Non-stationary disk accretion in Soft X-ray transients

K.L. Malanchev[1,2], A.V. Meshcheryakov[3], N.I. Shakura[1]

[1]Sternberg Astronomical Institute, Moscow State University
[2]Faculty of Physics, Moscow State University
[3]Space Research Institute, Russian Academy of Sciences

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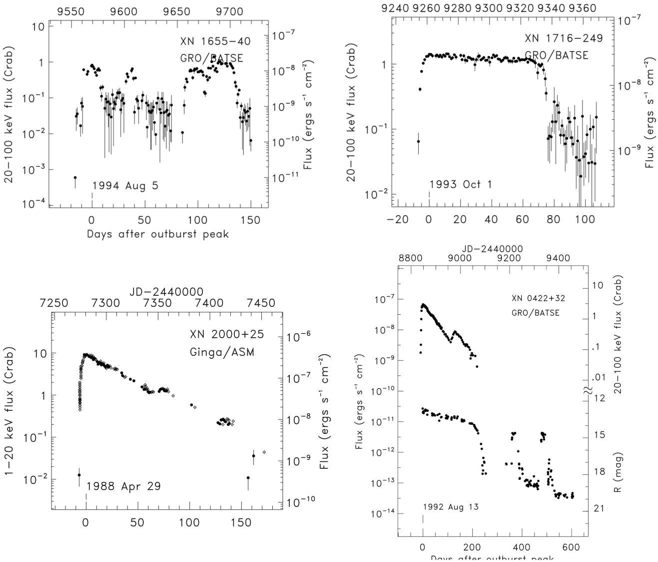
Light curves of X-ray transients



Chen Wan, Shrader C.R., Livio Mario, 1997, AJ 491:312-398

Non-stationary disk accretion

Continuity equation (We suppose that there are no inflows or outflows)

$$\frac{\partial \Sigma(r)}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}\left[\Sigma(r) v_r r\right] \tag{1}$$

Angular momentum transfer equation:

$$\Sigma(r) v_r \frac{\partial(\omega r^2)}{\partial r} = -\frac{1}{r}\frac{\partial}{\partial r}\left(W_{r\phi}(r) r^2\right) \tag{2}$$

Introduce new variables: viscous torque $2\pi F = 2\pi W_{r\phi} r^2$ and net angular momentum $h = \omega r^2$. We suppose that gas in the disk rotates with Keplerian velocity. Rewrite (2) with new variables:

$$\Sigma v_r r = \frac{1}{2\pi}\dot{M}(h,t) = -\frac{\partial F}{\partial h} \tag{3}$$

If we put equation (3) in (1) we obtain the diffusion equation:

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{2}\frac{(GM_x)^2}{h^3}\frac{\partial^2 F}{\partial h^2} \tag{4}$$

Solutions of the diffusion equation

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{2}\frac{(GM_x)^2}{h^3}\frac{\partial^2 F}{\partial h^2}$$

- When dynamic viscosity doesn't depend on surface density then $\Sigma(h) \sim F(h)/h^3$. In this case equation becomes linear and its solution is $F(t) \sim e^{-t/\tau}$.
- In 1987 Yu.E. Lyubarskii and N.I. Shakura derived relation between F and Σ from vertical structure equations. Then the diffusion equation has automodel solution.

$$\Sigma \sim \frac{F^{1-m}}{h^{3-n}} \Rightarrow F(t) \sim \frac{1}{(t-t_0)^{1/m}}, \quad \textit{where}$$

$$m = \frac{4+2\zeta}{10+3\zeta+2\nu}, \quad n = \frac{12+11\zeta-2\nu}{10+3\zeta+2\nu},$$

for opacities $\kappa \sim \rho^{\zeta}/T^{\nu}$

Vertical structure

Specific relation between $\Sigma(h)$ and $F(h)$ is determined from numerical solution of vertical structure equations. In real case opacity is a complicated function of its arguments. So we have used values from OPAL project.

- Along the z coordinate the hydrostatic equilibrium takes place:

$$\frac{dp}{dz} = -\rho \frac{GM_x}{r^2} \frac{z}{r} = -\rho \omega^2 z$$

- Energy transfer equation:

