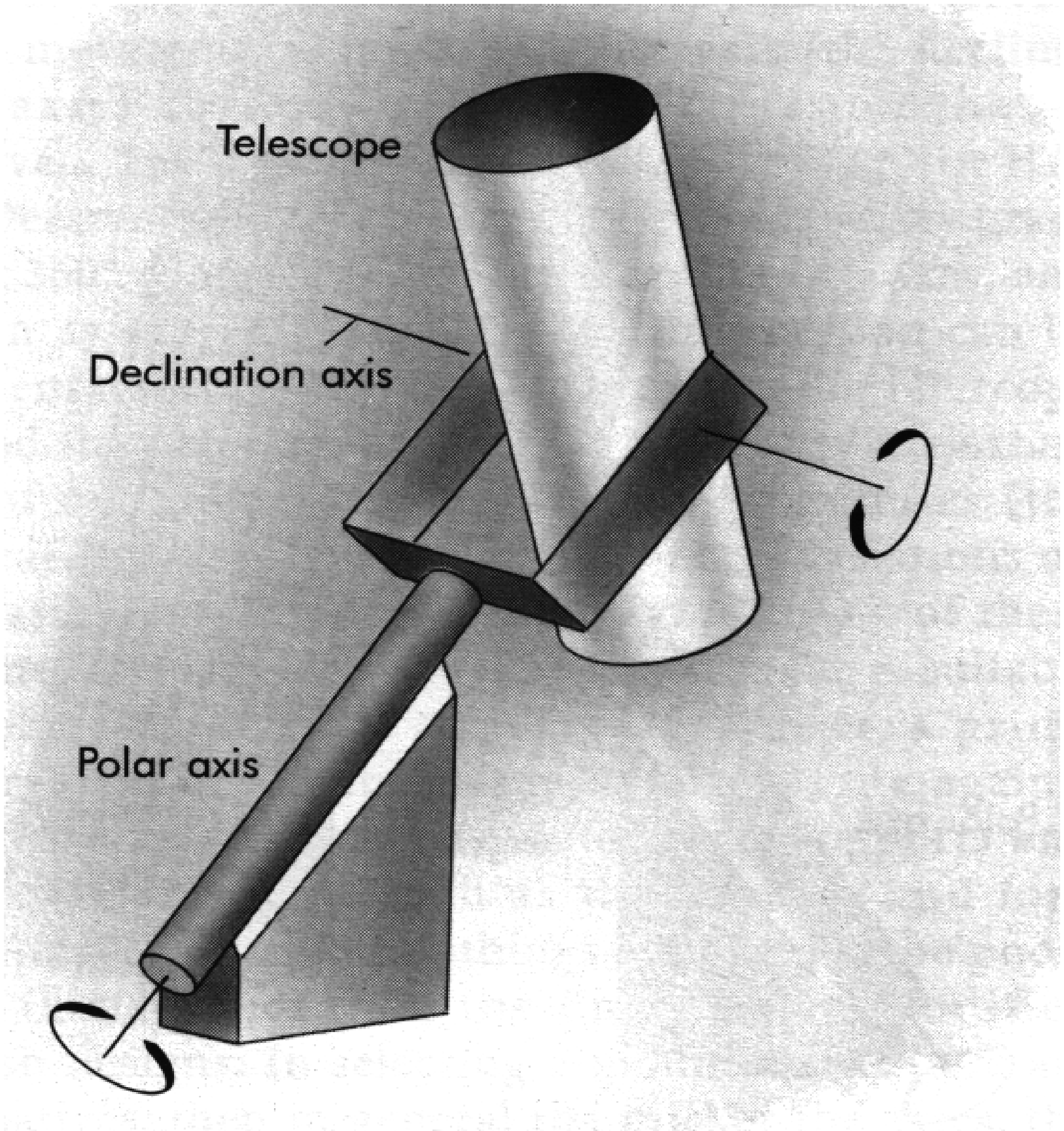
German equatorial mount

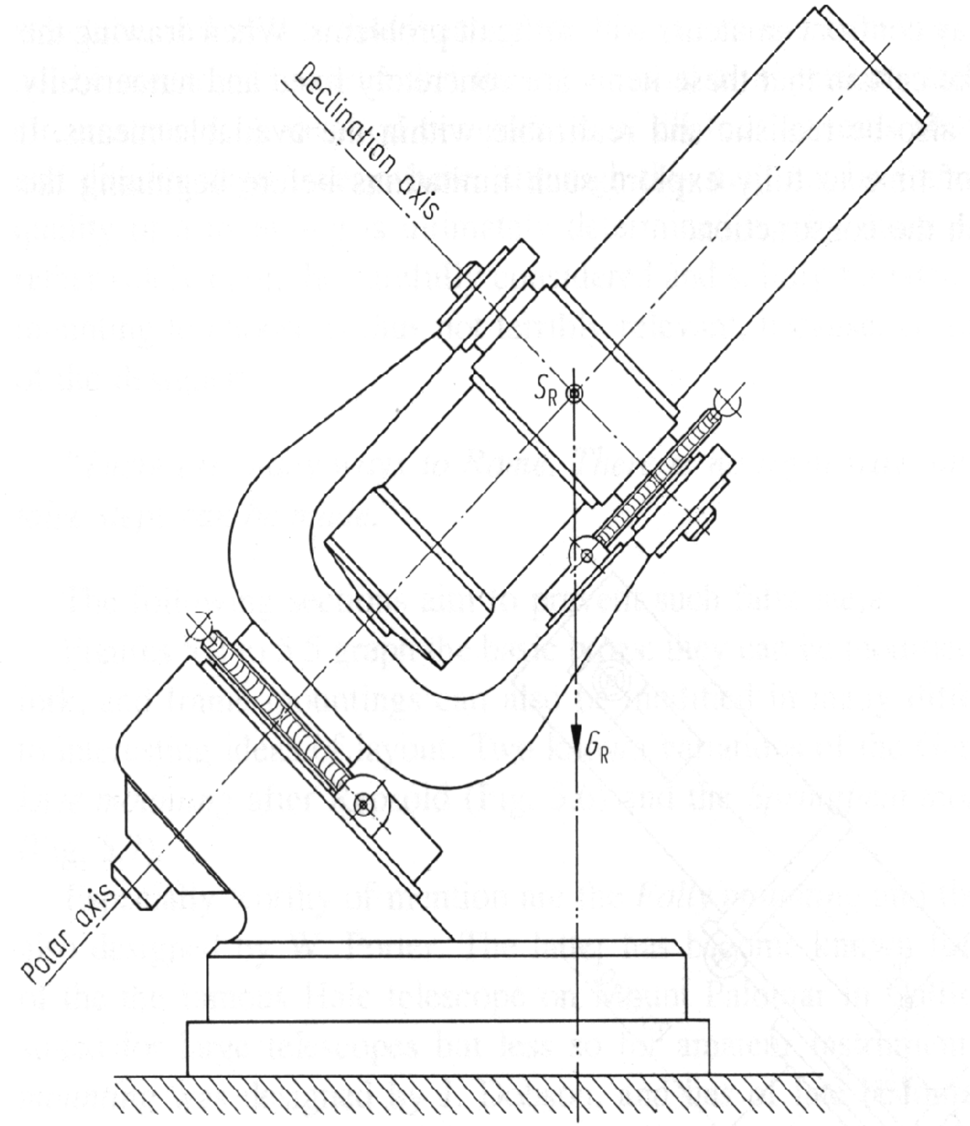




Telescope mounts

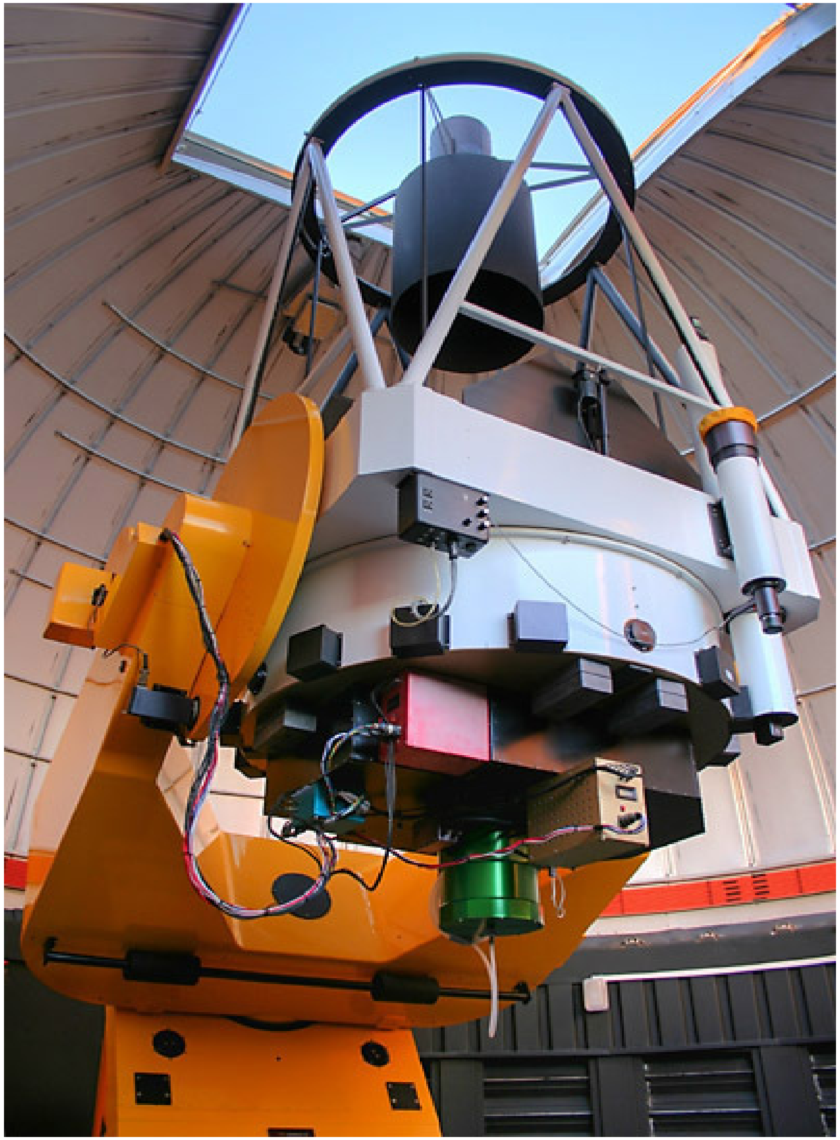
American (fork) mount





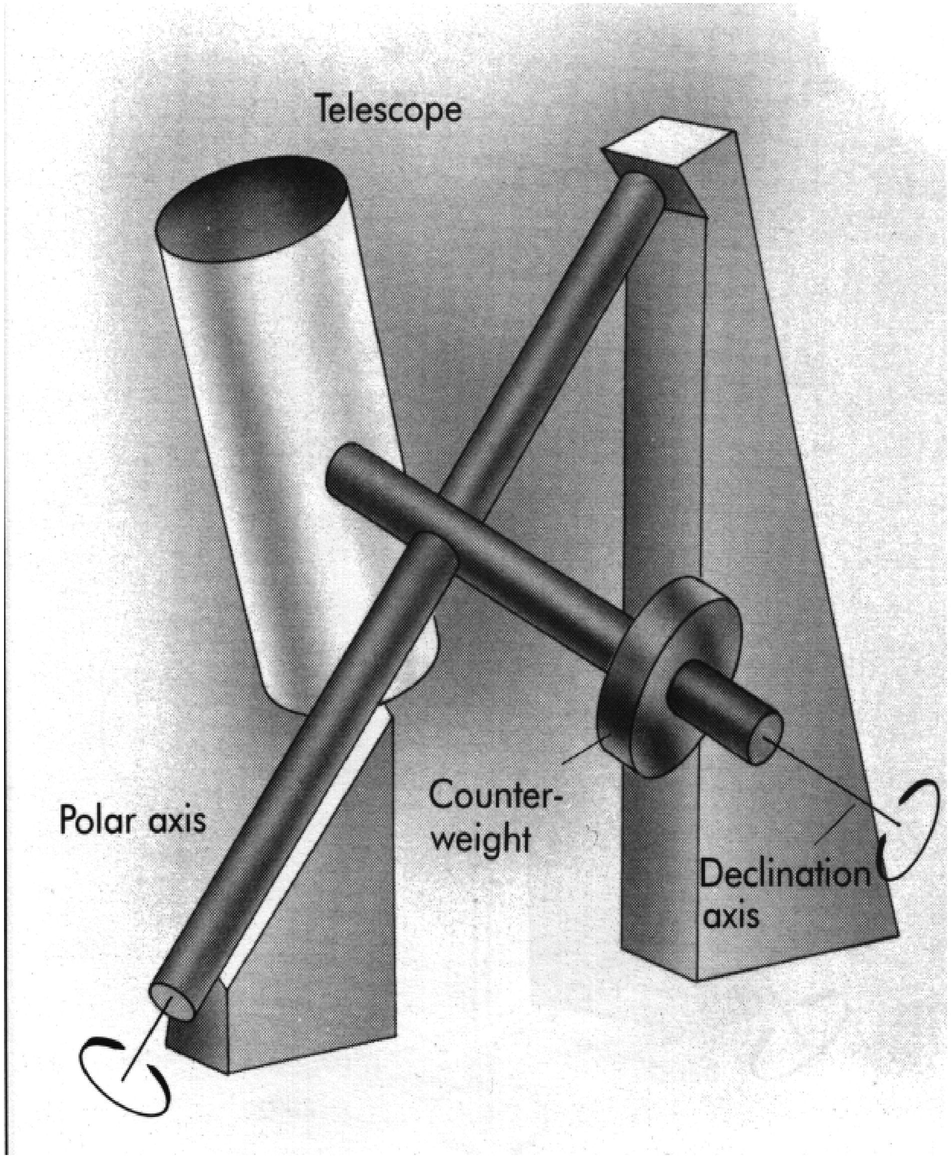
Telescope mounts

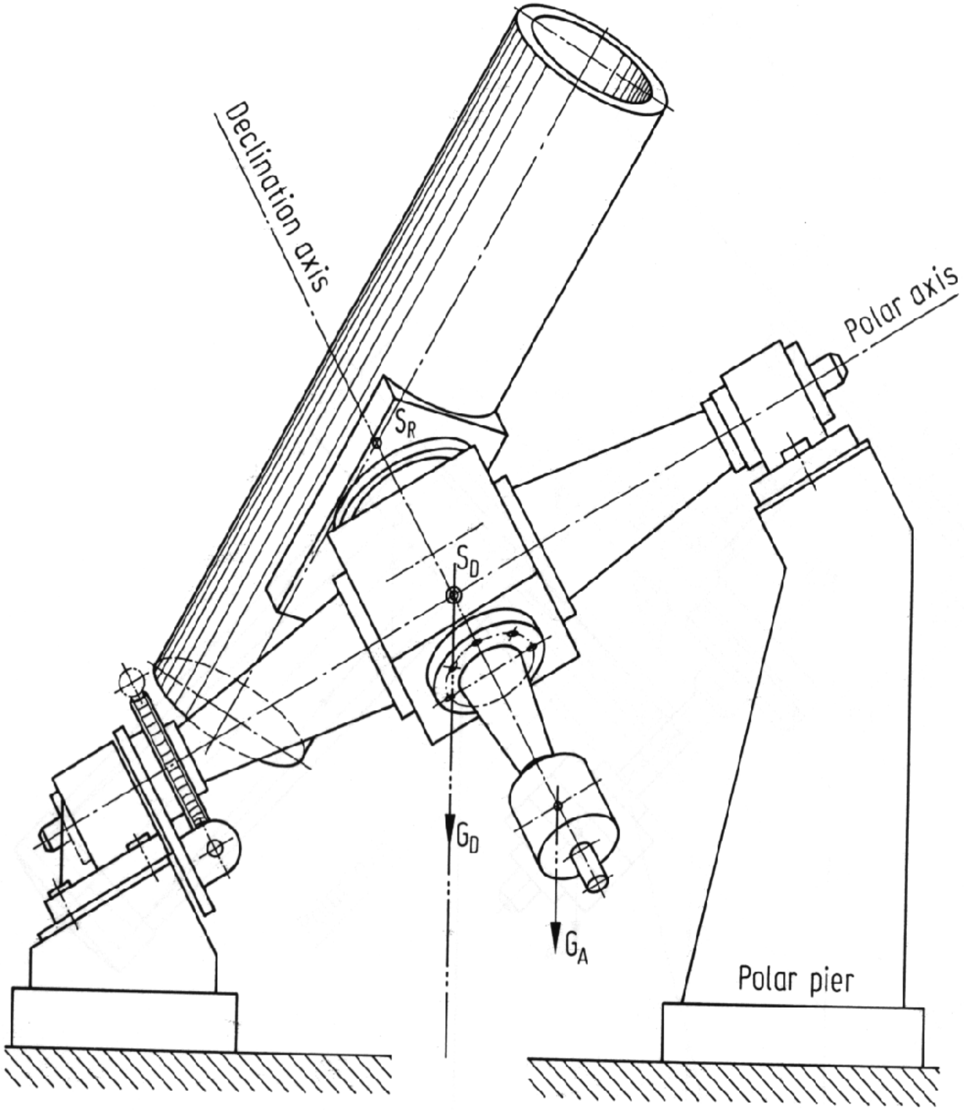
American (fork) mount → OGLE telescope



Telescope mounts

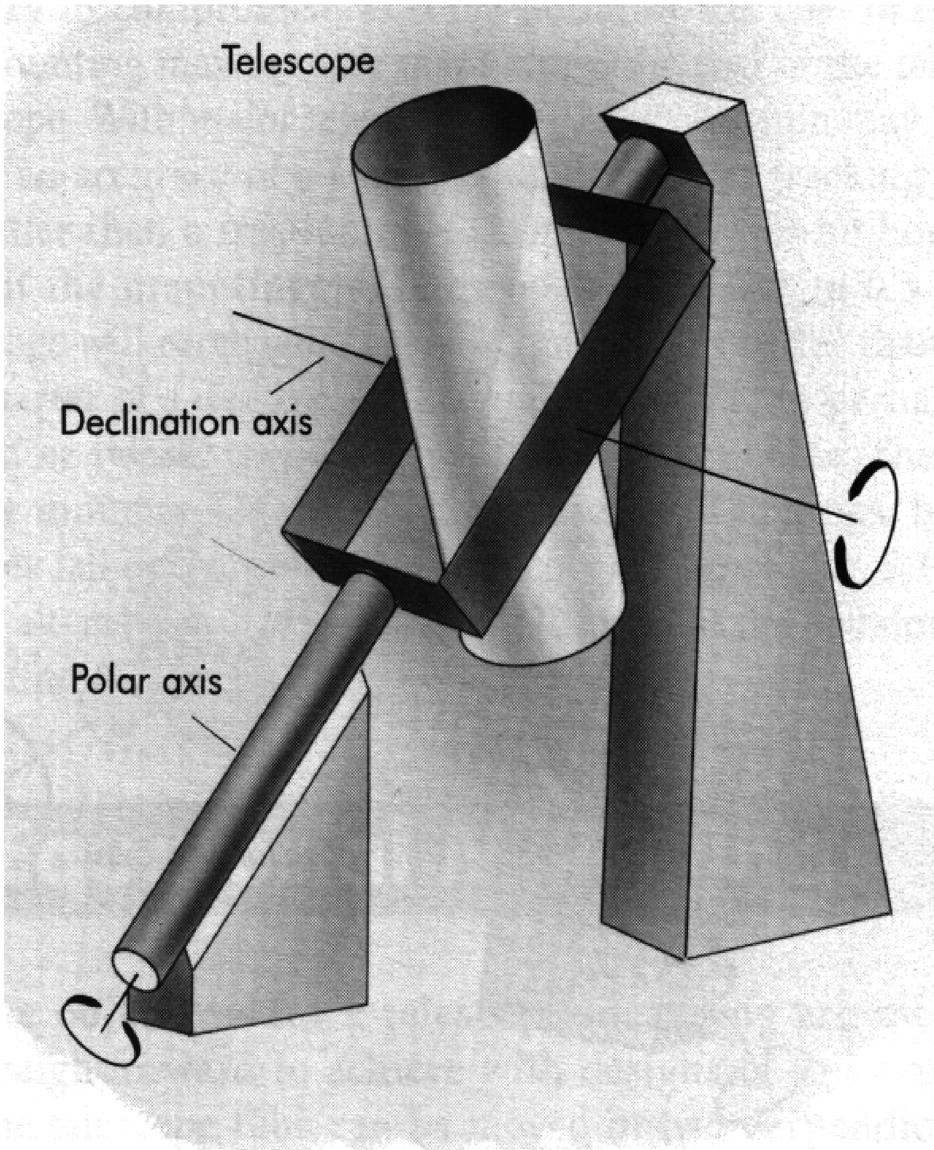
English mount

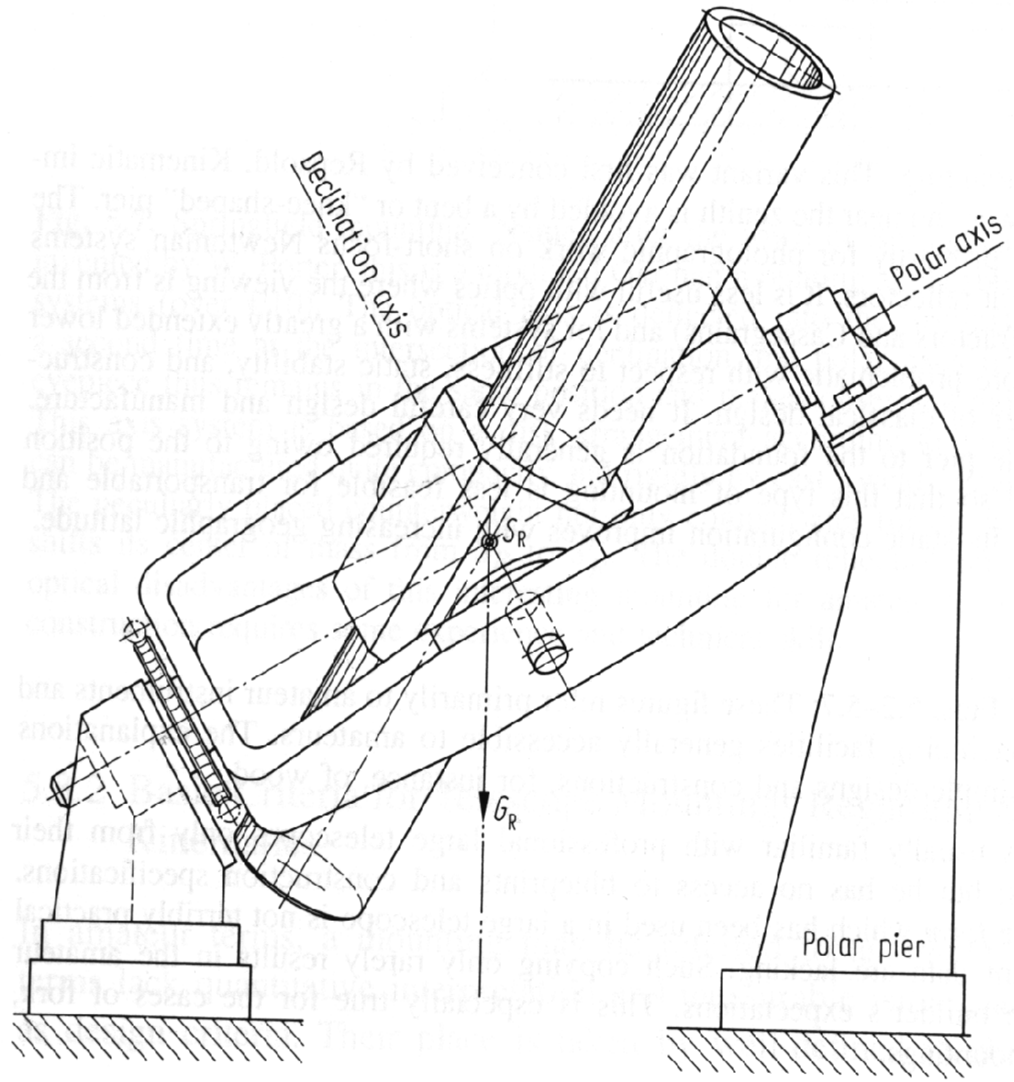




