

Numerical simulations of star-disk magnetospheric interaction

Miljenko Čemeljić

*With W. Kluźniak, V. Parthasarathy @CAMK, Warsaw
& with A.S.Brun @CEA, Saclay*

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Introduction- Star-disk problem in protostars- TOUPIES project

During the evolution from a pre-stellar core to protostar, the angular momentum decreases for about 4 orders of magnitude. The spin-up of a star is probably prevented by the magnetic interaction between the star and the disk, but the exact mechanism of this decrease is still not known. The angular momentum can be extracted from the system by violent outbursts, stable outflows & jets, or by an accretion column from the disk onto the star.

In CEA Saclay in 2014/15, I took part in the French ANR-TOUPIES (TOwards Understanding the sPIn Evolution of Stars) project with A.S. Brun, on rotational history of solar-like stars. In this collaboration, rotational velocities of several hundreds of stars in open clusters, at various evolutionary stages, are measured. Goal is to find scaling laws for exchange of angular momentum between the star and surrounding. Then the predicted stellar torques could be used in stellar evolution models.

My task was to investigate the influence of geometry of magnetic field on the transport of angular momentum between the star and the environment. I developed a successful setup and performed a parameter study, determining the torque in the system for different resistivities. I also checked the results with various strengths and topology of magnetic field. In the further study, the rotation rates from 2-10 days, accretion rates from 10^{-9} to 10^{-6} solar mass/year, with mass outflow of about 1/10 of the accretion rate, should be investigated.

Introduction- Star-disk problem in protostars- TOUPIES project

μ (B[kG])	α_v α_m P_m	$\theta=[0,\pi/2]$ Tempf t (P_*)	$\theta= [0,\pi]$ Tempf t (P_*)
1.4 (1.0)	1.0 0.1 6.7	D 800 100	Simulation S1 , run for comparison with C1 in ZF13
0.7 (0.5) Runs for our parameter study. Quadrupole and octupole runs only in $\theta= [0,\pi]$ case.			
	1.0 0.1 6.7	D 800 100	D 800 Q 200 O
	1.0 0.4 1.67	D 800 100 S2	D 800 Q 800 O
	1.0 0.7 0.95	D 800 86.9	D 800 Q 800 O
	1.0 1.0 0.67	D 800 85.2	D 800 Q 800 O
0.7 (0.5) Initial runs for wider parameter study.			
	0.7 0.1 4.67	D S6	D Q 800 O
	0.7 0.4 1.17	D 	D Q 800 O 400
			62

Table 1: Parameter space explored in my simulations.

Parameters of a typical protostar: $M=0.5M_{\text{sun}}$, $R=2R_{\text{sun}}$, $P_{*}=4.63$ days, $v_{K,0}=218\text{km/s}$, $\rho_0=8.5 \times 10^{-11} \text{ g/cm}^3$, $B_0=500 \text{ G}$, 1kG

Star-disk simulations setups

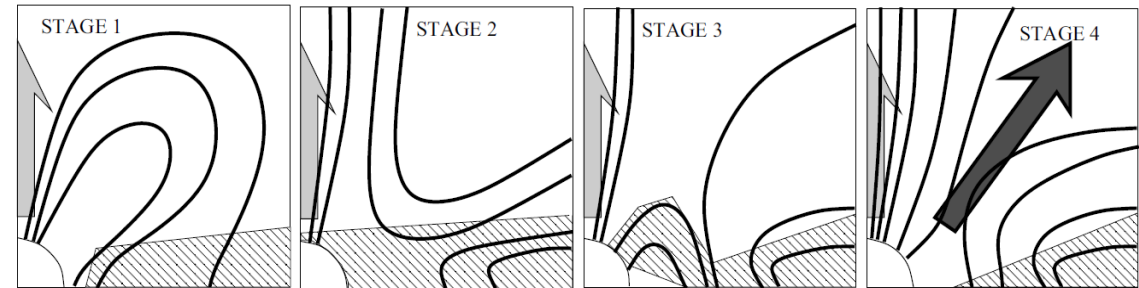
There exist only two sets of long-lasting star-disk MHD simulations—both before 2010, and no-one except their authors could repeat them. My goal is to obtain long-lasting quasi-stationary solutions, which could eventually be repeated by other researchers.

Tool: PLUTO, a finite volume/difference code. We solve viscous & resistive MHD equations, with split field method and constrained transport for $\text{div } \mathbf{B}=0$.

To avoid thermal thickening of the disk, I remove the viscous and Ohmic dissipative terms in the energy equation. Another method is to introduce the cooling source function which removes the heat.

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$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \left[\rho \mathbf{u} \mathbf{u} + \left(P + \frac{\mathbf{B} \cdot \mathbf{B}}{8\pi} \right) \mathbf{I} - \frac{\mathbf{B} \mathbf{B}}{4\pi} - \boldsymbol{\tau} \right] = \rho \mathbf{g}$$

$$\frac{\partial E}{\partial t} + \nabla \cdot \left[\left(E + P + \frac{\mathbf{B} \cdot \mathbf{B}}{8\pi} \right) \mathbf{u} - \frac{(\mathbf{u} \cdot \mathbf{B}) \mathbf{B}}{4\pi} \right] + \nabla \cdot [\eta_m \mathbf{J} \times \mathbf{B} / 4\pi - \mathbf{u} \cdot \boldsymbol{\tau}] = \rho \mathbf{g} \cdot \mathbf{u} - \Lambda_{\text{cool}}$$

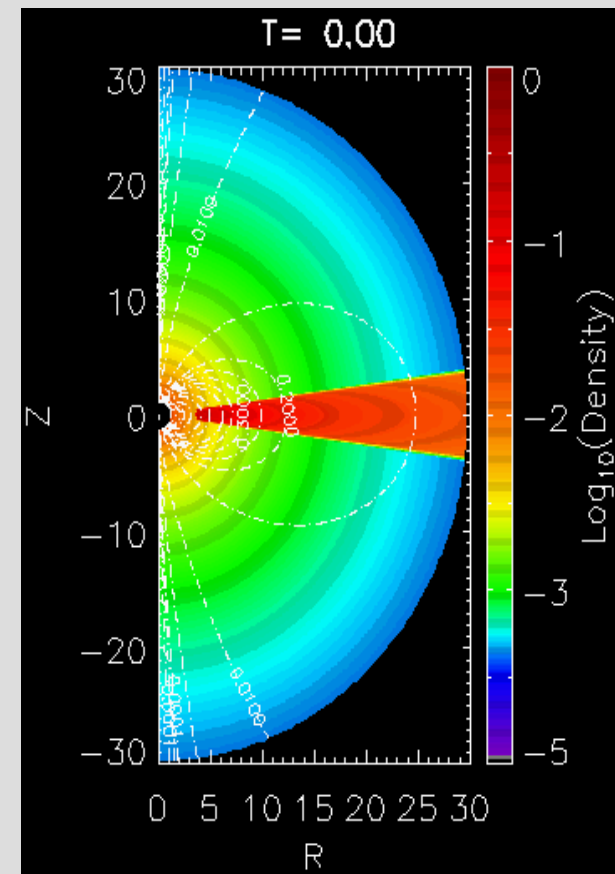
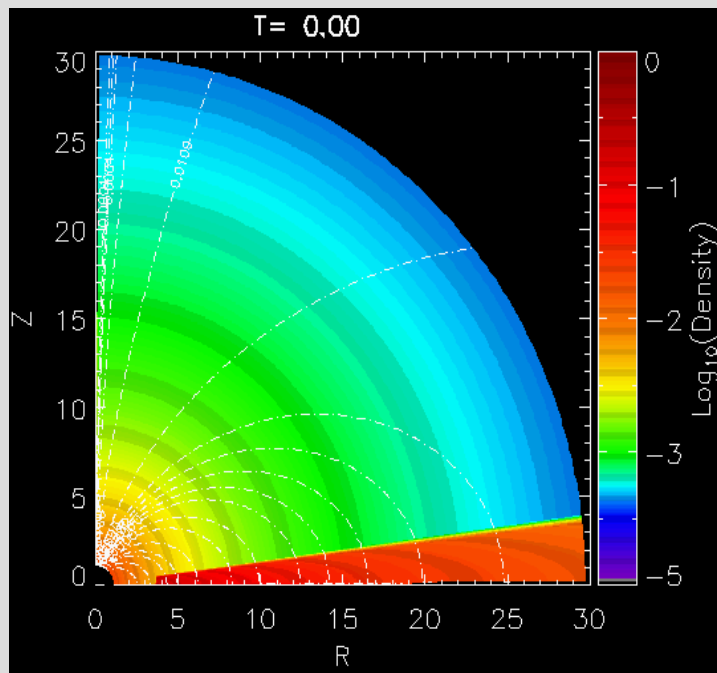
$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{B} \times \mathbf{u} + \eta_m \mathbf{J}) = 0.$$

Star-disk simulation setups

The disk is set by Kluźniak & Kita (2000) model.

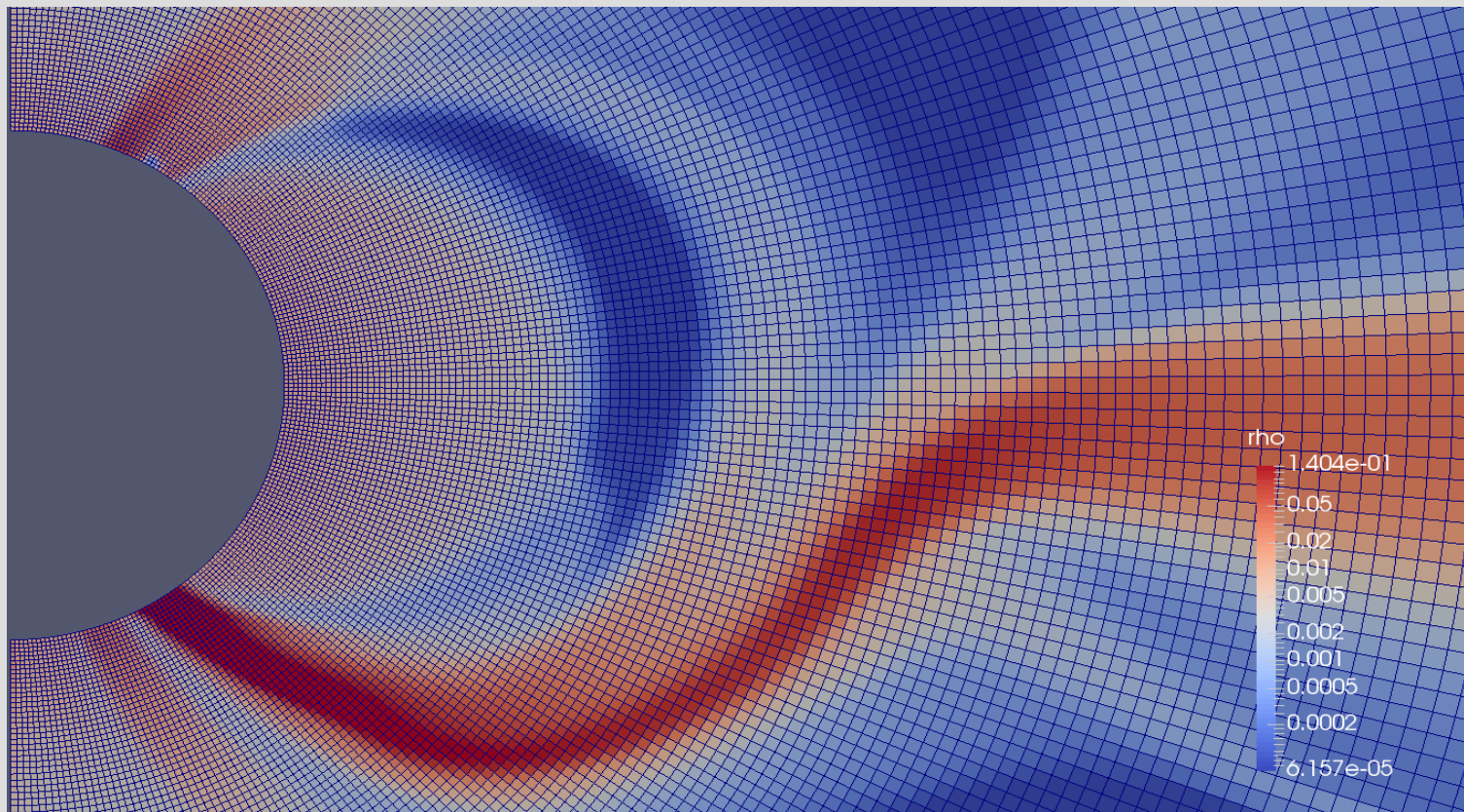
I set two kinds of 2D axi-symmetric simulations: **a)** in the half-plane $\vartheta=[0,\pi/2]$ and **b)** the full plane $\vartheta=[0,\pi]$, both to $R_{\text{max}}=30R_*$. I show the density in the logarithmic color grading.

