Gravitational waves and electrodynamics: New perspectives

Francisco Cabral* and Francisco S. N. Lobo†
Instituto de Astrofísica e Ciências do Espaço, Faculdade de Ciências da Universidade de Lisboa, Edifício C8, Campo Grande, P-1749-016 Lisbon, Portugal.
(Dated: March 29, 2016)

Given the recent direct measurement of gravitational waves (GWs) by the LIGO-VIRGO collaboration, the coupling between electromagnetic fields and GW have a special relevance since it might open new perspectives for future GW detectors and also potentially provide information on the physics of highly energetic GW sources. We explore such couplings using the field equations of electrodynamics on (pseudo) Riemann manifolds and apply it to the background of a GW, seen as a linear perturbation of Minkowski geometry. Electric and magnetic oscillations are induced that propagate as electromagnetic waves and contain information of the GW which generates them. We also show very briefly the generation of charge density fluctuations induced by GW and the implications for astrophysics.

Outline

- Introduction
- Electrodynamics and space geometry I: Foundations
- Electrodynamics and space geometry: Astrophysical applications
- Gravitational waves and electrodynamics: New perspectives
- Summary

Introduction

- Series of 3 papers:
- In the first one, presentation of electrodynamics in GR
- In the second one, discussion of possible astrophysical applications
- In the third one, discussion of gravitational waves and electrodynamics

Electrodynamics and spacetime geometry I: Foundations

Francisco Cabral[*] and Francisco S. N. Lobo[†]
Instituto de Astrofísica e Ciências do Espaço, Faculdade de Ciências da Universidade de Lisboa, Edifício C8, Campo Grande, P-1749-016 Lisbon, Portugal.
(Dated: February 5, 2016)

We explore the intimate connection between spacetime geometry and electrodynamics. This link is already implicit in the constitutive relations between the field strengths and excitations, which are an essential part of the axiomatic structure of electromagnetism, clearly formulated via integration theory and differential forms. We briefly review the foundations of electromagnetism based on charge and magnetic flux conservation, the Lorentz force and the constitutive relations which introduce the spacetime metric. We then proceed with the tensor formulation by assuming local, linear, homogeneous and isotropic constitutive relations, and explore the physical, observable consequences of Maxwell's equations in curved spacetime. The field equations, charge conservation and the Lorentz force are explicitly expressed in general (pseudo) Riemanian manifolds. The generalized Gauss and Maxwell-Ampère laws, as well as the wave equations, reveal potentially interesting astrophysical applications. In all cases new electromagnetic couplings and related phenomena are induced by spacetime curvature. The implications and possible applications for gravity waves detection are briefly addressed. At the foundational level, we discuss the possibility of generalizing the vacuum constitutive relations, by relaxing the fixed conditions of homogeneity and isotropy, and by assuming that the symmetry properties of the electro-vacuum follow the spacetime isometries. The implications of this extension are briefly discussed in the context of the intimate connection between electromagnetism and the geometry (and causal structure) of spacetime.

- The concept of a physical field was introduced in the works of Faraday and Maxwell.
- Electrodynamics of moving objects inspired Einstein's Special Relativity theory.
- The form of Maxwell's equations led Lorentz, Poincare and Einstein to Lorentz and Poincare spacetime transformations, and Minkowski completed space-time unification.
- Since electro-magnetic fields have energy-momentum, they gravitate, affecting spacetime geometry.
- On the other hand, light rays propagate along null geodesics-this express a link between the causal structure of spacetime and the propagation of electromagnetic fields. Photons seem to be the only massless particles of the Standard Model of elementary particles => they are the only ones that can provide experimental study of the null cones.

- Formally, Maxwell's theory can be derived from the following axioms (postulates):

  1) Charge conservation

  2) Magnetic flux conservation

  3) Lorentz force

  4) Linear constitutive (space-time) relations.