Telescope mounts

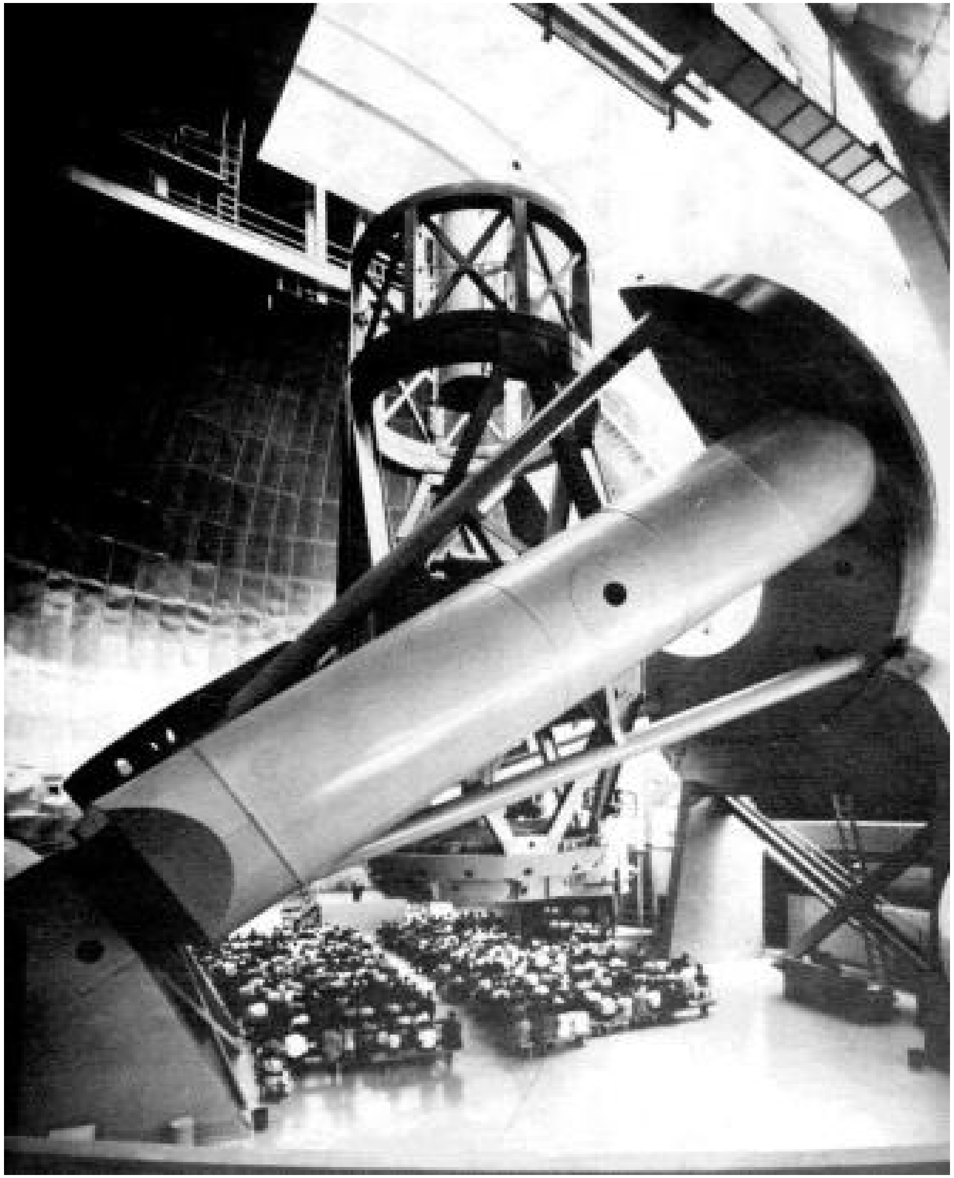
Yoke mount





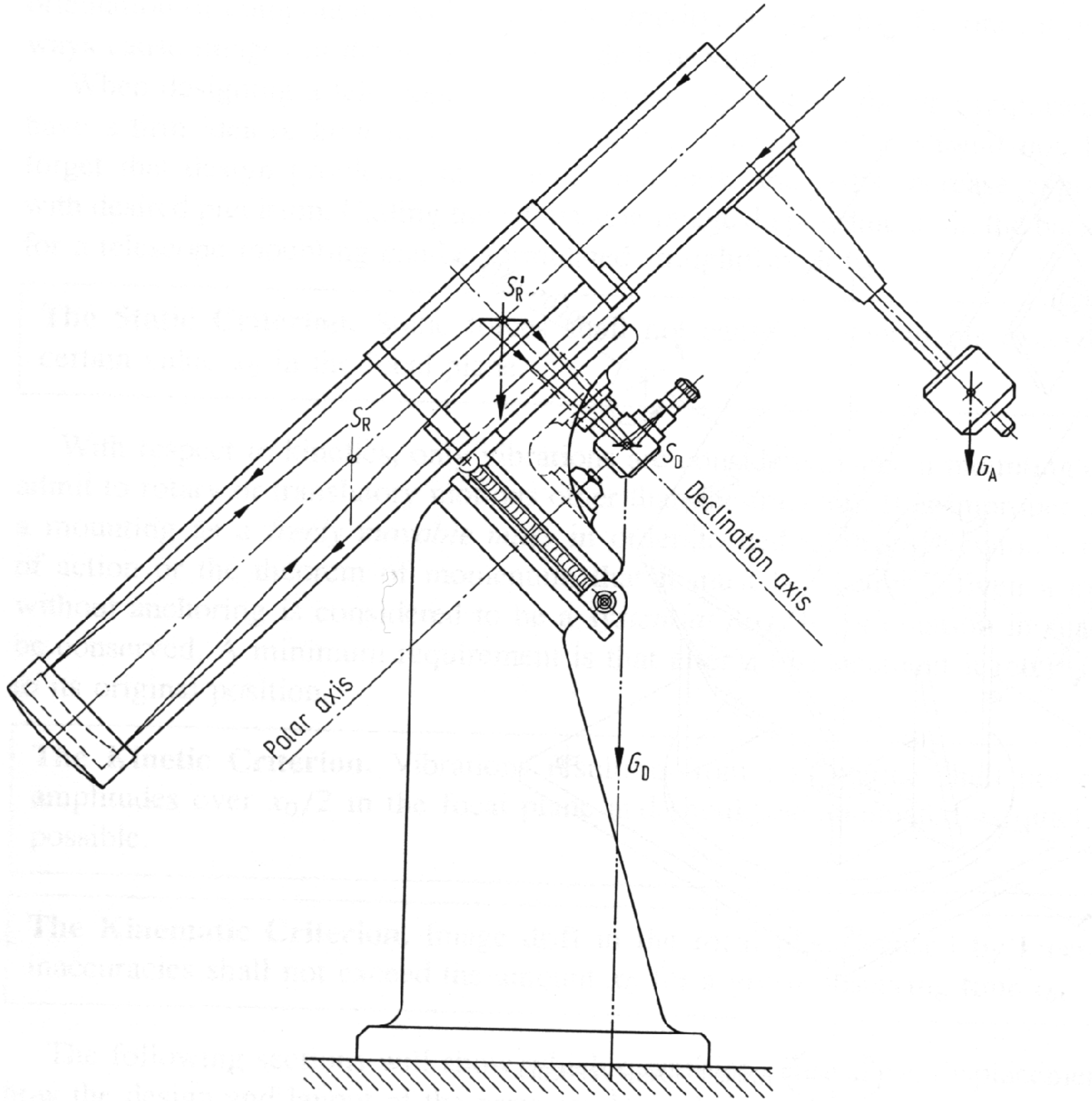
Telescope mounts

Yoke mount → 5-m Mt. Palomar telescope



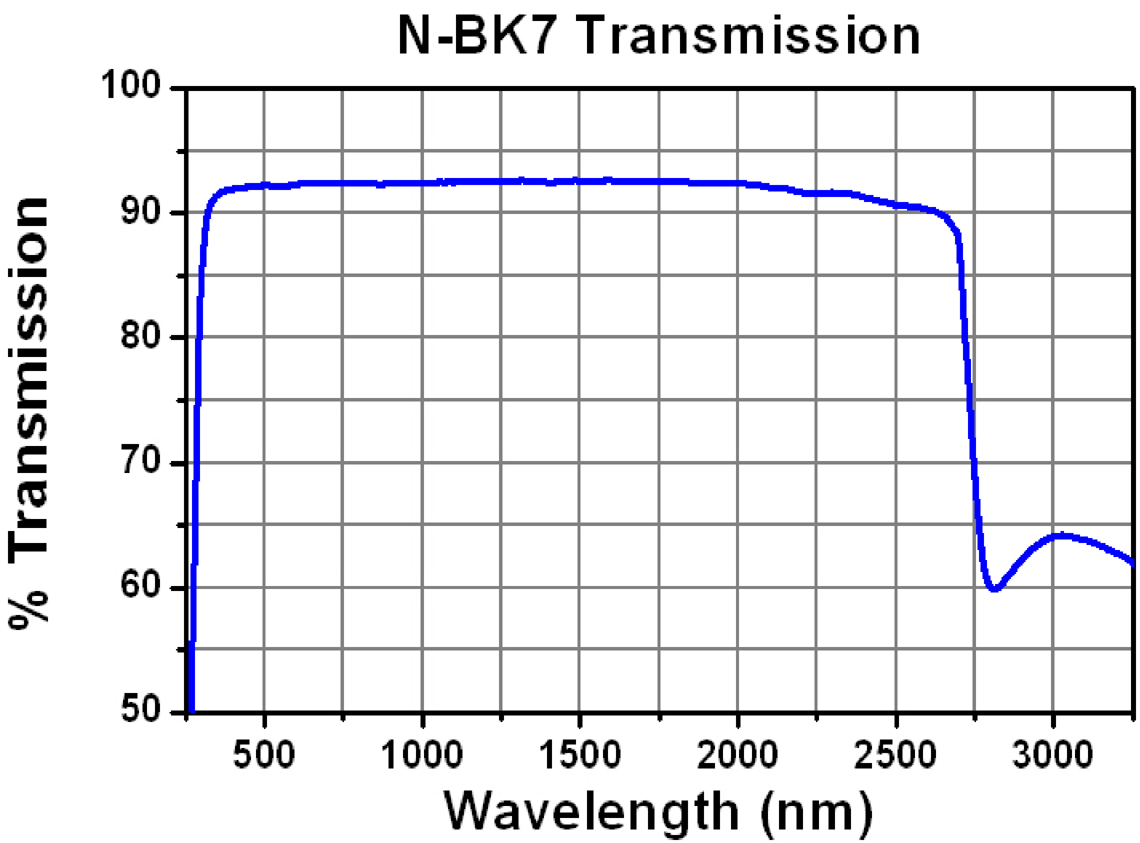
Telescope mounts

Springfield mount



Antireflection coatings

At each air-to-glass surface there is not only refraction of the light but also reflection.



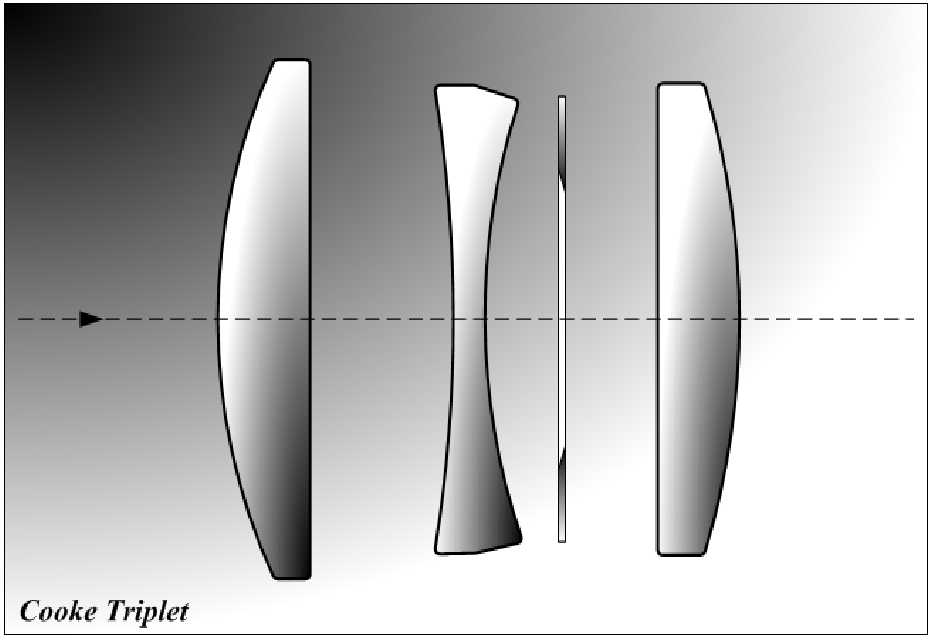
Typical Bk7 glass reflects about 4-4.5% of light at one surface.