In the case **b)**, we do not prescribe the disk equatorial plane as a boundary condition, so that a more complete disk evolution is obtained.



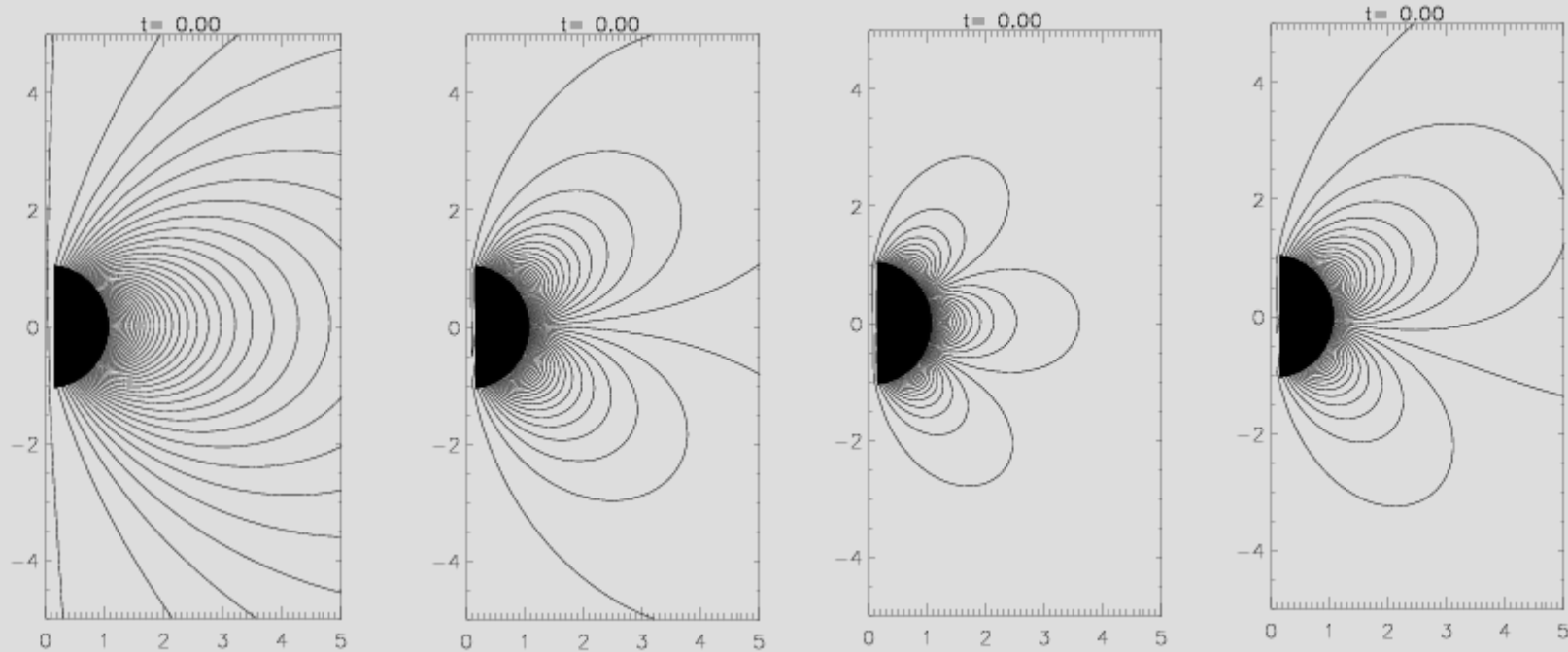
Star-disk simulation setups

Resolution is $R \times \vartheta = [217 \times 200]$ grid cells in $\vartheta = [0, \pi]$, with a logarithmic grid spacing in the radial direction. In a zoom close to the star after $T = 25$ stellar rotations, for the dipole magnetic field case, I show that the accretion column is well resolved. Star in my simulations typically rotates at about 1/10 of the breakup rotational velocity.



Star-disk simulation setups

I am investigating solutions with the different geometries of a stellar magnetic field: dipole, quadrupole, octupole and combinations of those (multipole).



Stellar surface as a boundary condition

Special care is needed for matching of stellar and rotation of the magnetic field lines. Star is assumed to be a perfect, rotating conductor:

$$\mathbf{E}_{\Omega=\Omega_{\star}} = \mathbf{B} \times (\mathbf{u} - \Omega_{\star} \times \mathbf{R}) = 0$$

$$u_{\phi} = r\Omega_{\star} + u_p B_{\phi} / B_p$$

In addition to this, we need to set the correct magnetic torque to drive the plasma rotation atop the star. We measure the matching by the comparison of the stellar angular velocity and the effective rotation rate of the field lines:

$$\Omega_{\text{eff}} = \Omega - u_p B_{\phi} / r B_p$$

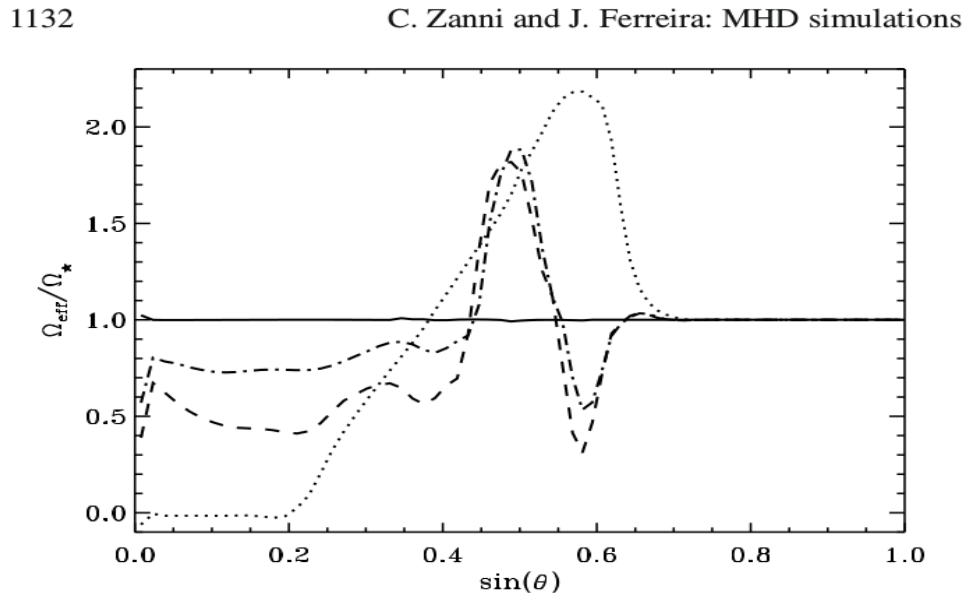
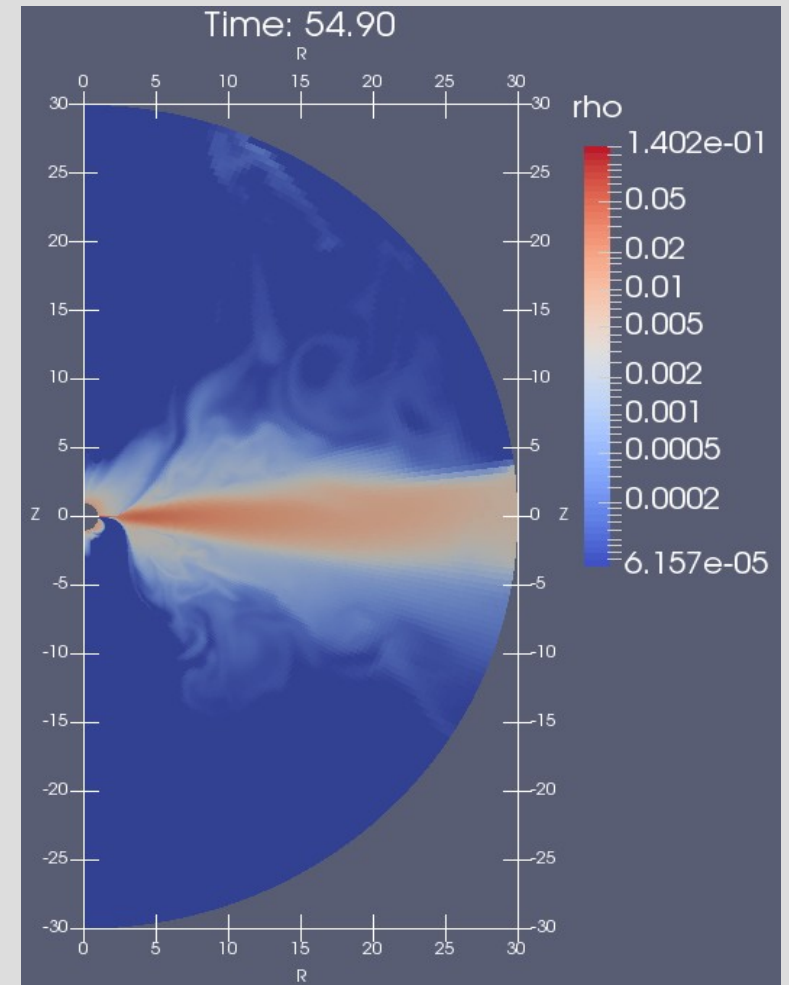
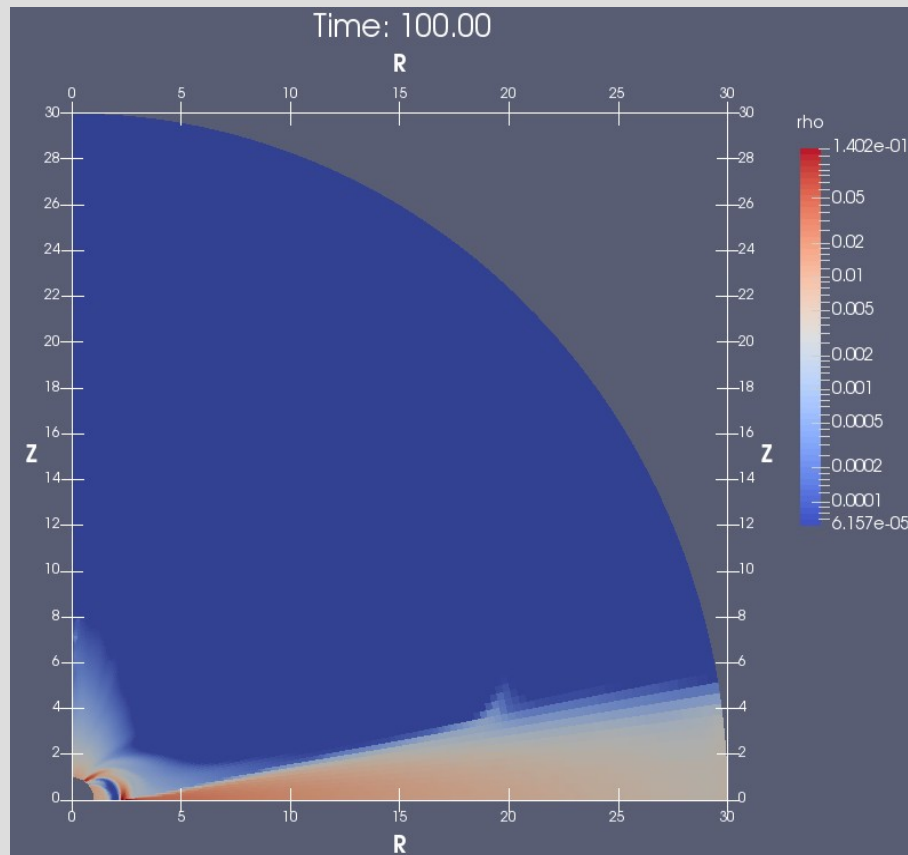


