

Practical observational astronomy

Lecture 2

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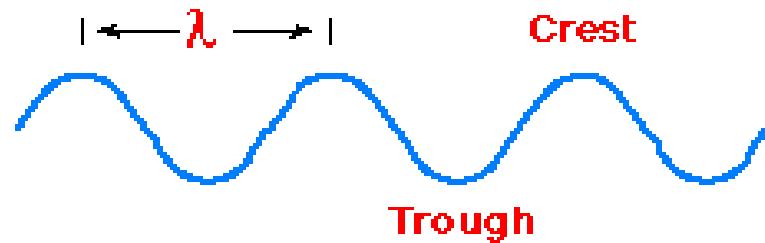
Based on lectures by Arkadiusz Olech

Warszawa, 2019-05-16



Electromagnetic radiation

Wave-particle duality



$$\lambda = \frac{c}{\nu}$$

$$E = h\nu = \frac{hc}{\lambda}$$

$$c = 3 \times 10^8 [m \cdot s^{-1}]$$

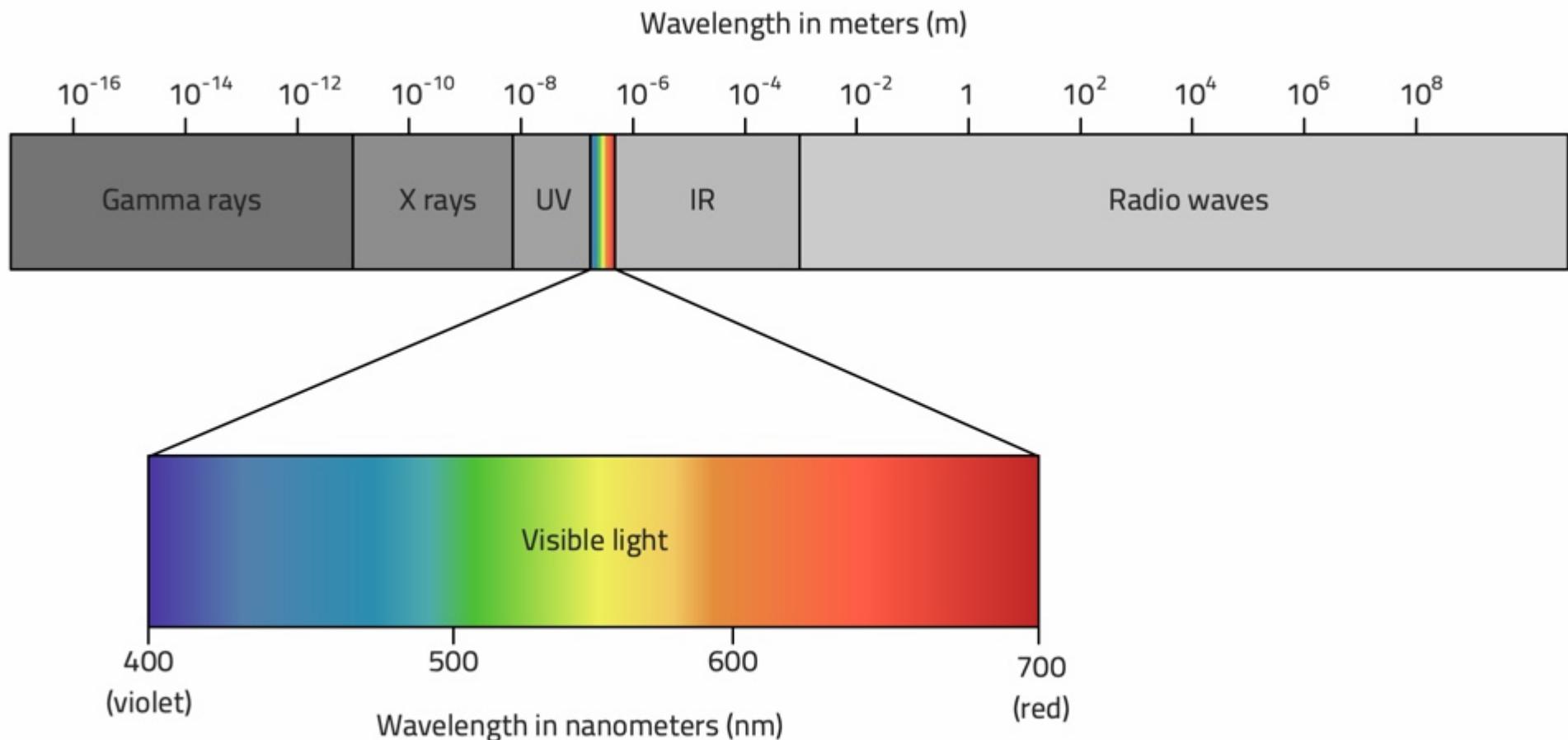
$$h = 4.135 \times 10^{-15} [eV \cdot s] = 6.625 \cdot 10^{-27} [erg \cdot s]$$



Electromagnetic Radiation Spectrum

Region	Wavelength [Amstroems]	Wavelength [centimeters]	Frequency [Hz]	Energy [eV]
Radio	$>10^9$	>10	$< 3 \cdot 10^9$	$< 10^{-5}$
Microwave	$10^9 - 10^6$	$10 - 0.01$	$3 \cdot 10^9 - 3 \cdot 10^{12}$	$10^{-5} - 0.01$
Infrared	$10^6 - 7000$	$0.01 - 7 \cdot 10^5$	$3 \cdot 10^{12} - 4.3 \cdot 10^{14}$	$0.01 - 2$
Visible	$7000 - 4000$	$7 \cdot 10^{-5} - 4 \cdot 10^{-5}$	$4.3 \cdot 10^{14} - 7.5 \cdot 10^{14}$	$2 - 3$
Ultraviolet	$4000 - 10$	$4 \cdot 10^{-5} - 10^{-7}$	$7.5 \cdot 10^{14} - 3 \cdot 10^{17}$	$3 - 10^3$
X-rays	$10 - 0.1$	$10^{-7} - 10^{-9}$	$3 \cdot 10^{17} - 3 \cdot 10^{19}$	$10^3 - 10^5$
Gamma-rays	<0.1	$<10^{-9}$	$>3 \cdot 10^{19}$	$>10^5$

Electromagnetic Radiation Spectrum



Electromagnetic radiation

Black body - black body is an idealized physical body that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence.

Planck's law:

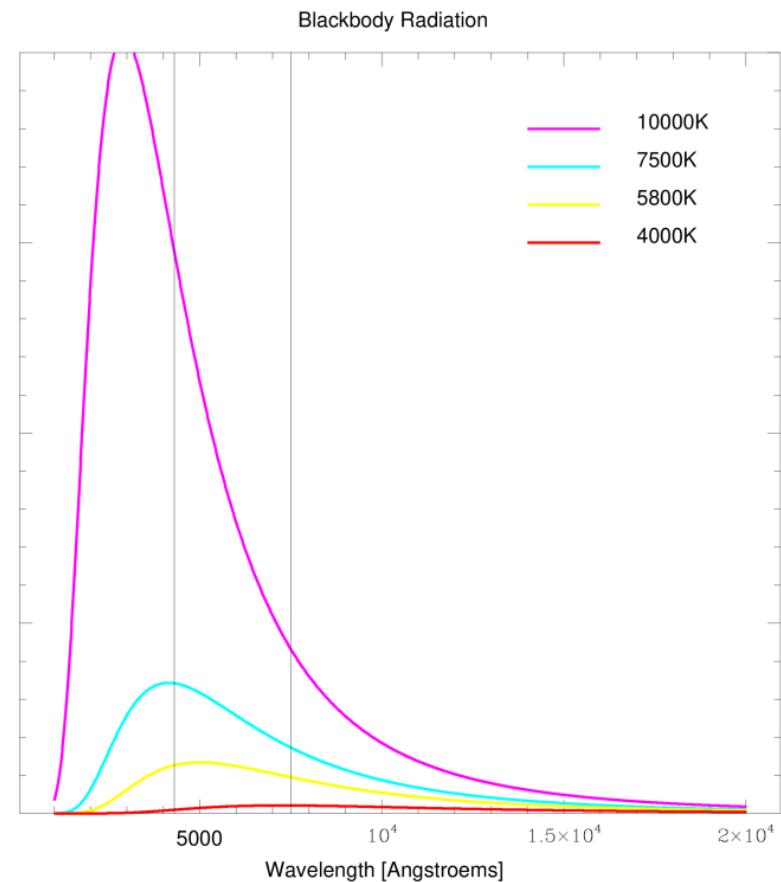
$$E(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right) - 1}$$

$$k_B = 1.38 \times 10^{-16} \text{ [erg K}^{-1}\text{]}$$

Stefan-Boltzman law:

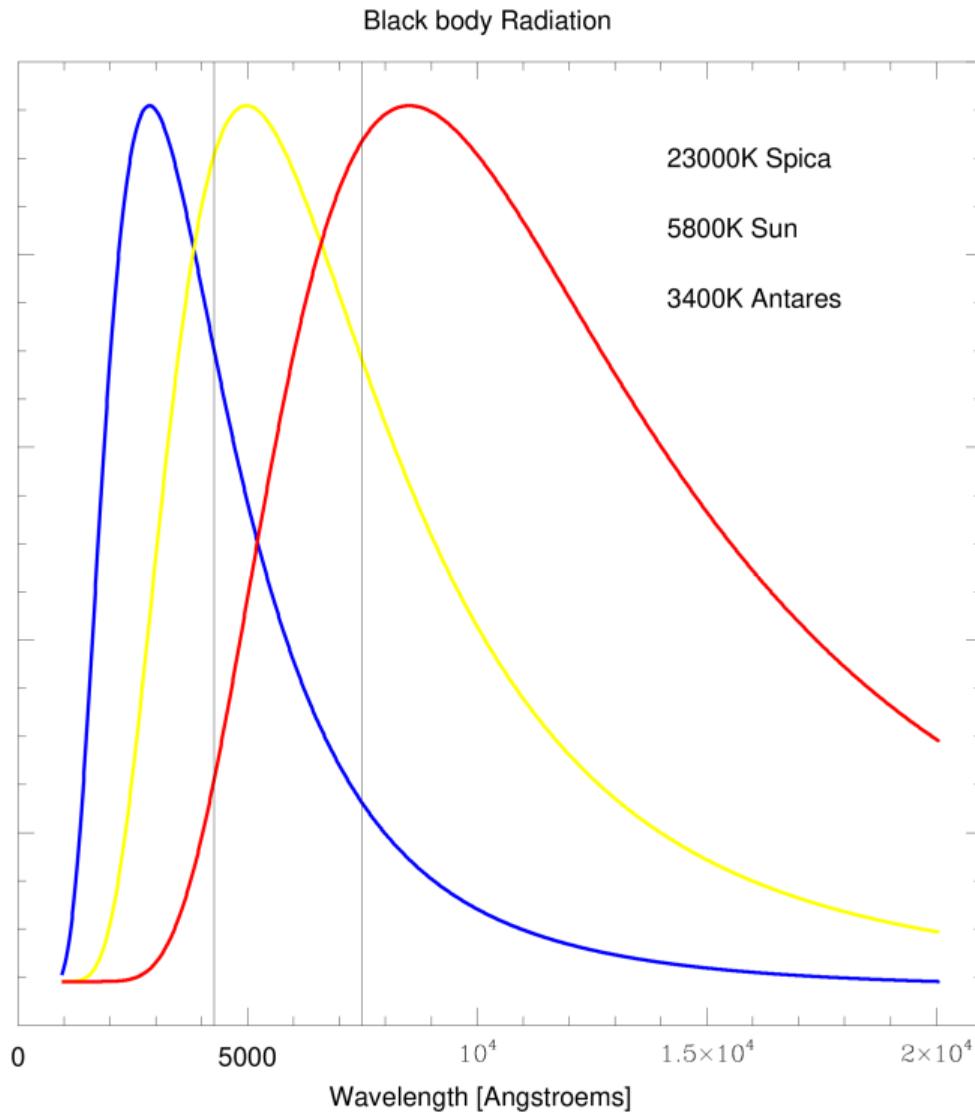
$$E(T) = \int_0^{\infty} E(\lambda, T) d\lambda = \sigma T^4$$

$$\sigma = 5.6705 \times 10^{-5} \text{ [erg} \cdot \text{cm}^2 \cdot \text{K}^{-2} \cdot \text{s}^{-1}\text{]}$$



Electromagnetic radiation

Wien's displacement law



$$\lambda_{max} = \frac{b}{T}$$



Electromagnetic radiation

Energy emitted from unit area of the star (BB radiation) from Stefan-Boltzman law:

$$E = \sigma T_{eff}^4$$

Energy emitted from the whole star (luminosity):

Po polsku dobrze jest mówić moc promieniowania zamiast jasność.

$$L = 4\pi R^2 E = 4\pi R^2 \sigma T_{eff}^4$$

Flux (intensity) observed on the Earth:

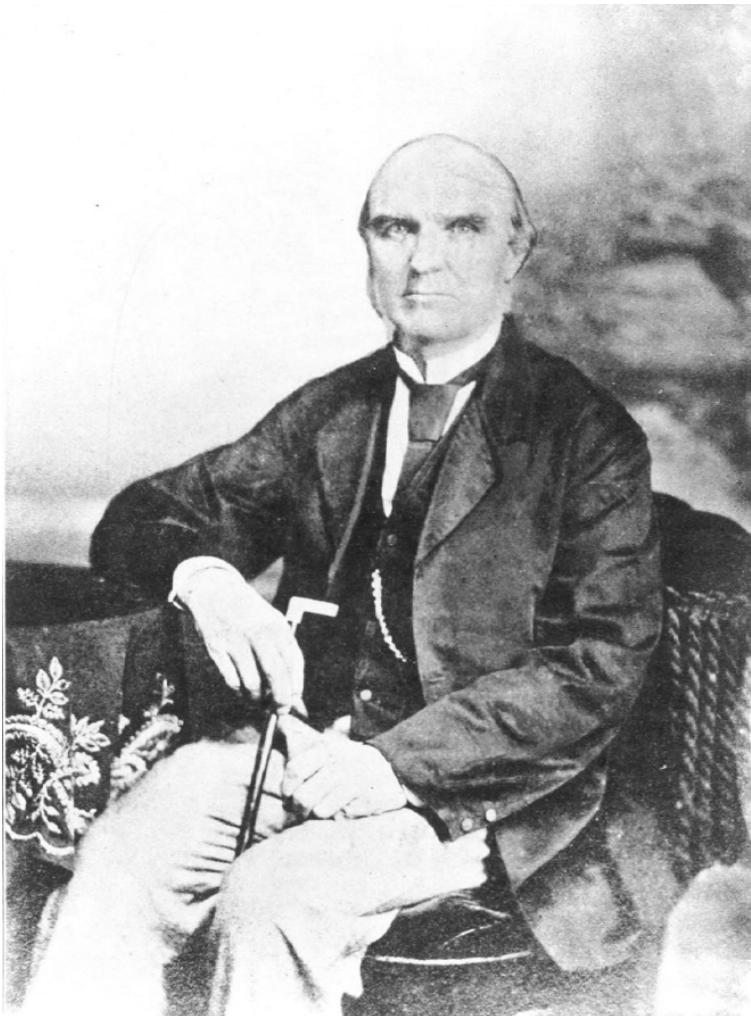
$$I = \frac{L}{4\pi d^2} = \sigma T_{eff}^4 \cdot \left(\frac{R}{d}\right)^2$$



Magnitude scale

In 1856 N.R. Pogson proposed the precise magnitude scale. There were only two assumptions:

1. One magnitude difference produces constant difference in brightness
2. 5 magnitude difference is equivalent to 100 ratio in brightness



$$\frac{I_m}{I_{m+1}} \approx 2.512$$

$$\frac{I_m}{I_{m+5}} = 100$$

Magnitude scale

Pogson law – relation between magnitudes m_1 and m_2 and intensities I_1 and I_2 :

$$m_2 - m_1 = -2.5 \log_{10} \left(\frac{I_2}{I_1} \right)$$

For one star:

$$m = -2.5 \log(I) + C$$

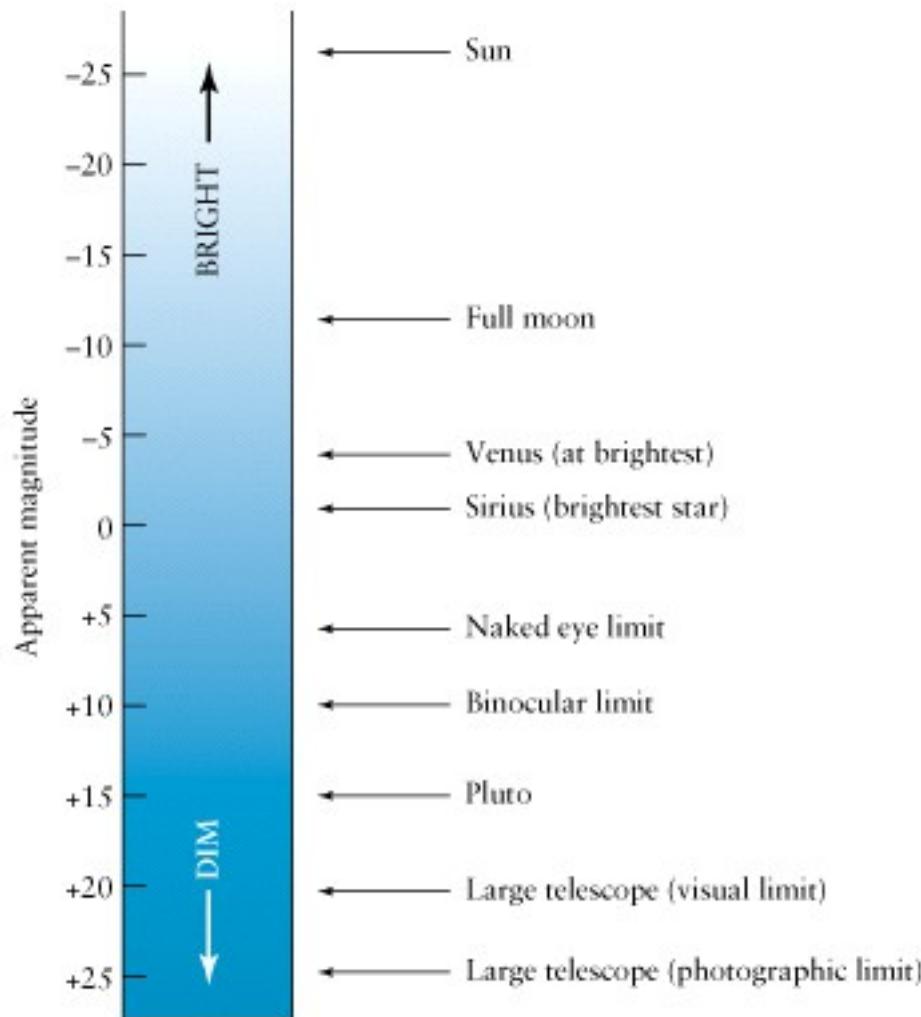
where C is the zero-point offset.

