Magnetohydrodynamic pseudo-Newtonian simulations of an accretion disk around a Reissner–Nordström naked singularity

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We present the first magnetohydrodynamic simulations of a thin accretion disk around a Reissner–Nordström naked singularity, modeled with our pseudo-Newtonian potential. This potential precisely reproduces the orbital frequency, including the radius at which it reaches a maximum and eventually vanishes. The latter one defines the zero-gravity sphere, below which the net force is repulsive. Our results show that matter accumulates around the singularity, with increased density near the zero-gravity sphere, in agreement with the general relativity analytical solutions. Naked singularity outflows are launched across the entire disk midplane, with significant differences in their characteristics compared to those from black holes. These differences could provide a basis for determining whether the compact objects such as M87* and Sgr A*, imaged by the Event Horizon Telescope, are indeed supermassive black holes.

Introduction:

The Reissner–Nordström metric arises as a solution to the Einstein–Maxwell field equations, describing the gravitational field of a spherically symmetric, non-rotating object with mass *M* and electric charge *Q:*

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right),$$

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}.$$

For $Q \leq M$, a black hole forms with two event horizons, which coincide when $Q = M$, while for $Q > M$, the horizons disappear giving rise to a naked singularity. Near a singularity, the effective potential diverges, leading to a repulsive force—a feature absent in black hole scenarios.

To identify potential observational differences between naked singularities and black holes, we perform numerical simulations of a thin accretion disk using a pseudo-Newtonian potential for the Reissner–Nordström metric (*Čemeljić et al 2025*):

$$V_{RN} = -\frac{M}{r} + \frac{Q^2}{2r^2}.$$

This potential exactly reproduces the orbital frequency derived for the Reissner–Nordström spacetime

$$\Omega_{\rm RN}(r) = \sqrt{\left(1 - \frac{r_0}{r}\right)\frac{M}{r^3}},$$

along with a radius where it vanishes, $r_o = Q^2/M$, as well as a maximum at $r_{max} = 4r_o/3$. The first radius defines the zero-gravity sphere (*Vieira & Kluźniak 2023*), where the net gravitational force is repulsive.

Numerical setup:

Simulations were performed using the 2D MHD module of the PLUTO code (*Mignone et al. 2007*). The initial state of the accretion disk and corona was set according to the analytical solution of *Kluźniak & Kita* (*2000*). The initial magnetic field was defined as a weak field with an hourglass shape (*Mishra et al. 2020*). We applied distinct inner boundary conditions for both types of compact objects: reflective for naked singularities and outflow for black holes. Bremsstrahlung cooling was implemented using units appropriate for a SMBH. Viscosity and resistivity were parametrized by the Shakura-Sunyaev *α* parameters (α_v for viscosity and α_m for resistivity).

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References:

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Results:

Accretion around naked singularities stops near the zero-gravity sphere due to the repulsive net force in this region (Fig. 1a-c). In contrast to black holes (Fig. 1d), matter accumulates around the singularity, with an enhanced density near the zero-gravity sphere. The repulsive effect is most pronounced for cases with *Q/M* = 1.5 and 1.9, where the density is considerably reduced on the inner side of the zero-gravity sphere.

The radial distributions reveal different characteristics of outflows between black holes and naked singularities. Outflows from naked singularities remain relatively weak and are more evenly distributed across the entire angular domain (Fig. 2a-c). In contrast, black holes produce significantly stronger and coherent outflows, with streams concentrated in a broad region along the axis perpendicular to the disk midplane (Fig. 2d).

