Analytical solution for magnetized thin accretion disk in comparison with numerical simulations

Miljenko Čemeljić, Varadarajan Parthasarathy, Włodek Kluźniak

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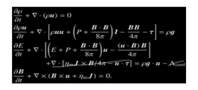
Abstract: A magnetic extension of hydrodynamical solution for a thin accretion disk around a central star is obtained by using the method of asymptotic approximation. We compare the analytical results with the long-lasting resistive and viscous MHD axi-symmetric numerical simulations of the quasi-stationary disk.

Introduction: The gravitational infall of matter onto the rotating central object naturally forms a rotating accretion disk. The matter from the disk is then fed inwards through the accretion column. Kluźniak & Kita (2000) gave a hydro-dynamical model of the accretion disk, with viscosity and resistivity parameterized by Shakura & Sunyaev α-prescription.

We extended the model to the non-ideal MHD, and added the magnetosphere of a star-disk system. Examples of such systems are young stellar objects and white dwarfs or neutron stars (see Čemeljić et al., 2017) in close binary systems with a donor star and a disk.

Numerical setup: We use the PLUTO v.4.1 code (Mignone et al. 2007, 2012), with logarithmically stretched grid in radial direction in spherical coordinates to perform axisymmetric 2D star-disk simulations in resistive and viscous MHD, following Zanni & Ferreira (2009). The resolution is RxΘ=[217x100] grid cells, stretching the domain to 30 stellar radii. The equations we solve are: **HERE>** 

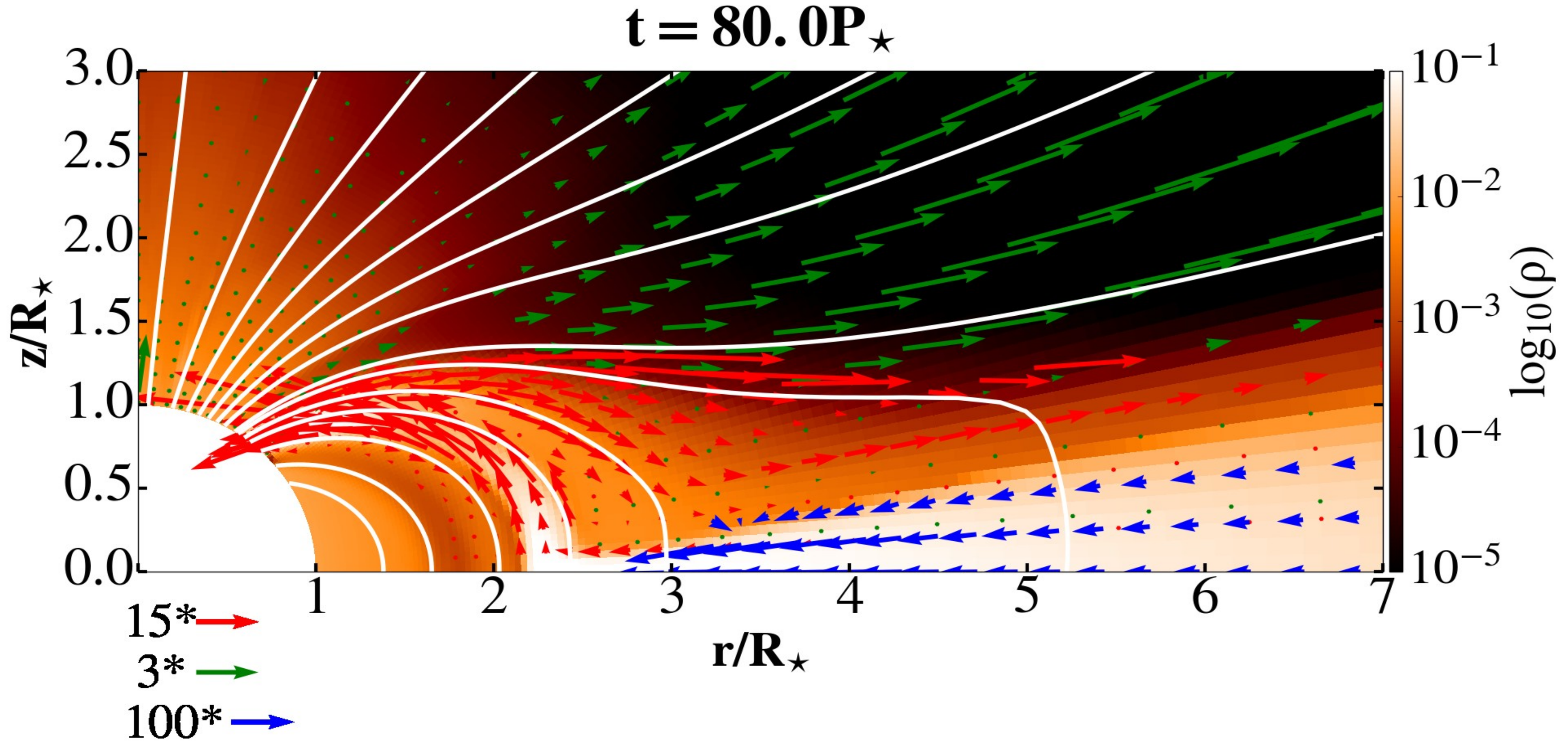
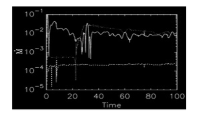
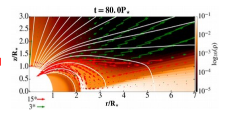
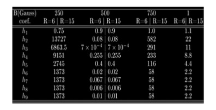
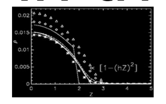


