MRI Simulations
and the Thermal-Viscous Instability

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Accretion Flow Instabilities
3rd NCAC Symposium
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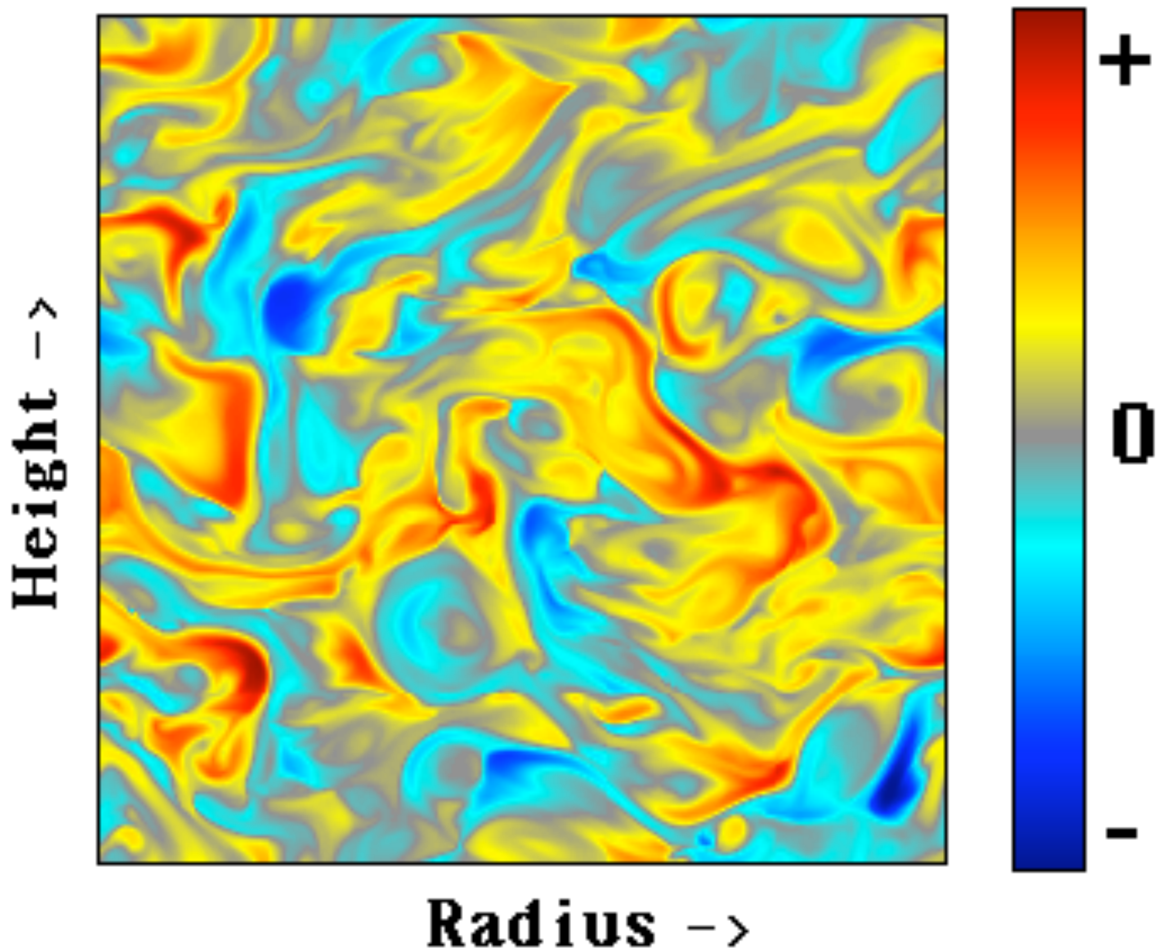


