CFD-MHD Seminar

Relaxation of initial conditions in MHD simulations

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Outline

-Time evolution of MHD equations

-Ideal MHD ver. resistive and viscous MHD

-Numerical resistivity and viscosity

-Magnetic Prandtl number

-Relaxation phase in MHD simulation

-Dangers of too nice methods

-Is the solution realistic?

Introduction

In numerical simulations, there is always transition between the initial conditions and time-evolved initial phase of the simulation. It is the *relaxation* of initial conditions.

What ensures a good relaxation?

-appropriate initial and boundary conditions

-well chosen parameters for physical problem we are solving

Resistive MHD equations

-in addition to physical resistivity, hydrostatic, viscous dissipation term could be added-but we investigate effects of resistivity

-we mimic viscosity with von Neumann-Richtmyer artificial viscosity, which is significant only for part of the flow with shocks-good for relaxation phase

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (1)$$

$$\rho \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] + \nabla p + \rho \nabla \Phi - \frac{\boldsymbol{j} \times \boldsymbol{B}}{c} = 0 \quad (2)$$

$$\frac{\partial \boldsymbol{B}}{\partial t} - \nabla \times \left(\mathbf{v} \times \boldsymbol{B} - \frac{4\pi}{c} \eta \boldsymbol{j} \right) = 0 \quad (3)$$

$$\rho \left[\frac{\partial e}{\partial t} + (\mathbf{v} \cdot \nabla) e \right] + p(\nabla \cdot \mathbf{v}) = 0 \quad (4)$$

$$\boldsymbol{j} = \frac{c}{4\pi} \nabla \times \boldsymbol{B} \ . \quad (5)$$

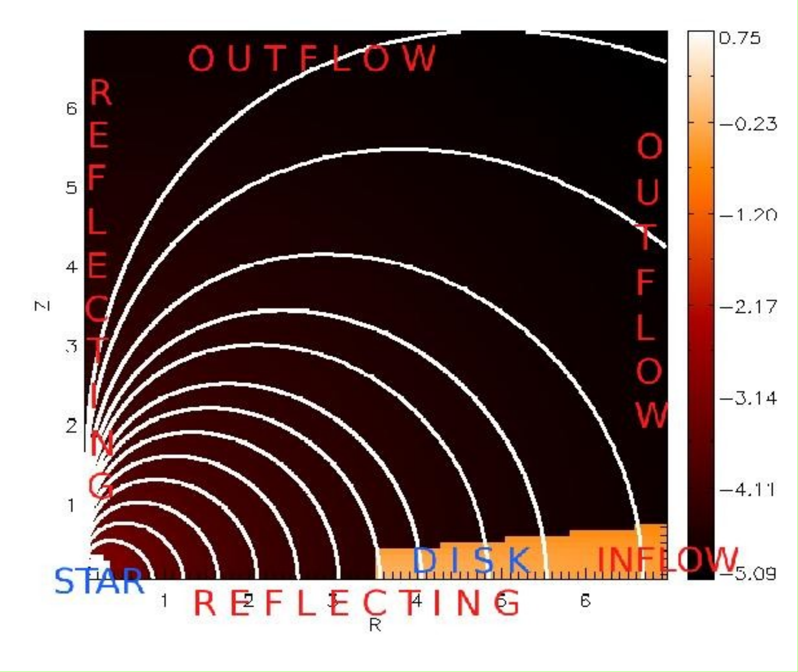
$$\frac{\partial \mathbf{v}'}{\partial \tau} + (\mathbf{v}' \cdot \nabla') \mathbf{v}' = \frac{2\, \boldsymbol{j}' \times \boldsymbol{B}'}{\delta_0\, \beta_0\, \rho'} - \frac{\nabla' p'}{\delta_0\, \rho'} - \nabla' \Phi' , \quad (6)$$

with $\nabla' = R_0 \nabla$, $\rho' = \rho/\rho_0$, $\boldsymbol{B}' = \boldsymbol{B}/\boldsymbol{B}_0$ and $\Phi' = -1/\sqrt{R'^2 + z'^2}$.

entropy S=$\ln(p/\rho^\gamma)$, with adiabatic index γ=5/3. The internal energy (per unit volume) is then $e = p/(\gamma - 1)$.