- From 1) follow the inhomogeneous Maxwell equations, the Gauss and Maxwell-Ampere laws
- From 3) and 4) follow the two homogeneous Maxwell equations.
- For a macroscopic description of electromagnetism, we would need two more axioms: 5) Specification of the energy-momentum tensor, 6) Splitting of the total electric charge and currents in a bound or material component which is conserved, and a free or external component.

$$d\mathbf{D} = \varrho, \qquad d\mathbf{H} = \mathbf{j} + \partial_t \mathbf{D}, \tag{2.19}$$

$$d\mathbf{E} + \partial_t \mathbf{B} = 0, \qquad d\mathbf{B} = 0, \tag{2.20}$$

are fully general pre-metric, covariant equations, coming directly from charge conservation, magnetic flux conservation and the Lorentz force. As previously mentioned, to solve this set of equations one requires the (spacetime) constitutive relations relating the electric and magnetic fields to the excitations. Assuming linear, homogeneous and isotropic constitutive relations, without mixing electric and magnetic properties, these relations, in the language of forms are achieved via the Hodge star operator in 3-dimensional space which maps k-forms to $(d-k)$-forms (where d is the dimension of the manifold under consideration), and are given by

$$\mathbf{D} = \varepsilon_0 \star \mathbf{E}, \qquad \mathbf{H} = \mu_0^{-1} \star \mathbf{B}, \tag{2.21}$$

which introduces the spacetime metric

$$D_{jk} = \varepsilon_0 \sqrt{|h|} \epsilon_{ijk} h^{im} E_m, \tag{2.22}$$

$$E_i = \frac{1}{2\varepsilon_0 \sqrt{|h|}} \epsilon_{ijk} D_{mn} g^{mj} g^{nk}, \tag{2.23}$$

- The constitutive equations in matter are more complicated. For a general linear magneto-electric medium they are:

$$D^i = \left(\varepsilon^{ij} - \epsilon^{ijk} n_k\right) E_j + \left(\gamma^i_j + \tilde{s}^i_j\right) B^j + (\alpha - s)\, \delta^i_j B^j, \qquad (2.50)$$

$$H_i = \left(\mu^{-1}_{ij} - \epsilon_{ijk} m^k\right) B^j + \left(-\gamma^j_i + \tilde{s}^j_i\right) E_j + (\alpha + s)\, \delta^j_i E_j,$$

In fact, after contracting μ with λ, the third term in Eq. (3.8) vanishes, since the (symmetric) connection is contracted with the components of an anti-symmetric tensor and on the other hand, for (pseudo) Riemann manifolds we can use the following relation $\Gamma^{\varepsilon}_{k\varepsilon} = \partial_k\left(\log(\sqrt{-g})\right)$, where a contraction is assumed in ε. Therefore, the inhomogeneous equations are given by

$$\partial_\mu F^{\mu\nu} + \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g})F^{\mu\nu} = \mu_0 j^\nu, \tag{3.10}$$

therefore,

$$g^{\alpha\mu}g^{\beta\nu}\partial_\mu F_{\alpha\beta} + F_{\alpha\beta}\Big[\partial_\mu(g^{\beta\nu}g^{\alpha\mu}) + g^{\alpha\mu}g^{\beta\nu}\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g})\Big] = \mu_0 j^\nu, \tag{3.11}$$

and more explicitly, in terms of the electric and magnetic components, we have

$$\frac{1}{c}\partial_\mu E_j\left(g^{0\mu}g^{j\nu} - g^{j\mu}g^{0\nu}\right) + \frac{1}{c}E_j\left[\partial_\mu\left(g^{0\mu}g^{j\nu} - g^{j\mu}g^{0\nu}\right) + \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g})\left(g^{0\mu}g^{j\nu} - g^{j\mu}g^{0\nu}\right)\right]$$
$$-\partial_\mu B^k g^{m\mu}g^{n\nu}\epsilon_{kmn} - B^k\epsilon_{kmn}\left[\partial_\mu(g^{m\mu}g^{n\nu}) + \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g})(g^{m\mu}g^{n\nu})\right] = \mu_0 j^\nu. \tag{3.12}$$