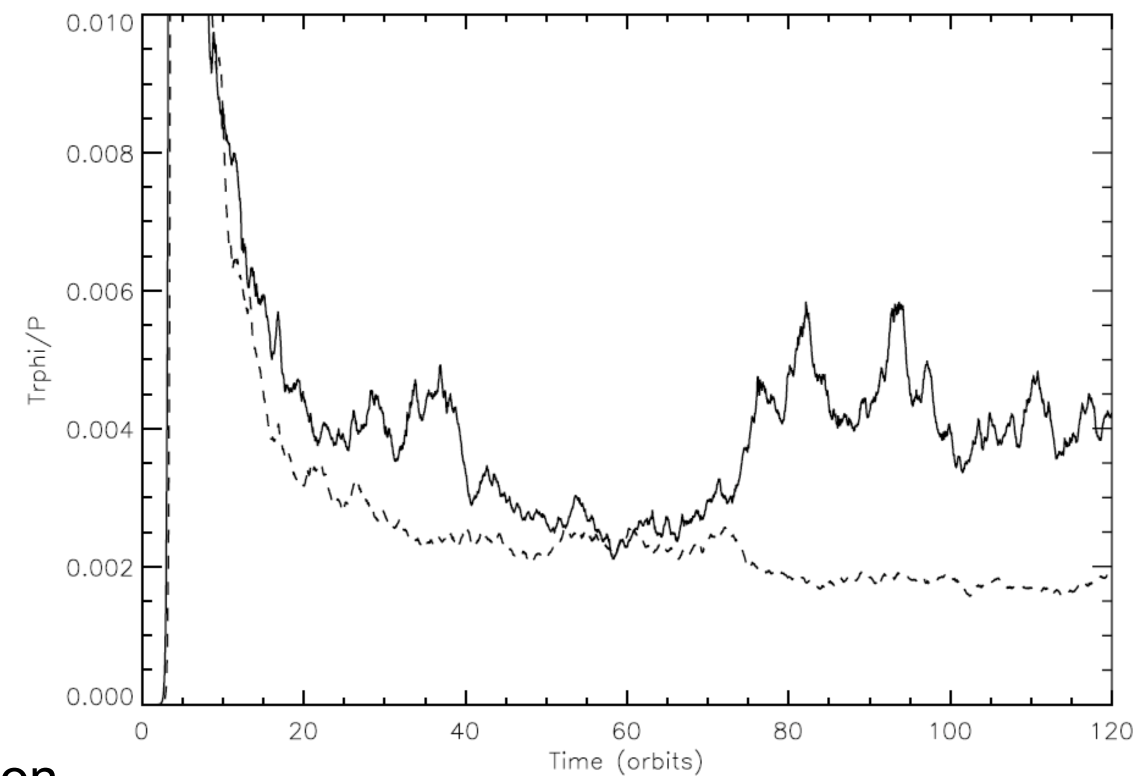
Snapshot of angular momentum per unit mass in MRI turbulence.

$$\tau_{R\phi} = \overline{\rho} < v_R v_\phi - B_R B_\phi /(4\pi\rho) >$$

-Hawley & Balbus (1992)