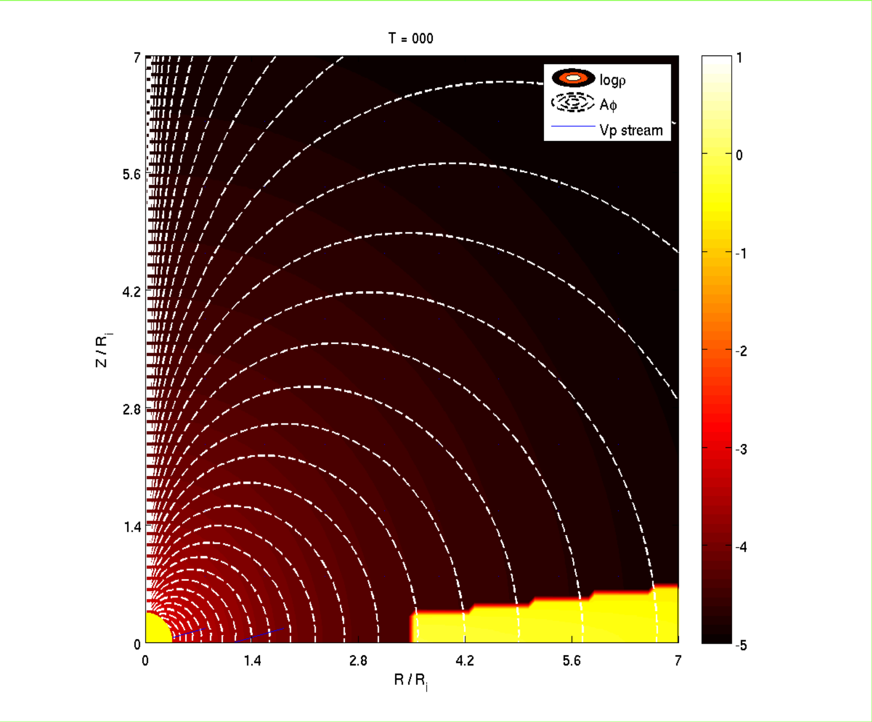
Straightforward simulations-setup

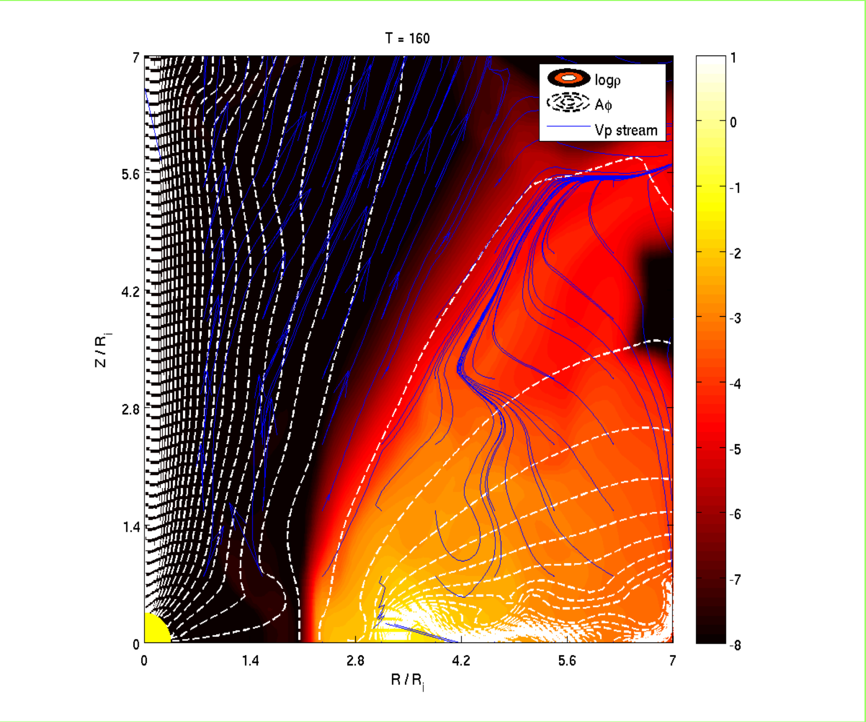
-example of setup for simulation when there is no “tricks”. We set the initial and boundary conditions and leave the code to deal with physical and numerical instabilities, large pressures....