It is small problem for simple constructions (cemented achromats, triplets).

Antireflection coatings

The problem is more significant for more complicated designs.

Cooke triplet (invented in 1893) comprises a negative flint glass element in the center with a crown glass element on each side.



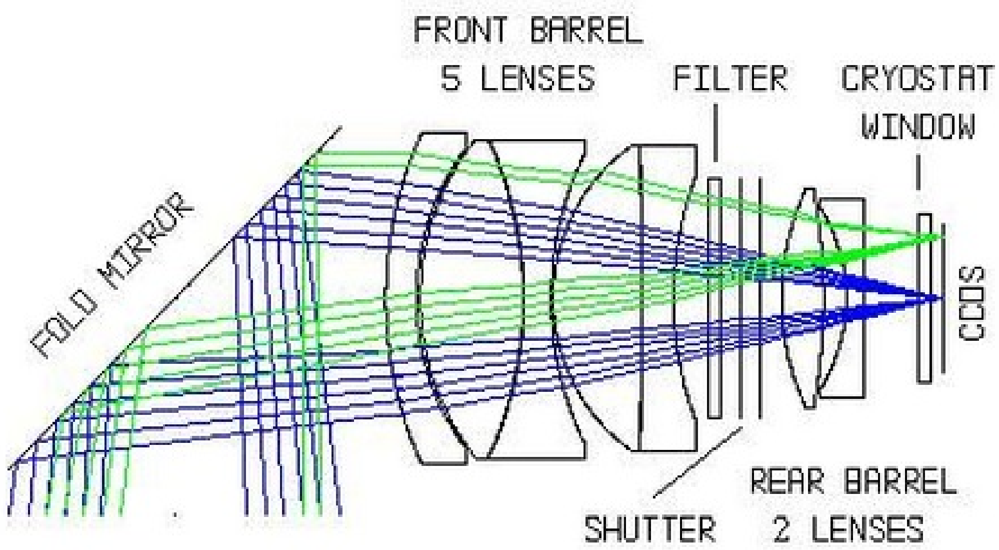
It is very well corrected for almost all aberrations and allows to construct lenses as fast as f/3.5.