Figure 1: *A zoom into our simulation result after T=80 rotations of the underlying young star. Shown is the density in logarithmic color grading and magnetic field lines in white line. Velocity in the disk, column and in the stellar wind are shown in different normalizations. The quasi-stationarity of our result is illustrated* ***HERE>*** 

Animation is shown ***HERE>*** 

Analytical solutions ver. numerical solutions

We extend the asymptotic approximation from the Kluzniak & Kita (2000) to the magnetic thin accretion disk case, and compare the results with the outcome of the numerical simulations. In the region far from the stellar surface, the MHD solution does not differ much from the hydro-dynamical numerical solution. Difference is only in the proportionality coefficients in the corresponding power laws.

We write the expressions and tabulate **HERE>**  the coefficients. The solutions are given in the region in the middle of the simulated disk, at R=15R_star. **HERE>**  is shown the trend in the density of the disk with the different strengths of the stellar magnetic field.

$$\rho(r,z) = \frac{h_1}{r^{3/2}}\left[1-(0.4z)^2\right],$$

$$v_r(r,z) = -\frac{h_2}{r^{3/2}}\left[1+(0.5z)^2\right],$$

$$v_z(r,z) = \frac{h_3}{r}z^2,$$

$$v_\varphi(r,z) = \frac{h_4}{\sqrt{r}},$$

$$B_r(r,z) = -\frac{h_5}{r^3}z,$$

$$B_z(r,z) = -\frac{h_6}{r^3}z,$$

$$B_\varphi(r,z) = -\frac{h_7}{r^2}z,$$

$$\eta(r,z) = \frac{h_8}{r}\left[1-(0.45z)^2\right],$$

$$\eta_{\rm m}(r,z) = h_9\sqrt{r}\left[1-(0.3z)^2\right].$$

Conclusions:

We present results of the numerical simulations of a star-disk system with magnetospheric interaction. In combination with the equations obtained from the asymptotic approximation, we provide expressions for the description of a thin magnetic accretion disk.

References:

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Zanni, C., Ferreira, J., 2009, A&A, 508, 1117