Straightforward simulations-time evolution

-this is referent case for other trials, when we will try to ease the relaxation. In step-by step animation we see the relaxation process. Here I show the initial and end stage.





Resistivity and viscosity

-physical and numerical resistivity and viscosity affect the simulation. Exact form of this effect depends on numerical methods used in code. We measure the effect by the magnetic Prandtl number, Pr=viscosity/resistivity

-two important regimes, when Pr<1 or Pr>1

-for Pr>1 viscosity affects the solutions, one more degree of freedom for solutions

-for Pr>1 m-angular momentum flux is more effectively extracted from the disk=>larger F_l

Mass and angular momentum fluxes

-for straightforward "simulation"

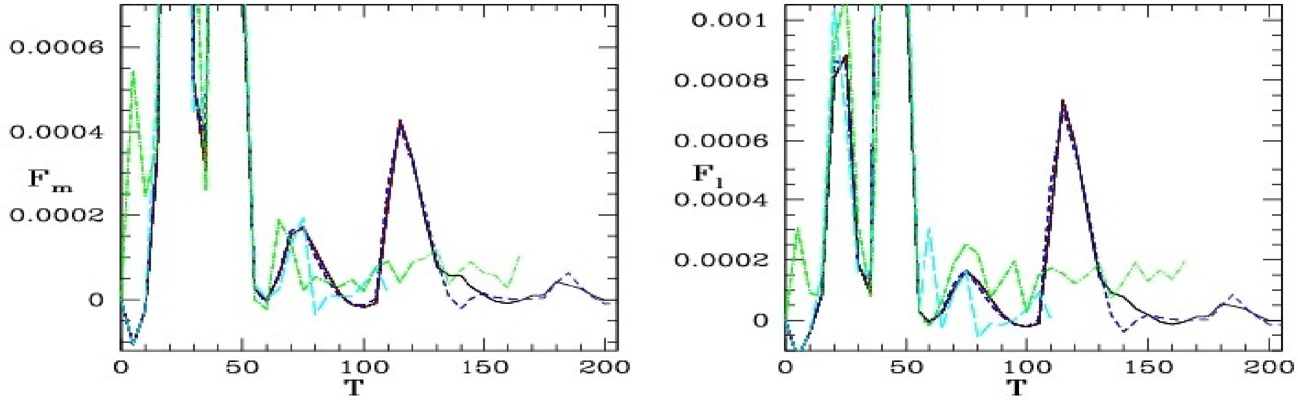


Fig. 7.— The mass fluxes (*left* panel) and and angular momentum flux (*right* panel) parallel to the axis, across the Z_{max} boundary for increasing stellar magnetic field in our typical setup. In solid (black), dotted (red), dashed (blue), long-dashed (cyan) and dot-dashed (green) lines depicted are the solutions with B_*=(0,3,10,30,100) Gauss, respectively.