It has never became popular → 6 air-to-glass surfaces → $T=0.955^6=0.758$

Antireflection coatings

But current optical devices are even more complicated.

For example SALT telescope SALTICAM CCD optical path.

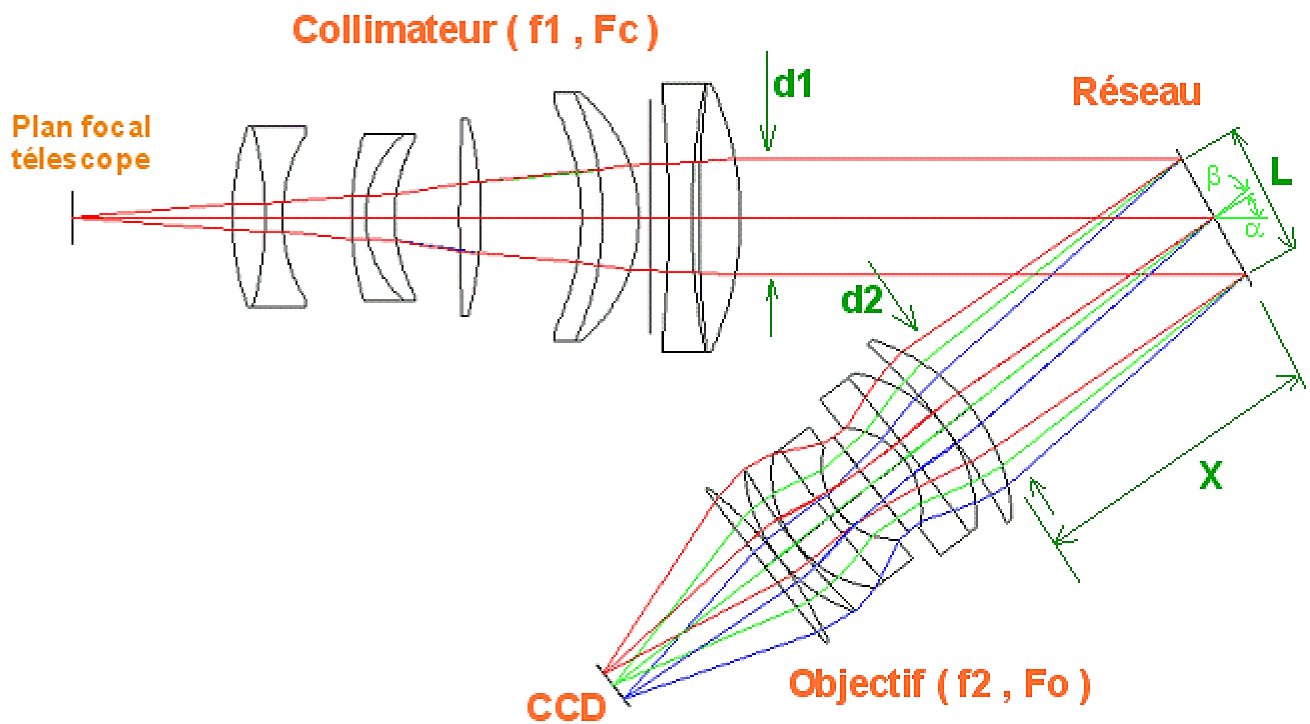


We have mirror + 5-lens corrector + filter + 2-lens corector.

Antireflection coatings

But current optical devices are even more complicated.

In case of astronomical spectrographs we have to deal with many lenses.

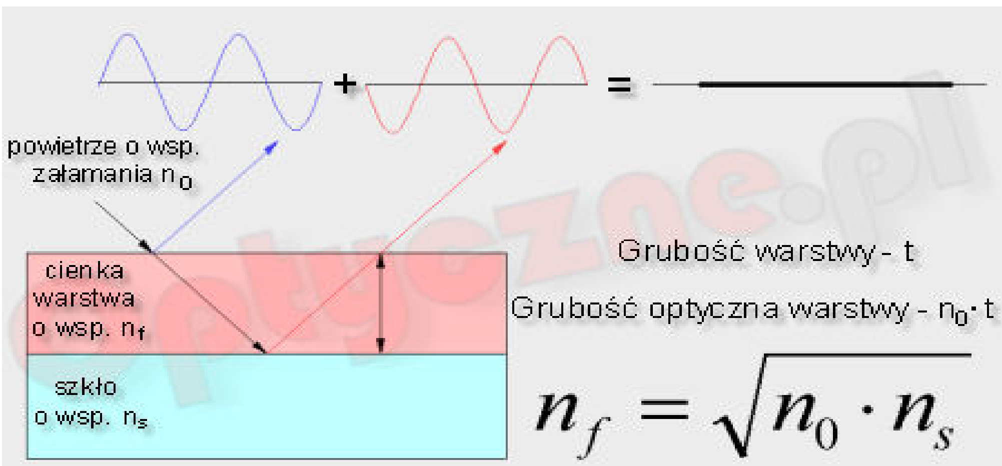


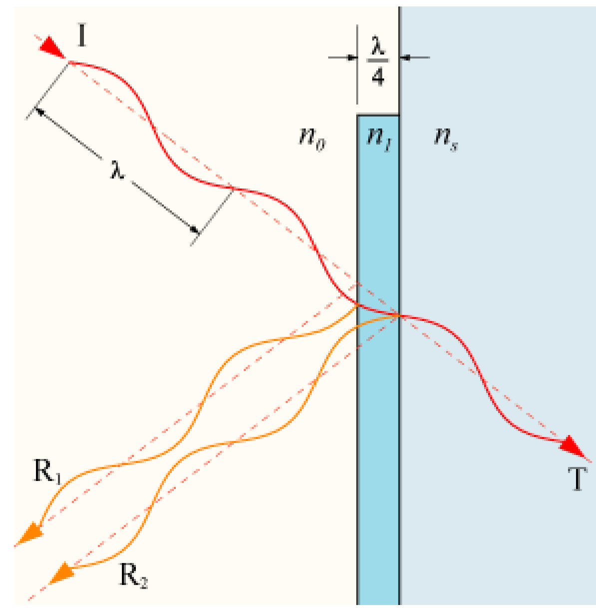
This constructio

If left uncoated the transmission is ~36%.

Antireflection coatings

The solution is to put thin layer of substance on surface of the glass.
The refraction index of this substance should be between the refraction index of air and lens.



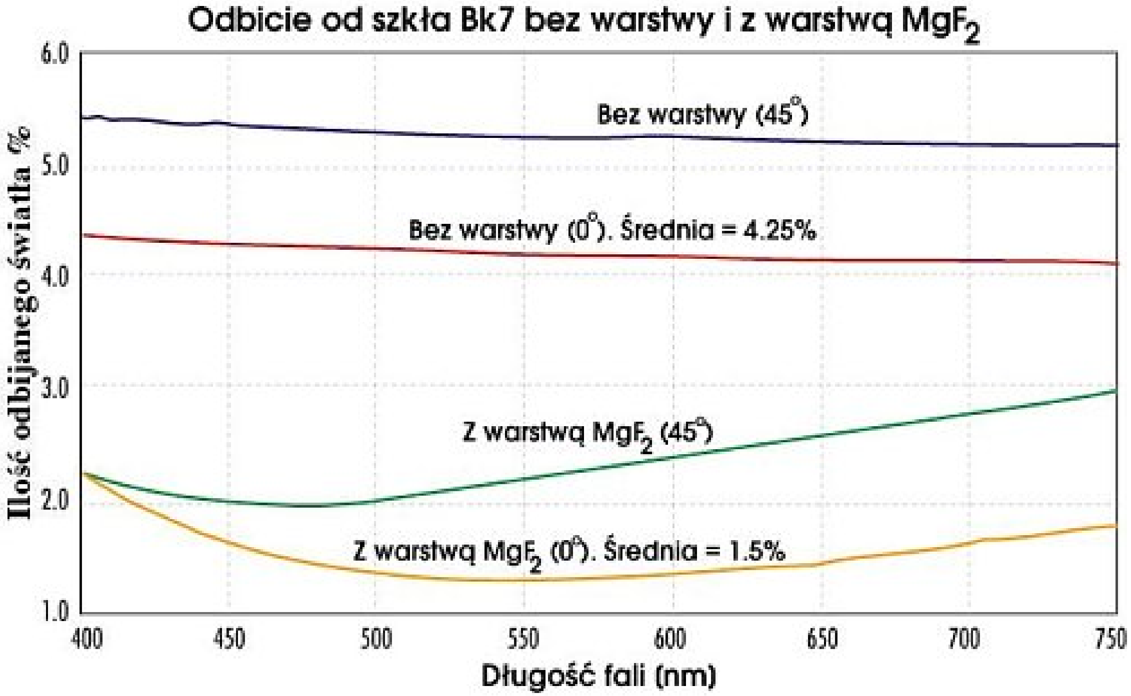


Two effects:

1. there are 2 shallow boundaries instead of one steep → reduction of light loss
2. thickness of the antireflection layer is choosen to cancel out the waves reflected from two boundaries

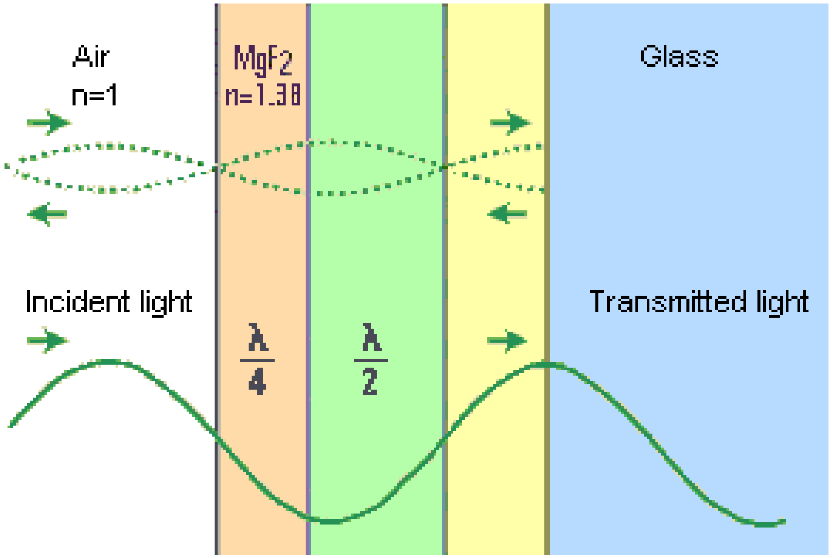
Antireflection coatings

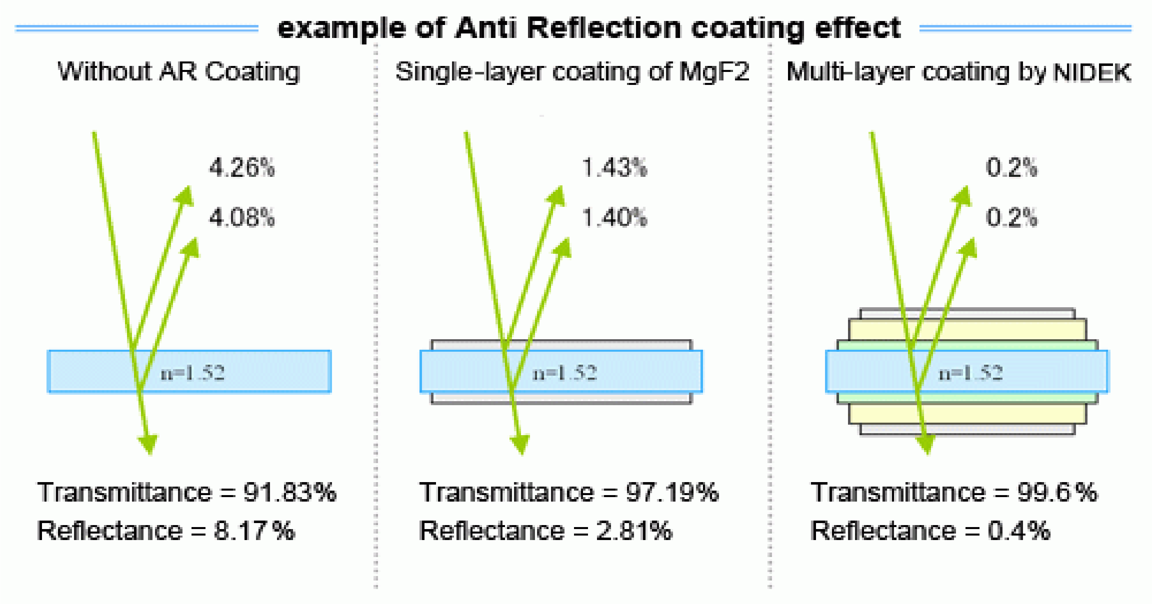
Single layer of magnesium fluoride (n=1.38)