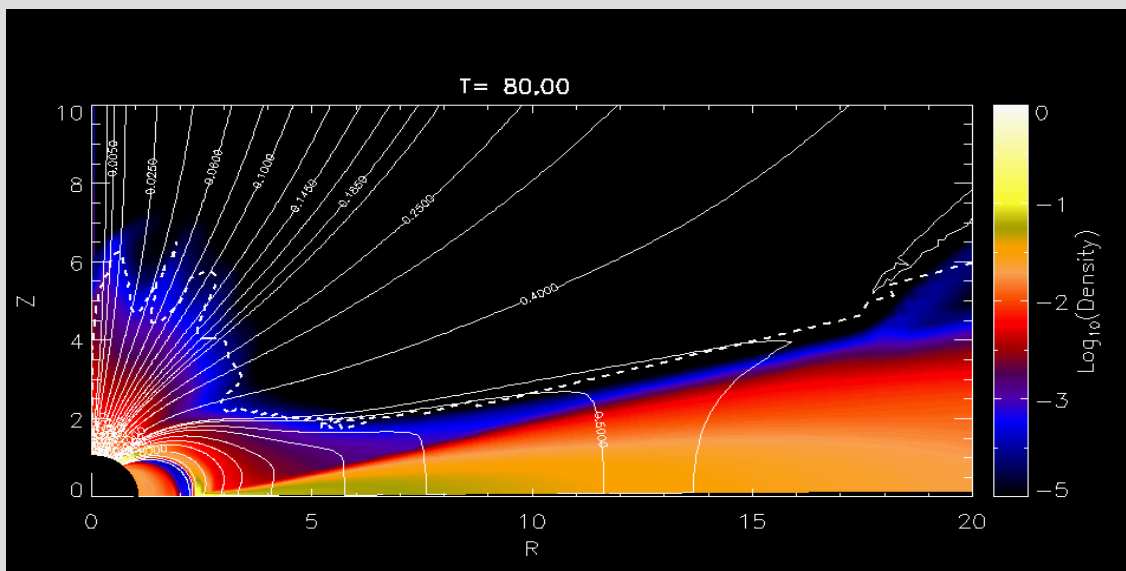
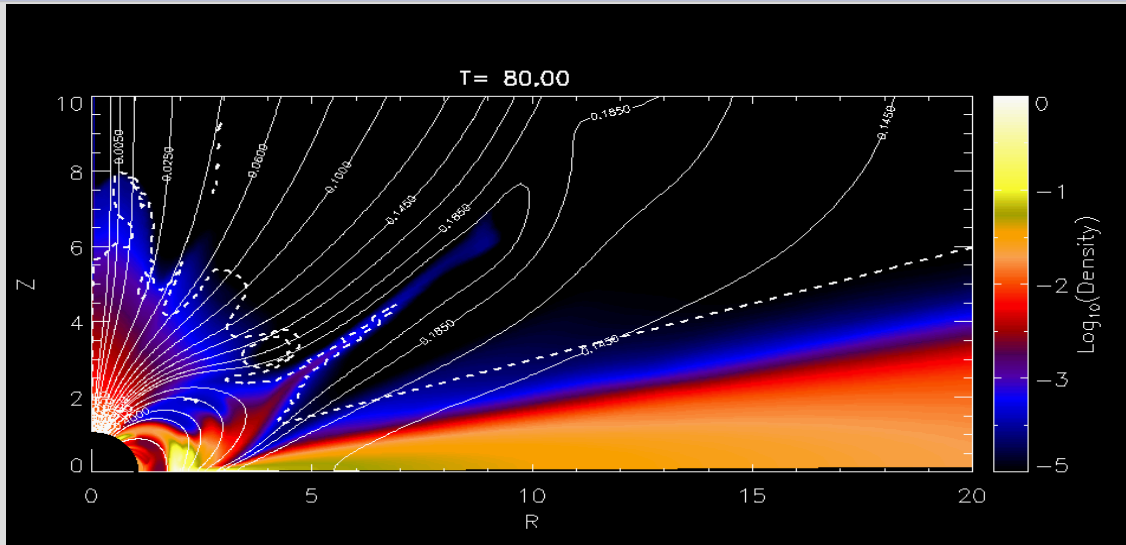
Fig. A.1. Effective rotation rate of the magnetic surfaces measured on the surface of the star as a function of the polar angle θ . The curves correspond to different boundary conditions on the toroidal field: the boundary condition used in this paper (solid line), $\partial(RB_{\phi})/\partial R = 0$ condition (dot-dashed line), “outflow” boundary condition (dashed line), and $B_{\phi} = 0$ condition (dotted line). The snapshots are taken after ~ 64 periods of rotation of the central star.

Preliminary results: YSO

I show the preliminary results with the **dipole** and **quadrupole** stellar magnetic field. Shown is the density in the whole computational box.



Preliminary results: YSO in $\vartheta=[0,\pi/2]$

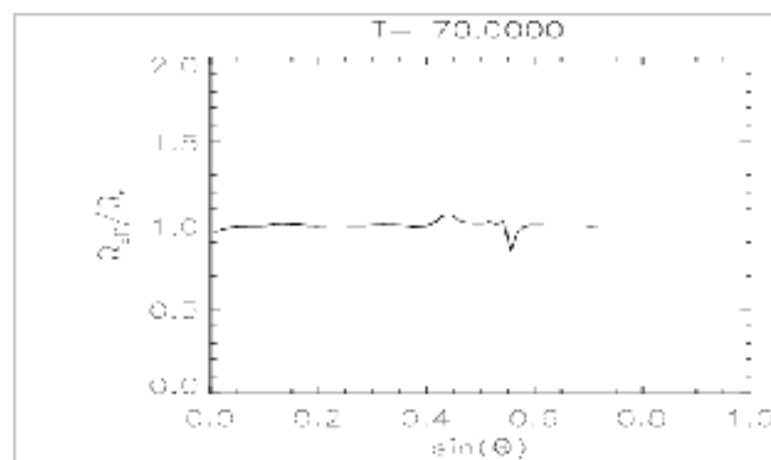
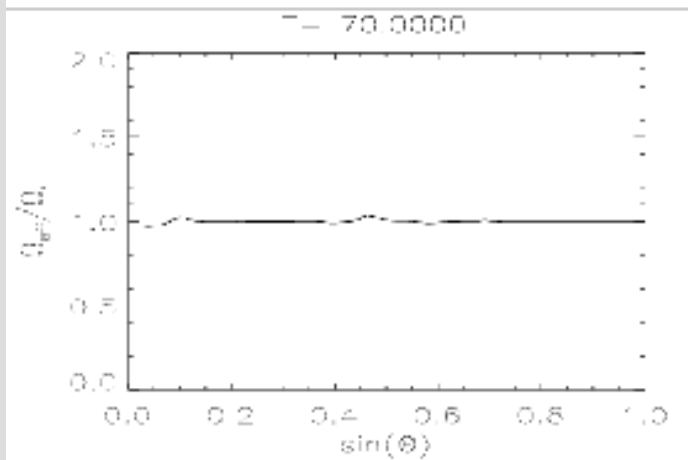
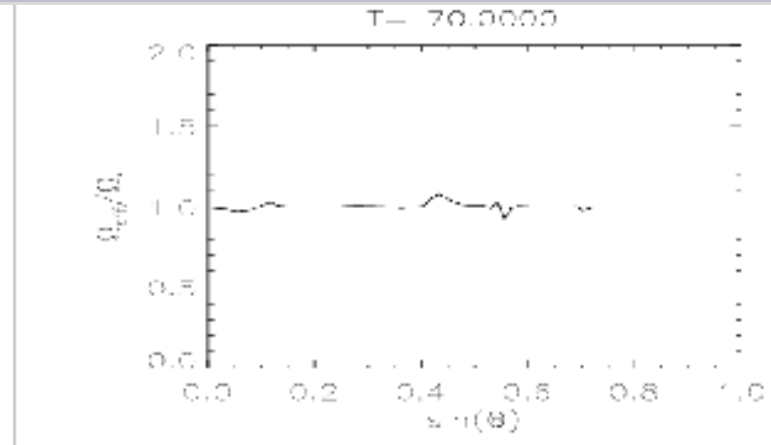
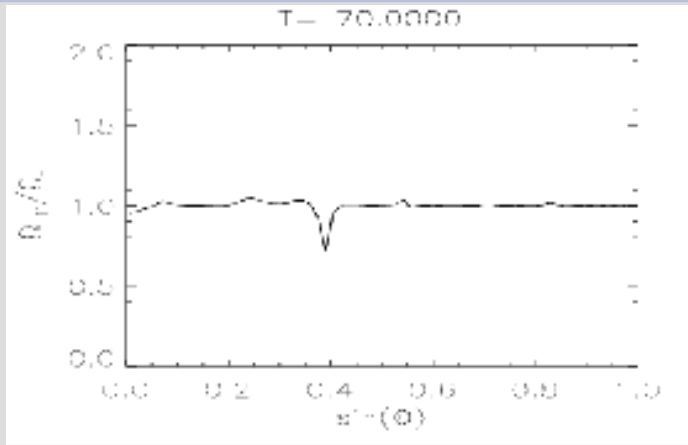


Zoom into the preliminary results with the **dipole** stellar magnetic field of 500 G (my simulations S2), in simulations with different resistivity. In the top panel, $\alpha_m=0.1$, in the bottom, it is 1.0

In the more resistive simulations, magnetic field lines connecting the star and the disk, extend well beyond the corotation radius, which is $R_{cor}=4.65 R_*$ in our setup.