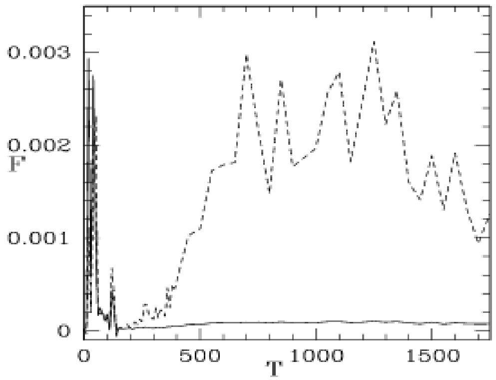
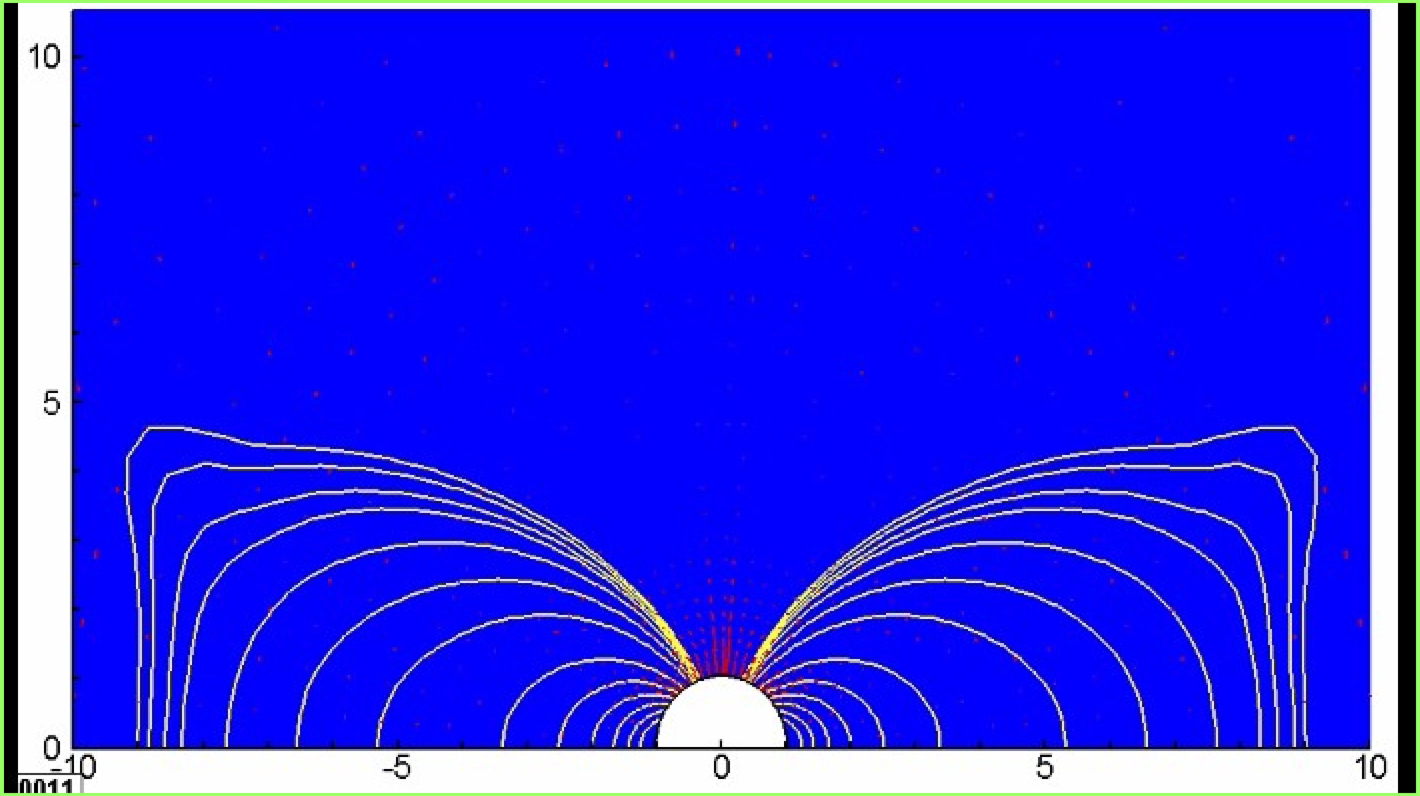


Fig. 8.— Results for the simulation with the magnetic Prandtl number Pr>1, and the stellar magnetic field of 50 Gauss. Depicted are the time evolution of mass flux (solid line) and angular momentum flux (dashed line) parallel to the axis, across the Z_{max} boundary.