Antireflection coatings

Multilayer coatings



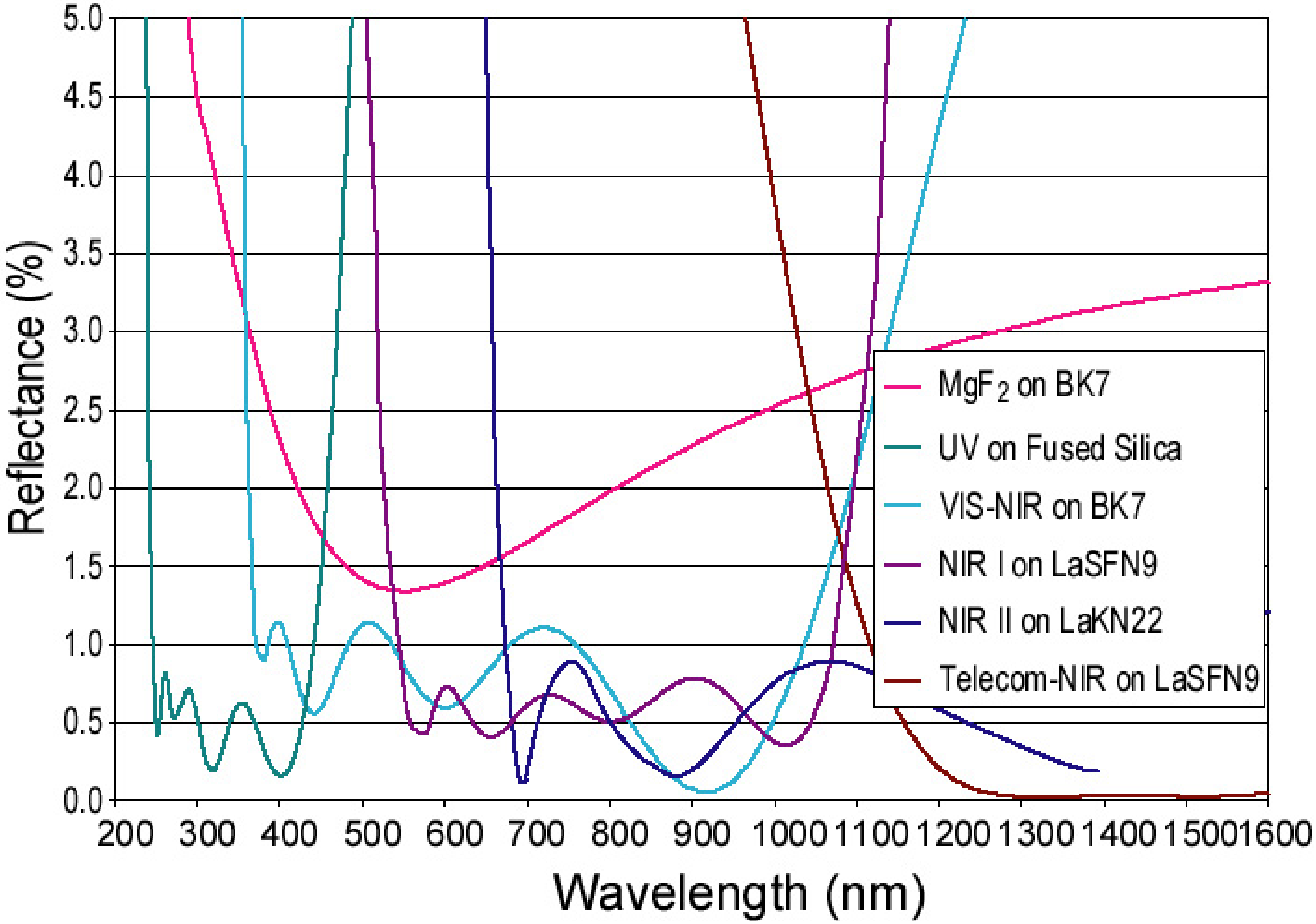


Best multilayer coatings have 0.2-0.3% light loss at one surface.

Antireflection coatings

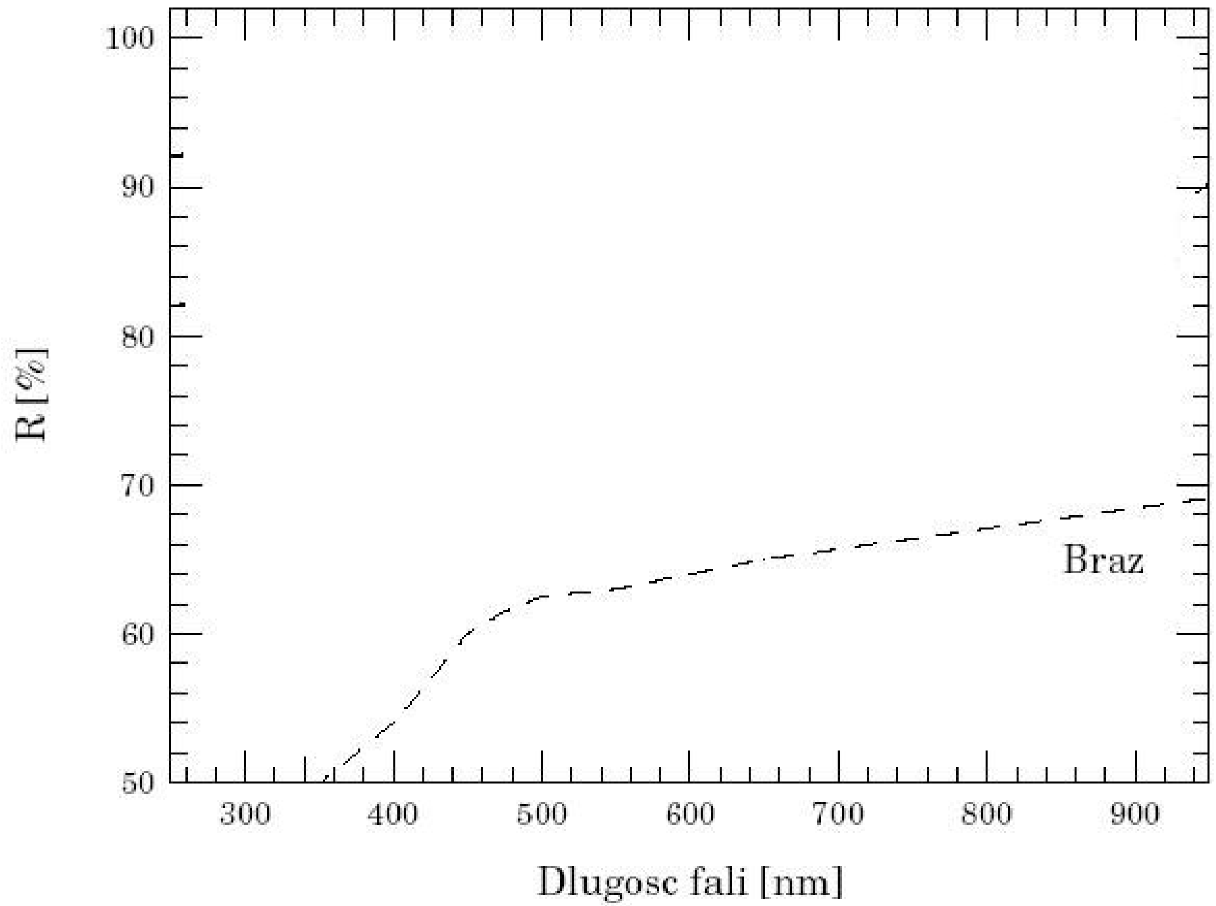
Multilayer coatings

Best multilayer coatings have 0.2-0.3% light loss at one surface.



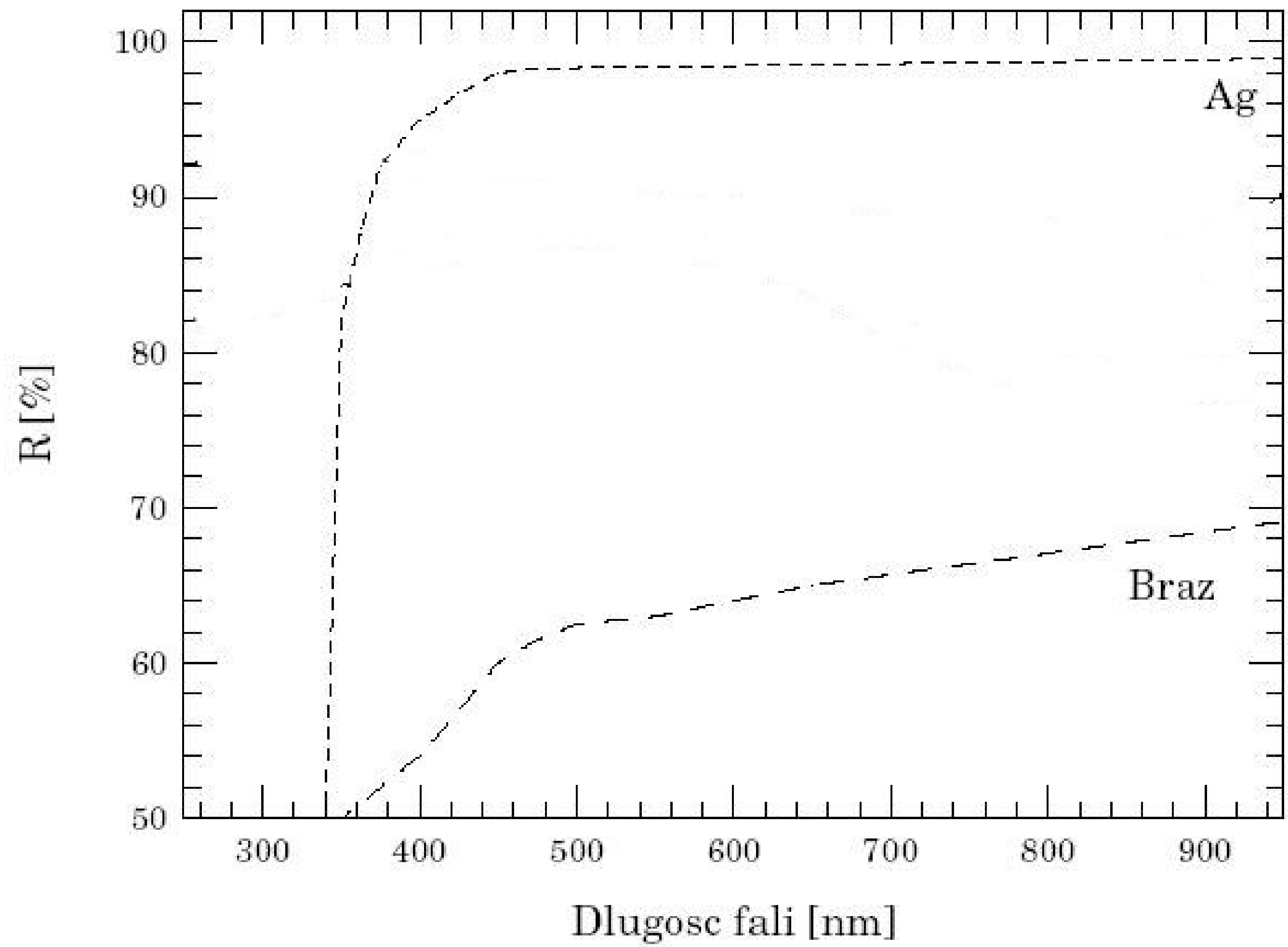
Reflection coatings

For mirrors we want to have highest possible reflectance.
The first mirrors were made of polished bronze with R~60%
For two mirrors in the telescope about 60% of light was lost.