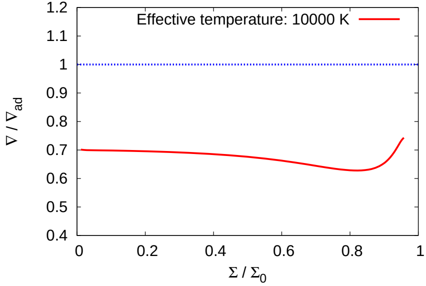
$$\begin{cases} \dfrac{dT}{dz} = -\dfrac{3\kappa\rho}{4acT^3} Q, & \left|\dfrac{dT}{dz}\right| < \left|\left(\dfrac{dT}{dz}\right)_{ad}\right| \\ \dfrac{dT}{dz} = \left(\dfrac{dT}{dz}\right)_{ad}, & \left|\dfrac{dT}{dz}\right| \geq \left|\left(\dfrac{dT}{dz}\right)_{ad}\right| \end{cases}$$

$$Q = \frac{1}{2} \omega W_{r\phi} r \frac{\partial \omega}{\partial r} = \frac{3}{4} \frac{(GM_x)^2 \omega F}{h^4}$$

- Vertical gradient of the radiation flux:

$$\frac{dQ}{dz} = \frac{3}{2} \alpha p \omega$$

Secondary peak



Vertical structure of the accretion disk at some radius. ∇/∇_{ad} — ratio of current vertical temperature gradient to adiabatic one. When the ratio equals 1 convection takes place. Σ/Σ_{tot} — vertical mass coordinate, it equals 0 in the plane of symmetry of the disk and equals 1 in the photosphere.

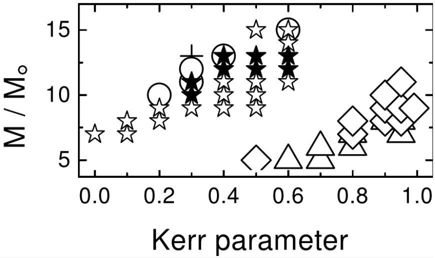
During the evolution of the accretion disk its temperature at fixed radius decreases.
So, at some moment, a zone of partial ionized hydrogen appears in outer part of the disk.
This zone becomes convective very fast.
The convection increases parameter α and consequently accreation rate and X-ray luminosity of the object.

Simulation of A0620-00: Choise of system parameters

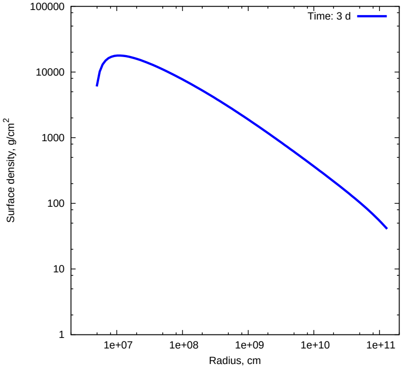
Researchers get a wide range of system parameters:

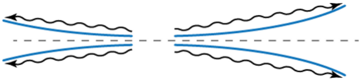
Authors	Mass of black hole	Distance
Haswell et al. 1993	$4.6 \pm 1.1 \quad M_\odot$	
Shahbaz et al, 1994	$10 \pm 7 \quad M_\odot$	$1.0 \pm 0.4 \quad kpc$
Gelino et al., 2001	$11 \pm 1.9 \quad M_\odot$	$1.1 \pm 0.1 \quad kpc$
Cantrell et al., 2010	$6.61 \pm 0.25 \quad M_\odot$	$1.06 \pm 0.12 \quad kpc$

Suleimanov et al. (2008) used parameters close to work Gelino et al., 2001.



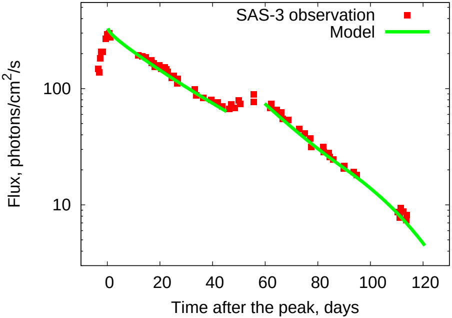
Simulation of A0620-00: evolution of surface density





Bump appears in convection zone. Direct X-Ray radiation cannot illuminate outer part of the disk anymore. This part becomes cold and mass transfer slows down rapidly in it. We assume that this cold part conserves its surface density without accretion.

Simulation of A0620-00: results



1.5-6keV light curve of A0620-00.
Red squares — observed light curve
(E. Kuulkers, 1998).
Green line — modeled light curve.

System parameters:

$$\alpha_1 = 0.4$$
$$\alpha_2 = 0.7$$
$$\dot{M}_0 = 1.9 \times 10^{18} g/s \simeq 0.2 \dot{M}_{Ed}$$
$$a_{Kerr} = 0.4$$

Conclusions

- Observations are explained satisfactory by the model of non-stationary disk accretion.
- Zone with partial hydrogen ionization appeares 40 days after the primary peak. It becomes convective and accretion rate increases. Then it moves inside the disk.
- Values of the parameter α are greater than 0.1, this result is in agreement with the previous works. It is 0.4 before the secondary peak and 0.7 after it.

Thank you for your attention.