Local Shearing Box Simulations with no Vertical Stratification

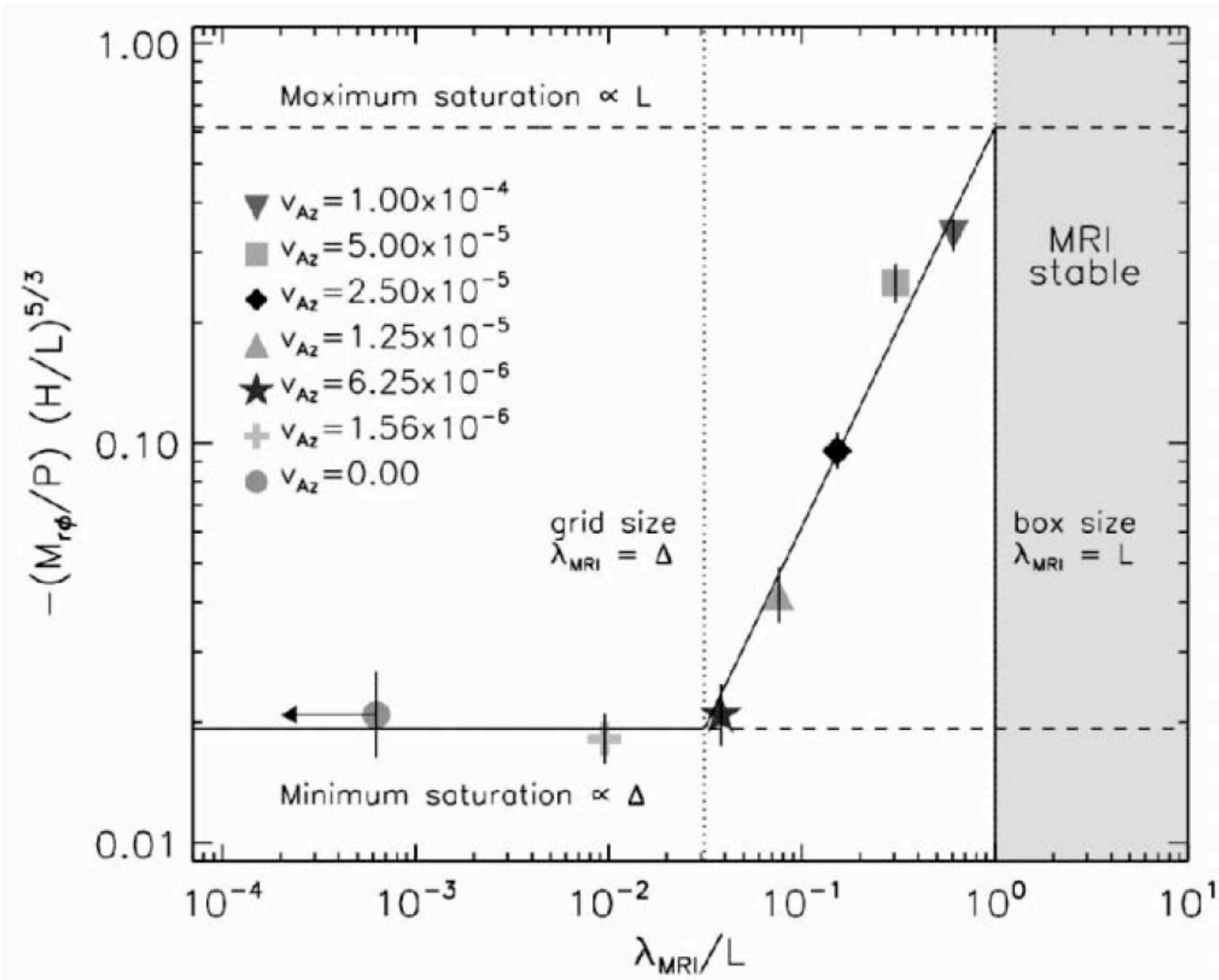
In simulations with no net flux and with no explicit viscosity and resistivity, turbulent stresses decrease with increasing numerical resolution (Pessah et al. 2007; Fromang & Papaloizou 2007).



Fromang & Papaloizou (2007)

BUT including net flux, or including explicit dissipation with sufficiently high Reynolds and magnetic Prandtl numbers leads to converged turbulence (Fromang et al. 2007), though the turbulent stress depends on these numbers.