Reflection coatings

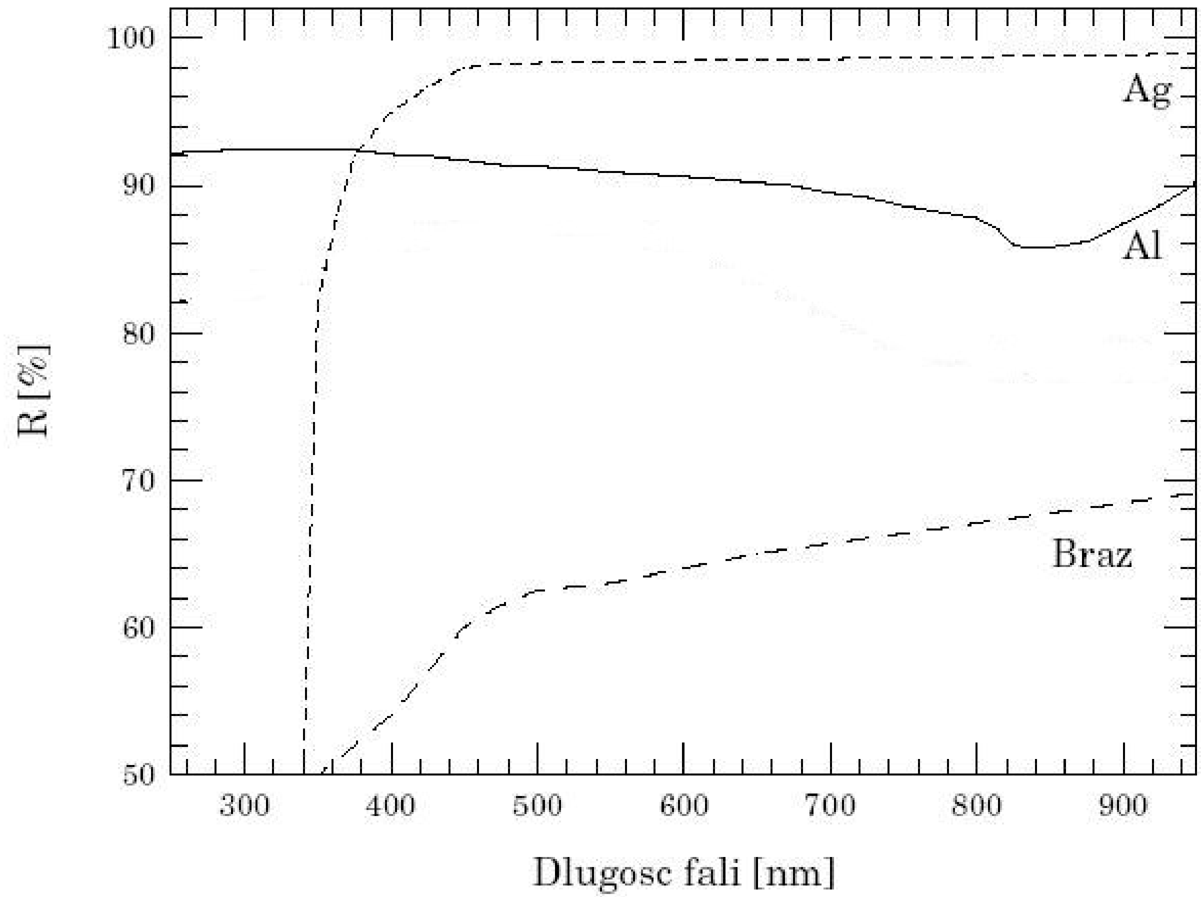
In the middle of XIX century silver coated mirrors appeared.



Much better r ... kly became yellowish) and UV light loss.

Reflection coatings

In 1932 the method of vacuum covering by aluminium.



Lower r

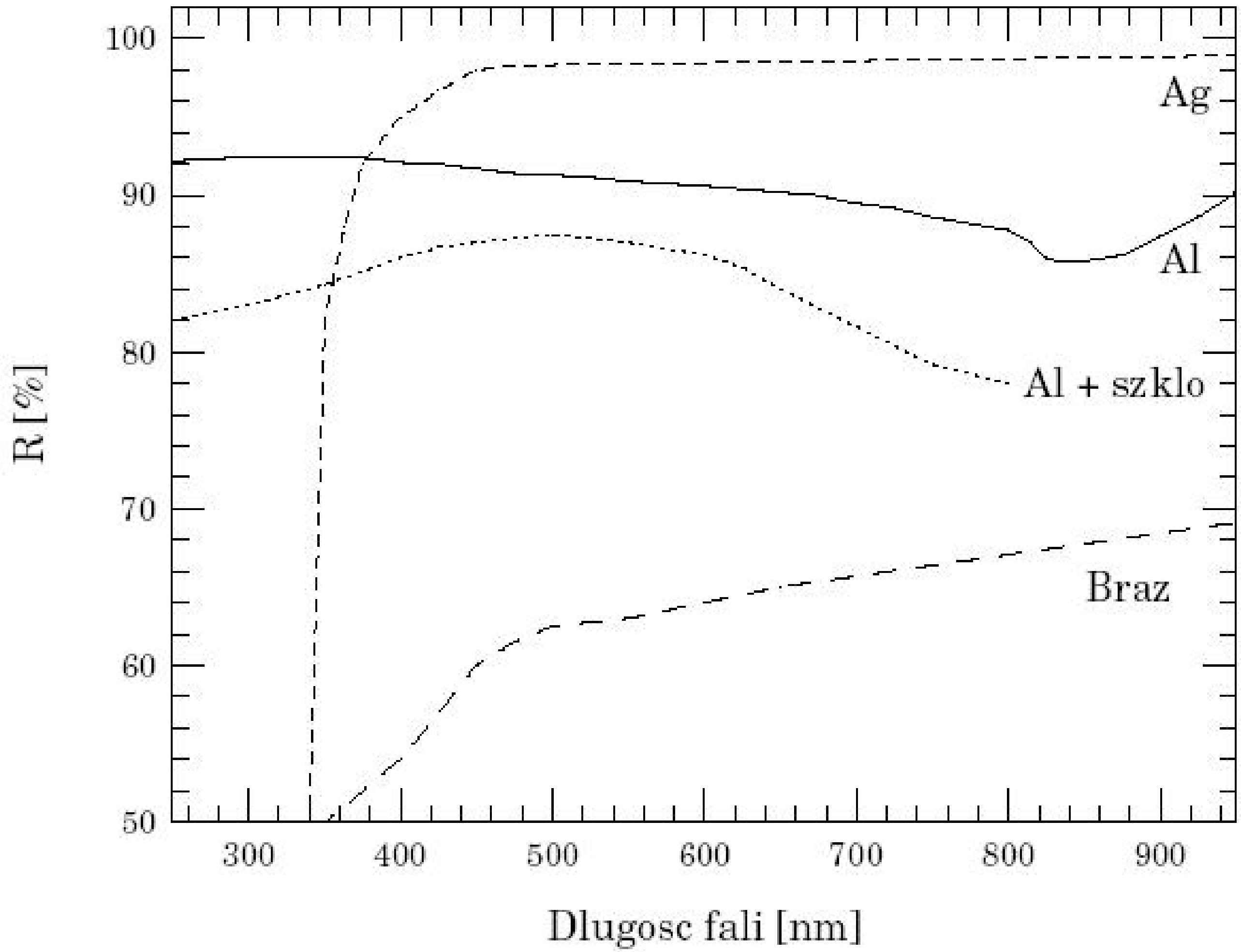
Renovation of the mirror coating is needed every 2-3 years.

No UV light loss.

Reflection coatings

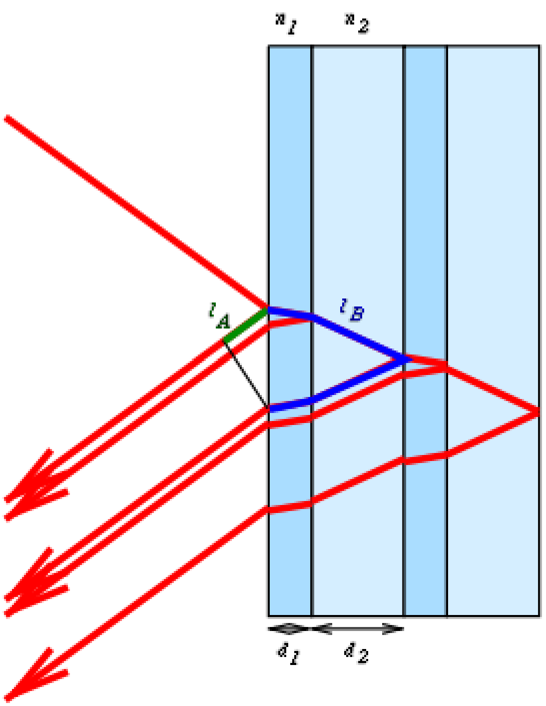
Aluminium layer can be covered by thin (½ lambda) glass layer → protected aluminium with extreme durability. No need for aluminization every 2-3 years.

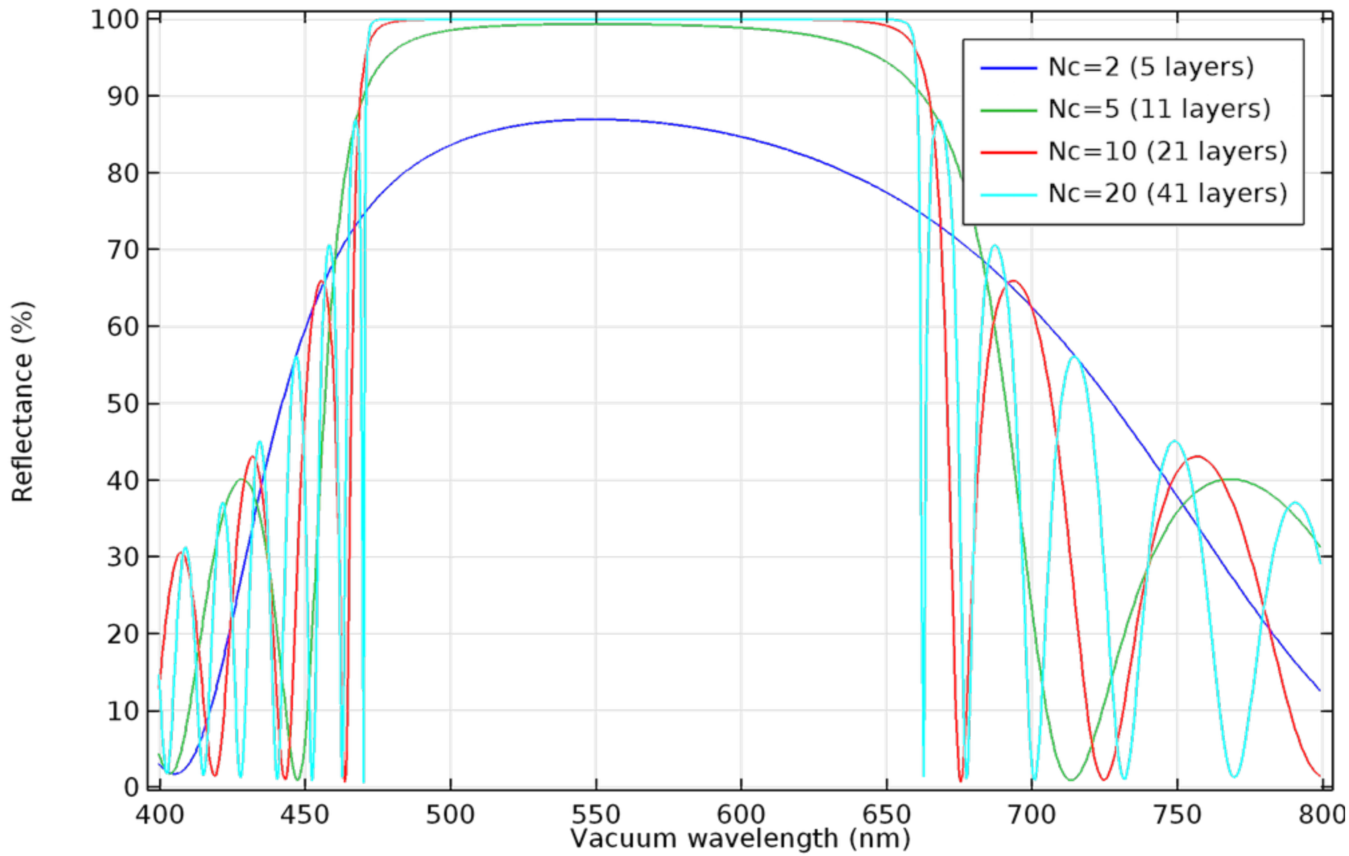
But reflectance of about 82-86%.



