Backflow in Accretion Disk

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Analytical solution for a thin hydrodynamic accretion disk shows that for some values of the viscosity parameter part of the accretion flow in the disk is not towards the star, but in the opposite direction. We study thin disks by performing hydrodynamic simulations and compare the numerical with analytical results. We confirm that for viscosity coefficient smaller than a critical value, there is a backflow in simulations near the disk mid-plane. The distance from the star to the starting point of backflow is increasing with viscosity, as predicted by the analytical solution. When the viscosity coefficient is larger than critical, there is no backflow in the disk.

1 Introduction

Backflow is a flow in the mid-plane of accretion disk directed away from central gravitating object, opposite to the direction of accretion flow. Backflow in the accretion disk was first studied by Urpin (1984a,b). Considering average inflow velocity he obtained backflow in the mid-plane of accretion disk for all values of viscosity parameter. Kley & Lin (1992) confirmed similar kind of backflow as Urpin in their numerical studies, but only for small values of viscosity parameter. Backflow was also found in works by Rozyczka et al. (1994) and Igumenshchev et al. (1996).

Full three dimensional analytical solution for a thin accretion disk was given by Kluzniak & Kita (2000, hereafter KK00). They derived the equations of a polytropic, viscous hydrodynamical (HD) accretion disk using the Taylor expansion in the small parameter $\epsilon = H/R$, the disk aspect ratio. Backflow near the disk mid-plane was obtained for all the values of viscous coefficient $\alpha < 0.685$. The point where the backflow started, stagnation radius, was found to be a function of viscosity parameter.

We shortly review the backflow in HD analytical solutions from KK00 in §2. Results from our numerical simulations are presented in §3, and a preliminary result in magnetic case is described in §4 .

2 Backflow in the analytical solution

From KK00 solution, the equation for radial velocity V_r in the equatorial plane in cylindrical coordinates is given by :

$$V_r(r,0) = -\alpha\epsilon^2 R\left(\frac{h^2}{r^{5/2}}\right)\left[2\left(\frac{d\ln h}{d\ln r}\right) - \Lambda\left(1+\frac{32}{15}\alpha^2\right)\right] \quad (1)$$

where

$$\Lambda = \frac{11}{5}/(1+\frac{64}{25}\alpha^2).$$

Here h is disk height, r is radial distance, α is the viscosity parameter and Λ is a function of α.

When $\alpha = 0$ then Λ reduces to 11/5 and the terms in the square bracket of Eq.(1) becomes negative, so that $V_r(r, 0) > 0$. Positive radial velocity indicates outflow in the equatorial plane away from the central object. When $\alpha = 1$ then $V_r(r, 0) < 0$, which indicates that the equatorial flow is directed towards the central object for all radii. The critical value over which there is no backflow is $\alpha_{cr} \approx 0.685$.

The radius for which the radial velocity is zero is the starting point of backflow. It is called the stagnation radius. The equation for stagnation radius is given by:

$$\frac{r_{stag}(\alpha)}{r_+} = \frac{[1 + 6(\Lambda(1 + \frac{32}{15}\alpha^2) - 2)]^2}{[6(\Lambda(1 + \frac{32}{15}\alpha^2) - 2)]^2} \tag{2}$$

A natural length scale is defined as $r_+ = \Omega_{\rm m}^2 r_{\rm m}^4/(GM_\star)$, with $\Omega_{\rm m}$ being the Keplerian rotation rate at a distance where the viscous torque is vanishing, $r_{\rm m}$.

The stagnation radius position is a function of viscous coefficient, α. With increasing α, the stagnation radius is increasing. When α approaches α_{cr}, the stagnation radius becomes infinite, indicating there is no backflow in the disk.