In 1950s visual magnitude and color indices of Vega (A0 main sequence star) were set to zero.

Modern zero-point calculations make use of synthetic colors from model atmospheres and spectrophotometric observations. As a result, the visual magnitude of Vega is now +0.03 mag.



Magnitude scale



Magnitude scale

Absolute magnitude M - brightness of the star as seen from 10 parsecs distance.

From Pogson law:

$$m - M = -2.5 \log \left(\frac{I_1}{I_2} \right)$$

$I \sim 1/D^2$, then:

$$m - M = 5 \log \left(\frac{D_1}{D_2} \right)$$

$D_2 = 10$ pc, then:

$$m - M = 5 \log(D_1) - 5$$

Value $m - M$, which depends only on distance is called **distance modulus**.

$$(m - M) = 5 \log_{10}(d) - 5$$



Magnitude scale

Distance modulus

Distance Modulus	
Distance in Magnitude ($m_v - M_v$)	Distance (pc)
0	10
1	16
2	25
3	40
4	63
5	100
6	160
7	250
8	400
9	630
10	1000
15	10000
20	100000



New magnitude scale

Lupton, Gunn, and Szalay, 1999, AJ, 118, 1406

„asinh magnitudes”

Used by Sloan Digital Sky Survey (SDSS)

$$\mu = (m_0 - 2.5 \log b) - a \sinh^{-1} \left(\frac{f}{2b} \right)$$

where:

$$a = 2.5 \log e$$

b - softening parameter

f - measured flux

New magnitude scale

Asymptotic behavior

$$\lim_{x \rightarrow \infty} \mu(x) = -a \ln x = m$$

$$\lim_{x \rightarrow 0} \mu(x) = -a \left(\frac{x}{2} b + \ln b \right)$$

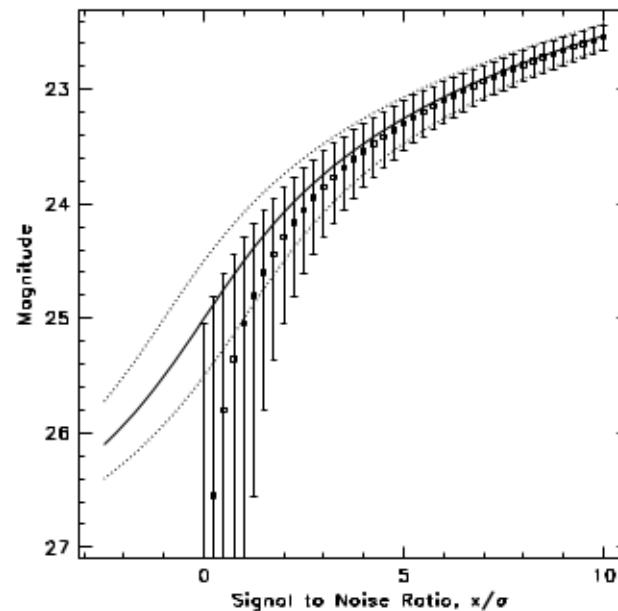


FIG. 2.—Behavior of m and μ and their respective errors as a function of signal-to-noise ratio x/σ . The solid line is the value of μ , and the region between the dotted lines its $\pm 1\sigma$ error region; the points with error bars are the classical magnitudes m . We have arbitrarily chosen a zero point of $\mu = 25.0$ for an object with no flux. One other feature of our modified

Photometric systems

The flux which we observe consists of:

$$f_{obs} = \int_0^{\infty} f(\lambda) p_1(\lambda) p_2(\lambda) p_3(\lambda) d\lambda$$

where:

$f(\lambda)$ flux over the atmosphere

$p_1(\lambda)$ atmospheric transmission

$p_2(\lambda)$ telescope characteristic

$p_3(\lambda)$ detector characteristic

The characteristic of the telescope may consist of optics properties and filters used.



Photometric systems

In 1953 H.L. Johnson and W.W. Morgan established a new photometric system based on three filters *UBV* with:

- B* – corresponding to the characteristic of typical photographic film
- V* – corresponding to visual magnitude
- U* – collecting light in violet and ultraviolet border.

Exact description in Johnson (1963, *Basic Astronomical Data*):

- telescope with aluminium covered mirrors,
- detector is photomultiplier 1P21,
- for *V* Corning 3384 filter is used,
- for *B* Corning 5030 + Schott CG13 filters are used,
- for *U* Corning 9863 filter is used.

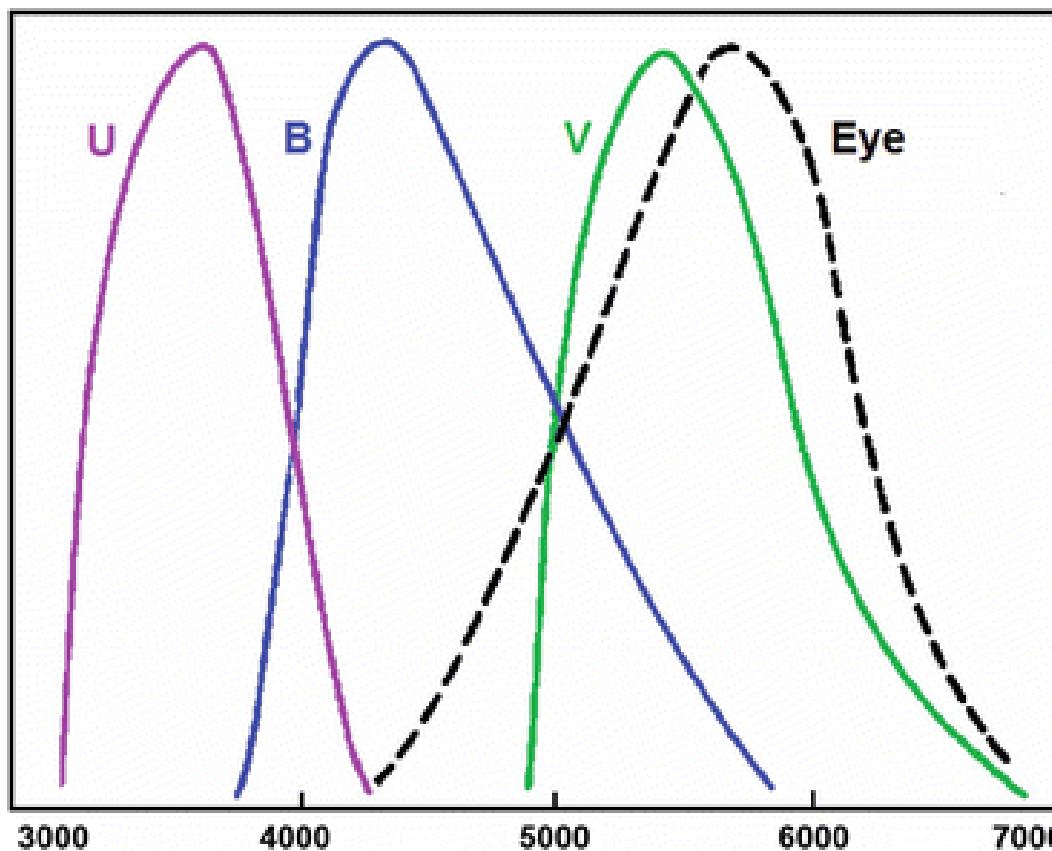
Telescope at altitude of >2000 meters to allow the detection of sufficient amount of UV light.



Photometric systems

Johnson-Morgan *UBV* system:

- *U* with max at 365 nm and FWHM=70 nm
- *B* with max at 440 nm and FWHM=100 nm
- *V* with max at 550 nm and FWHM=90 nm

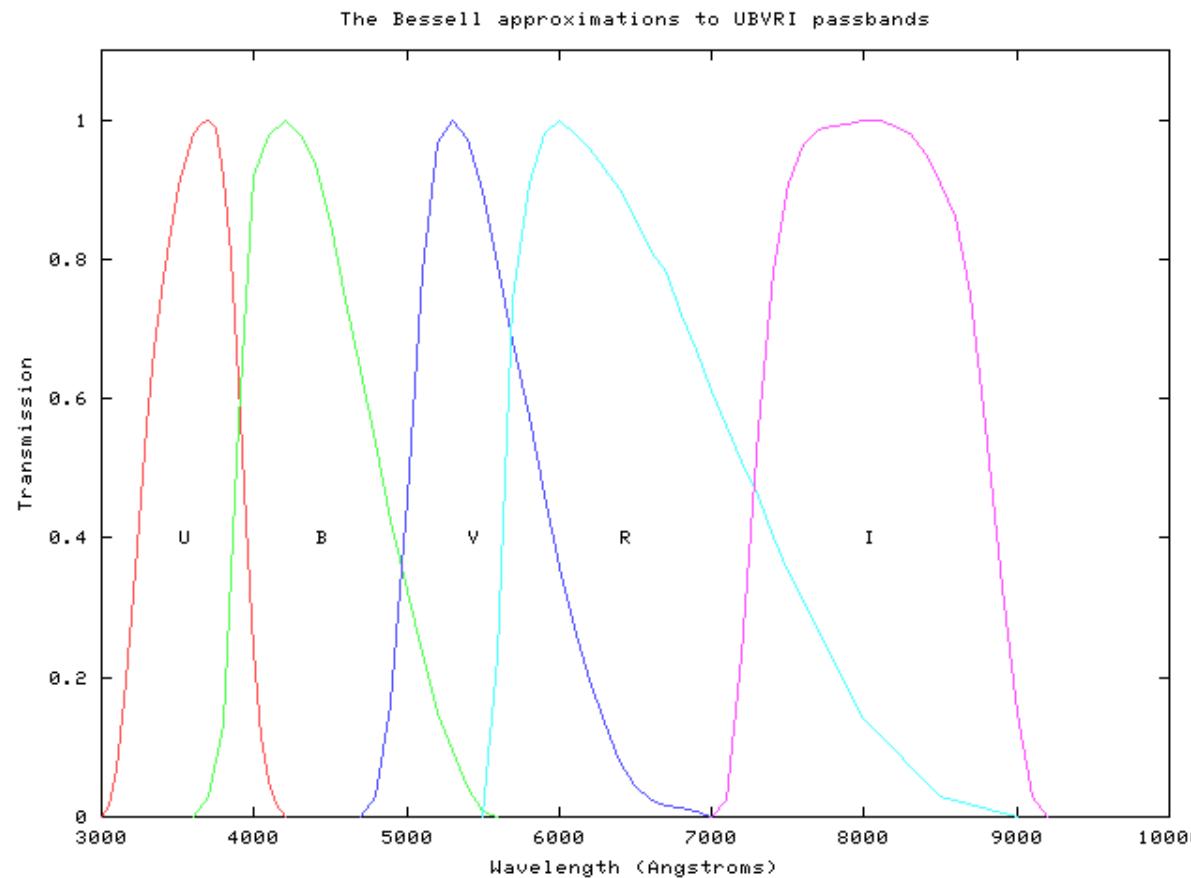


Photometric systems

Johnson-Morgan proposed also R and I filters but were too wide (FWHM ~ 220 - 240 nm).

Kron-Cousins R and I filters were narrower and became popular due to high sensitivity of electronic detectors to red light.

- R with maximum at 650 nm and FWHM of 100 nm
- I with maximum at 800 nm and FWHM of 150 nm



Colors

Having the photometric system one can define the **color**:

$$(m_B - m_V) = (B - V) = 2.5 \log \left(\frac{f_V}{f_B} \right)$$

You can also introduce so called **bolometric correction** which is the difference in magnitude between brightness measured in whole spectrum and particular filter:

$$m_{bol} - m_V = BC = M_{bol} - M_V$$

Zero point of BC scale is adopted for main sequence star with T=6500 K (F5 V).

Then the luminosity of the star is:

$$L_* = 3 \times 10^{28} \times 10^{-0.4 M_{bol}} \text{ [W]}$$

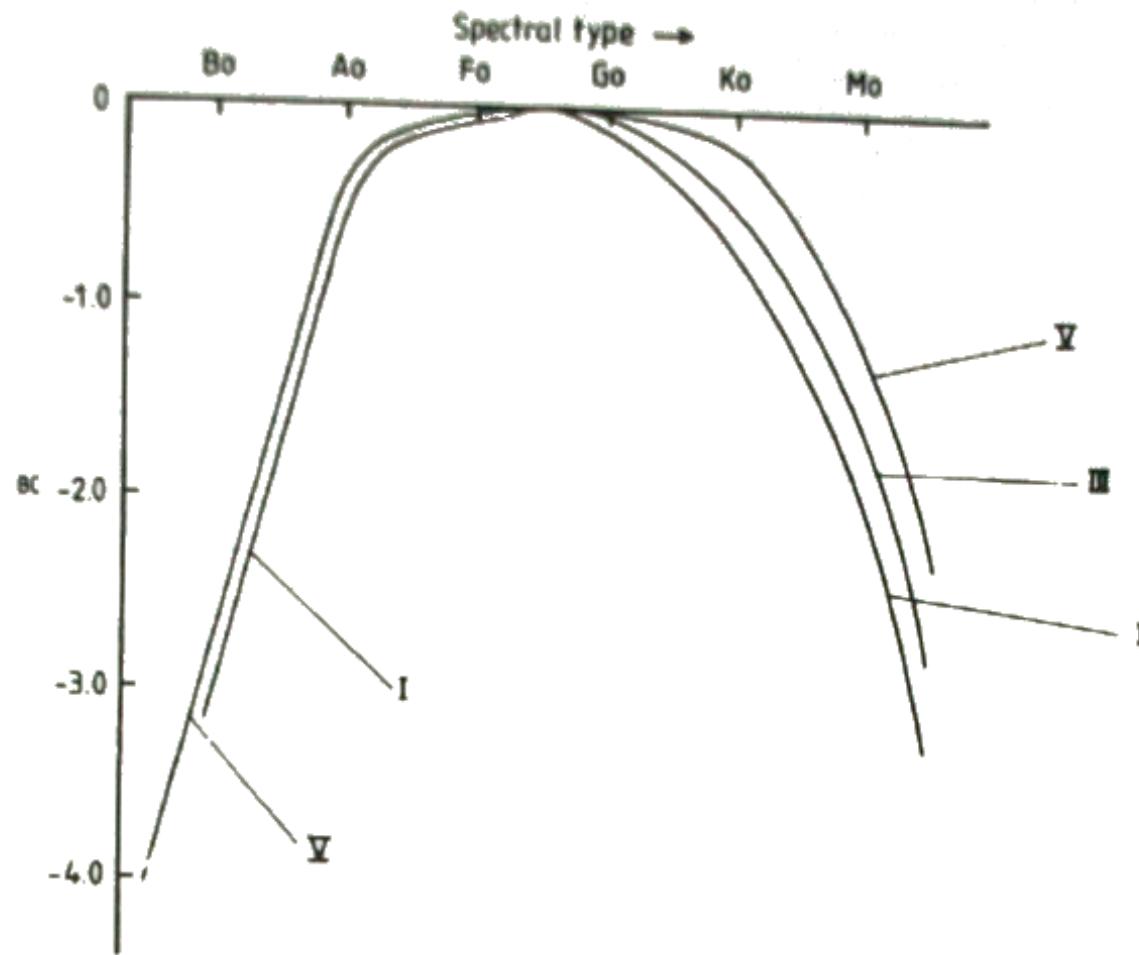
And flux over the atmosphere:

$$f_* = 2.5 \times 10^{-8} \times 10^{-0.4 m_{bol}} \text{ [W m}^{-2}\text{]}$$



Colors

Bolometric correction



Colors

$B-V$ index is good measure of temperature (the same is for $V-I$).