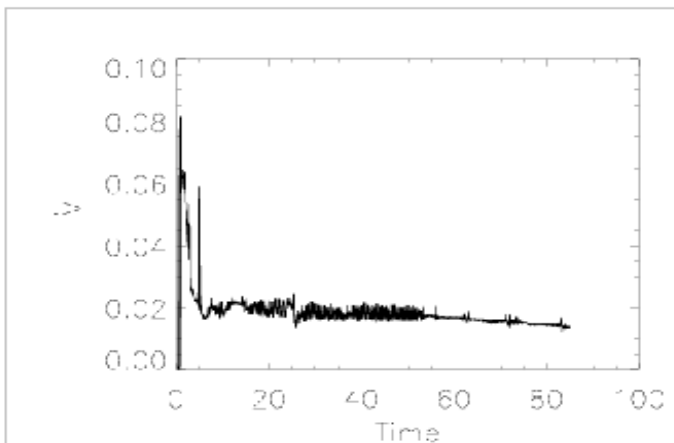
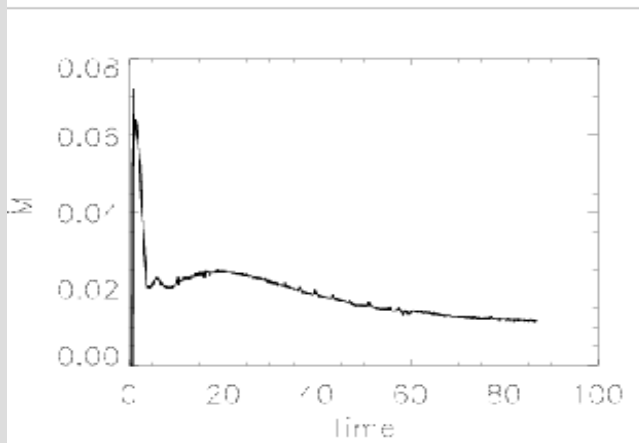
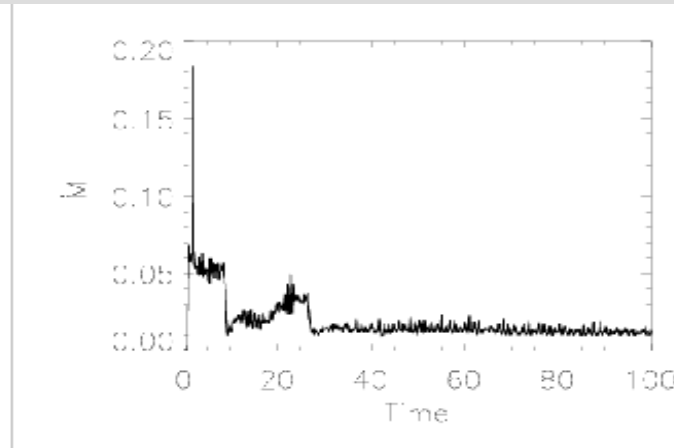
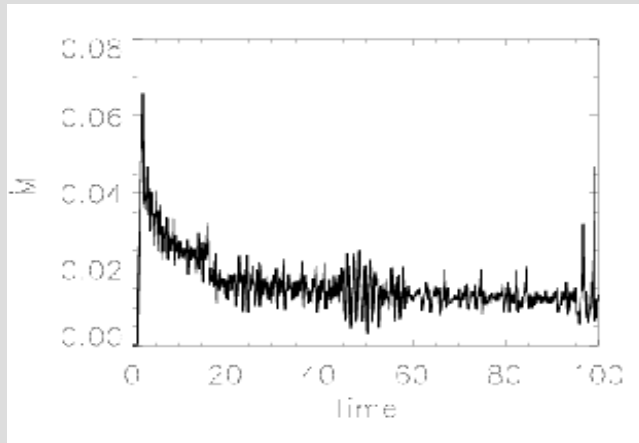
Torque exerted on the star by the infalling material is different in both amount and sign.

Preliminary results: YSO in $\vartheta=[0,\pi/2]$



$\Omega_{\text{eff}}/\Omega_*$ in S2 with $\alpha_v=1, 0.4, 0.7, 1.0$, respectively.

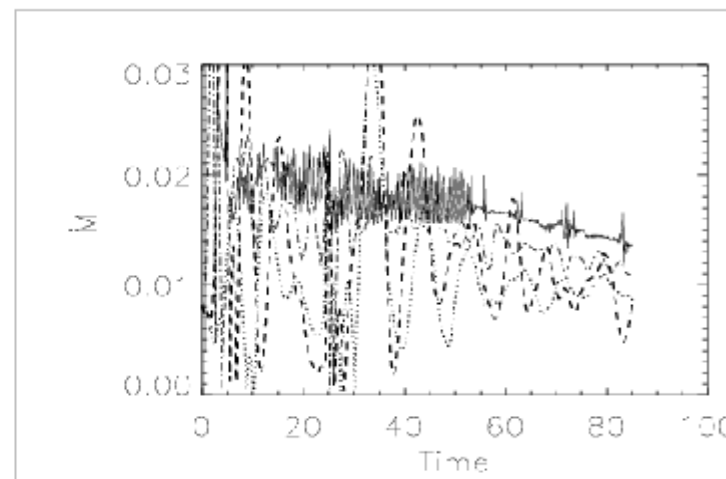
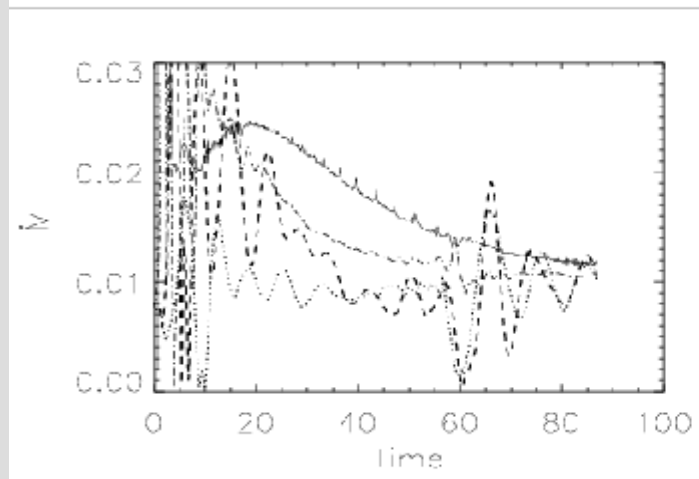
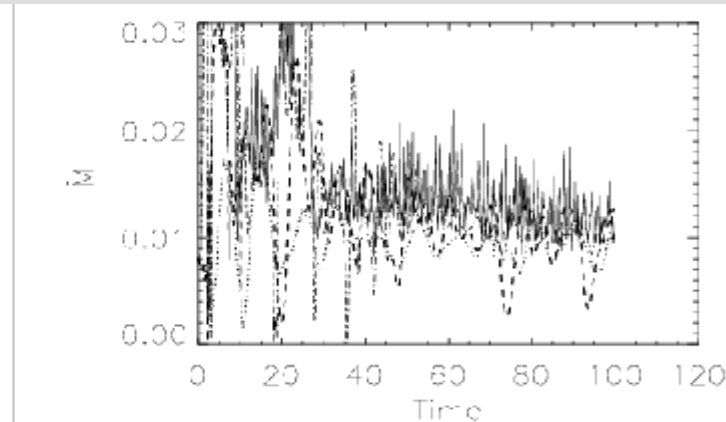
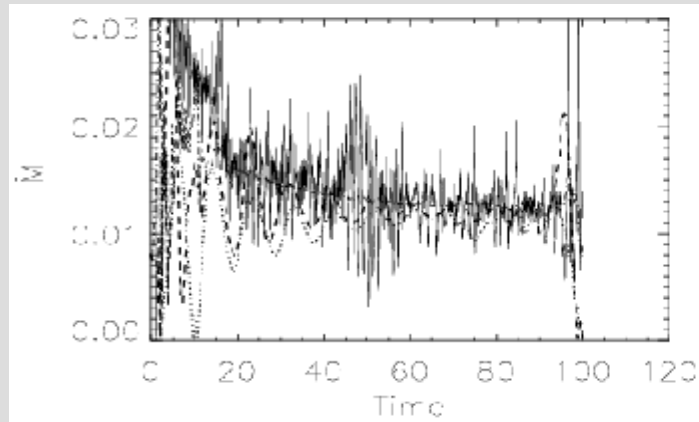
Preliminary results: YSO in $\vartheta=[0,\pi/2]$



Mass accretion rates in S2 with $\alpha_v=1$, $\alpha_m=0.1, 0.4, 0.7, 1.0$, respectively. It matches the result for

KK00 disk: $\dot{M}_d \sim 0.014 \alpha_v \left(\frac{\epsilon}{0.1}\right)^3 \dot{M}_0.$

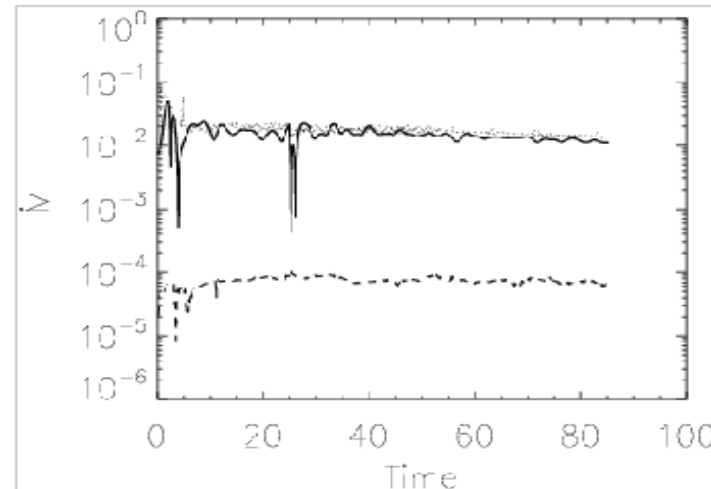
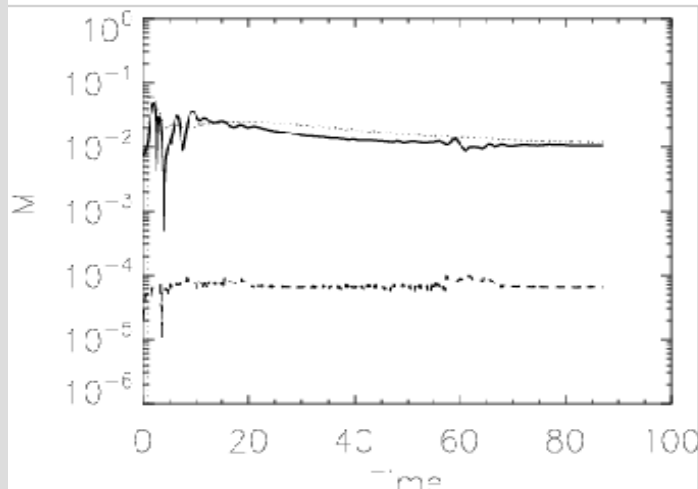
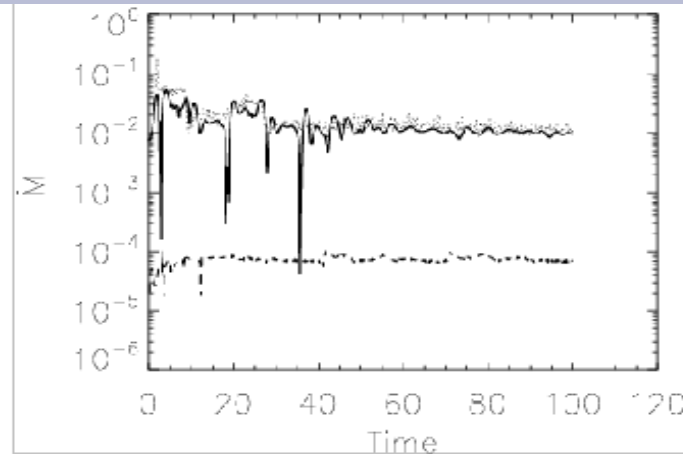
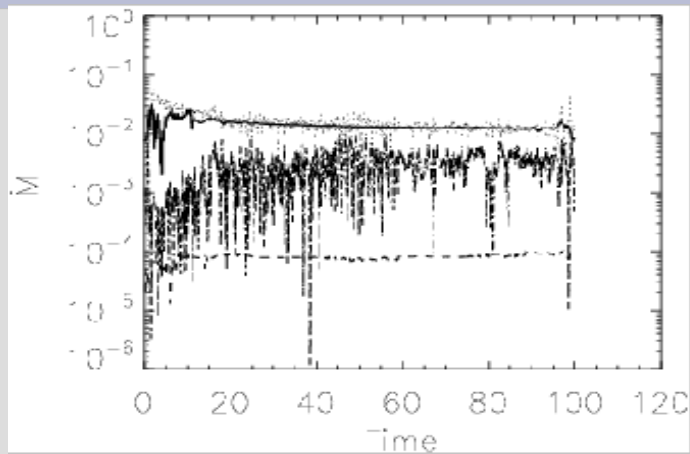
Preliminary results: YSO in $\vartheta=[0,\pi/2]$



Additional information about the stability of the simulations, and the amount of mass in the system which passes on to the star, and which is diverted into the outflows is given in the figures here.