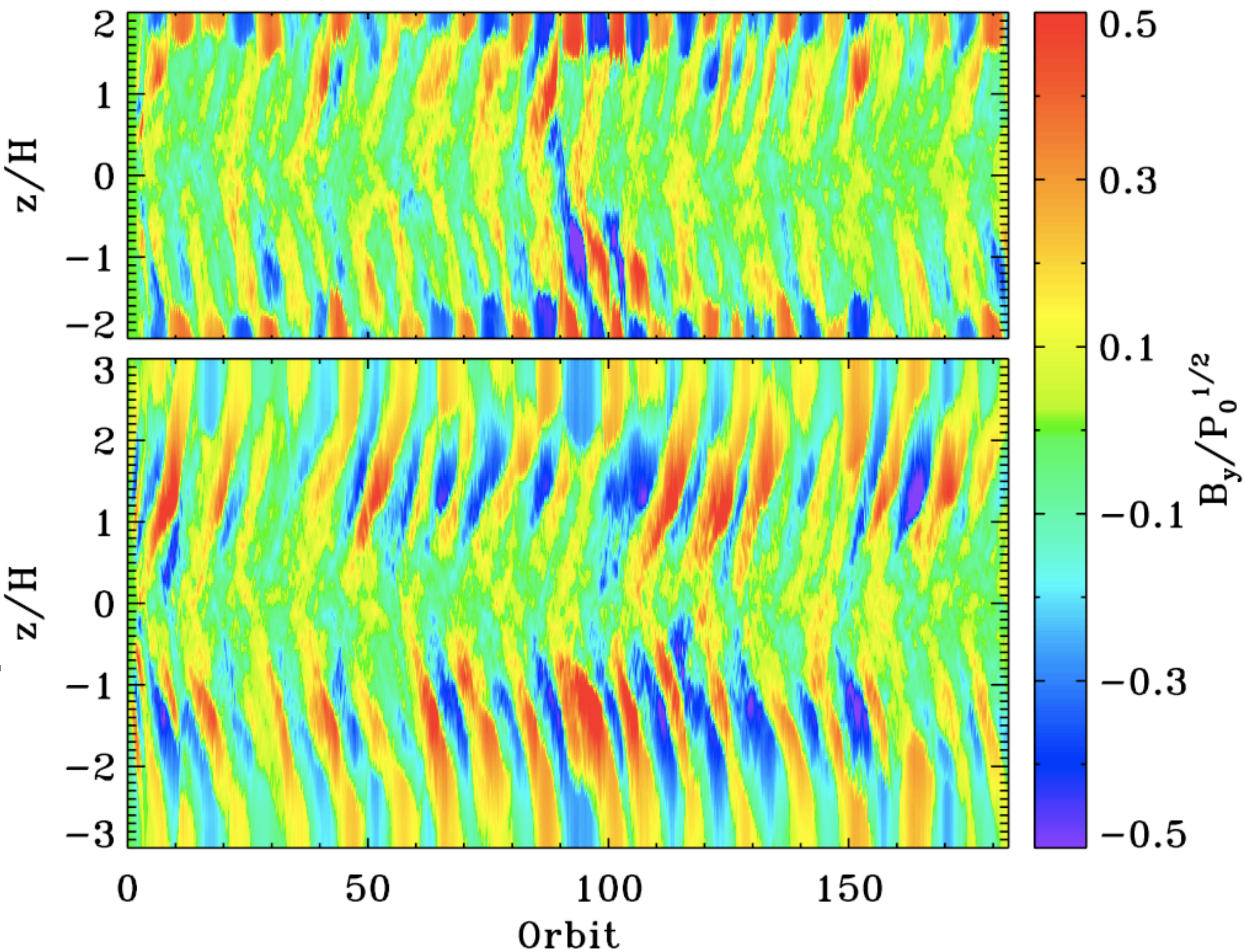
Adding net vertical magnetic flux should increase alpha



-Pessah, Chan, & Psaltis (2007)

Vertical Stratification Removes the Problem of No Net Flux, But the Microscopic Dissipation Might Still Matter