Reflection coatings

Thin multilayer dielectric coatings (up to even 60 layers!) with R>99%

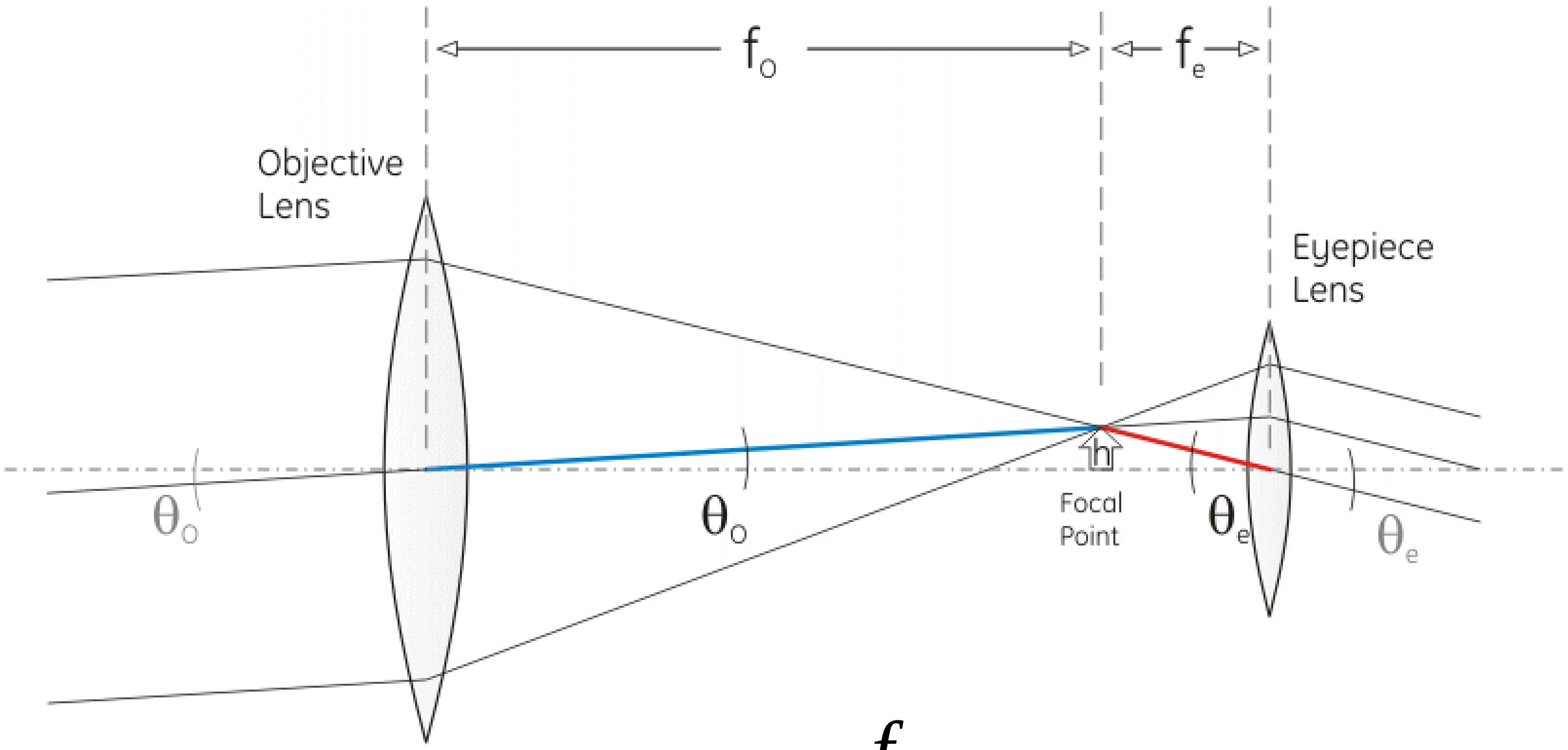




Dielectric coatings are hard → even harder than glass and difficult to scratch.

Useful equations

Magnification

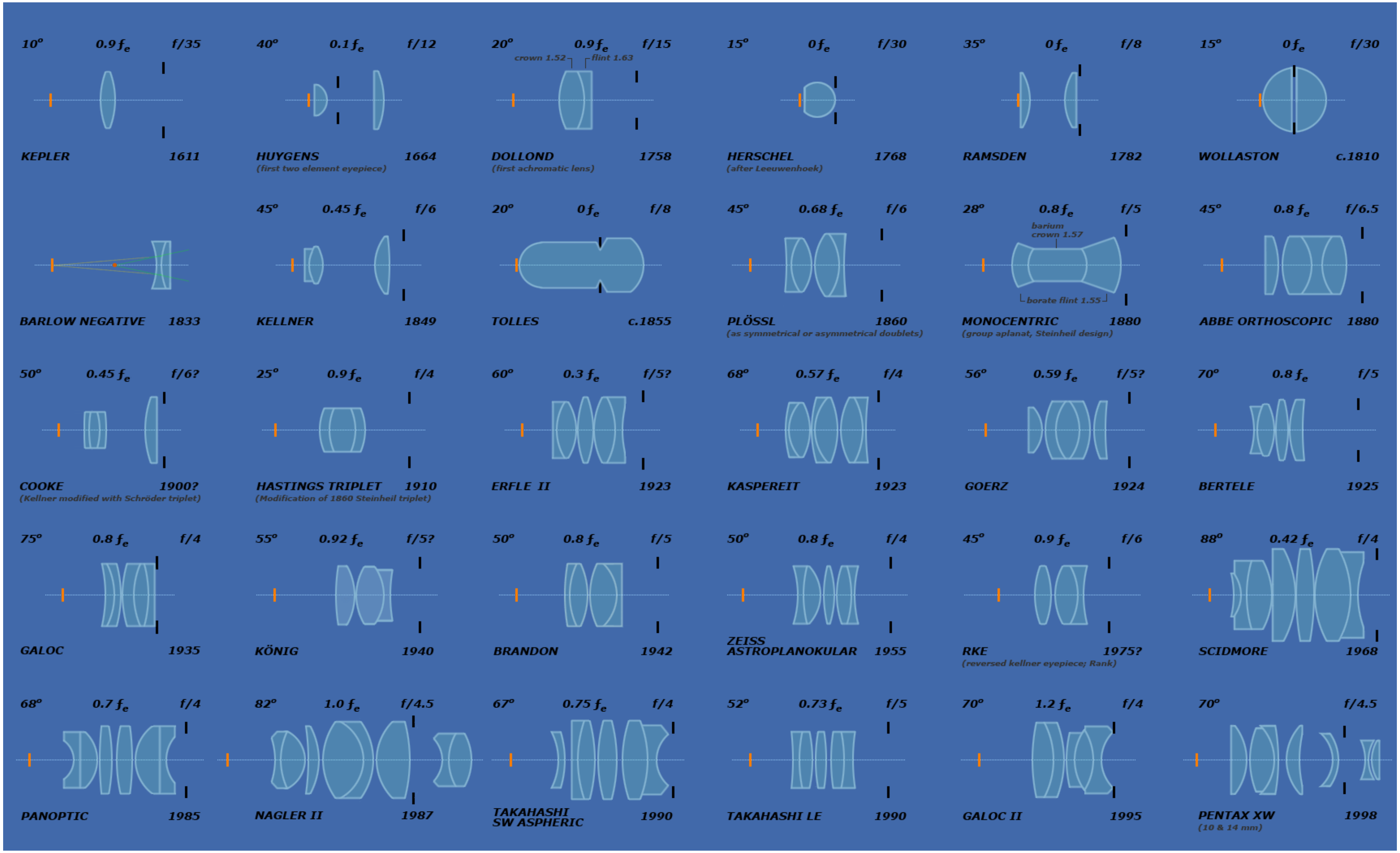


$$M = \frac{f_o}{f_e}$$

Useful equations

Field of view. Apparent field of view of the eyepiece (AFOV) → typically 50-80 deg.

$$FOV_{scope} = \frac{FOV_e}{M}$$

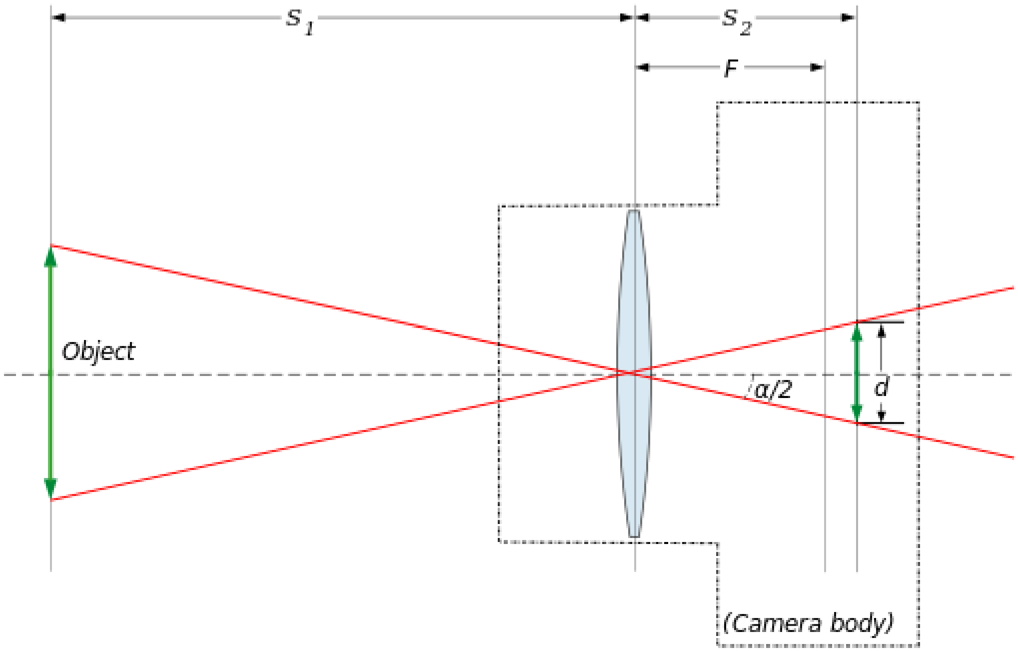


Useful equations

Angle of view →
rectalinear projection (no distrortion).

$$\tan\left(\frac{\alpha}{2}\right)=\frac{d}{2S_2}$$

$$\alpha=2\arctan\frac{d}{2S_2}$$

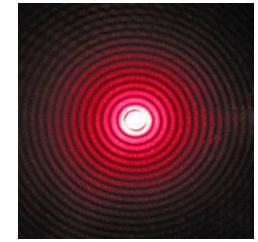


To project a sharp image of distant objects, S2 needs to be equal to the focal length *f*

$$\alpha=2\arctan\frac{d}{2f}$$

Useful equations

Resolution. Diffraction on the circular aperture.



$$I(\theta)=I_0\left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right]^2$$

where:

$$k=\frac{2\pi}{\lambda}$$

a - radius of the aperture

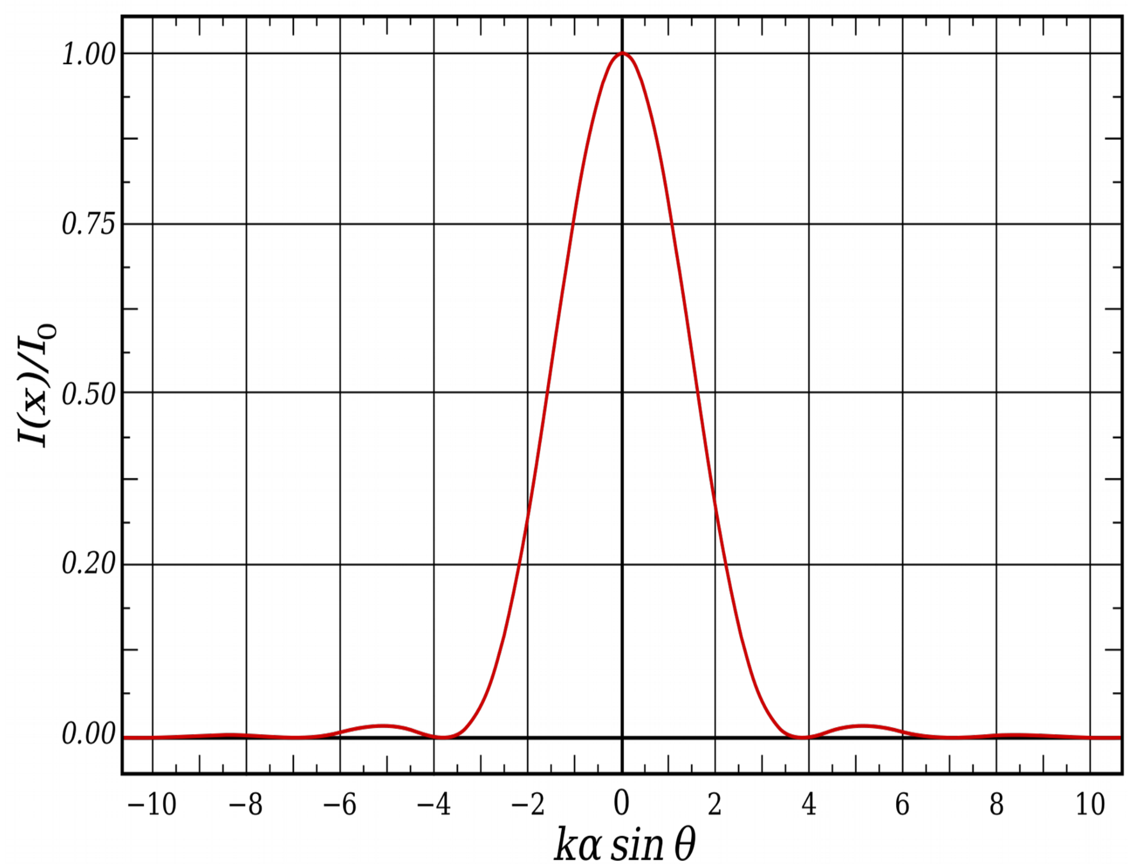
Zeros for:

$$\sin\theta=\frac{1.916\lambda}{\pi r},\ \frac{3.508\lambda}{\pi r},\ \frac{5.087\lambda}{\pi r},\ \ldots$$

$$\theta\simeq\frac{1.220\lambda}{d},\ \frac{2.233\lambda}{d},\ \frac{3.238\lambda}{d}.\ \ldots$$