Comparison of mass accretion rates at the stellar surface (solid line) and at various radii in the disk in simulations S2, left to right, top to bottom, respectively. $R=(29, 20, 12) \cdot R_$ in dotted, dashed and dot-dashed line, respectively. Amount of matter reaching the stellar surface is less than the total accretion through the disk. Depending on the disk resistivity, some portion of matter accreted through the disk is launched into the stellar wind and outflow.*

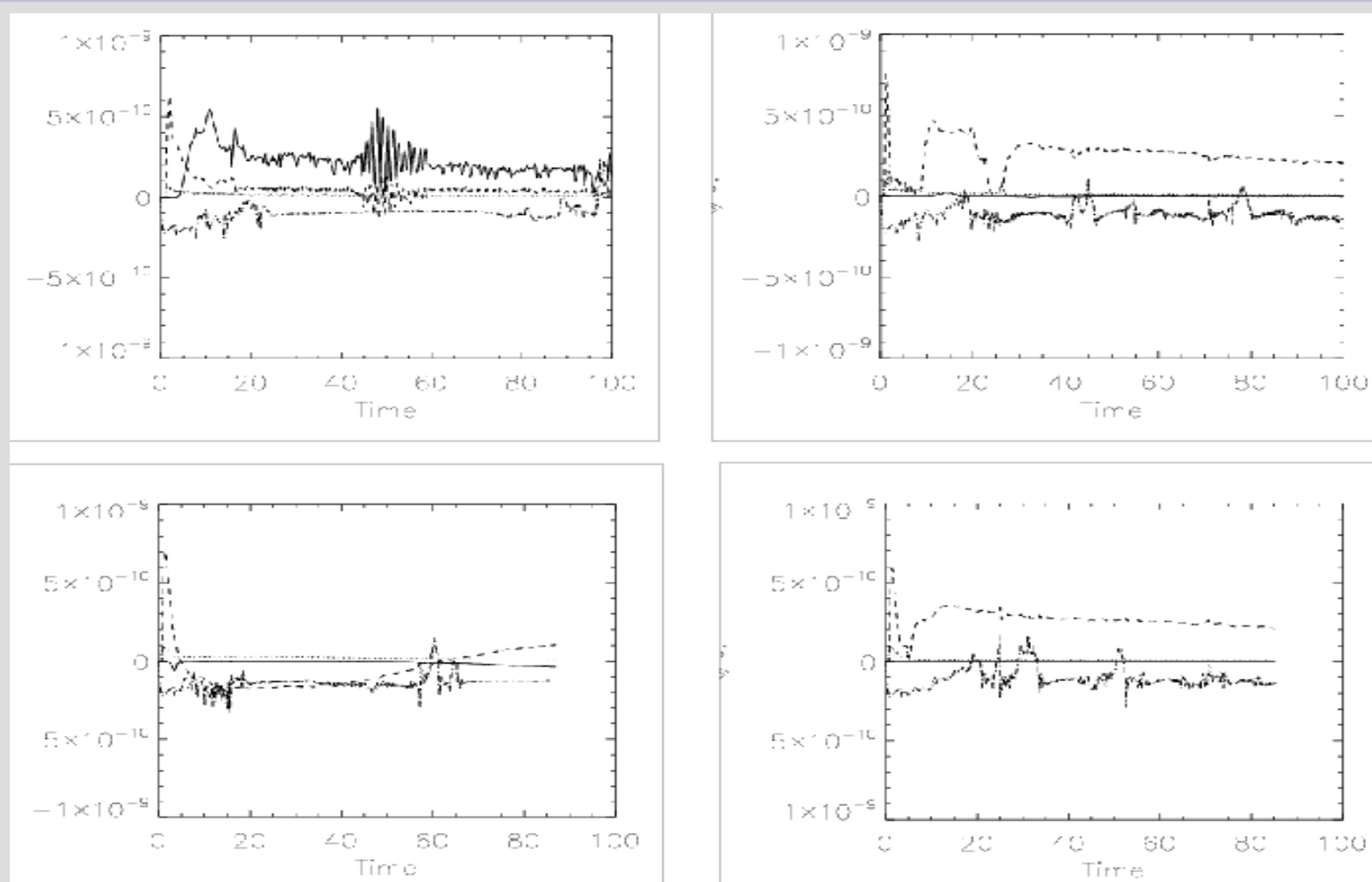
Preliminary results: YSO in $\vartheta=[0,\pi/2]$



Another useful information is comparison of the mass accreted through the disk with the mass accreted onto the star, into the stellar wind and, if such exists, into the outflow.

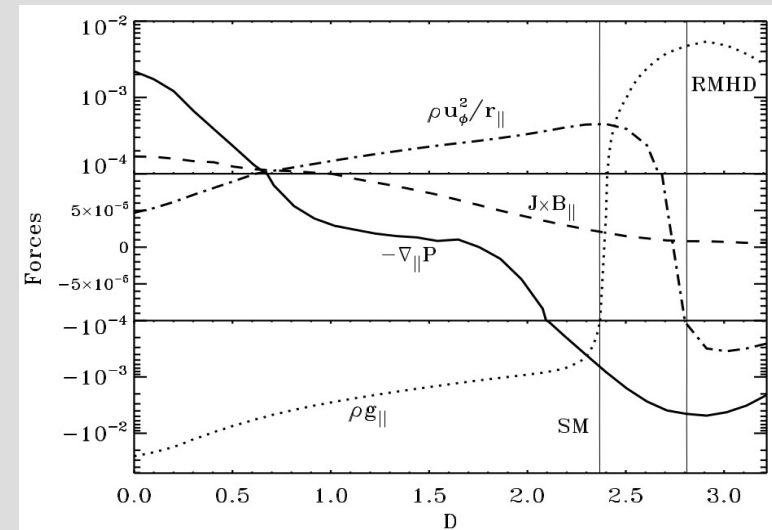
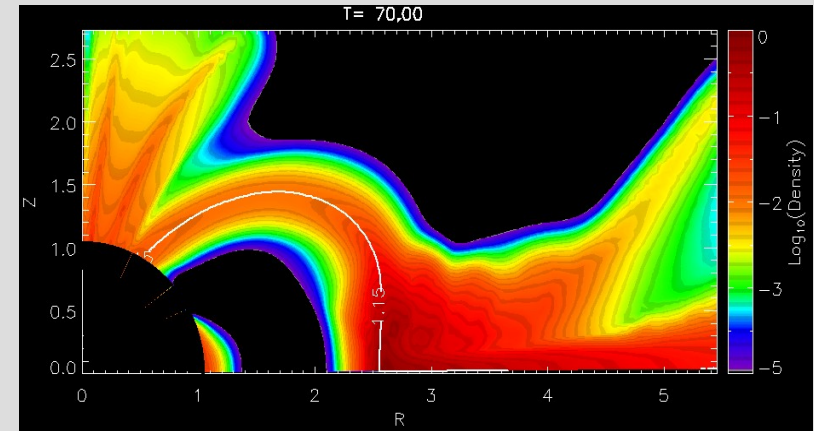
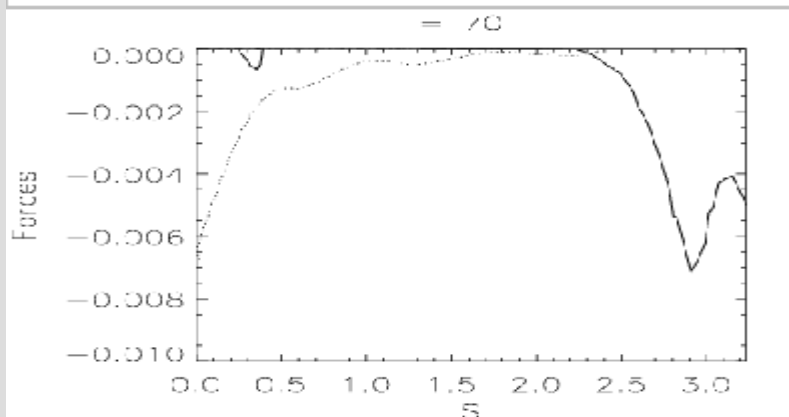
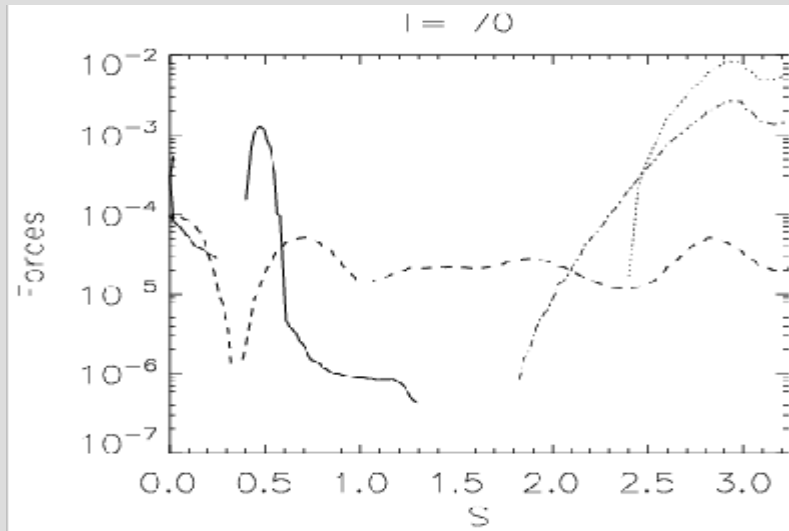
The mass fluxes in simulations S2, with $\alpha_m=0.1,0.4,0.7,1.0$, left to right, top to bottom, respectively, through the disk at $R=12 R_$ (dotted line), onto the star (solid line), stellar wind (dashed line). In the $\alpha_m=0.1$ case, additionally is shown the mass flux loaded into the MEs in a thick long dashed line. Most of the mass accreted through the disk is in-falling onto the star. About two orders of magnitude less matter flux ends loaded into the stellar wind.*

Preliminary results: YSO in $\vartheta=[0,\pi/2]$



Torques onto the star in S2 with $\alpha_v=1$, $\alpha_m=0.1, 0.4, 0.7, 1.0$, respectively, as in the previous figures. Kinetic torque (dotted line), the magnetic torque acting along the opened field lines (dot-dashed line), along the magnetosphere connected to the disk below (solid line) and beyond (dashed line) the corotation radius.