We now take the case where $\nu = 0$, which gives the extended Gauss law in curved spacetime

$$\partial_\mu E_j\left(g^{0\mu}g^{j0} - g^{j\mu}g^{00}\right) + E_j\left[\partial_\mu\left(g^{0\mu}g^{j0} - g^{j\mu}g^{00}\right) + \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g})\left(g^{0\mu}g^{j0} - g^{j\mu}g^{00}\right)\right]$$
$$-\partial_\mu B^k c g^{m\mu}g^{n0}\epsilon_{kmn} - B^k c\epsilon_{kmn}\left[\partial_\mu(g^{m\mu}g^{n0}) + \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g})(g^{m\mu}g^{n0})\right] = \frac{\rho}{\varepsilon_0}. \tag{3.13}$$

Electrodynamics and spacetime geometry: Astrophysical applications

Francisco Cabral[*] and Francisco S. N. Lobo[†]
Instituto de Astrofísica e Ciências do Espaço, Faculdade de Ciências da Universidade de Lisboa, Edifício C8, Campo Grande, P-1749-016 Lisbon, Portugal.
(Dated: March 29, 2016)

After a brief review of the foundations of (pre-metric) electromagnetism in differential forms, we proceed with the tensor formulation and explore physical consequences of Maxwell's equations in curved spacetime. The generalized Gauss and Maxwell-Ampère laws, as well as the wave equations, reveal potentially interesting astrophysical applications. The physical implications of these equations are explored and some solutions are obtained. In all cases new electromagnetic couplings and related phenomena are induced by the spacetime curvature. The applications of astrophysical interest considered here correspond essentially to the following geometries: the Schwarzschild spacetime and the spacetime around a rotating spherical mass in the weak field and slow rotation regime. In the latter, we use the Parameterised Post-Newtonian (PPN) formalism. In general, new electromagnetic effects induced by spacetime curvature include the following: Gravitational contributions for the decay of electric and magnetic fields in spherically symmetric spacetime, magnetic contributions to Gauss law due to the gravitomagnetic character of the spacetime around rotating objects, additional electric contributions to Maxwell's displacement current in the Maxwell-Ampére law induced by gravitomagnetism (and also for dynamical geometries), and the coupling between the wave dynamics for different electric and magnetic components (affecting polarization), amongst others.

- Due to the presence of extra electromagnetic couplings induced by spacetime curvature, we have new electromagnetic phenomena.

1) ***Electrostatics:***

The magnetic terms in the Gauss law are only present for non-vanishing off-diagonal time-space components, which in linearized gravity correspond to the gravitomagnetic potential-they are typical of axially symmetric geometries.

-For the Schwarzschild solution of Einstein equations, the resulting radial dependence in the static radial electric field, from the Gauss law, becomes r^{-6}, in contrast to r^{-2} from the usual electrostatics.

-Also, the deviation between the magnitudes of the electric field due to charged spherical bodies, for the cases of flat and curved spacetime, is largest near the surface of the charged body.

-The electric field decays more strongly outside a (spherical) charged body and increases more slowly inside, in comparison with the same physical situation in flat spacetime.

-Another effect follows from the fact that electric field due to charged plates in equatorial orbit in the (gravitational) weak field limit is no longer uniform: imagine a plane capacitor, with electric field given by only one component, say E_x. In the vicinity of a spherical mass, such field changes its magnitude due to spacetime curvature, dependent on direction.

2) ***Magnetostatics:***

Suppose that in some reference frame we have a static magnetic field due to a stationary current, and no electric field.

From the generalized Maxwell-Ampere law we obtain additional terms because of the non-vanishing space-time components.

- Magnetic field created by a ring of circulating plasma around a spherical mass (e.g. a BH): consider a stationary current loop around the mass on the equatorial plane, e.g. from a disk of very hot plasma. It would create an axially symmetric magnetic field. For very strong gravitational fields, the extra terms due to curved spacetime result in a faster decay of the magnetic field with the radial distance, than in the flat (Minkowski) space.

- Magnetic field created by an astrophysical jet: charged particles moving along the axis create a magnetic field along the perpendicular, φ direction. This field is dependent on curvature of the space-time.