Smoothed initial conditions: slow introduction of matter

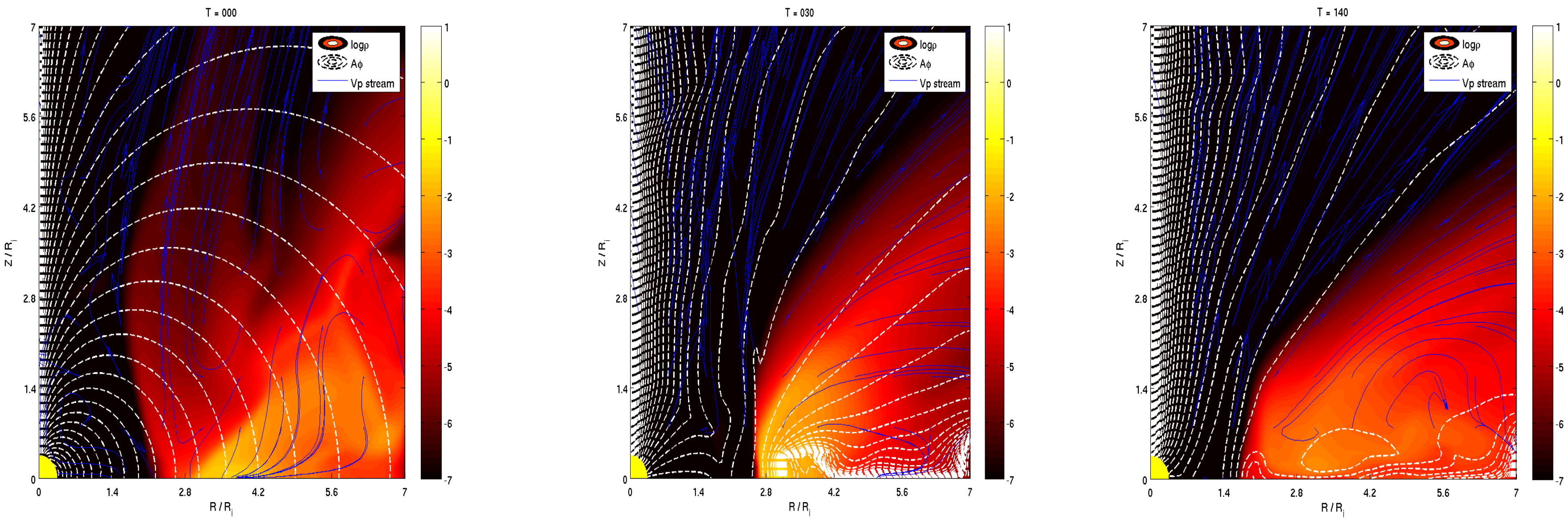
-if we start the simulation without the matter in the computational box, and then allow the matter to slowly enter the box, we avoid chaotic relaxation phase-Romanova et al. 2009a,b

-but, do we really solve the same mathematical and physical problem?



Smoothed initial conditions: slow increasing of B

- we leave the code to deal with HD relaxation, as it shows to be good with it (in simulations without/with small B), but we gradually increase the magnetic field, reaching the magnitude we want in few steps.
- should be more realistic, but there is no guarantee.



Mass and angular momentum fluxes

- we are interested in mass and angular momentum fluxes, so we compare them.
- angular momentum fluxes are few times larger with such “trick”-as in more dissipative setup-not necessarily wrong.

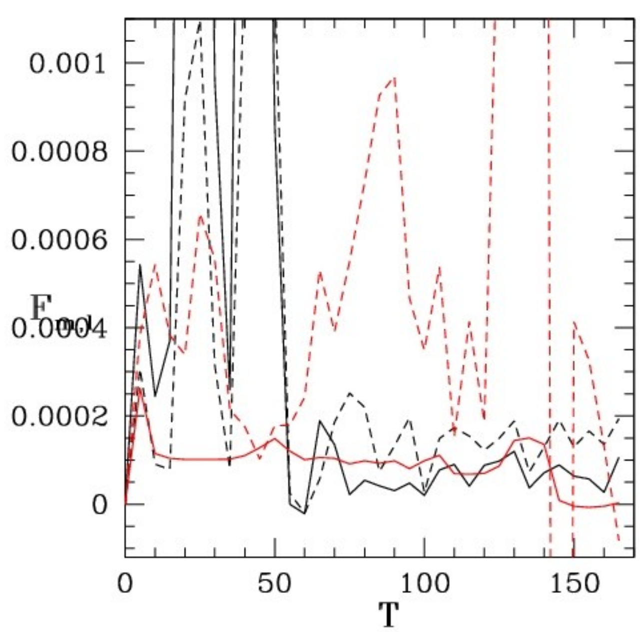


Fig. 1.— Comparison plots for mass (F_m) (solid lines) and angular momentum (F_l) fluxes (dashed lines) for typical simulation from paper, with B_*=100G for the case with usual evolution (black) and evolution when B_* has been gradually increased from 0 to 100 Gauss, in order-of-magnitude steps (red).

Summary

-even in ideal MHD simulations, there is always some numerical resistivity and viscosity, hence results will depend on resolution

-magnetic Prandtl number measures influence of viscosity and resistivity

-relaxation phase of simulation might define the result

-sometimes coarser grid gives more realistic results, because of numerical viscosity and resistivity.