3 Backflow in numerical simulations

We perform axisymmetric 2D star-disk simulations in viscous HD and in resistive magneto-hydrodynamics (MHD) following Zanni & Ferreira (2009). Our setup with the publicly available PLUTO code (v.4.1) (Mignone et al., 2007, 2012) is presented in detail in Čemeljić (2019). A logarithmic stretched grid in radial direction in spherical coordinates is used, with uniformly spaced latitudinal grid. Resolution is $R \times \theta$=[217 × 100] grid cells, stretching the domain to 30 stellar radii(for some runs 50 stellar radii, to prevent influence of the outer boundary) in the half of the meridional plane. Typically we run simulations for 100 stellar rotations.

The equations solved by the PLUTO code are, in the cgs units:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{\rm v}) = 0 \tag{3}$$

$$\frac{\partial \rho \vec{\rm v}}{\partial t} + \nabla \cdot \left[\rho \vec{\rm v}\vec{\rm v} + \left(P + \frac{B^2}{8\pi} \right) \vec{\rm I} - \frac{\vec{\rm B}\vec{\rm B}}{4\pi} - \vec{\tau} \right] = \rho \vec{g} \tag{4}$$

$$\frac{\partial E}{\partial t} + \nabla \cdot \left[\left(E + P + \frac{B^2}{8\pi} \right) \vec{\rm v} - \frac{(\vec{\rm v} \cdot \vec{\rm B})\vec{\rm B}}{4\pi} \right] + \nabla \cdot \left[\eta_{\rm m} \vec{J} \times \vec{B}/4\pi - \vec{\rm v} \cdot \vec{\tau} \right] = \rho \vec{g} \cdot \vec{\rm v} - \Lambda \tag{5}$$

$$\frac{\partial \vec{\rm B}}{\partial t} + \nabla \times (\vec{\rm B} \times \vec{\rm v} + \eta_{\rm m} \vec{J}) = 0 \tag{6}$$

where the symbols have their usual meaning: ρ and v are the matter density and velocity, P is the pressure, B is the magnetic field and $\eta_{\rm m}$ and τ represent the resistivity and the viscous stress tensor, respectively.