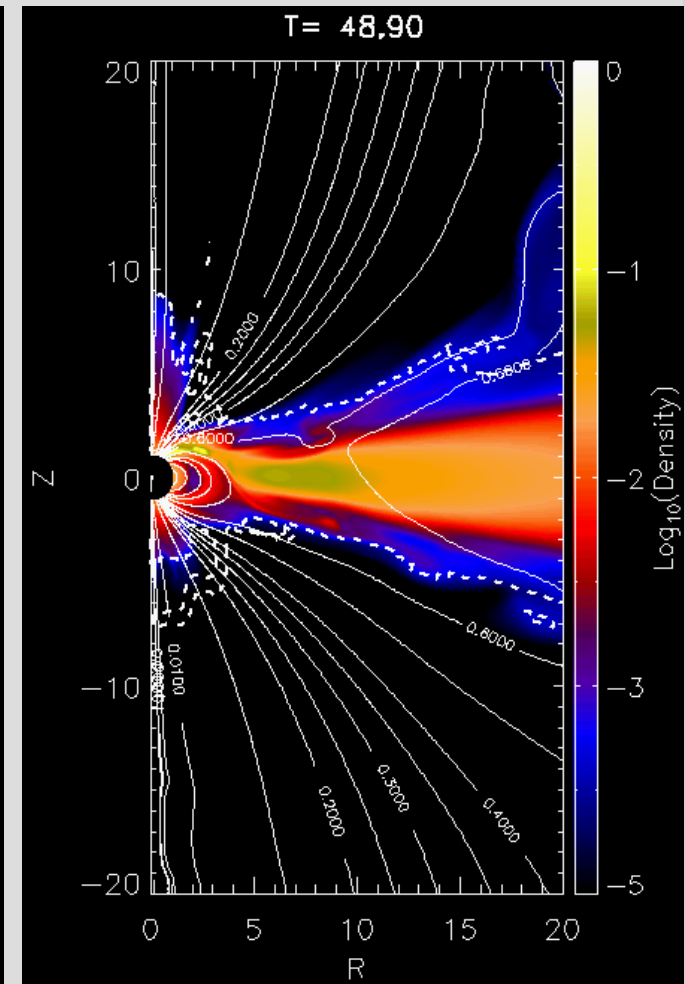
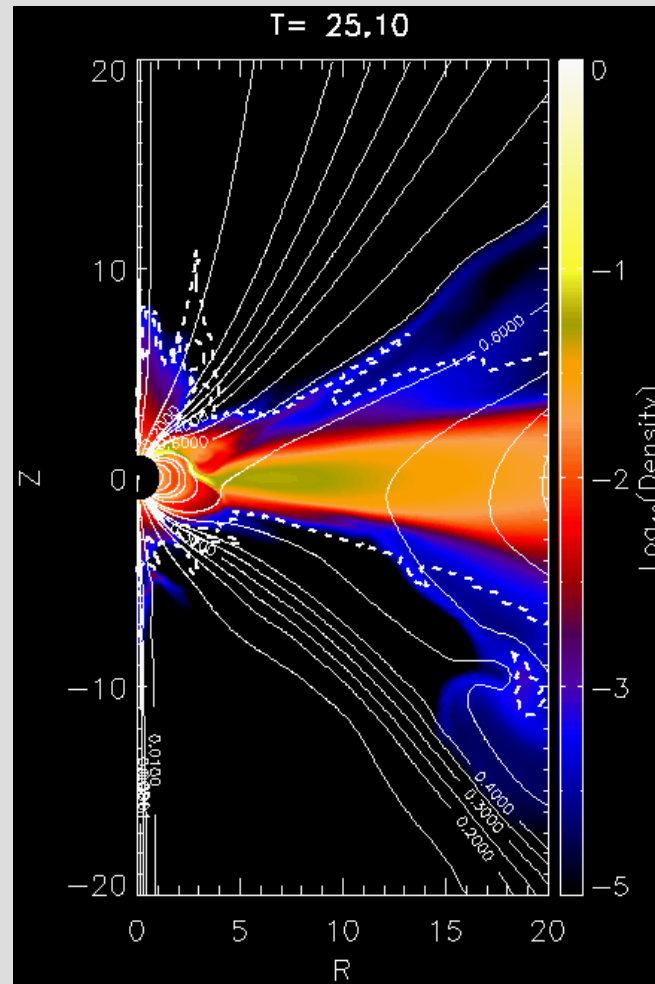
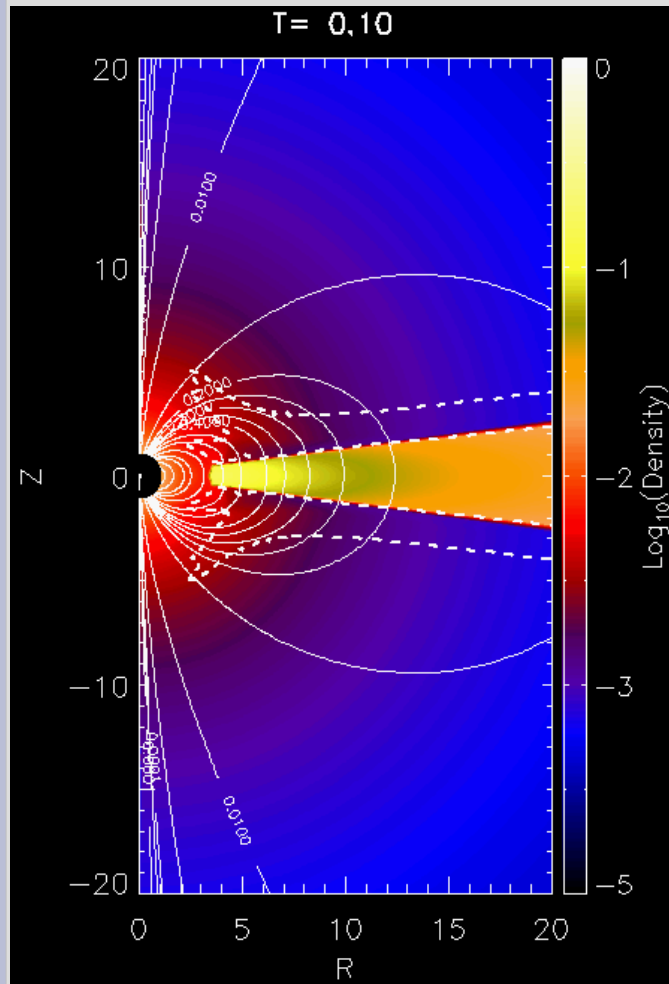
Preliminary results: YSO in $\vartheta=[0,\pi/2]$



In the left panel shown are forces along the line passing through the middle of the accretion column in simulation S1 ($B=1kG$), which is shown in the right panel. The positive values of projections onto the B_p line are plotted in logarithmic scale, and below is the plot for negative values of force projections, in linear scale. Line types in the left panels assign the same forces as marked in the reference plot from ZF09, in the right bottom panel.

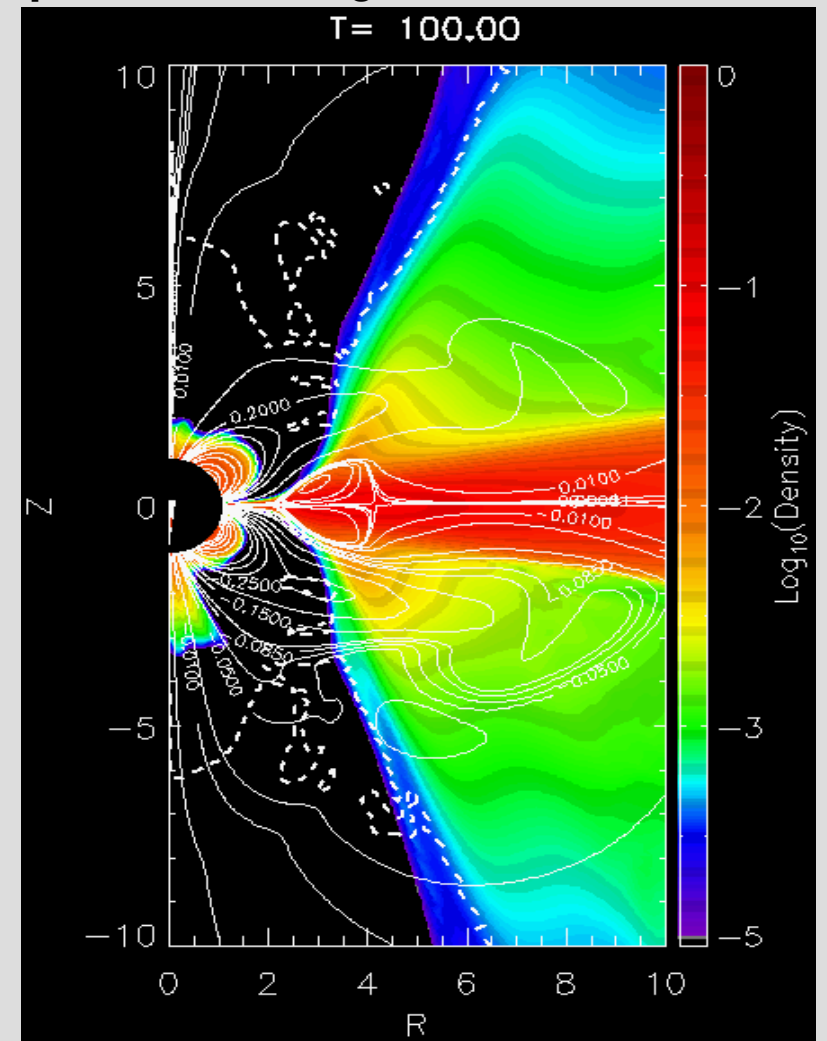
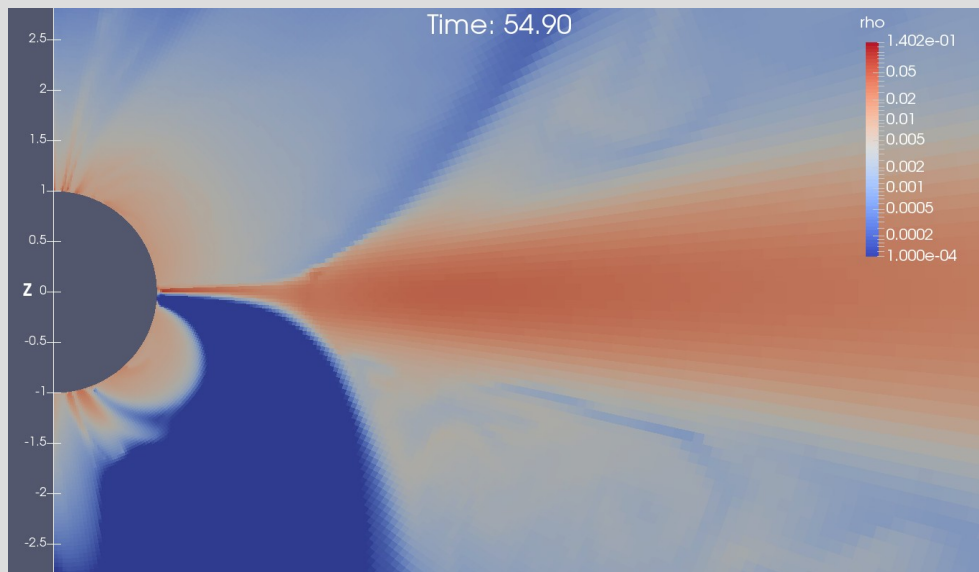
Preliminary results: YSO in $\vartheta=[0,\pi]$

Stills from the animation of results with the **dipole** magnetic field.