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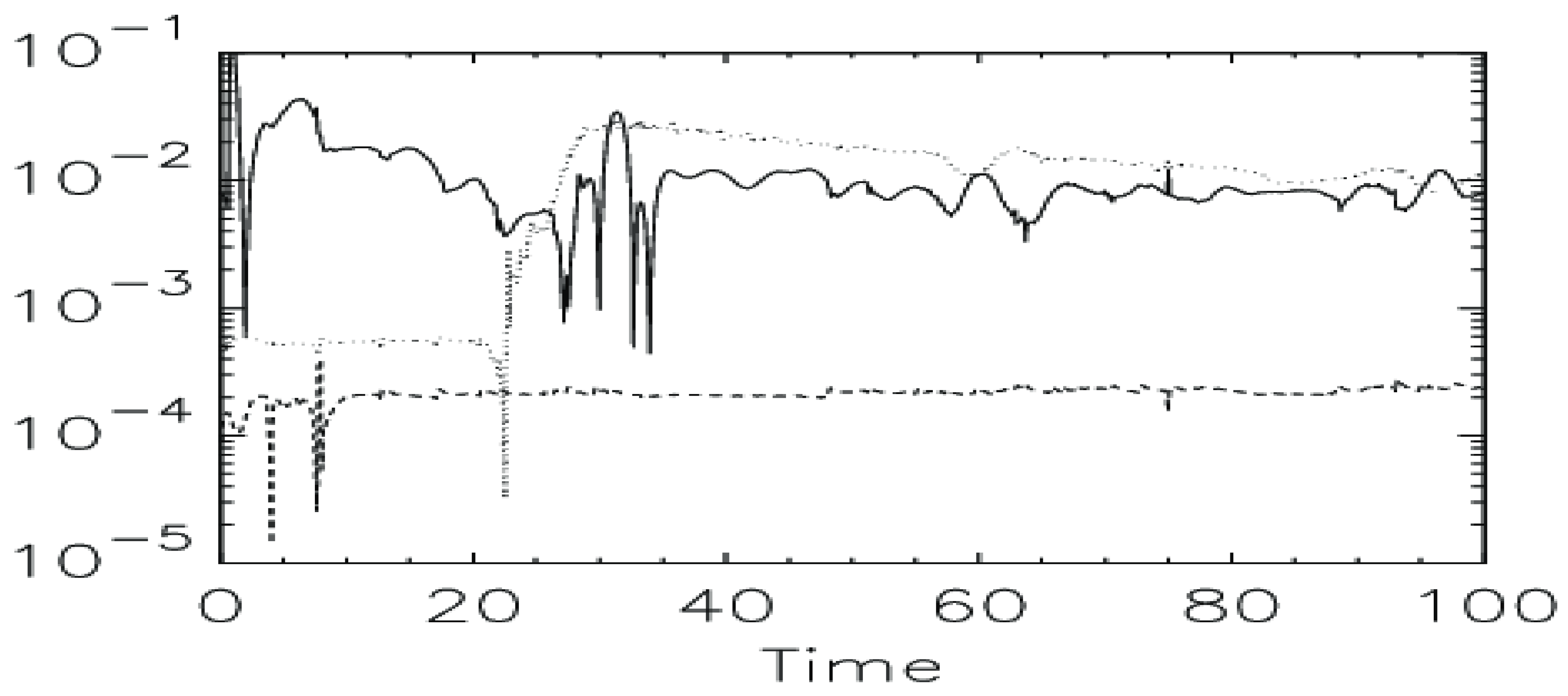
$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\boldsymbol{u}) = 0$$

$$\frac{\partial\rho\boldsymbol{u}}{\partial t} + \nabla\cdot\left[\rho\boldsymbol{u}\boldsymbol{u} + \left(P + \frac{\boldsymbol{B}\cdot\boldsymbol{B}}{8\pi}\right)\boldsymbol{I} - \frac{\boldsymbol{B}\boldsymbol{B}}{4\pi} - \boldsymbol{\tau}\right] = \rho\boldsymbol{g}$$

$$\frac{\partial E}{\partial t} + \nabla\cdot\left[\left(E + P + \frac{\boldsymbol{B}\cdot\boldsymbol{B}}{8\pi}\right)\boldsymbol{u} - \frac{(\boldsymbol{u}\cdot\boldsymbol{B})\,\boldsymbol{B}}{4\pi}\right]$$
$$+\nabla\cdot[\eta_{\rm m}\boldsymbol{J}\times\boldsymbol{B}/4\pi - \boldsymbol{u}\cdot\boldsymbol{\tau}] = \rho\boldsymbol{g}\cdot\boldsymbol{u} - \Lambda_{\rm cool}$$

$$\frac{\partial\boldsymbol{B}}{\partial t} + \nabla\times(\boldsymbol{B}\times\boldsymbol{u} + \eta_{\rm m}\boldsymbol{J}) = 0.$$

Equations we solve by the PLUTO code. We assume the disk radiates away all the heat created by the dissipative processes (viscous and resistive heating), so we neglect those terms in the energy equation. We keep dissipative terms in the momentum and induction equation.



Quasi-stationarity of our result in simulations is illustrated with the mass flux in code units, in the various parts of the flow. Time is given in the number of stellar rotations. After the initial relaxation, system reaches quasi-stationarity.