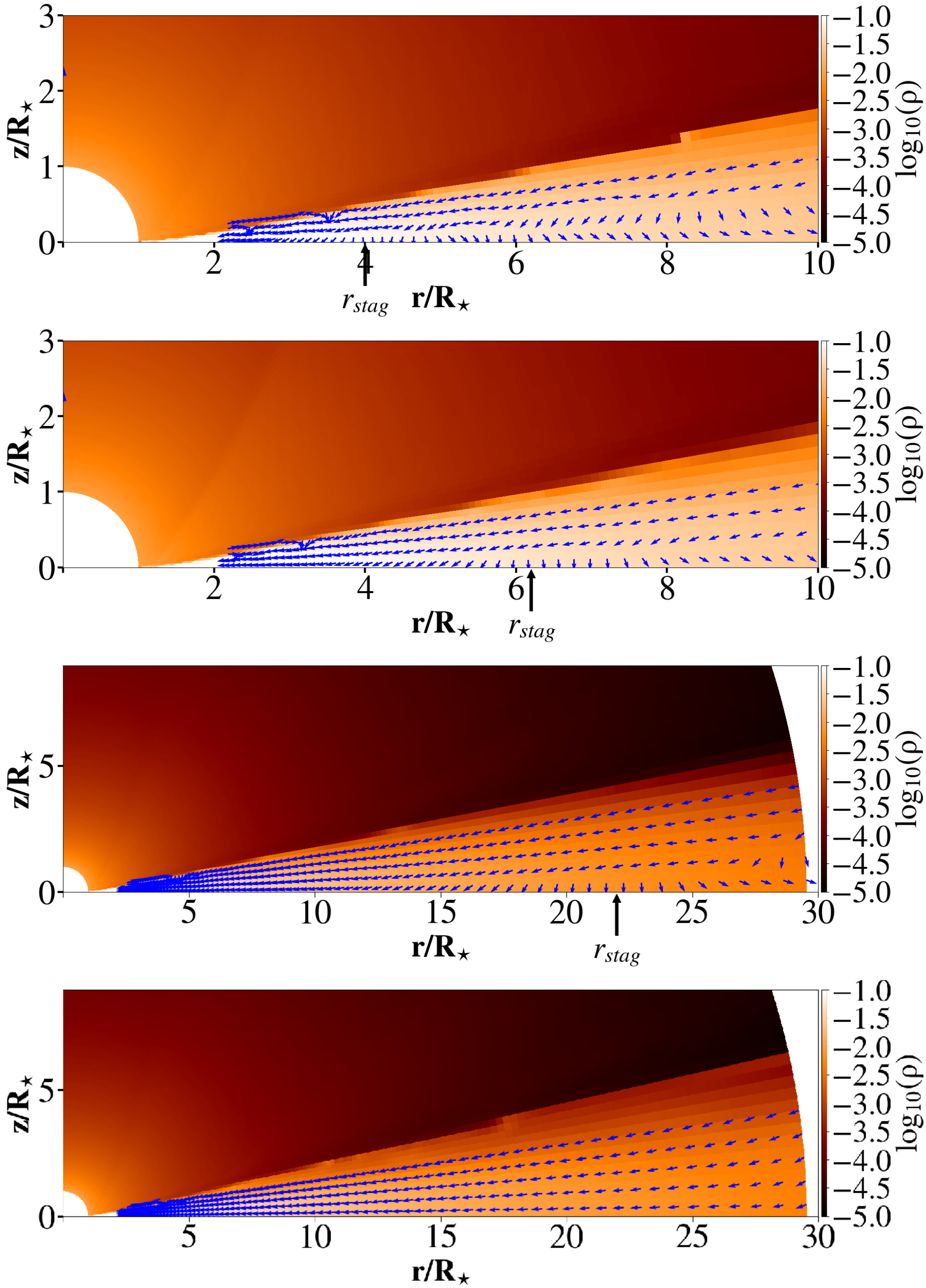

Fig. 1: The density in a logarithmic grading in purely HD solutions in our simulations with α=0.1, 0.2, 0.4 and 1, top to bottom panels, respectively. Vectors show the velocity in the disk. The position of stagnation radius is marked r_{stag}.

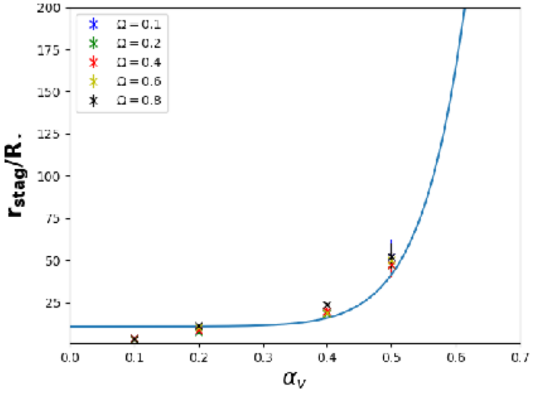

Fig. 2: Position of the stagnation radius r_{stag} in simulations with the different viscosity coefficients α. The cases with different rotation rates are represented with the red, blue and green symbols for simulations with Ω_*=0.1, 0.2, 0.4 and 0.6, respectively, with shown estimate of the error bars in positions.

3.1 Backflow in the HD disk

Purely HD simulations are obtained for different stellar rotation rates Ω (scaled with Keplerian break-up velocity) and α. For all the values $\alpha < 0.6$, we find backflow in our simulation. The stagnation radius for different α is marked r_{stag}. When $\alpha > 0.6$, there is no backflow in the disk. In the Fig.1 we have presented our simulations with different α, indicating the stagnation radius.

We find that stagnation radius is a function of α. It follows the same trend as predicted by analytical model in KK00.

4 Conclusions

We present results of HD and MHD simulations of thin accretion disks with backflow. The result from numerical simulations is in agreement with the analytical solution: with viscous coefficient $\alpha < 0.6$, backflow occurs in the mid-plane of the disk. The starting point of backflow, stagnation radius, is a function of α, as predicted by analytical model. For future work we will extend our study to magnetic cases.

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