Starting from Planck and Pogson laws and assuming that B and V are monochromatic filters with 440 and 550 nm.

$$F(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right) - 1}$$

$$B-V = -2.5 \log \left(\frac{(5.5 \times 10^{-7})^5 \left[\exp\left(\frac{6.62 \times 10^{-34} \times 3 \times 10^8}{5.5 \times 10^{-7} \times 1.38 \times 10^{-23} \times T}\right) - 1 \right]}{(4.4 \times 10^{-7})^5 \left[\exp\left(\frac{6.62 \times 10^{-34} \times 3 \times 10^8}{4.4 \times 10^{-7} \times 1.38 \times 10^{-23} \times T}\right) - 1 \right]} \right)$$

$$B-V = -2.5 \log \left(3.05 \frac{\exp\left(\frac{2.617 \times 10^4}{T}\right) - 1}{\exp\left(\frac{3.27 \times 10^4}{T}\right) - 1} \right)$$



Colors

For $T < 10000$ K we can make approximation:

$$B-V \approx -2.5 \log \left(3.05 \frac{\exp\left(\frac{2.617 \times 10^4}{T}\right)}{\exp\left(\frac{3.27 \times 10^4}{T}\right)} \right) = -1.21 + \frac{7090}{T}$$

Zero point has to be changed that $B-V=0$ for A0 star (Vega)

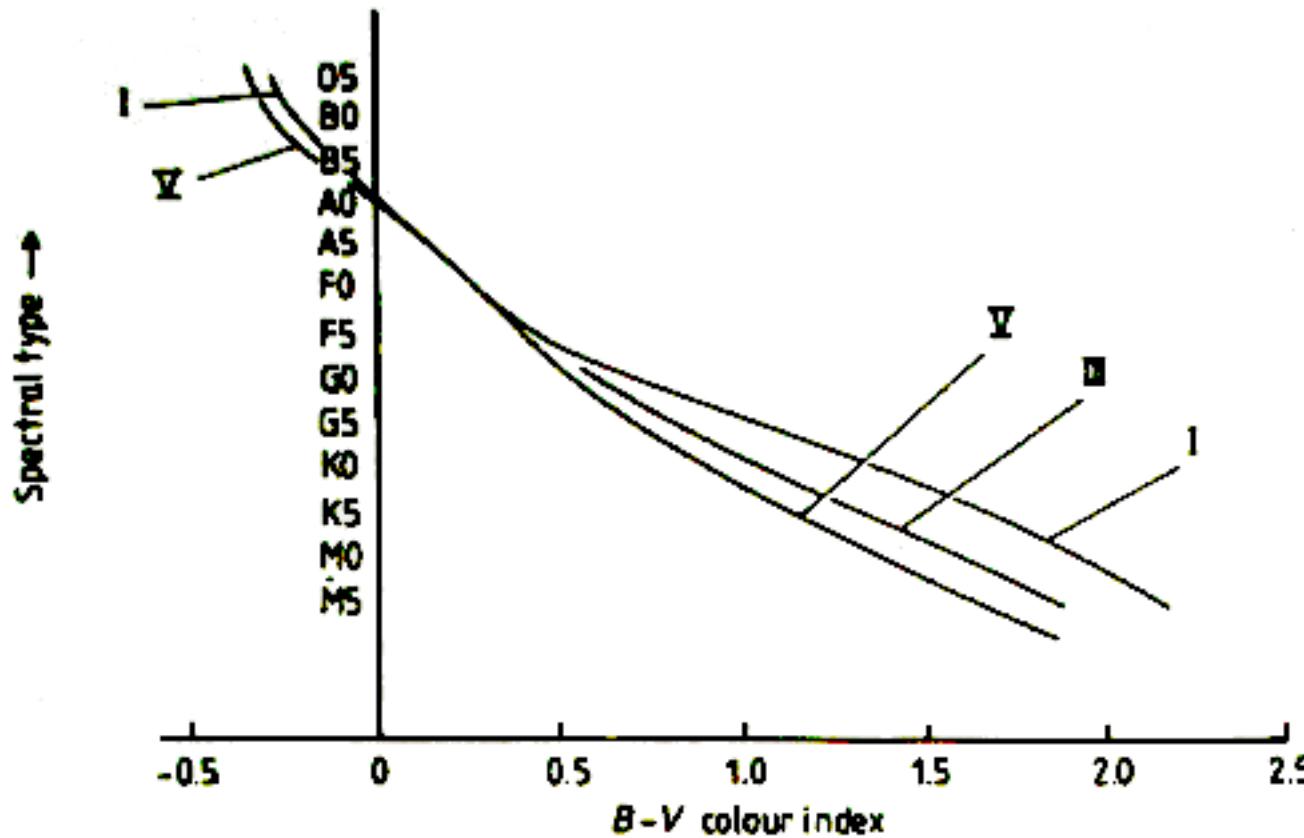
$$B-V = -0.71 + 7090 T^{-1}$$

$$T = \frac{7090}{(B-V) + 0.71} \quad [K]$$



Colors

In general the relation between B-V and temperature is:



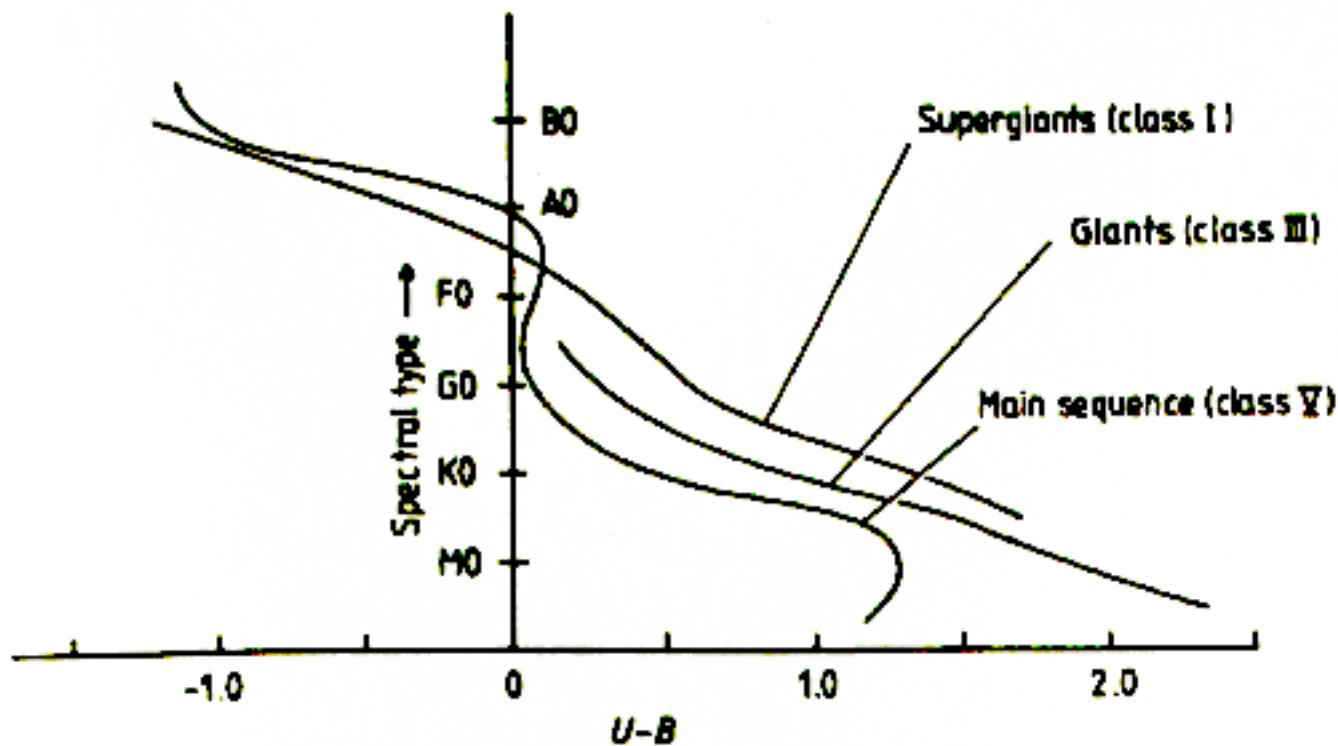
And can be approximated for temperature range 4000-10000 K with:

$$T = \frac{8540}{(B-V)+0.865} \text{ [K]}$$



Colors

$U-B$ index is not good for measuring the temperature:



Colors

$U-B$ index was introduced to measure the Balmer discontinuity

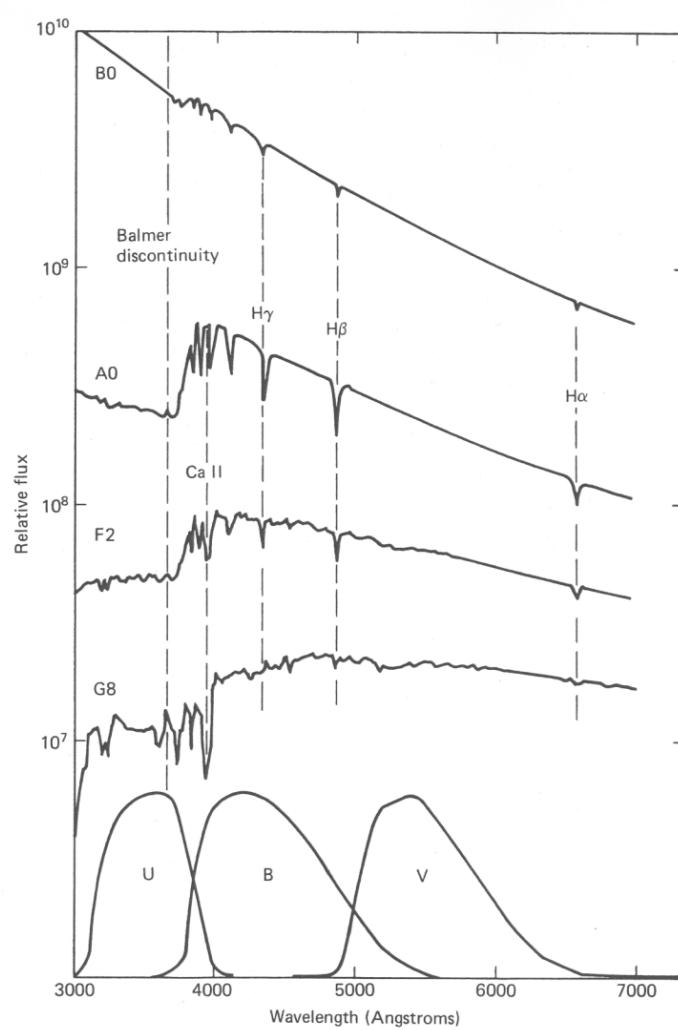


Figure 2.3 Spectra of some main sequence stars.

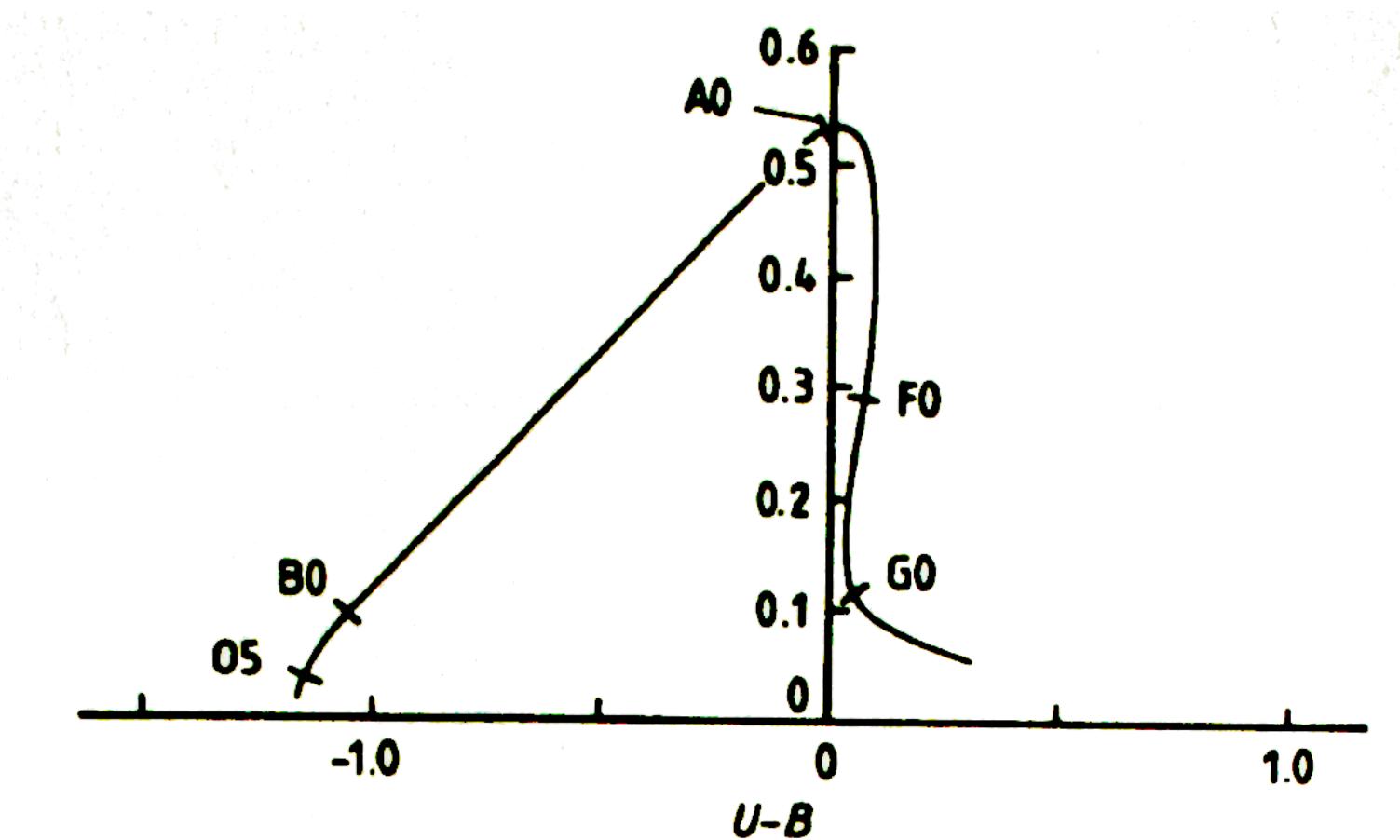


Colors

But $U-B$ is good indicator of Balmer discontinuity.

$$D = \log \left(\frac{I_{365+}}{I_{365-}} \right)$$

Where $I_{365+/-}$ is energy emitted above/below 365 nm

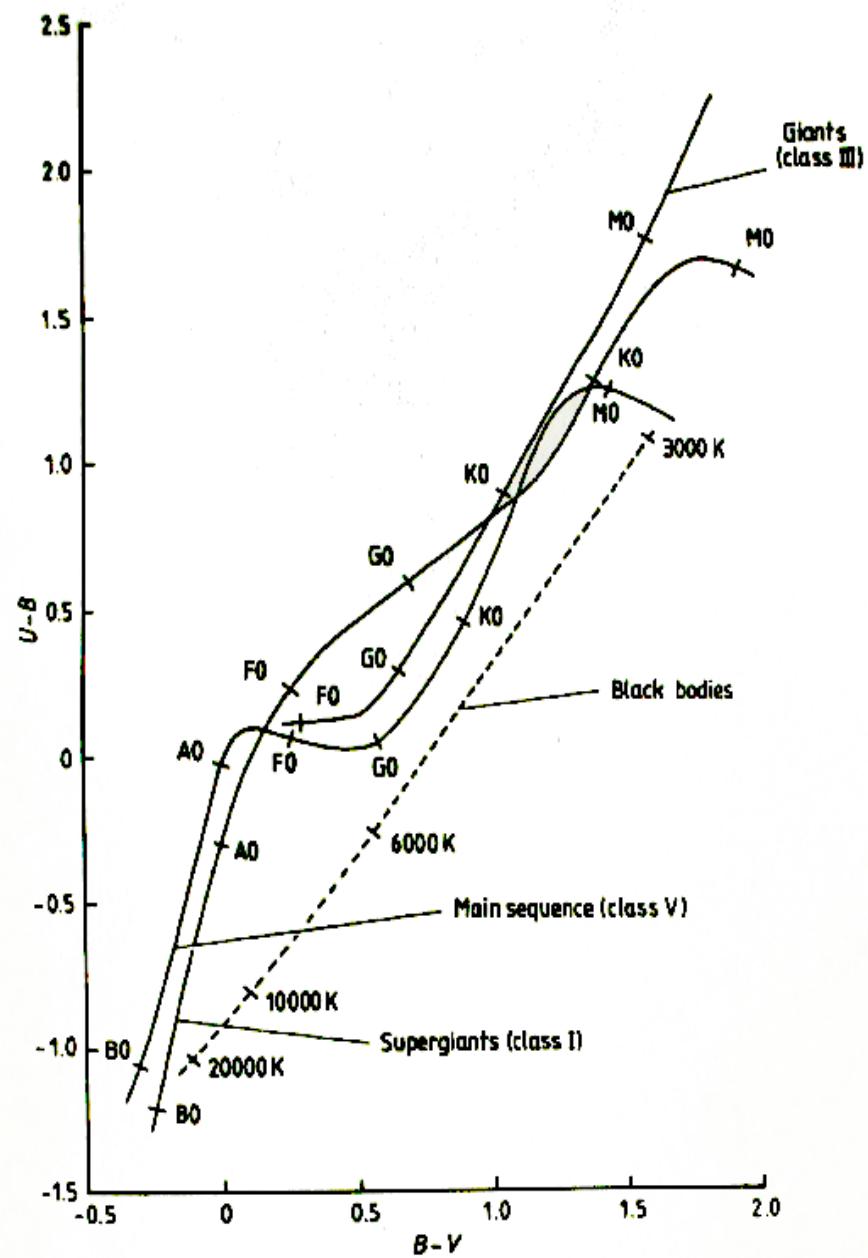
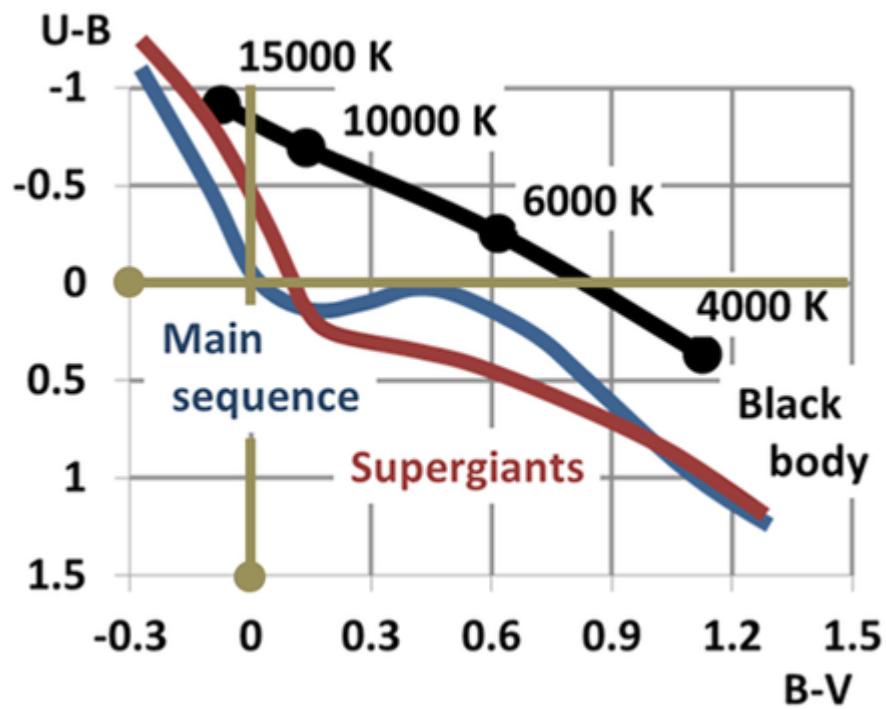