B(Gauss)	500
h_1	0.9
h_2	0.08
h_3	7×10^{-4}
h_4	0.255
h_5	0.4
h_6	0.02
h_7	0.067
h_8	0.006
h_9	0.01

Table of proportionality coefficients in expressions for the physical quantities in the disk.

Acknowledgement
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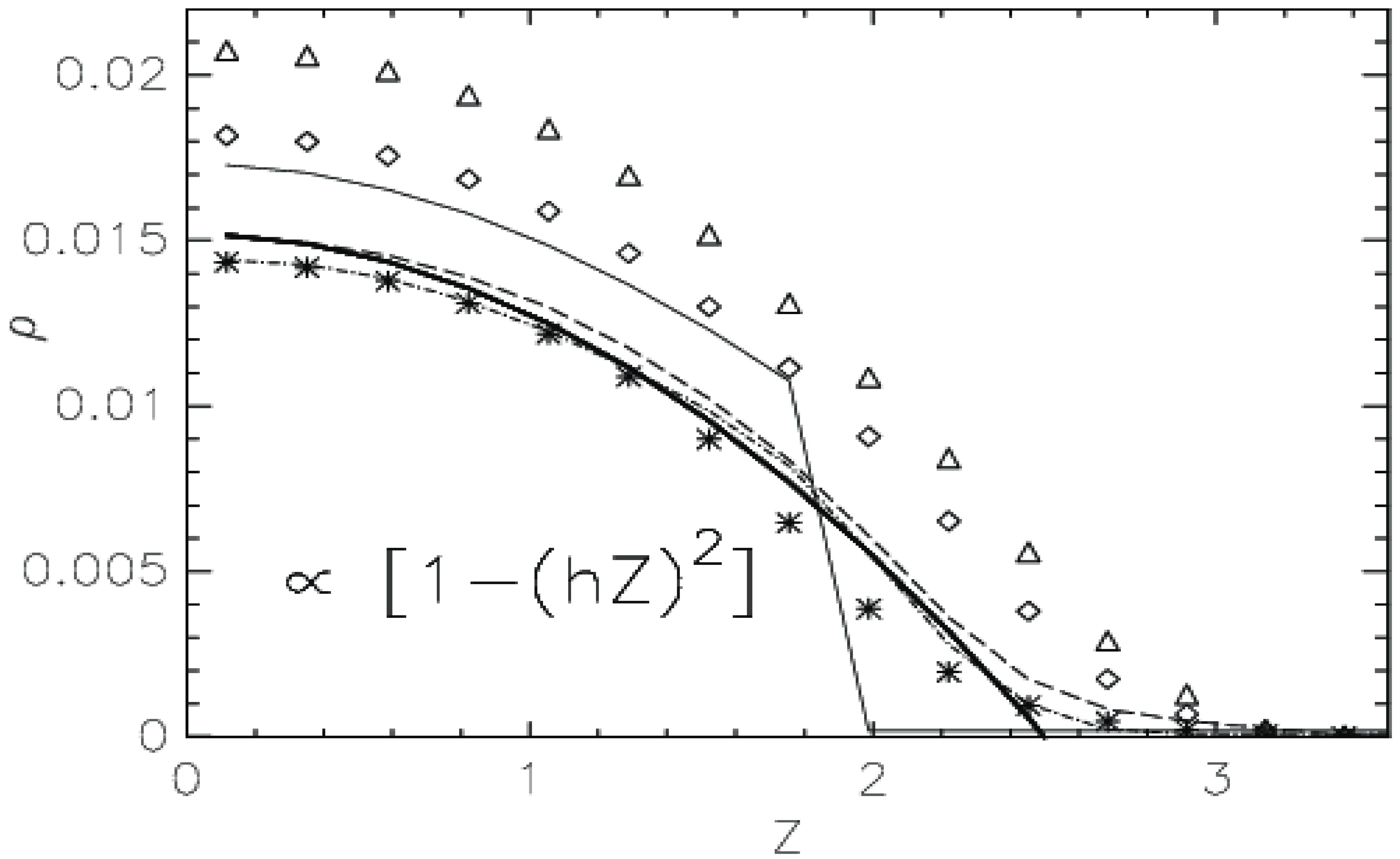

The matter density in the disk along the vertical line at R=15 ,in the cases with B=0.25, 0.5, 0.75 and 1kG is shown with the stars, dashed lines, diamonds and triangles, repectively, showing a trend in change. Thin solid line shows the analytical solution, and thick solid line shows a fit to the B=0.5kG case.