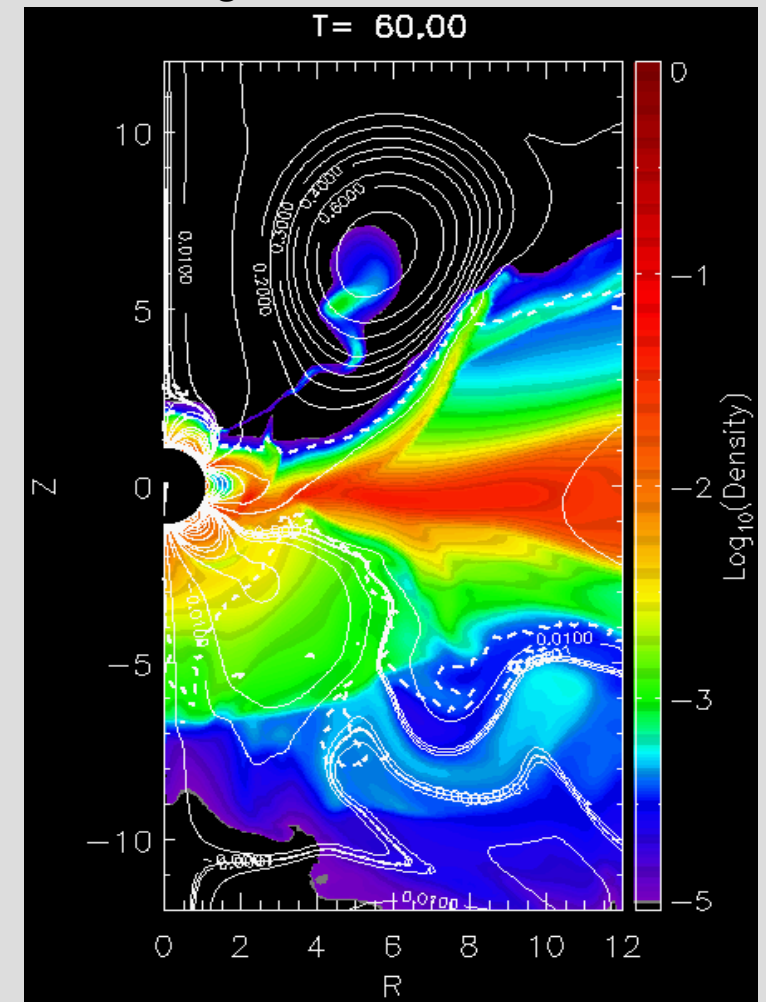
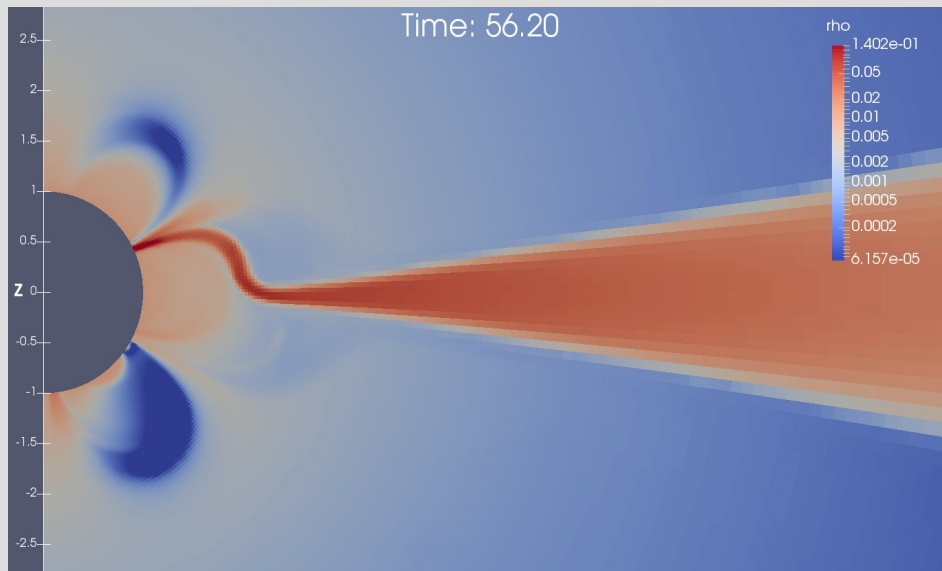
Preliminary results: YSO in $\vartheta=[0,\pi]$

Zoom into the preliminary results with the **quadrupole** stellar magnetic field.



Preliminary results: YSO in $\vartheta=[0,\pi]$

Zoom into the preliminary results with the **octupole** stellar magnetic field.



Star-disk problem in White Dwarfs

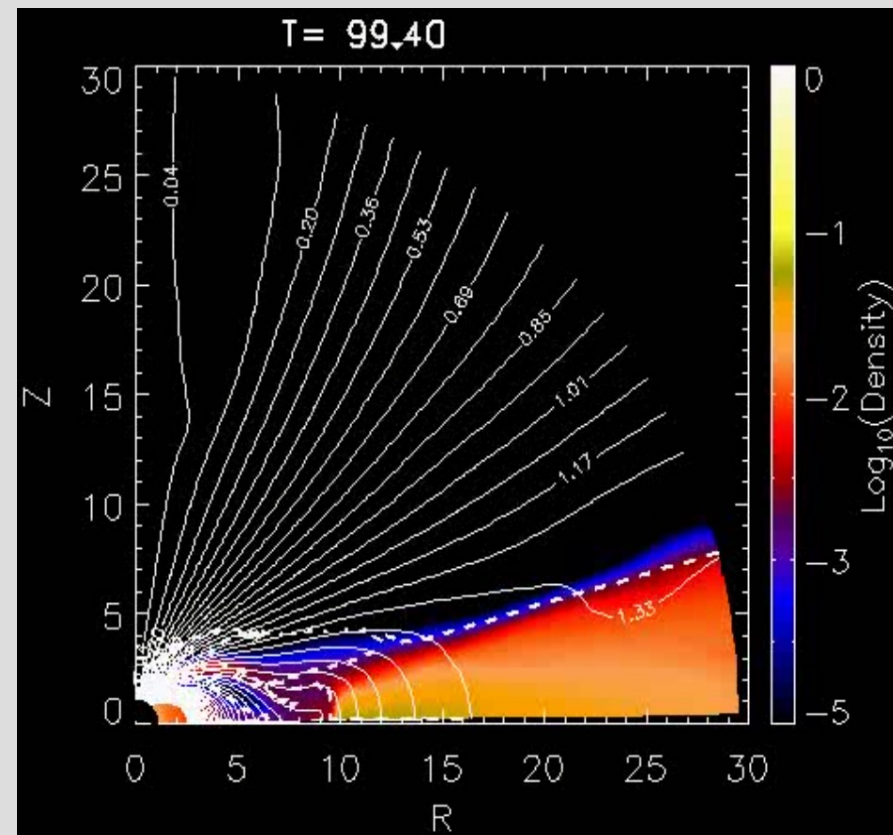
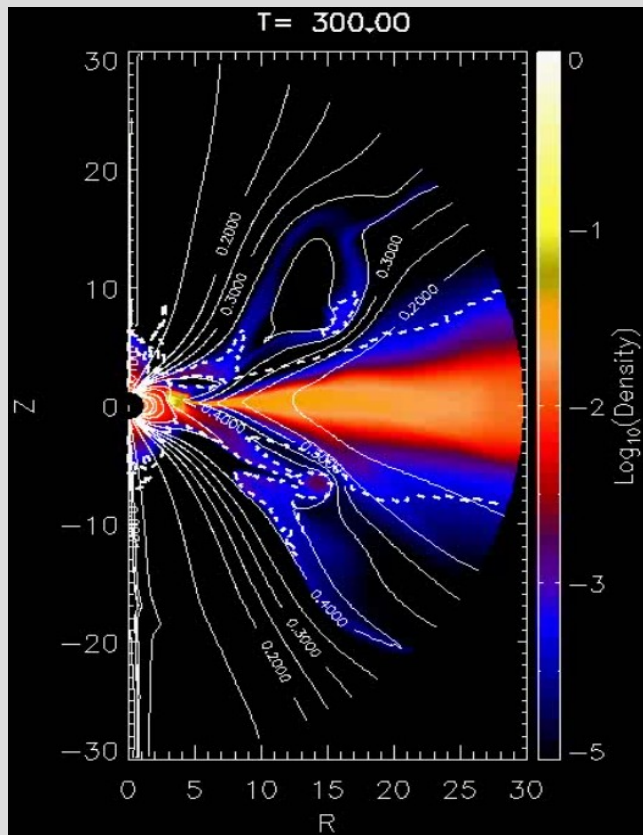
White dwarfs are Earth-radius stars of Solar mass.

In the WD case I set $M_* = M_{\text{sun}}$, $R_* = 5000 \text{ km}$, $P_* = 61 \text{ s}$, $B_* = 10 \text{ kG} / 100 \text{ kG}$,
 $\rho_0 = 9.4 \text{ e-}9 \text{ g/cm}^3$

$\dot{M}_0 = 1.9 \text{ e-}8 \text{ M}_{\text{sun}}/\text{yr}$, $V_{K,*} = 5.15 \text{ e}8 \text{ cm/s}$

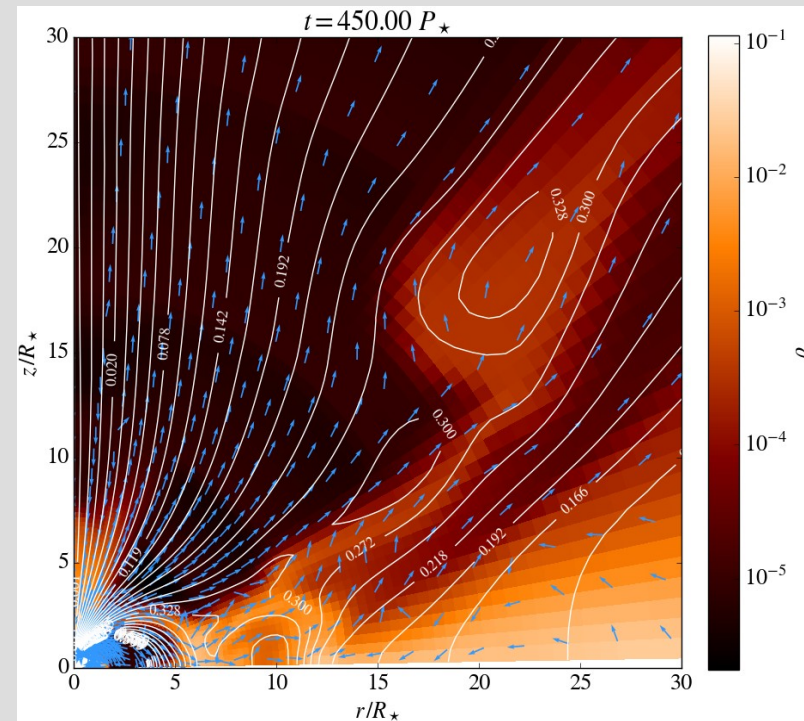
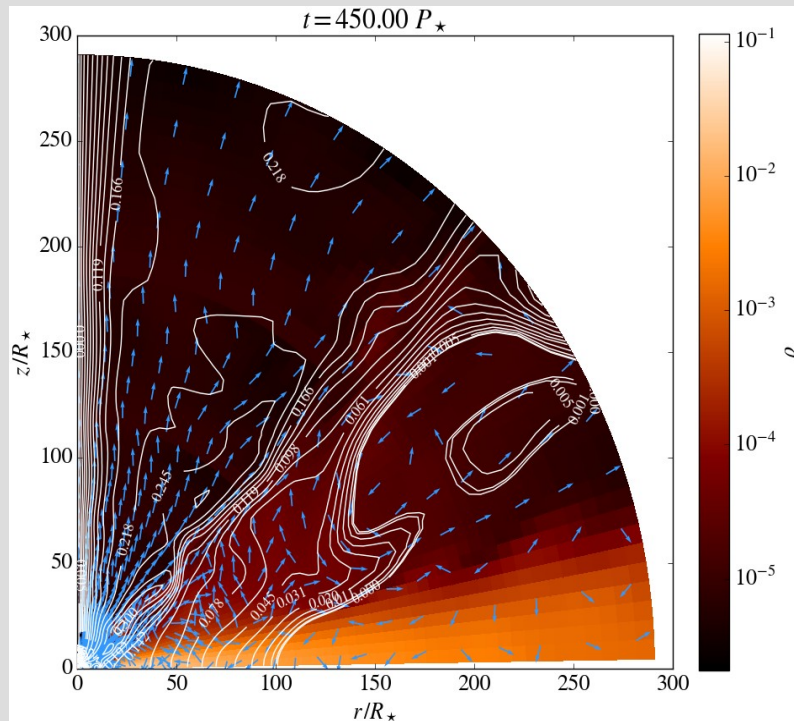
Preliminary results: WD

In the WD case, I set $M_* = M_{\text{sun}}$, $R_* = 5000 \text{ km}$, $P_* = 61 \text{ s}$, $B_* = 10 \text{ kG}/100 \text{ kG}$, $\rho_0 = 9.4 \times 10^{-9} \text{ g/cm}^3$, $\dot{M}_0 = 1.9 \times 10^{-8} M_{\text{sun}}/\text{yr}$, $V_{K,*} = 5.15 \times 10^8 \text{ cm/s}$.