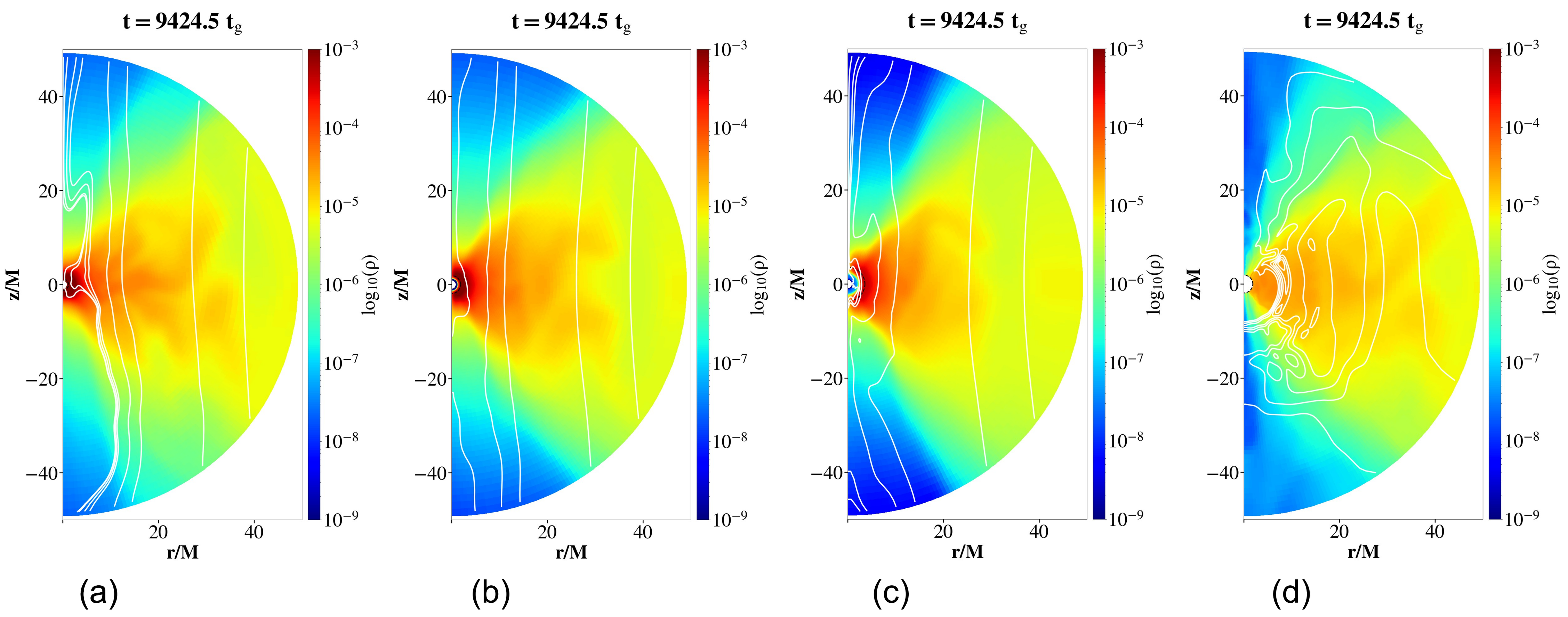

Figure 1: *Logarithmic gas density distribution at t = 9424.5 t_g obtained for simulations with α_v = 0.5 and α_m = 0.5. Black dashed lines mark the outer event horizon (black hole) and the zero-gravity sphere (naked singularity). White lines represent sample of magnetic field lines. (a) Naked singularity with charge-to-mass ratio Q/M = 1.1, (b) Q/M = 1.5, (c) Q/M = 1.9, (d) Black hole with Q/M = 0.5.*

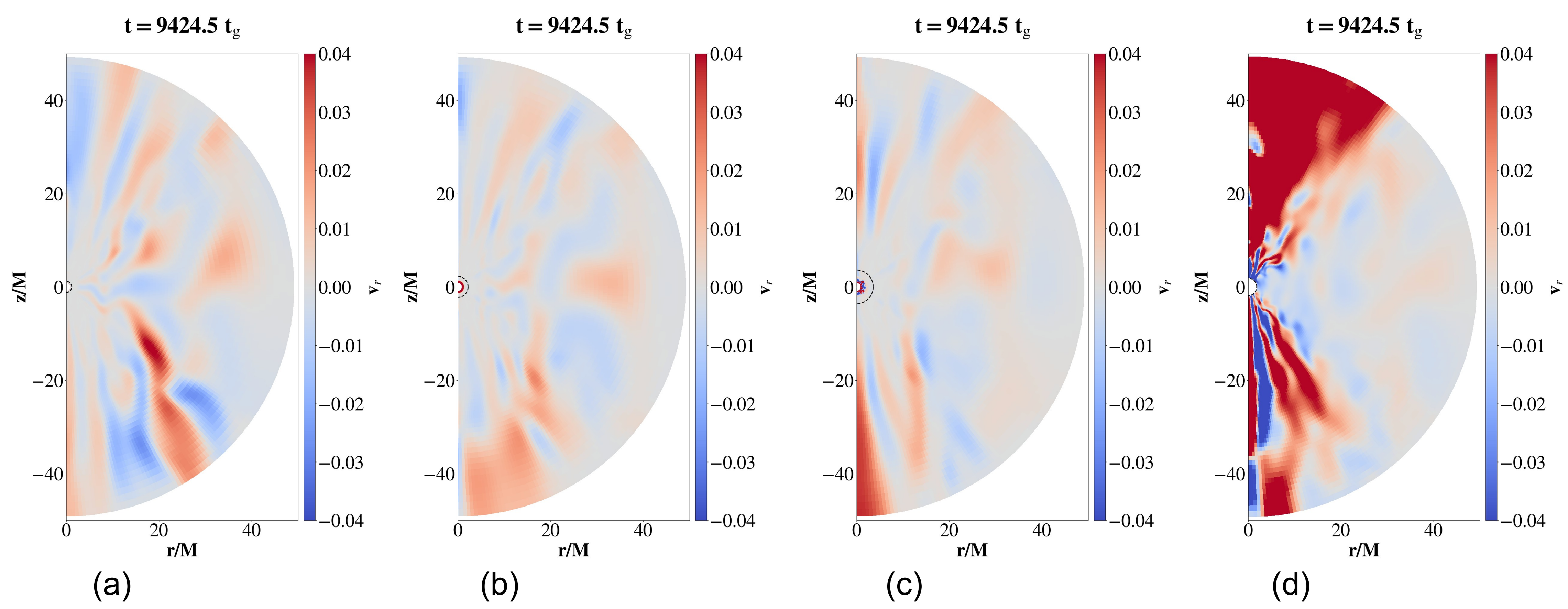

Figure 2: *Radial velocity distribution at t = 9424.5 t_g obtained for simulations with α_v = 0.5 and α_m = 0.5. Black dashed lines mark the outer event horizon (black hole) and the zero-gravity sphere (naked singularity). (a) Naked singularity with charge-to-mass ratio Q/M = 1.1, (b) Q/M = 1.5, (c) Q/M = 1.9, (d) Black hole with Q/M = 0.5. Velocity components were normalized to the range [-1, 1], where -1 and 1 represent the maximum and the minimum value of radial velocity, respectively. For more pronounced visual contrast the color scale was truncated to [-0.04, 0.04].*

Conclusions and future work:

Naked singularities and black holes in the Reissner–Nordström spacetime, in the presence of a magnetic field, exhibit distinct features evident in the geometry of the accretion disk density near the singularity or event horizon, and in the distribution of the resulting outflows.

In this study, we focus on the weak magnetic field, as a preliminary work for initial check of the magnetic effects on the flow. Our ongoing work aims to implement stronger magnetic fields, which require more advanced cooling prescription to keep the disk more stable.