Colors

Color-color diagram

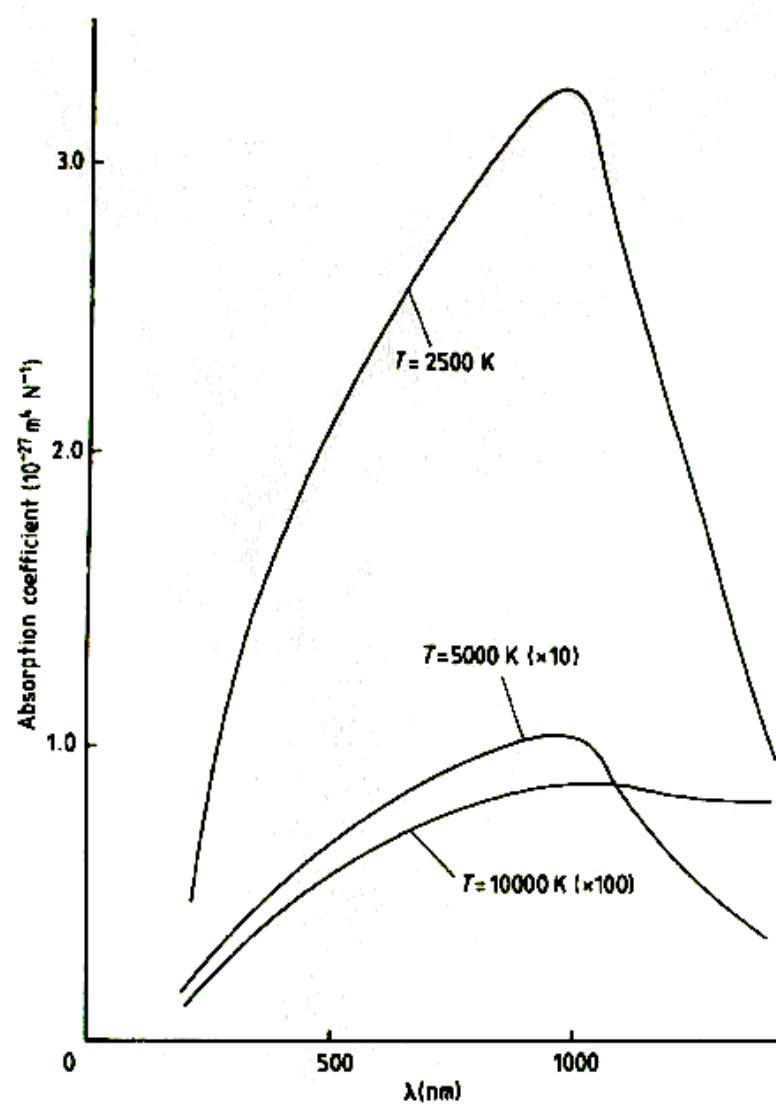
Clear differences between stars and BB.

Stars emit less UV radiation than BB



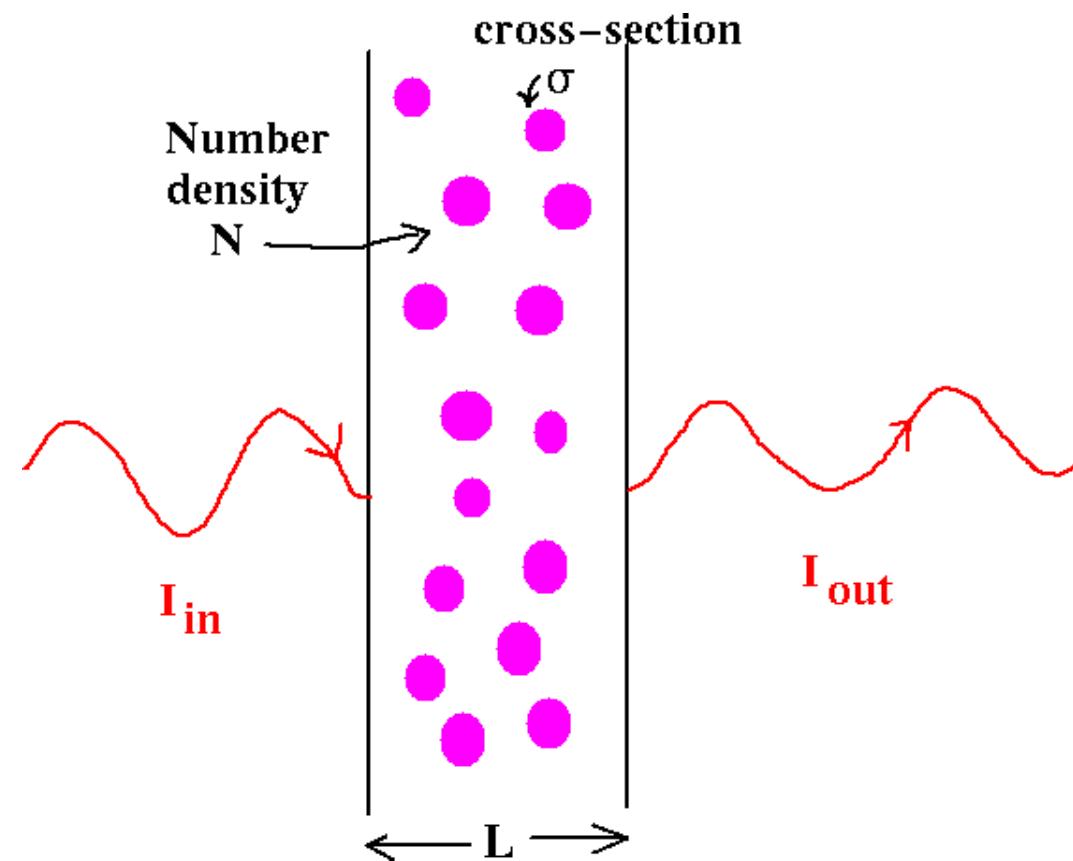
Colors

The differences are caused mainly due Balmer discontinuity and negative hydrogen ion absorption in cool stars.



Interstellar extinction/absorption

The absorption and scattering of electromagnetic radiation by dust and gas between an emitting astronomical object and the observer.



$$I_{out} = I_{in} e^{-\tau}$$
$$\frac{I_{out}}{I_{in}} = e^{-\tau}$$

where

$$\tau = N \sigma L$$

or, converting to magnitudes

$$m_{out} - m_{in} = -2.5 \log e^{-\tau}$$

$$m_{out} - m_{in} = -2.5(-\tau) \log e$$

$$m_{out} - m_{in} = 1.086 \tau$$



Interstellar extinction/absorption

The absorption can be defined as:

$$A = 1.086 \tau$$

Our magnitude equation becomes:

$$m - M = 5 \log d - 5 + A$$

Writing it in *UBV* system:

$$U = U_0 + A_u$$

$$B = B_0 + A_b$$

$$V = V_0 + A_v$$



Interstellar reddening

The color excess or reddening:

$$B - V = B_0 + A_b - V_0 - A_v = (B - V)_0 + A_b - A_v = (B - V)_0 + E_{B-V}$$

$$U - B = U_0 + A_u - B_0 - A_b = (U - B)_0 + A_u - A_b = (U - B)_0 + E_{U-B}$$

or:

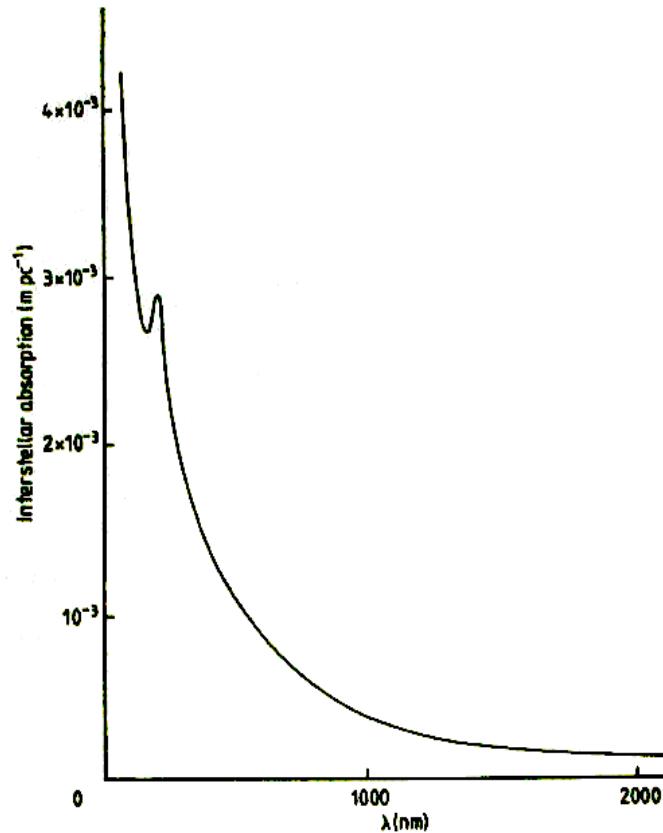
$$E_{U-B} = (U - B) - (U - B)_0$$

$$E_{B-V} = (B - V) - (B - V)_0$$



Interstellar reddening

Interstellar absorption is color sensitive.



For visible spectrum it can be approximated by:

$$A_\lambda = 6.5 \times 10^{-10} \lambda^{-1} - 2.0 \times 10^{-4} \text{ [mag pc}^{-1}\text{]}$$



Interstellar reddening

Using this equation and assuming that U and B filters are monochromatic with 365 nm and 440 nm effective wavelengths.

$$\frac{E_{U-B}}{E_{B-V}} = \frac{(U - U_0) - (B - B_0)}{(B - B_0) - (V - V_0)}$$

$$\frac{E_{U-B}}{E_{B-V}} = \left[\left(\frac{6.5 \times 10^{-10}}{3.65 \times 10^{-7}} - 2.0 \times 10^{-4} \right) D - \left(\frac{6.5 \times 10^{-10}}{4.4 \times 10^{-7}} - 2.0 \times 10^{-4} \right) D \right]$$

$$\times \left[\left(\frac{6.5 \times 10^{-10}}{4.4 \times 10^{-7}} - 2.0 \times 10^{-4} \right) D - \left(\frac{6.5 \times 10^{-10}}{5.5 \times 10^{-7}} - 2.0 \times 10^{-4} \right) D \right]^{-1}$$

$$\frac{E_{U-B}}{E_{B-V}} = \left(\frac{1}{365} - \frac{1}{440} \right) \left(\frac{1}{440} - \frac{1}{550} \right)^{-1}$$

$$\frac{E_{U-B}}{E_{B-V}} = 1.027$$

Color excess ratio is temperature and reddening independent.



Interstellar reddening

In fact the filters are not monochromatic and then:

$$\frac{E_{U-B}}{E_{B-V}} = (0.70 \pm 0.10) + (0.045 \pm 0.015) E_{B-V} \quad \text{at } 30\,000\,K$$

$$\frac{E_{U-B}}{E_{B-V}} = (0.72 \pm 0.06) + (0.05 \pm 0.01) E_{B-V} \quad \text{at } 10\,000\,K$$

$$\frac{E_{U-B}}{E_{B-V}} = (0.82 \pm 0.12) + (0.065 \pm 0.015) E_{B-V} \quad \text{at } 5\,000\,K$$

But the dependence on temperature and reddening is weak and for most cases one can adopt:

$$\overline{\left(\frac{E_{U-B}}{E_{B-V}} \right)} = 0.72 \pm 0.03$$



Interstellar reddening

Q factor which is reddening independent.

$$Q = (U - B) - \left(\frac{\overline{E_{U-B}}}{\overline{E_{B-V}}} \right) (B - V)$$

$$= (U - B)_0 + E_{U-B} - \left(\frac{\overline{E_{U-B}}}{\overline{E_{B-V}}} \right) [(B - V)_0 + E_{B-V}]$$

$$= (U - B)_0 - \left(\frac{\overline{E_{U-B}}}{\overline{E_{B-V}}} \right) (B - V)_0 - \left(\frac{\overline{E_{U-B}}}{\overline{E_{B-V}}} \right) E_{B-V} + E_{U-B}$$

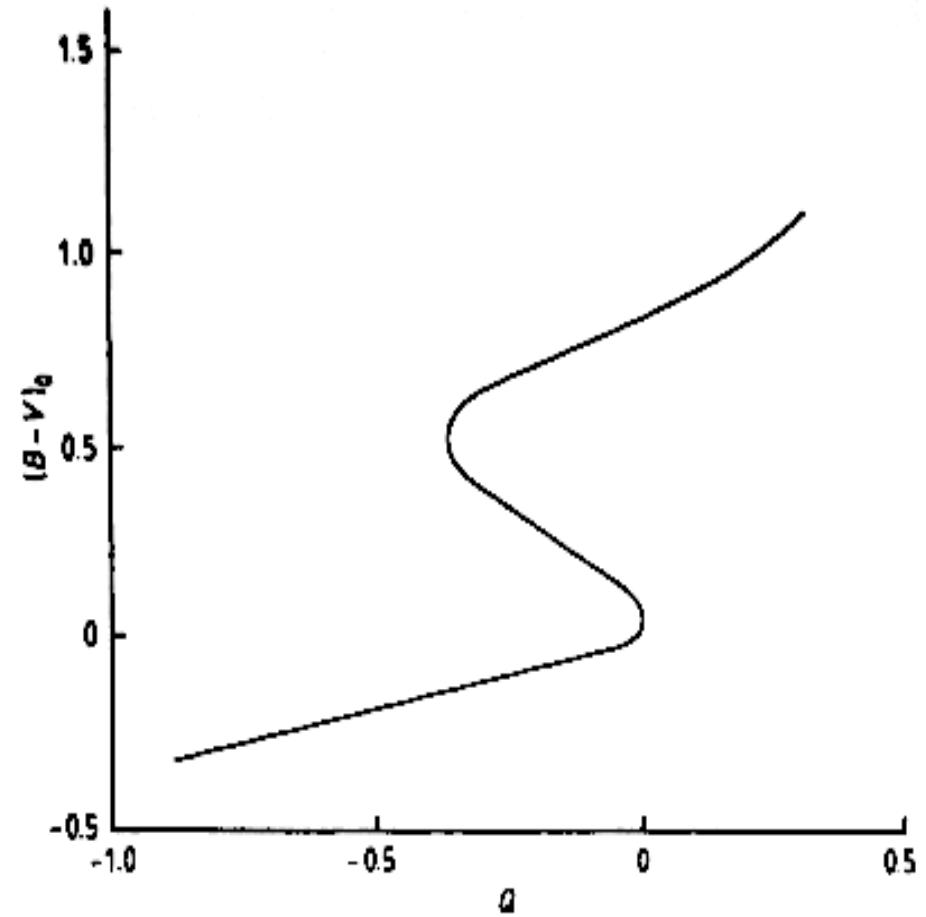
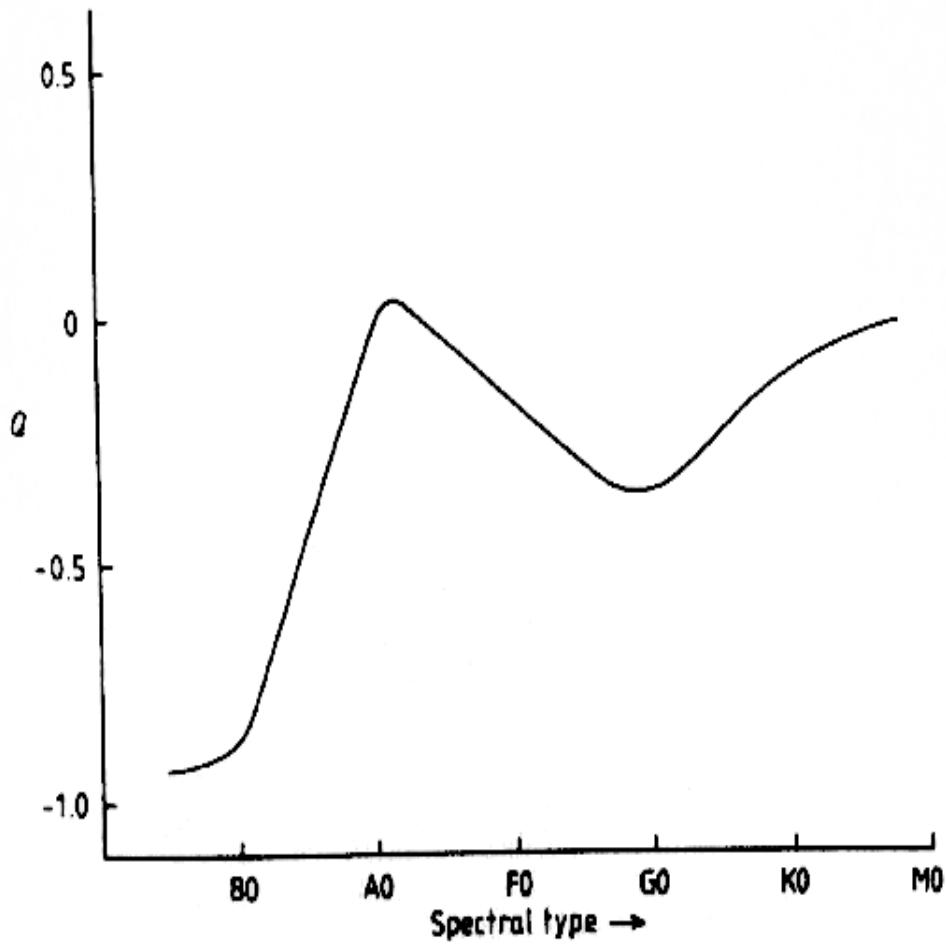
$$\simeq (U - B)_0 - \left(\frac{\overline{E_{U-B}}}{\overline{E_{B-V}}} \right) (B - V)_0$$

$$\simeq (U - B)_0 - 0.72 (B - V)_0$$



Interstellar reddening

Relation between Q and spectral type or color.