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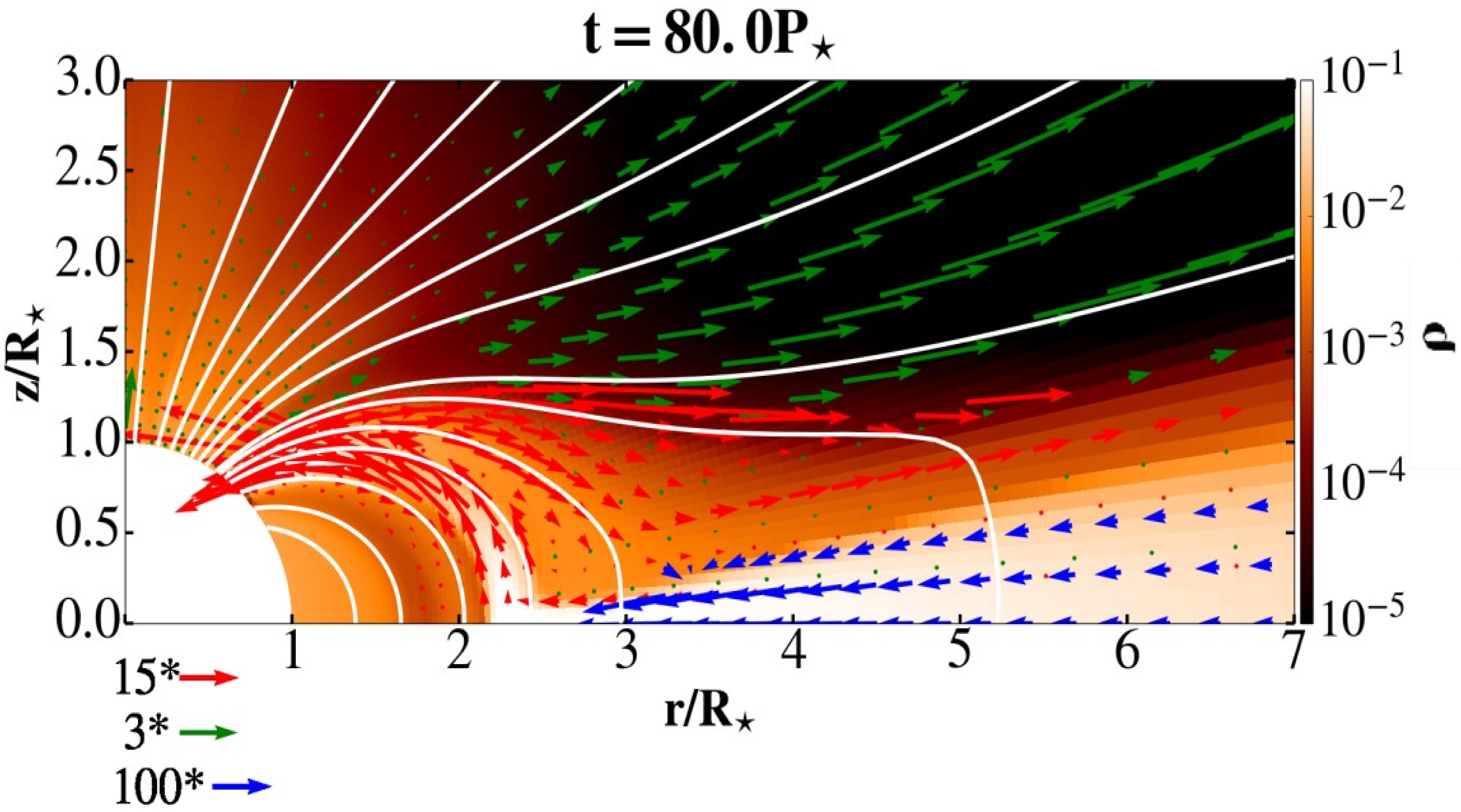

Animation of the results in our simulations with the strellar dipole field of 500 Gauss. Time is measured in the number of stellar rotations. Velocities in the different parts of the flow are shown with the different normalization.