Simulations in the case of WD show to be in the sweet spot in the parameter space, where the timestep is larger and we can obtain long-lasting results in realistic time, even in the $[0, \pi]$ case, shown in the left panel. We can also perform simulations with 10 times larger magnetic field, 100kG, shown in the right panel, when the disk inner radius is larger.

Preliminary results: WD



In the WD case I also could perform set of simulations with 10 times larger Rmax, up to Rmax=300, in $\vartheta=[0,\pi/2]$ case, in resolution $R_x \times \vartheta=[109 \times 50]$ grid cells, with $\alpha_v=0.1$. I shown the results in a full box and in a zoom to my usual domain with Rmax=30. This way I check that the solution is independent of boundary conditions, and I also trace the development of the magnetospheric outflow to larger distances.

Star-disk problem in Neutron Stars

In the interaction of the NS with close companion star, the accretion disk around the NS is formed.

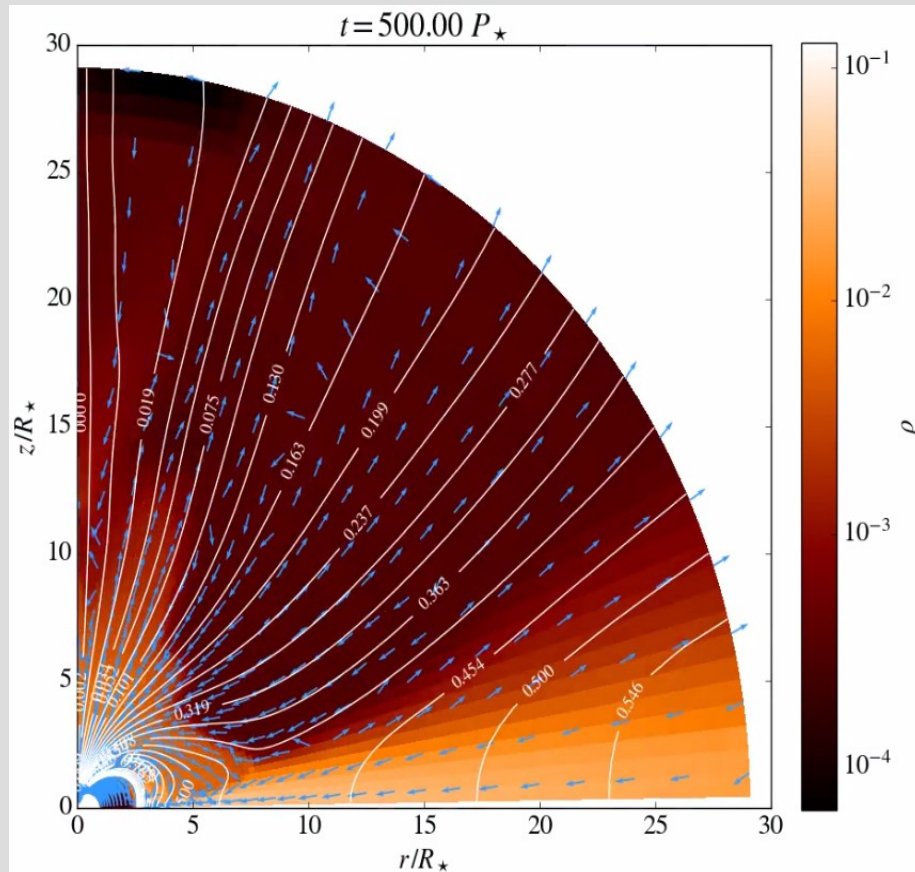
We observe various configurations of binary systems. Properties depend on the type of the companion star and the neutron star magnetic field strength and geometry.

Kluźniak & Kita (2000) gave a HD model of the accretion disk, with viscosity and resistivity parameterized by Shakura & Sunyaev (1973) as $\alpha c^2/\Omega$.

We wish to extend that model to the non-ideal MHD.

Typical parameters for NS in millisecond pulsars: $M=1.4M_{\text{sun}}$, $R\sim 10\text{km}$, $B\sim 10^8$ Gauss, $P=0.01$ sec (10 msec), $\rho_0=4.62\times 10^{-6}$ g/cm³.

Preliminary results: NS

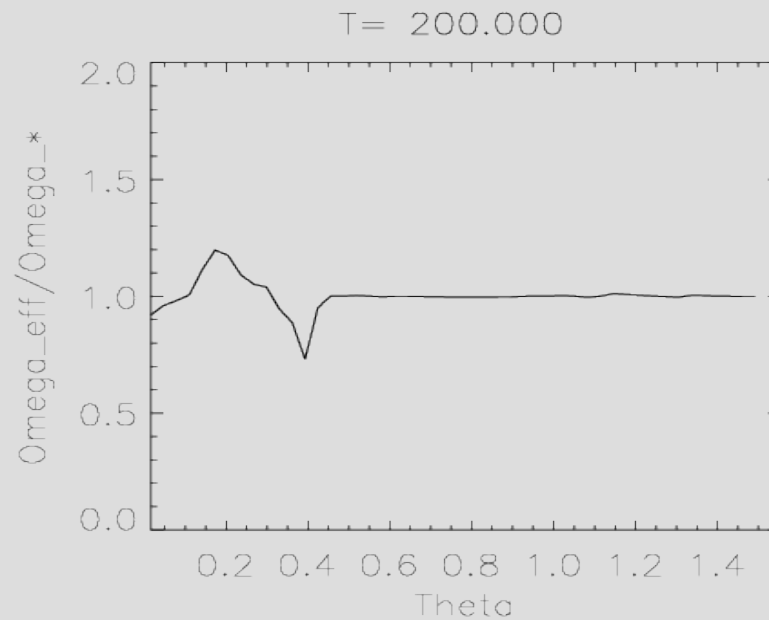


Solution for the NS case with millisecond pulsar parameters, up to $T=500$ stellar rotations= $5s$

By comparison of different resolution simulations I investigated which is the lowest resolution for the reliable MHD simulations. For the $[0, \pi/2]$ case it is enough to have $R \times \vartheta = 109 \times 50$ grid cells. Then I run the simulation for long time in this resolution.

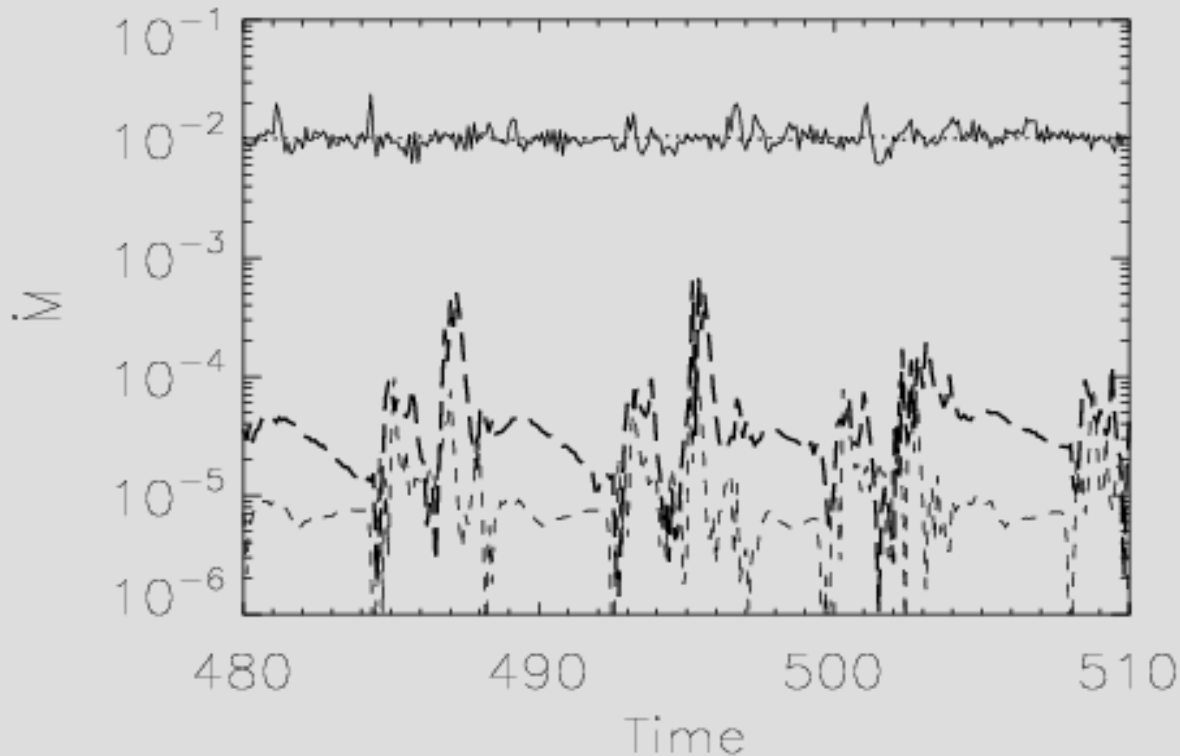
For other types of objects, we need to increase the stellar magnetic field. I did shorter simulations with 10^{10} Gauss, but something like magnetar field strength, $B=10^{14}$ Gauss, is still not reachable, because of too small computational timestep.

Preliminary results: NS



Omega_eff/Omega_* in NS case.

Preliminary results: NS



Time evolution of the mass flux in the different components of the flow in the quasi-stationary state, during the last 30 rotations. In the dotted line is shown the mass flux through the disk at $R=12R_*$. This mass flux is then distributed onto the star (solid line), into the stellar wind (thin dashed line), and in the magnetospheric ejection (thick long-dashed line). Most of the mass from the disk is accreted onto the star, about 1/100 of it goes into the magnetospheric ejection, and about 1/1000 goes into the stellar wind. Note the logarithmic \dot{M} scale.

Summary & Prospects

I obtained the long lasting star-disk simulations in 2D axi-symmetric case in PLUTO code, with viscous & resistive MHD, in both $[0, \pi/2]$ and $[0, \pi]$ domains of the ϑ -plane.

I performed the simulations with dipole, quadrupole, octupole and multipole magnetic field configurations.

Relaxed, quasi-stationary results were obtained for the setup with parameters of YSOs, WDs and NSs.

We are currently working on the resistive magnetic extension to Kluźniak-Kita HD disk equations, in the asymptotic expansions approach. The results of such analytical solutions will then be compared with the simulations results.