Interstellar reddening

For hot stars:

$$(B-V)_0 = 0.332 Q$$

$$E_{B-V} = (B-V) - 0.332 Q = 1.4 E_{U-B}$$

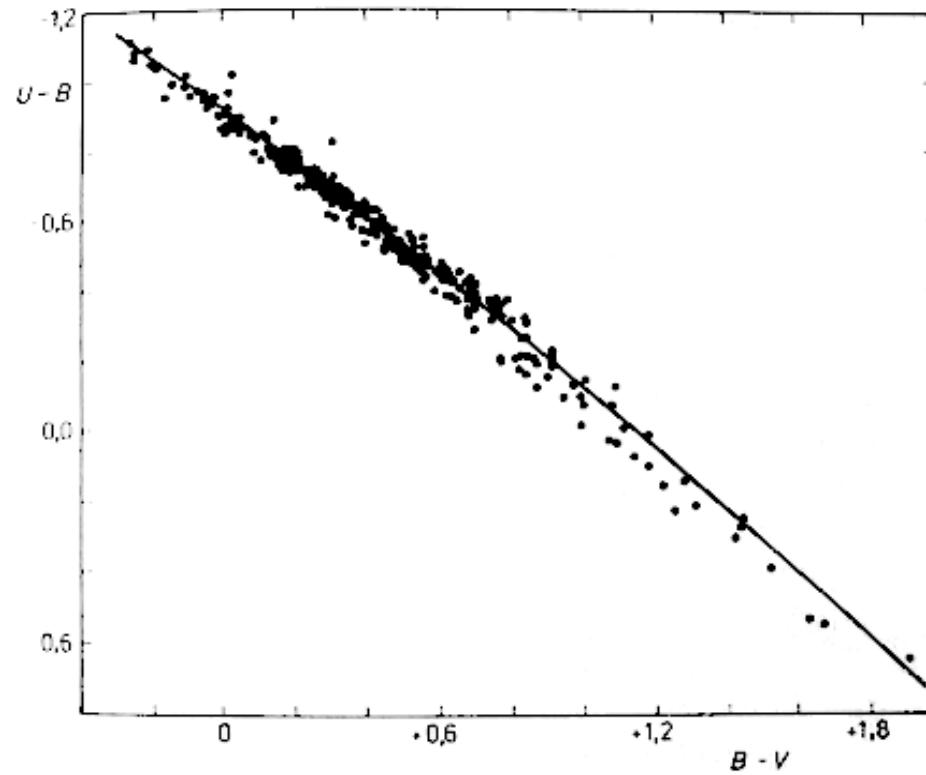
Performing precise UBV photometry for hot stars one can determine:

- Q
- extinction free colors
- reddening
- temperatures and spectral types
- true magnitudes



Interstellar reddening

Example: color-color diagram for stars with spectral type O



The same spectral type means the same Q. Then the slope is simply the ratio between E_{U-B} and E_{B-V}



Other photometric systems – Stromgren uvby

u – 350 nm, FWHM = 34 nm

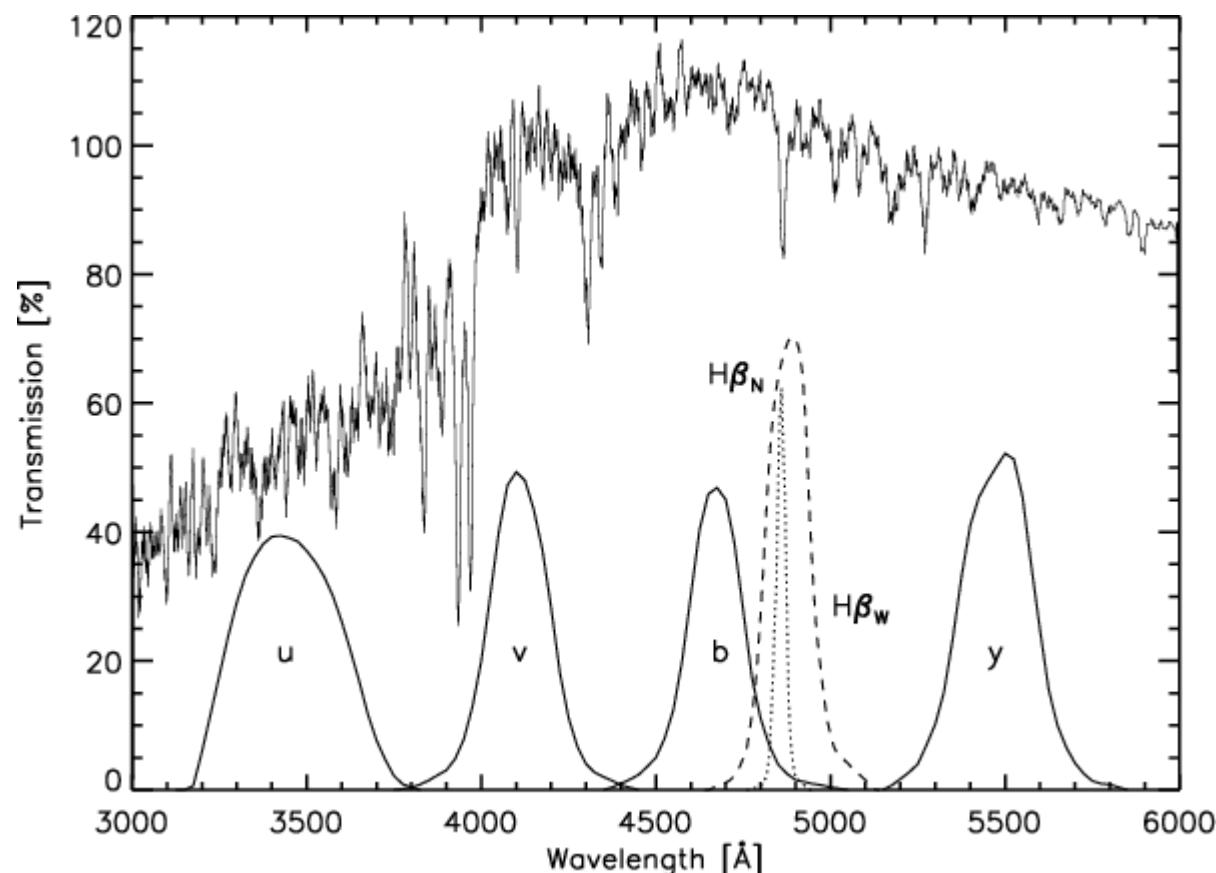
b – 467 nm, FWHM = 16 nm

H_{β} – 486 nm, FWHM = 3 nm

v – 411 nm, FWHM = 20 nm

y – 550 nm, FWHM = 24 nm

Comparison with spectrum of T=6000 K star.



Other photometric systems – Stromgren uvby

One can define the following colors:

$$b - y$$

$$c_1 = (u - v) - (v - b)$$

$$m_1 = (v - b) - (b - y)$$

From colors of O-type stars the reddenings are:

$$E(c_1) = 0.20 E(b - y)$$

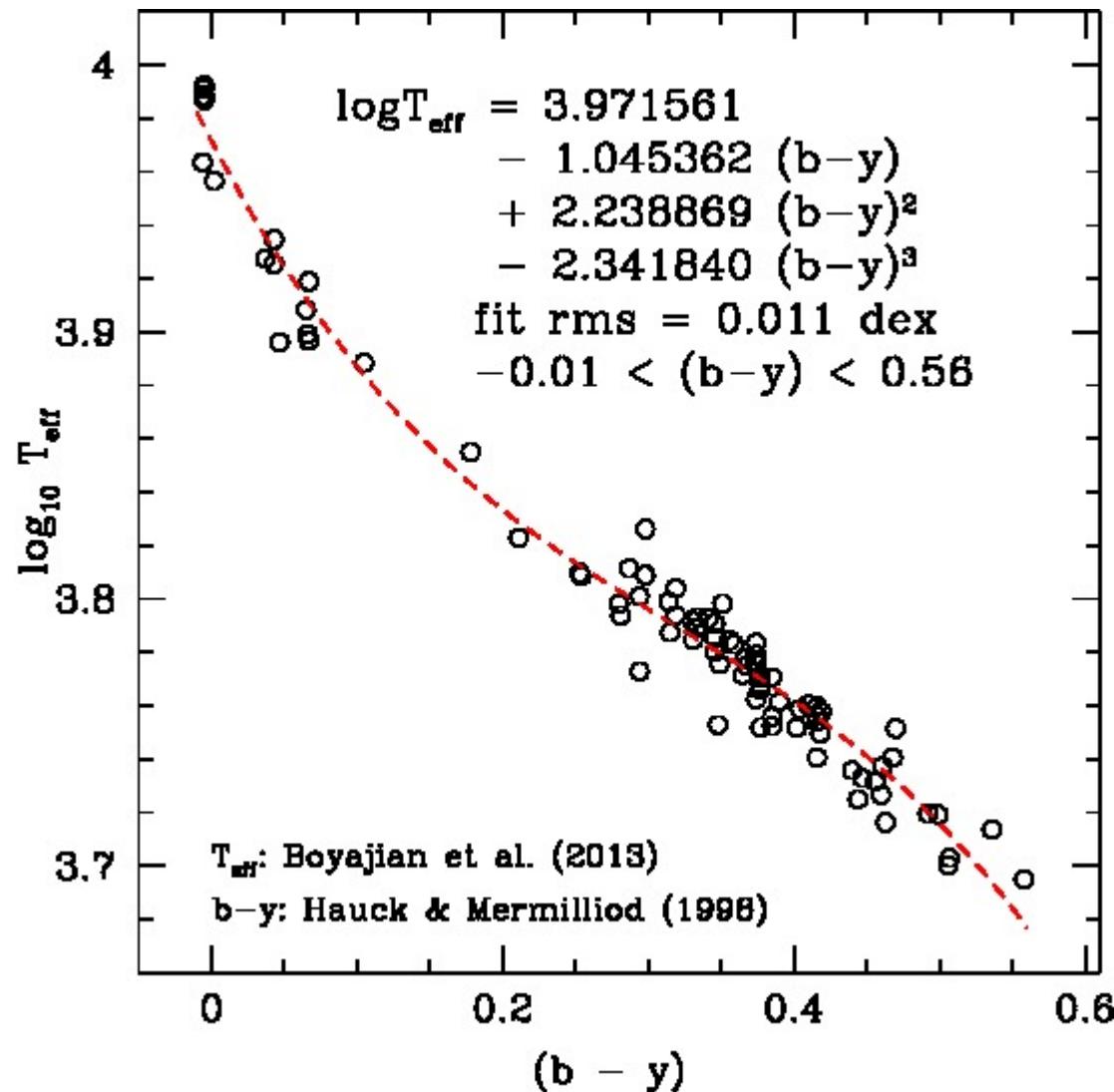
$$E(m_1) = -0.32 E(b - y)$$

$$E(b - y) = 0.74 E(B - V)$$



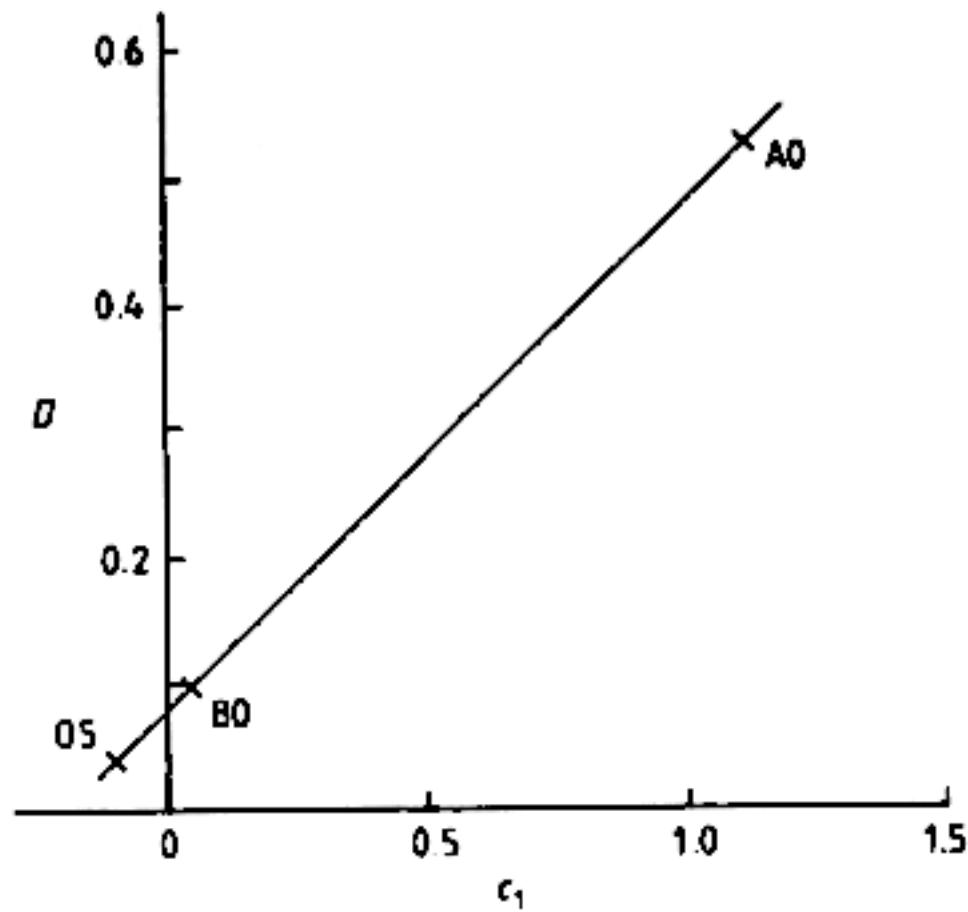
Other photometric systems – Stromgren uvby

The $b-y$ color is a good temperature (spectral type) idicator.