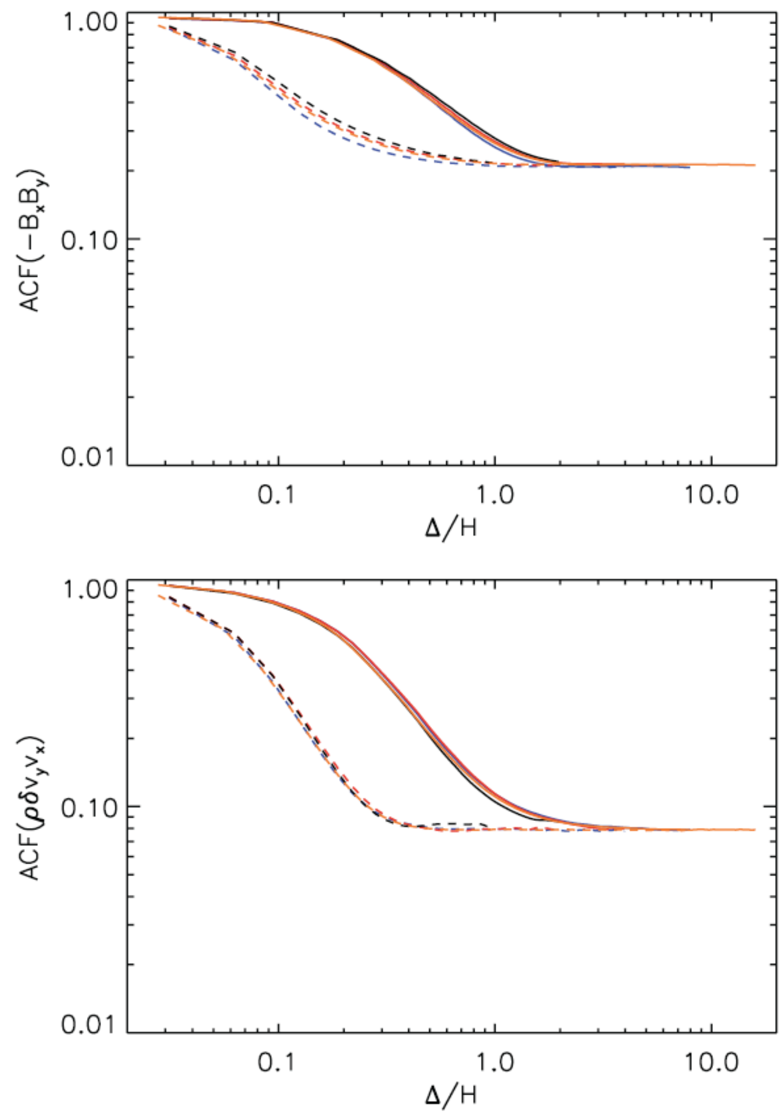
MRI dynamo with buoyancy creates alternating signs of net toroidal magnetic flux (the so-called MRI "butterfly diagram").



-Davis, Stone, & Pessah (2010)

What about radially wide shearing boxes – is MRI transport even local?

Not completely – there are significant large scale correlations (8% and 20% in Reynolds and Maxwell stresses,respectively).



-Simon, Beckwith, & Armitage (2012)

more direct astrophysical implications. The most direct way of determining α in real accretion disks is the analysis of dwarf nova outbursts and X-ray transient outburst. As pointed out by King et al. (2007), these estimates suggest $\alpha \sim 0.1$–0.4, whereas numerical simulations (including those presented here) typically obtain steady-state values of α that are an order of magnitude smaller. In light of our results, we note that an accretion disk that has just entered an outburst state may well go through a period of field growth that resembles the early transients seen in our simulations. During these transients, the effective value of α is substantially enhanced, and spatio-temporal gradients in α further enhance the angular momentum transport. Thus, it is interesting to conjecture that the large values of α inferred from outburst systems correspond to these transient phenomena. These issues will be explored in a future publication.