The first zero gives the equation for resolution:

$$\alpha=\frac{1.220\lambda}{d}$$

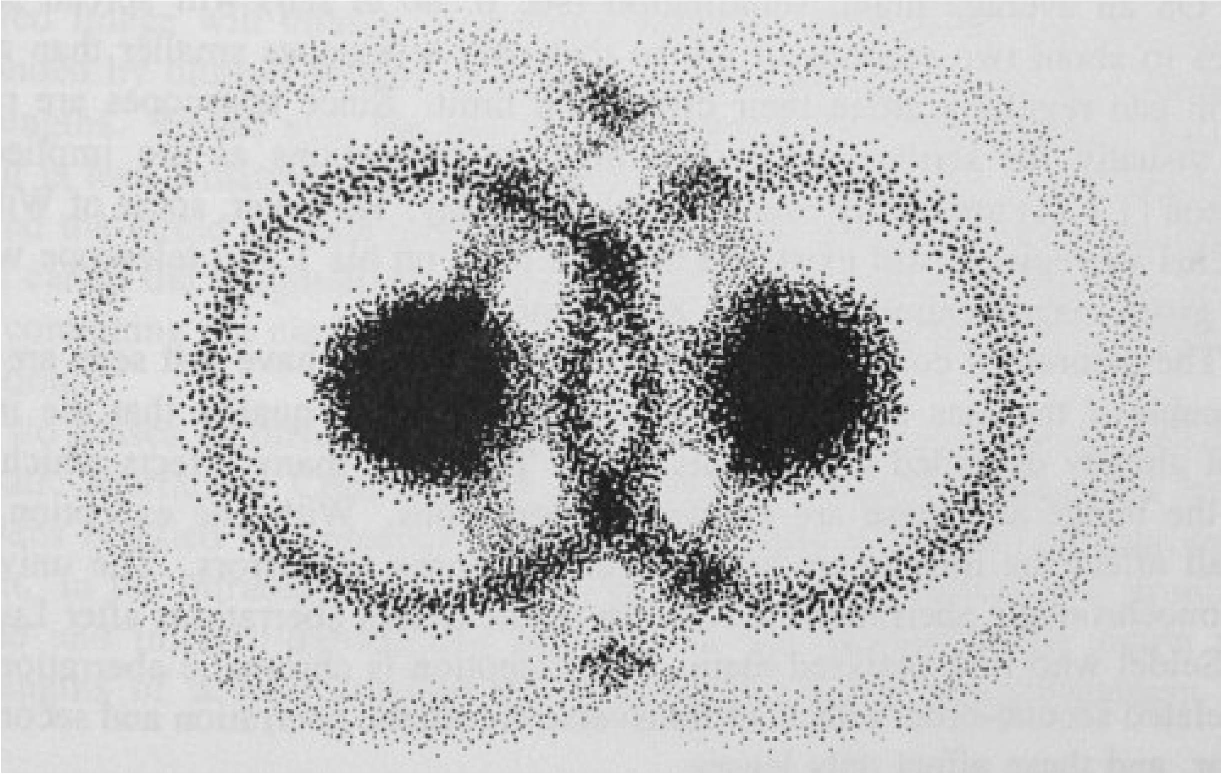


Useful equations

Resolution. The highest sensitivity of human eye is for 510 nm. This gives:

$$R = 0.128/D$$

Where R is resolution in arc seconds and D is diameter in meters.



Useful equations

Limiting magnitude

Amount of collected light is proportional to second power of diameter: $I \sim D^2$

m_1 = 6.5 mag is limiting magnitude for naked eye,

m_2 – limiting magnitude for the telescope,

d = 0.75 cm diameter of the human pupil in the darkness,

a – light loss caused by the optics.

From Pogson law:

$$m_1 - m_2 = -5 \log\left(\frac{d}{D}\right)$$

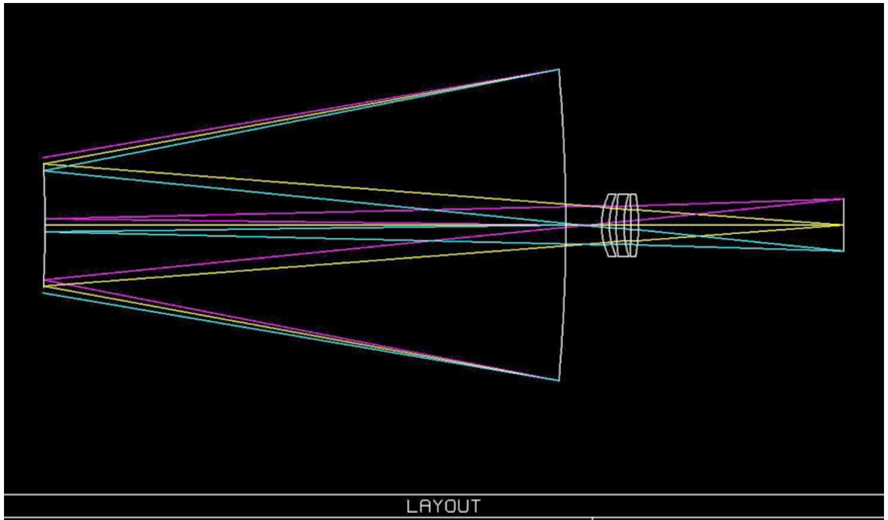
$$m_2 = 7.1 + 5 \log\left(\sqrt{a} \cdot D\right)$$

Useful equations

Limiting magnitude

Parameters of our telescope

Orion ODK (Optimized Dall-Kirkham).



ODK12
Optical design and light path through telescope. Large back focus and three element computer optimised corrector lens system shown which enable a flat field of over 50mm to be acheived.

Specification	ODK12
Primary mirror diameter	300mm
Focal length	2040mm
Focal Ratio	f6.8
Tube Weight (inc. mount plate/dovetail)	19kg
Tube Diameter	350mm
Tube Length	615mm
BFD (Back Focal Distance) from cell face*	218mm
Tube Material	Carbon fibre sandwich
Secondary Size	120mm
Mirror Cell	Honeycombe CNC machined alum. cell
Focuser	3" Crayford with 10:1 reduction
Corrector Design	Multi coated 3 lens
Flat Field Size	>52mm
Spot Size on Axis	1.8μ
Spot Size at Field Edge	5.6μ
Mirror / Tube Cooling	3 Controllable fans. Heaters optional
Attachment Method	CNC machined alum. dovetail with supports

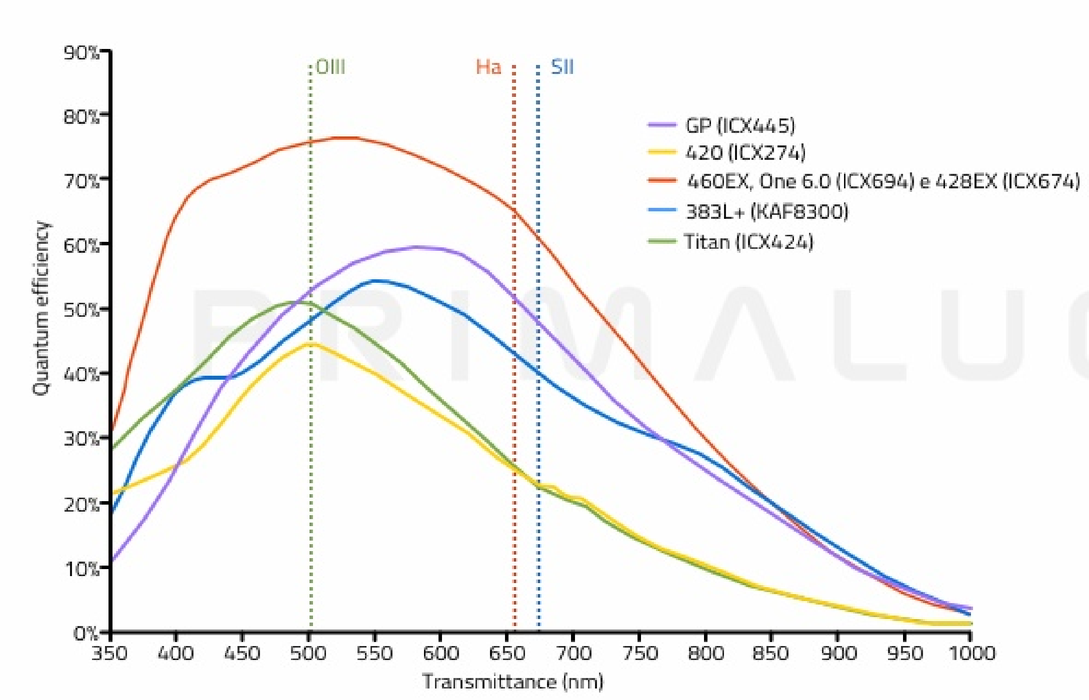
Parameters of our telescope

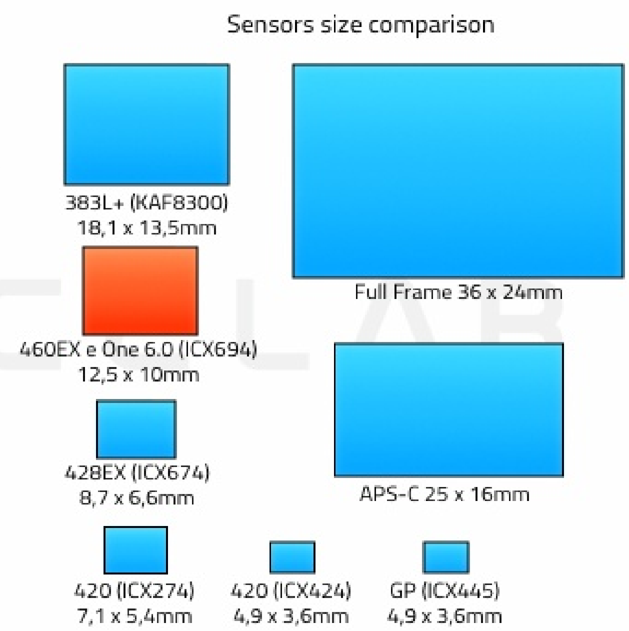
ATIK 460EX monochrome CCD camera.

	Atik 420	Atik 450	Atik 428EX	Atik 460EX
Sensor Type	Sony ICX274	Sony ICX655	Sony ICX674	Sony ICX694
Resolution	1620x1220 pixels	2448x2050 pixels	1932x1452 pixels	2750x2200 pixels
Pixel Size	4.4x4.4µm	3.45x3.45µm	4.54x4.54µm	4.54x4.54µm
ADC	16 bit	16 bit	16 bit	16 bit
Readout Noise (typical)	5 e^-	5 e^-	5e-	5e-
Interface	USB	USB	USB	USB
Power	12V DC 1A	12V DC 1A	12V DC 1A	12V DC 1A
Maximum Exposure	Unlimited	Unlimited	Unlimited	Unlimited
Minimum Exposure	1/1000 s	1/1000s	1/1000s	1/1000s
Cooling	Thermoelectric	Thermoelectric	Thermoelectric	Thermoelectric
	Delta T = 30ºC	Delta T = 30ºC	Delta T = 25ºC	Delta T = 25ºC
Weight	Approx. 400 g	Approx. 400g	Approx. 400g	Approx. 400g
Backfocus distance	13mm	13mm	13mm	13mm

Parameters of our telescope

ATIK 460EX monochrome CCD camera.





For f=2040 mm, 12.5x10 mm 2750x 2200 pix detector:

→ field of view = 21.04' x 16.83'

→ scale 0.46”/pixel

Parameters of our telescope

ASA DDM60 Pro equatorial mount of german type.

Easy transportable,
low current consumption

Fast polar alignment with
laser and software

Loading capacity 28kg
(with counterweights 56kg)

Connection to all
popular telescopes
(tube ring- and railsystem)

Connection to all
common tripods
(Losmandy, Berlebach, etc.)

DDM60 PRO:

USB-Hub, all power and
data connections from
the DEC-axis