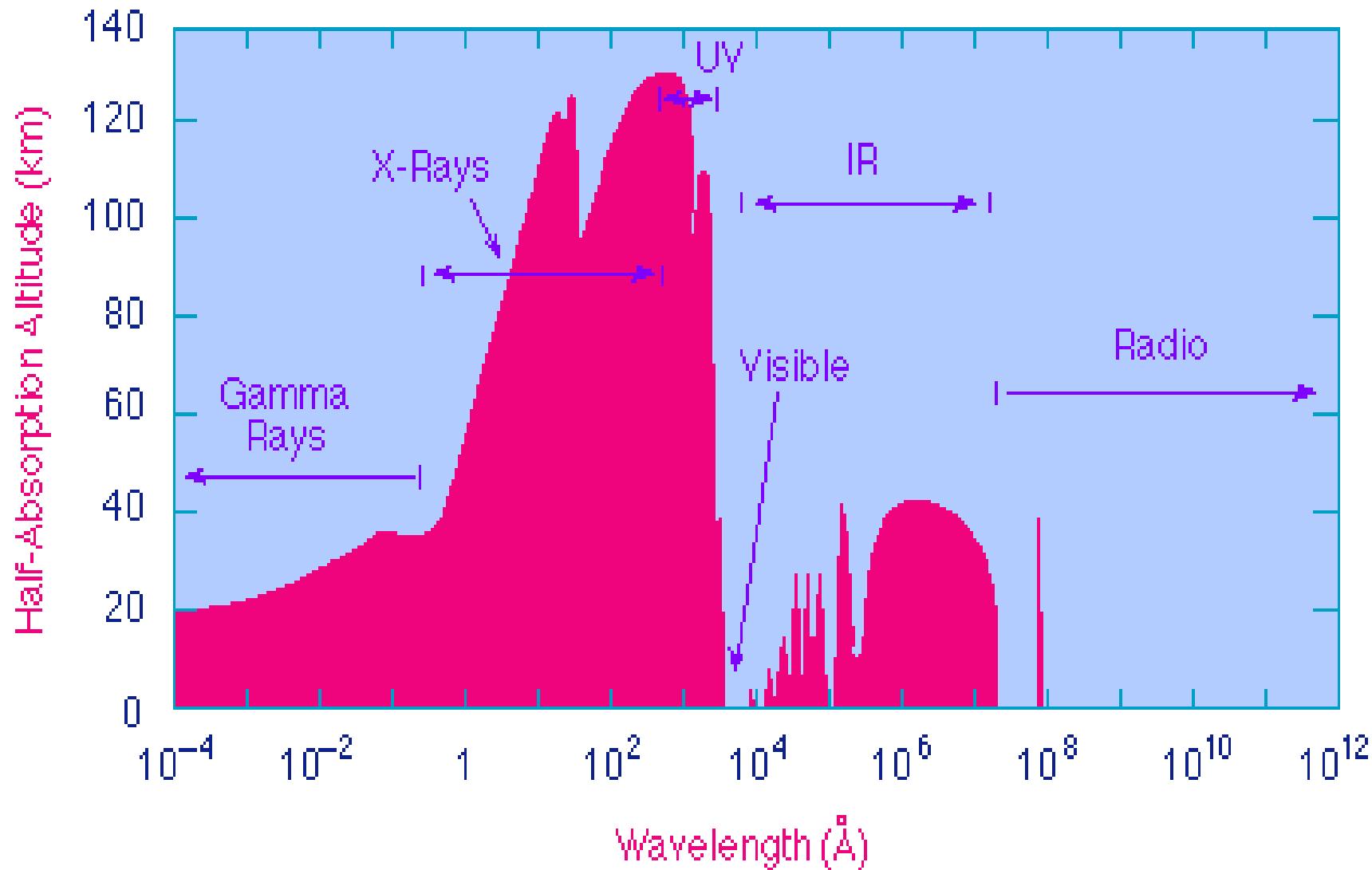
Other photometric systems – Stromgren uvby

c_1 – good indicator of Balmer discontinuity height



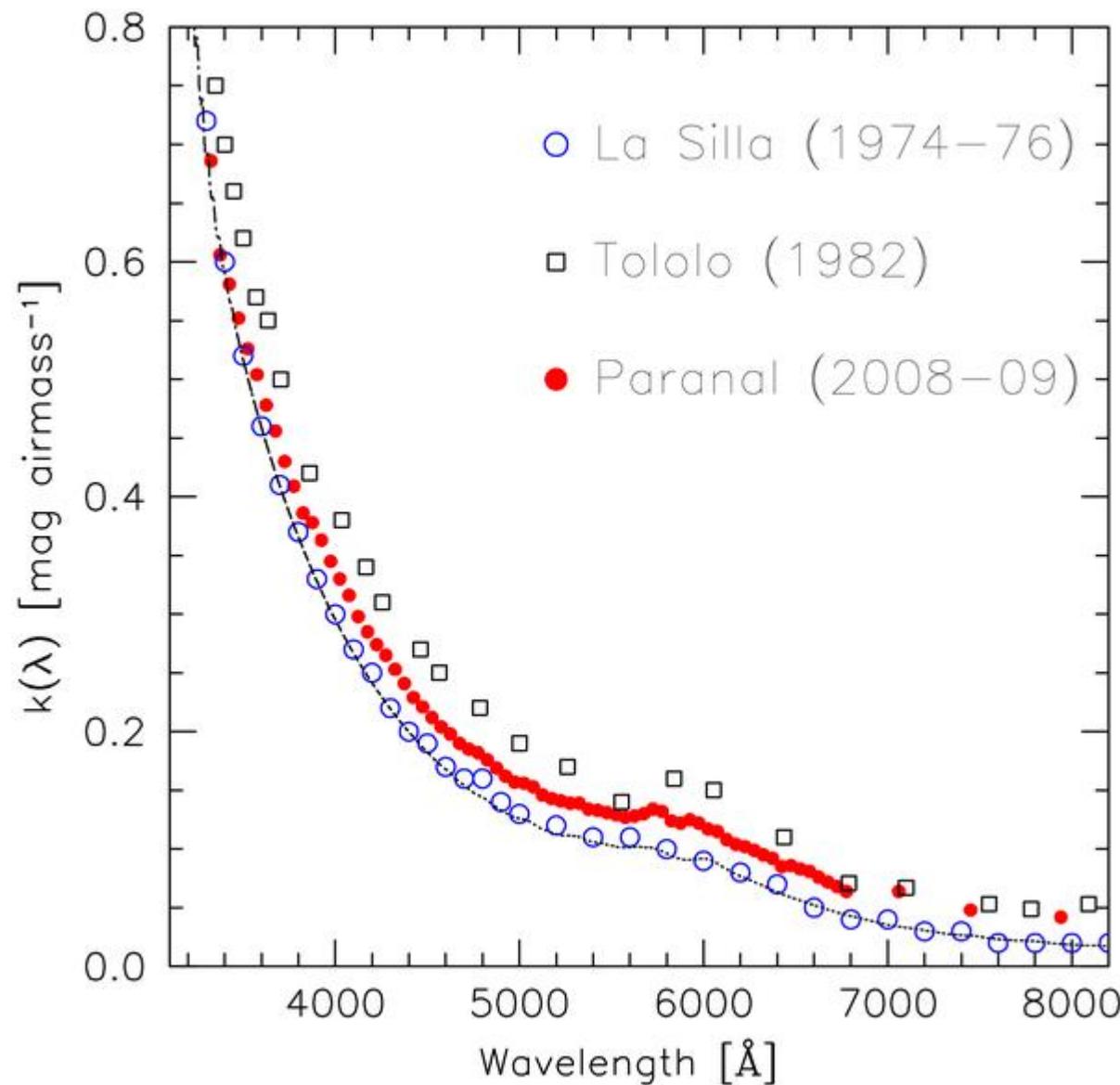
Atmospheric extinction

In whole spectrum:



Atmospheric extinction

In visible range:



Atmospheric extinction

Similar situation to interstellar absorption → similar solution.

$$dE = -E \tau dx' \rightarrow E = E_0 \exp\left(-\int_0^x \tau dx'\right)$$

Assuming for simplicity: $\tau(x) = \tau = \text{const}$

We can integrate

$$E = E_0 e^{-\tau x}$$

And take logarithm:

$$\ln E = \ln E_0 - \tau x$$



Atmospheric extinction

Transforming into magnitudes:

$$-2.5 \log E = -2.5 \log E_0 + k x$$

where

$$k = \frac{2.5 \tau}{\ln 10} = 1.086 \tau$$

Then:

$$m_\lambda = m_{\lambda,0} + k x$$

Normalization: for zenith ($z=0$) $x=1$

For flat atmosphere:

$$x = \sec z = \frac{1}{\cos z}$$

For spherical atmosphere:

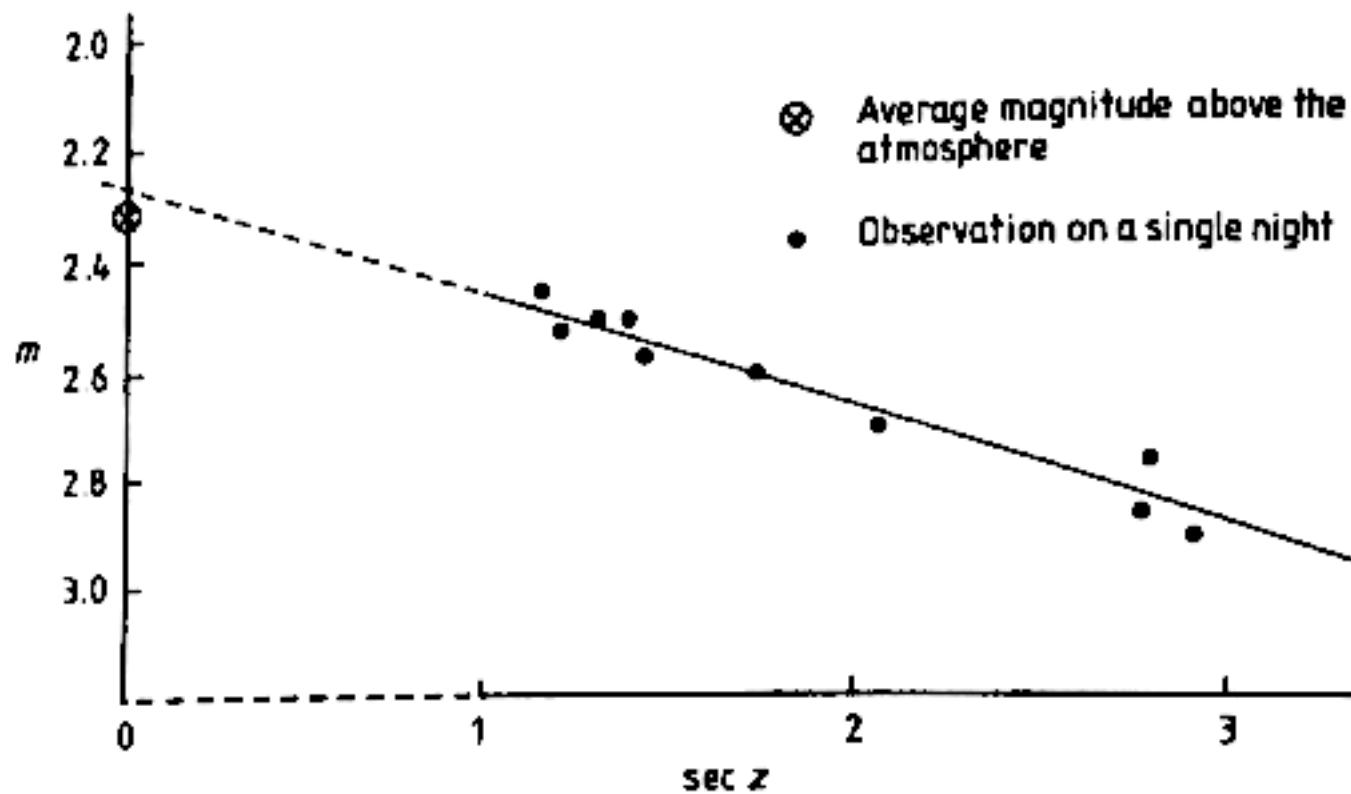
$$x = \sec z = 0.0018167(\sec z - 1) - 0.002897(\sec z - 1)^2 - 0.0008083(\sec z - 1)^3$$



Atmospheric extinction

Having: $m_\lambda = m_{\lambda,0} + k x$

During one night determine the magnitudes of one star at different elevations.



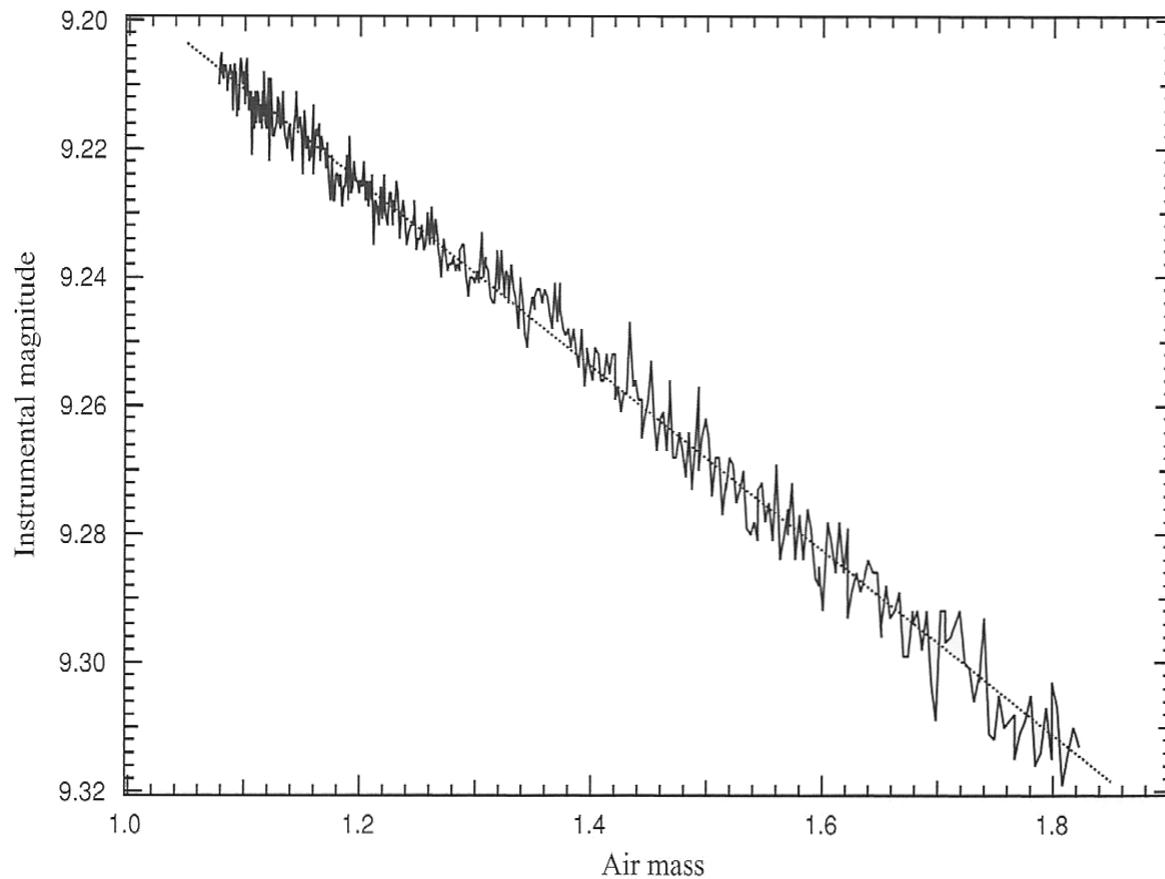
Slope gives you k , and zero point gives you magnitude above the atmosphere.



Atmospheric extinction

Having: $m_\lambda = m_{\lambda,0} + k x$

During one night determine the magnitudes of one star at different elevations.

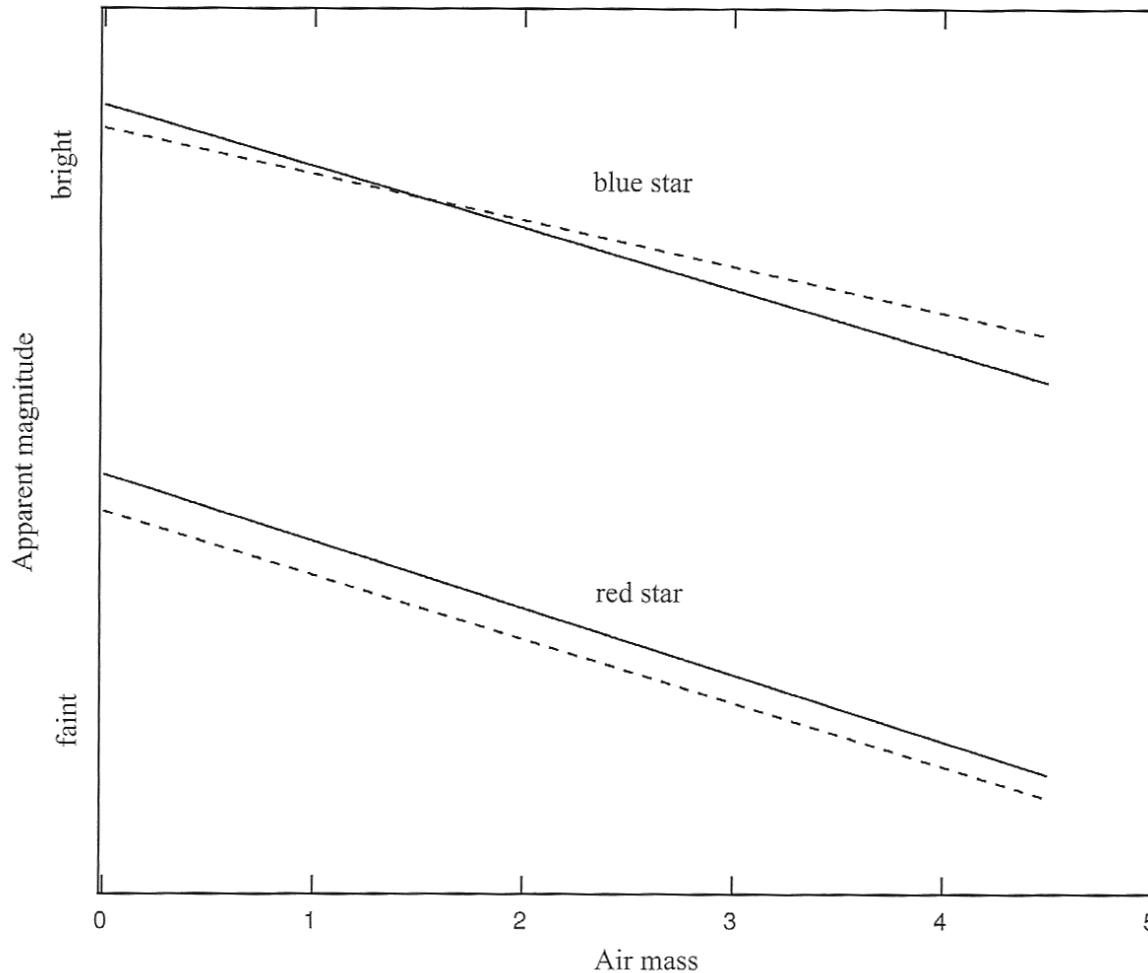


R filter
instrumental magnitudes for
a V = 8.33 magnitude star.
The data are spaced 30 s
apart. They were obtained on
a clear night in September
2004 with the 24 inch
telescope at Fick Observatory
in central Iowa. The dotted
line is a least-squares fit to the
data.



Atmospheric extinction

The problem is that k is also color sensitive...



The extinction coefficient depends on the color of the star. The solid lines are for blue magnitudes and the dashed lines are for visual magnitudes.



Atmospheric extinction

The problem is that k is also color sensitive... For each filter we can write:

$$c_0 = m_{0,1} - m_{0,2} = m_1 - m_2 - (k_1 - k_2)x = c - k_c x$$

For Johnson-Cousins system one can assume linear relation:

$$k = k' + k''c \quad k_c = k_c' + k_c''c$$

Where k' is extinction coefficient for star with color $c=0$.