- For the equations of electromagnetic potential and electromagnetic waves, we also obtain dependence on the space-time geometry. In general, gravity contributes to the decay of the spatial oscillations of the electric potential. Also, a richer wave dynamics is induced by the gravitational field, which should contribute to the radial decay of the oscillations.

This is the topic of the last article in the line, the “New perspectives”:

- Hope is that coupling of electromagnetic fields and gravitational waves could provide an alternative for GW detection and provide information on the physics of highly energetic GW sources.
- Question: are there measurable effects on electric and magnetic fields during the passage of a GW? In particular, could lensing provide a natural amplification of such effects? Effect of gravitational waves on polarization was, theoretically, already shown to exist.
- Due to huge cosmical distances, any GW reaching Earth should have a very small amplitude => it is possible to linearize gravity, and derive the wave equations. It is a (Minkowski) background dependent perturbative approach. GW can be seen as a manifestation of propagating spacetime geometry perturbations.
- GW is considered as a perturbation of Minkowski spacetime given by

$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$, with $h_{\alpha\beta} \ll 1$, so that

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 + h_{\alpha\beta} dx^\alpha dx^\beta$$

The perturbation corresponds to a wave travelling along the z-axis:

$$ds^2 = c^2dt^2 - dz^2 - [1 - f_+(z-ct)]dx^2$$
$$-[1 + f_+(z-ct)]dy^2 + 2f_\times(z-ct)dxdy$$

where (+) and (x) refer to the two independent polarizations characteristic of gravitational waves in General Relativity.

This metric is a solution of Einstein's field equations in the linear approximation, in the so-called Transverse-Traceless (TT) Lorentz Gauge.

E.g. for Gauss' law in vacuum, we obtain:

$$0 = [1 - f_+(z,t)]\partial_x E_x + [1 + f_+(z,t)]\partial_y E_y + \partial_z E_z$$
$$-2f_\times(z,t)\left(\partial_y E_x + \partial_x E_y\right) + \left[\frac{1}{\sqrt{-g}}\partial_z(\sqrt{-g})\right] E_z, (3.3)$$

We see that physical, possibly observable effects on the electric field are induced by the propagation of gravity waves.

- Similar effect is visible in the Maxwell-Ampere law. Conclusion is that GW effect on electric and magnetic fields is the generation of electromagnetic waves induced by gravitational radiation.
- During the passage of GW, initially static electric and magnetic fields become time dependent.

-**Effect on electric fields**:

-in the case of electric field aligned with the direction of the GW propagation, the gravitational wave induces electric oscillations, and with this el.mag waves propagating along the same direction and with the same frequency as the passing GW, in both (+) and (x) modes. The electromagnetic intensity of a signal is proportional to the square of the GW amplitudes in the (+) and (x) modes. Since the GW amplitudes are very small, detectors would have to be extremely sensitive.

-The similar result is obtained in the case of electric field perpendicular to the direction of GW propagation.

-**Effect on magnetic fields**:

-passage of a gravitational wave induces time-varying (oscillating) magnetic field, even for an initially static electric field, due to the terms containing the derivatives of the metric (in usual electromagnetism we obtain static B).

-here also the frequency of the oscillations of the magnetic field is the same as the deformations because of the GW passage.

- Gravitational waves also induce the charge density (i.e. currents) fluctuations along the propagation direction of GWs. Astrophysical sources of gravitational waves as GRBs, or coalescing binaries surrounded by plasmas in accretion disks or stellar atmospheres, might generate density waves induced by the GW propagation.

- Effects of GW on electromagnetic waves are similar as above. Eventually they could be measurable by Long Base interferometry, in order to be amplified-simulations are needed to see the feasibility of such methods.

Summary

- The result from the treatment of electro-magnetism in the background of gravitational waves, is that among other effects, the coupling between gravity and electromagnetism produces electromagnetic waves. This is not evident from the geometrical optics limit in curved spacetime.
- Reverse is also true, so that in the full theory, the contribution of the electromagnetic stress-energy tensor to the gravitational field should be taken into account.