Finally:

$$m_0 = m - k' - k''cx$$

$$c_0 = c - k_c'x - k_c''cx$$



Atmospheric extinction

For stars which are close on celestial sphere:

$$\Delta m_0 = \Delta m - k'' \Delta c x$$

$$\Delta c_0 = \Delta c - k''_c \Delta c x$$

Doing it for many pairs at different x one can plot Δm and $\Delta c x$, and determine k'' and k''_c coefficients.



Atmospheric extinction

Observing standard UBV stars from Landolt (1992) list at different elevations we have many instrumental ubv magnitudes obtained for many x.

$$v_0 = v - k_v X$$

$$(b-v)_0 = (b-v) J_x - k'_{BV} x$$

$$(u-b)_0 = (u-b) G_x - k'_{UB} x$$

$$J_x = 1 - k''_{BV} x$$

$$G_x = 1 - k''_{UB} x$$

We can determine all k coefficients.

For photometric sites they are quite constant with time.



Transformation to standard system

Now we know extinction coefficients for our site and instrumental magnitudes and colors above atmosphere. But we want to know standard UBV magnitudes.

We need one more transformation.

$$V = v_0 + \epsilon(B - V) + \xi_V$$

$$B - V = \mu(b - v)_0 + \xi_{BV}$$

$$U - B = \nu(u - b)_0 + \xi_{UB}$$

Transformation is assumed to be good if:

$$|\epsilon| < 0.15$$

$$0.9 < \nu, \mu < 1.1$$



Transformation to standard system

Combining the equations from two previous slides:

$$V = v - k_v x + \epsilon(B-V) + \xi_V$$

$$B-V = \mu(b-v)J_x - \nu k'_{BV} x + \xi_{BV}$$

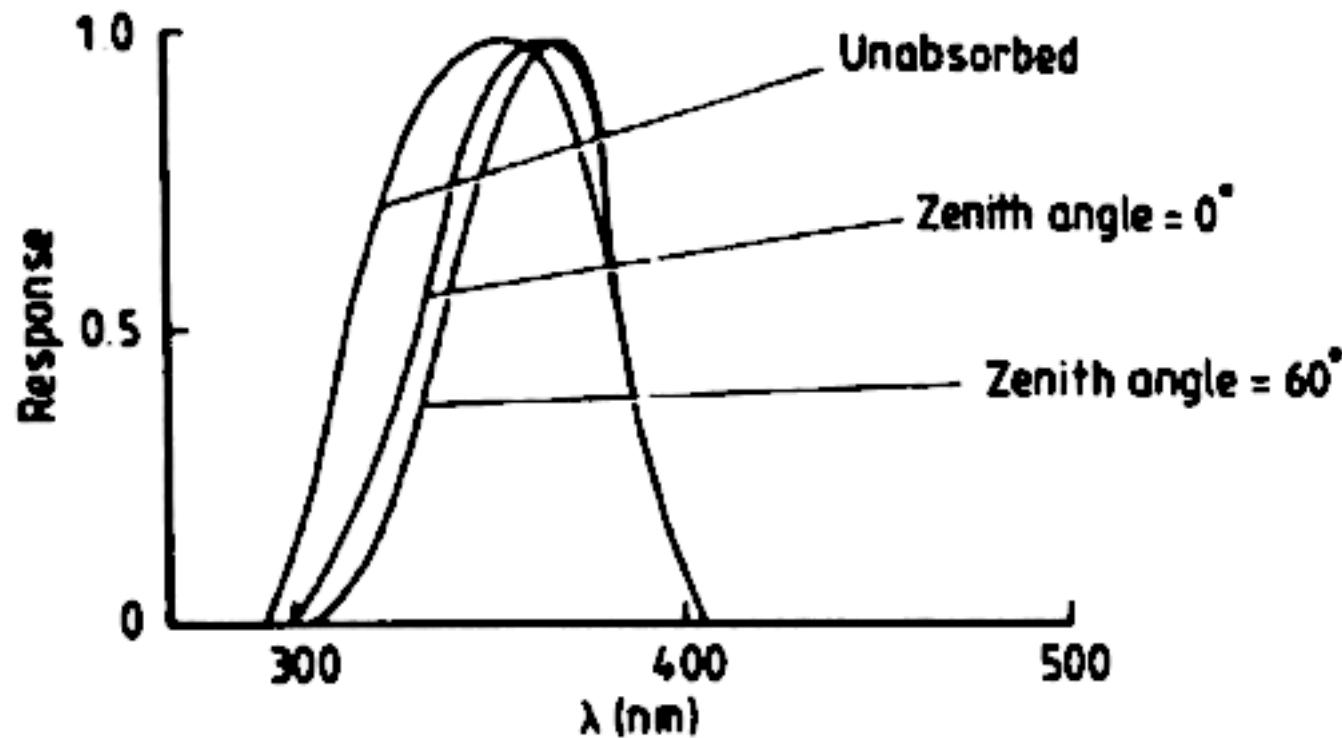
$$U-B = \nu(u-b)G_x - \nu k'_{UB} x + \xi_{UB}$$

Having many instrumental ubv magnitudes for standard Landolt (1992) stars with known UVW values obtained for different x , we are able to determine all coefficients.



Transformation to standard system

At sites with low elevation there is problem with UV absorption.



The solution is to use BVRI instead of UBV.



Differential photometry

It is much easier to do the differential photometry of stars within one field of view.

We measure:

$$V_{var}, V_{comp}$$

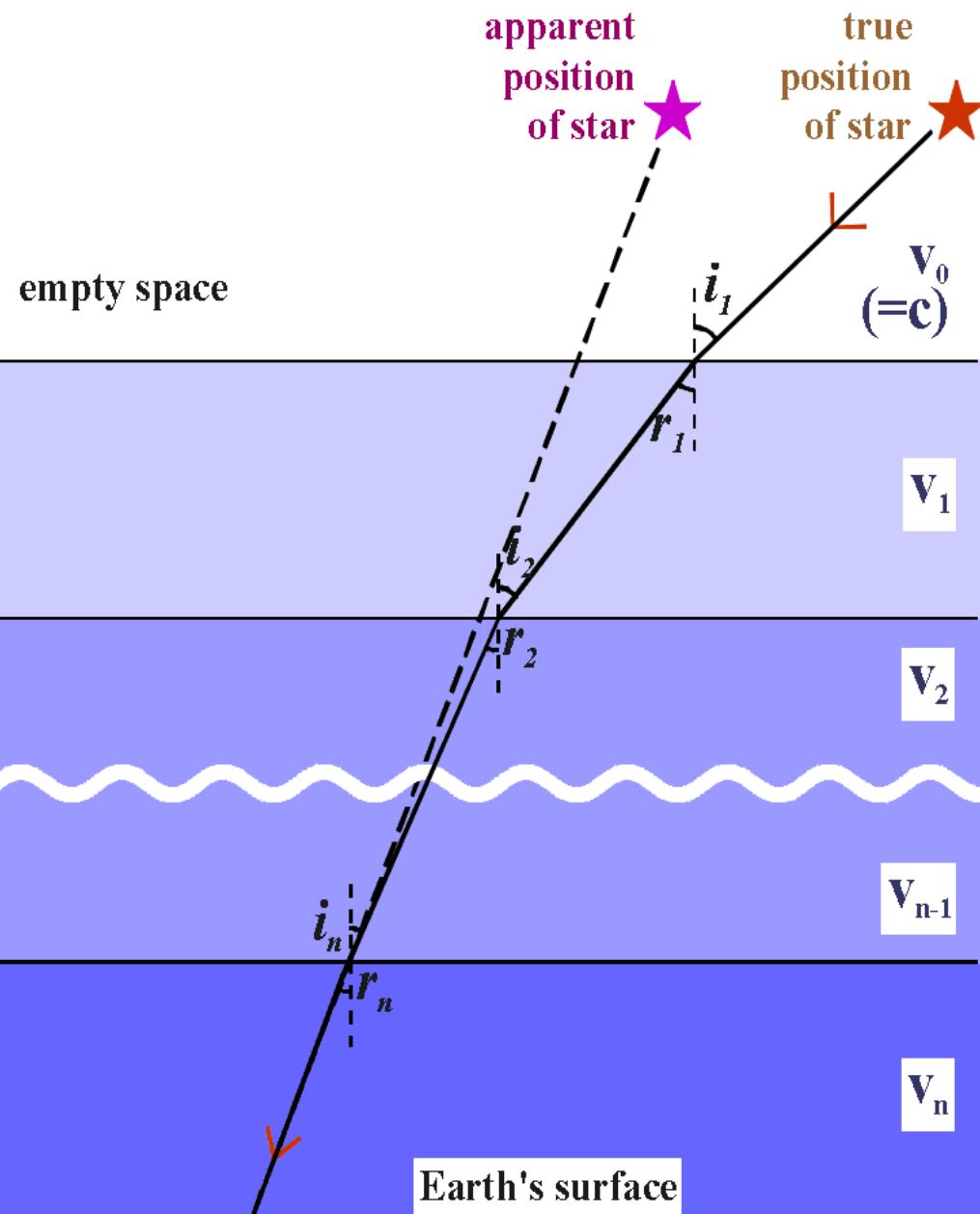
And then:

$$\Delta V = \Delta v + \epsilon \Delta(B - V)$$

During good weather conditions and for good system of filters: $\epsilon < 0.10-0.15$.
Choose for comparison star the object with similar color to your variable.



Atmospheric refraction



Atmospheric refraction is the deviation of light or other electromagnetic wave from a straight line as it passes through the atmosphere due to the variation in air density as a function of altitude.

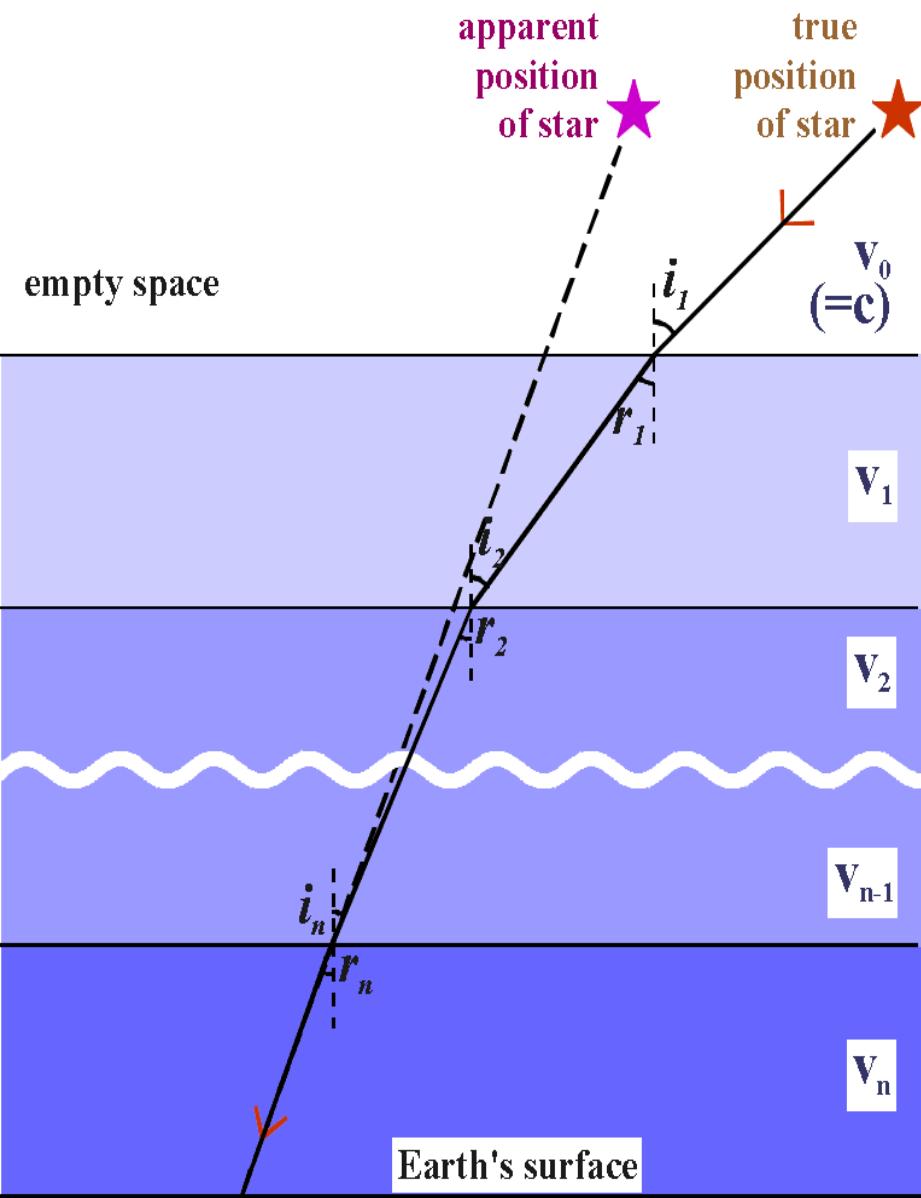
Astronomical refraction deals with the angular position of celestial bodies, their appearance as a point source, and through differential refraction, the shape of extended bodies such as the Sun and Moon.

Refraction has no effect if a star is at the zenith ($z=0$).

But at any other position, the star is apparently raised; the effect is greatest at the horizon.



Atmospheric refraction



$$\text{At the first boundary: } \frac{\sin(i_1)}{\sin(r_1)} = \frac{v_0}{v_1}$$

$$\text{At the next boundary: } \frac{\sin(i_2)}{\sin(r_2)} = \frac{v_1}{v_2} \quad \text{and so on.}$$

But, by simple geometry: $r_1=i_2, r_2=i_3$ and so on.

So we have:

$$\sin(i_1) = \frac{v_0}{v_1} \sin(r_1)$$

$$= \frac{v_0}{v_1} \sin(i_2)$$

$$= \frac{v_0}{v_1} \frac{v_1}{v_2} \sin(r_2)$$

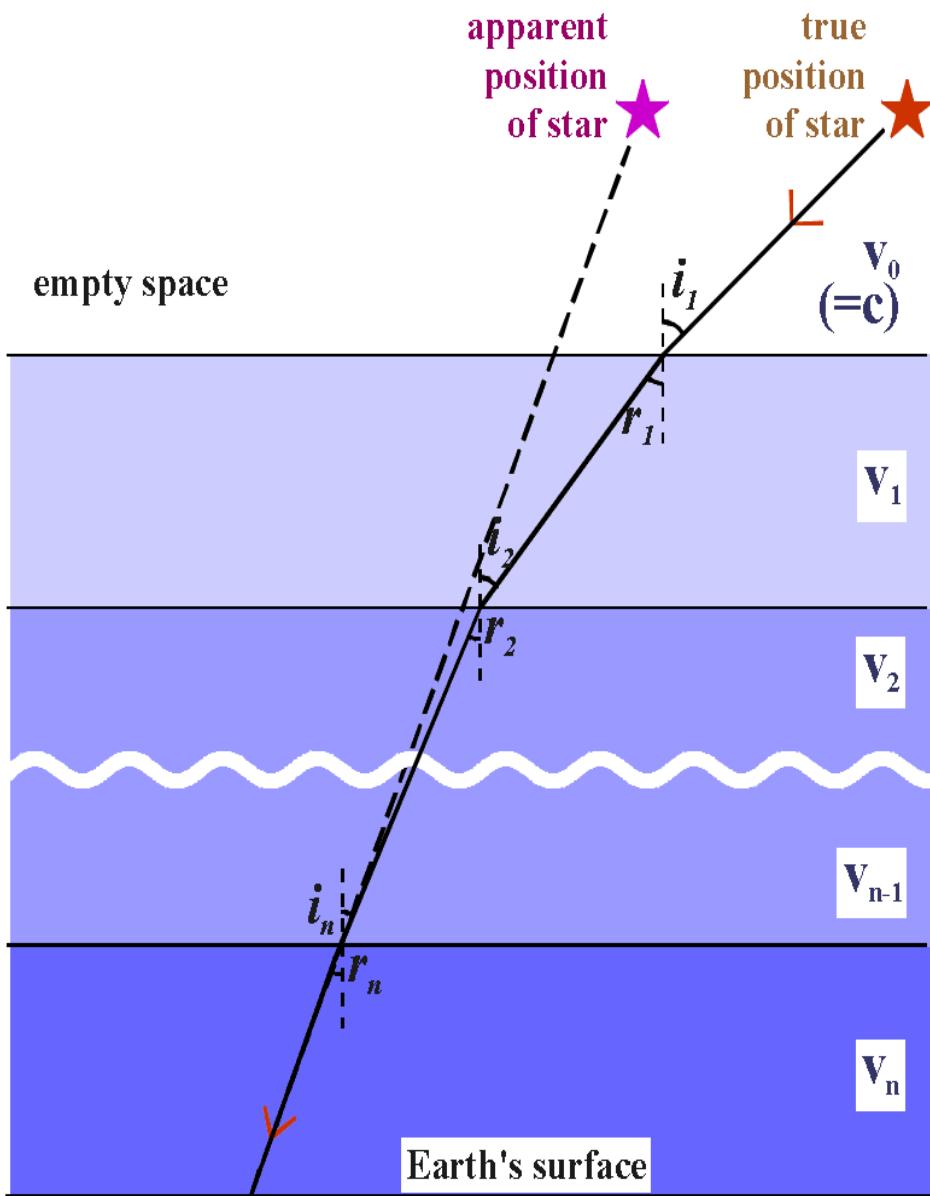
$$= \frac{v_0}{v_2} \sin(r_2)$$

=.....

$$= \frac{v_0}{v_n} \sin(r_n)$$



Atmospheric refraction



Define the angle of refraction R by: $R = z - z'$

Rearrange as: $z = R + z'$

Then:

$$\sin(z) = \sin(R) \cos(z') + \cos(R) \sin(z')$$

We assume R to be small, so approximately:

$$\sin(R) = R \text{ [radians]}, \quad \text{and } \cos(R) = 0$$

Thus, approximately:

$$\sin(z) = \sin(z') + R \cos(z')$$

Divide throughout by $\sin(z')$ to get:

$$\frac{\sin(z)}{\sin(z')} = 1 + \frac{R}{\tan(z')}$$

which is to say:

$$\frac{v_0}{v_n} = 1 + \frac{R}{\tan(z')}$$

So we can write:

$$R = \left(\frac{v_0}{v_n} - 1 \right) \tan(z')$$

$$R = k \tan(z')$$

$$\text{where } k = \frac{v_0}{v_n} - 1$$

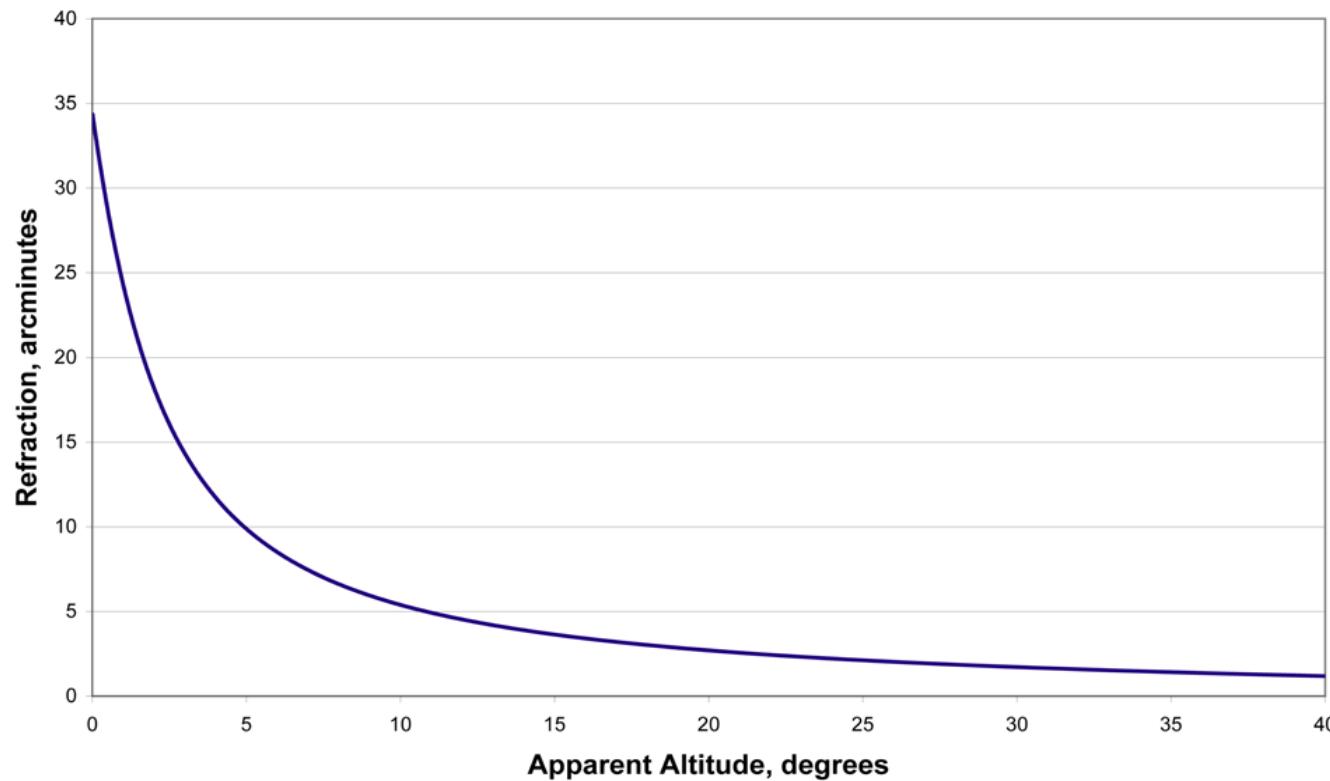


Atmospheric refraction

More precise Bennett's formula (R [arcmin], h_a [degrees]):

$$R = \cot\left(h_a + \frac{7.31}{h_a + 4.4}\right)$$

Refraction vs. Altitude, Bennett (1982)



Atmospheric refraction

Comstock's formula taking into account the temperature and pressure.

$$r = 60.4 \left(\frac{b/760}{1+t/273} \right) \tan(z') \quad [arcsec]$$

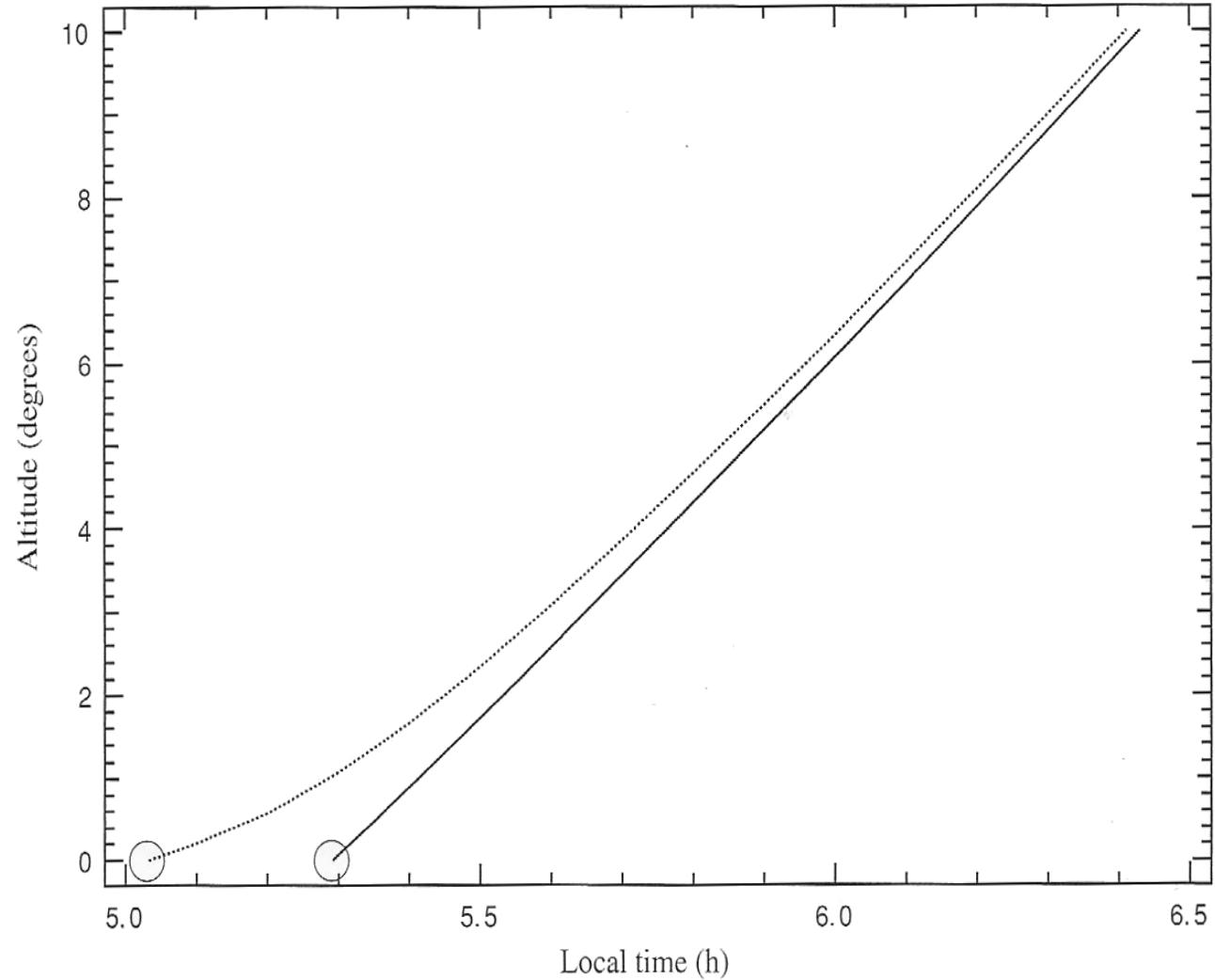
Where b is the barometric pressure in milimeters of mercury and t is the temperature in degrees Celsius.



Atmospheric refraction

Thanks to the refraction effect the day can be even almost one hour longer!

Path of the Sun near the horizon for Seattle on the morning of June 25. Two cases are shown: unrefracted (solid line), and refracted (dotted line). Note the lengthening of the time the Sun spends above the horizon for the refracted case.



Atmospheric refraction

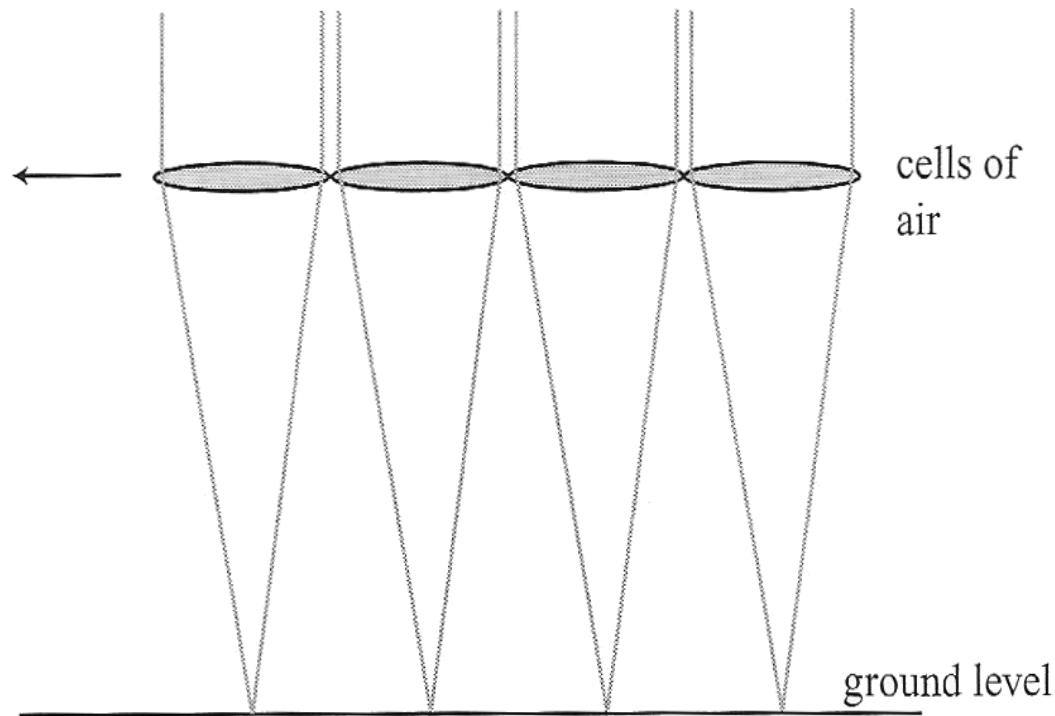
Thanks to the refraction we have nice sunsets...



Seeing

Seeing - blurring and twinkling of astronomical objects such as stars caused by turbulent mixing in the Earth's atmosphere varying the optical refractive index.

The effect at the ground caused by cells of air of different density moving in the lower atmosphere.

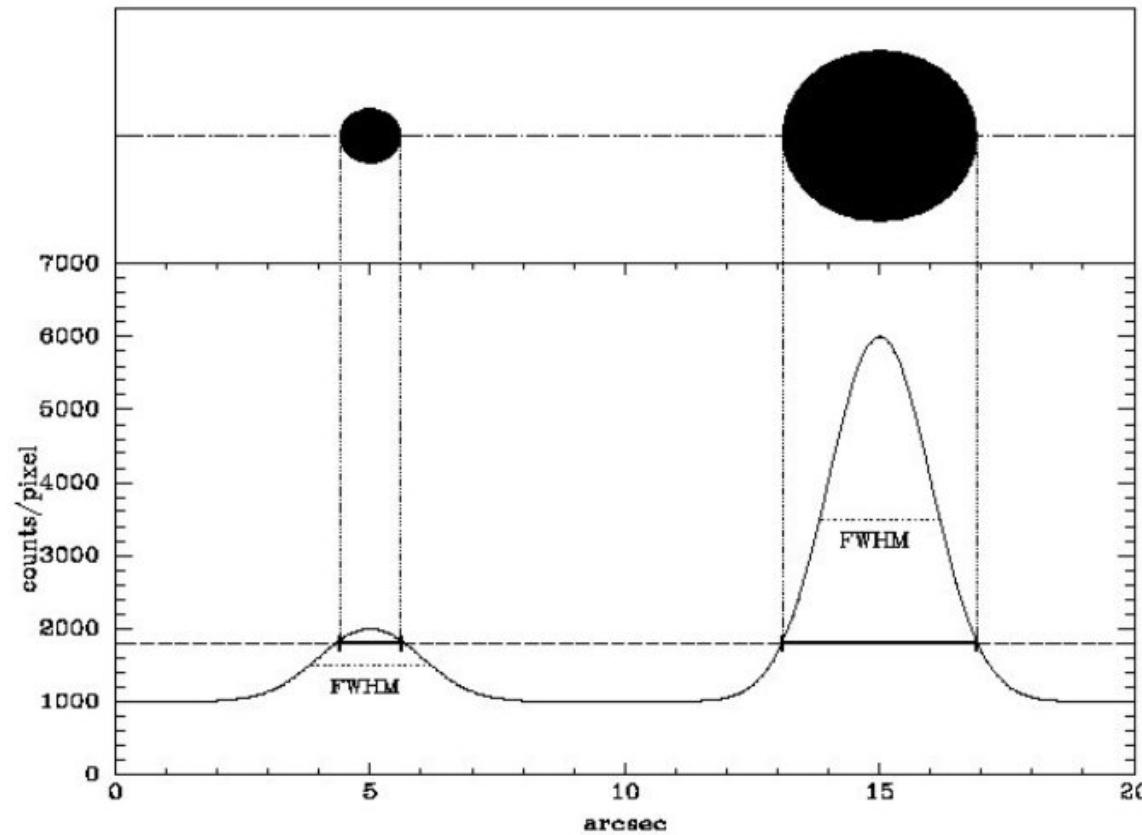


Seeing

Measuring the seeing:

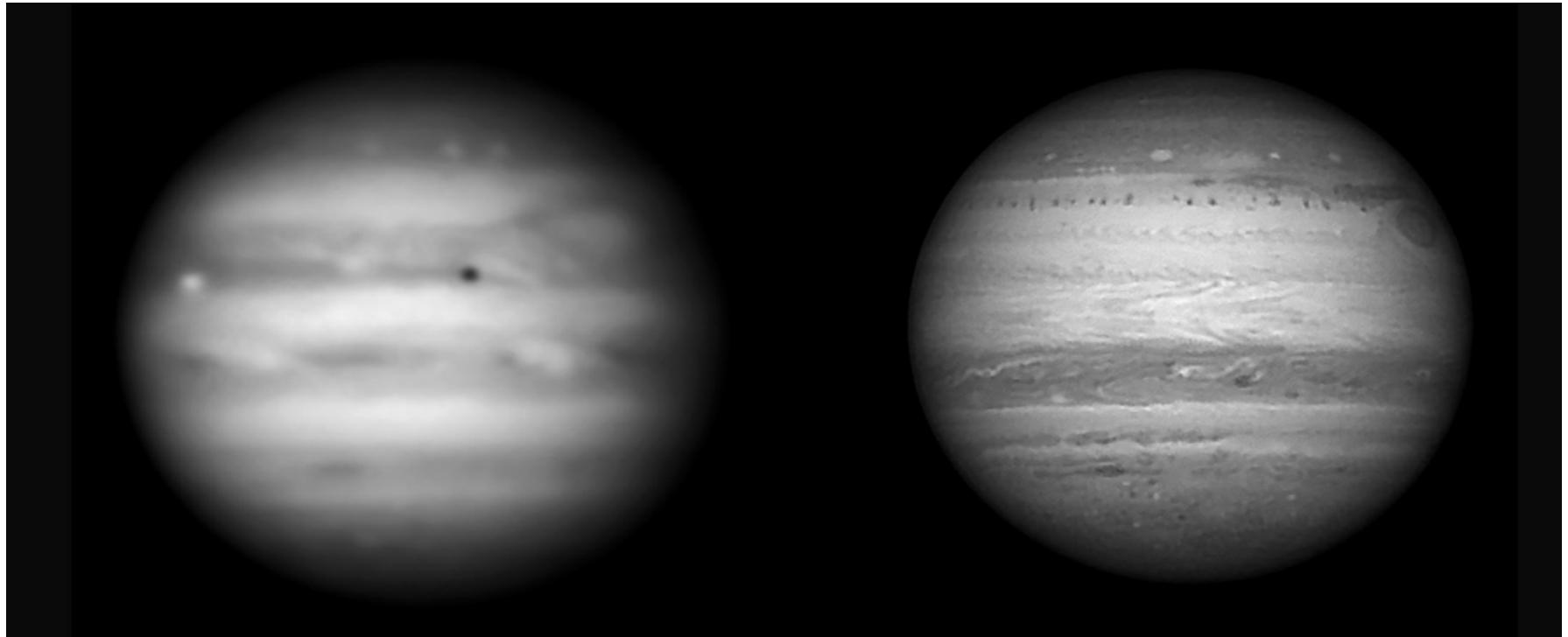
$$s = s_0 X^{\frac{3}{5}}$$

where s is the seeing at some air mass X , and s_0 is the seeing at the zenith.



Seeing

Images of Jupiter taken on two nights with different seeing conditions.



Good seeing conditions $\rightarrow \text{FWHM} < 1''$



Seeing

Seeing at good sites.