-Sorathia et al. (2012)

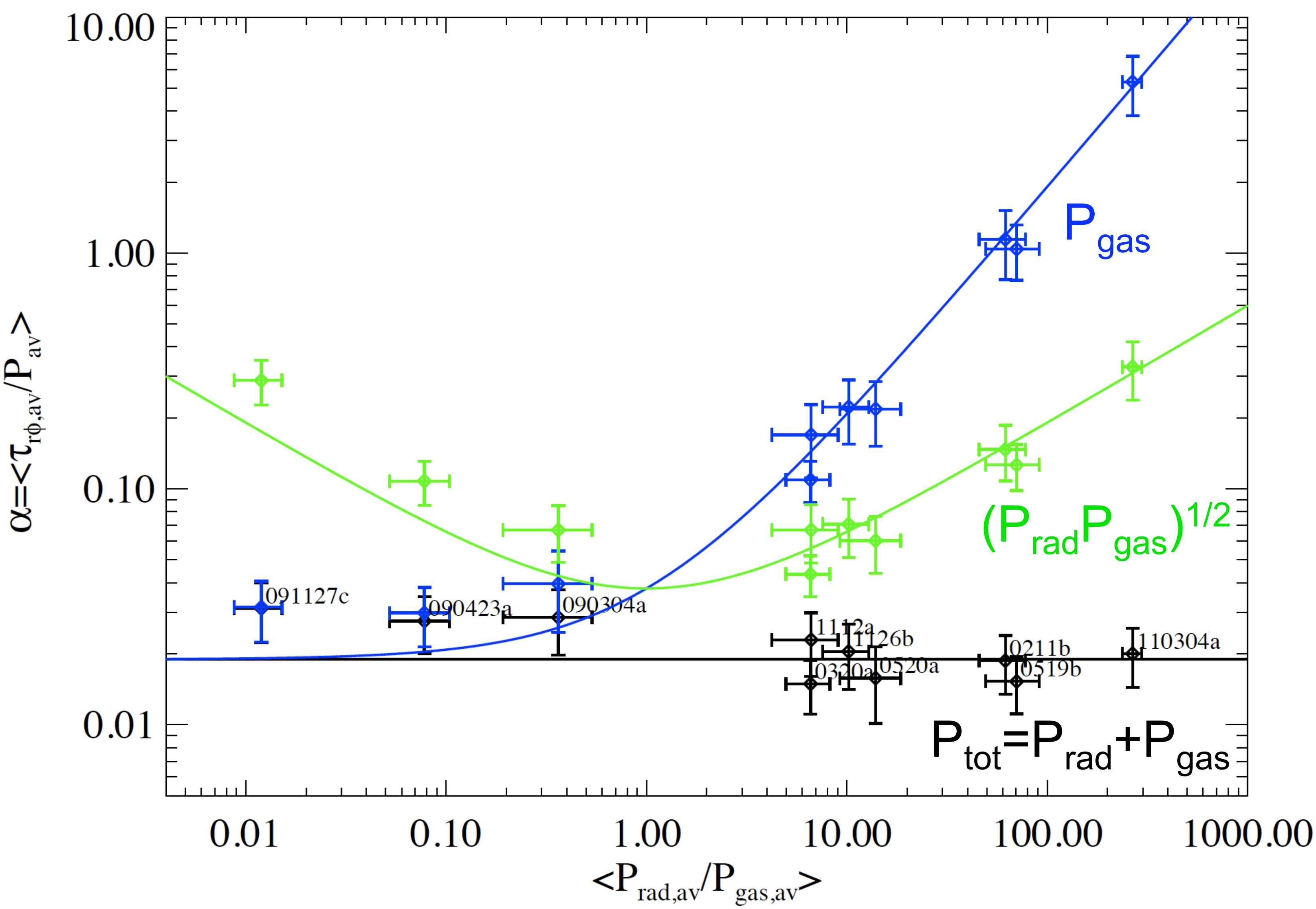
The Alpha Prescription in the Radiation Pressure Dominated Regime

$$\tau_{r\phi} = \alpha P_{\mathrm{thermal}}$$

$\tau_{r\phi}$ and P_{thermal} are *vertically-averaged quantities*.

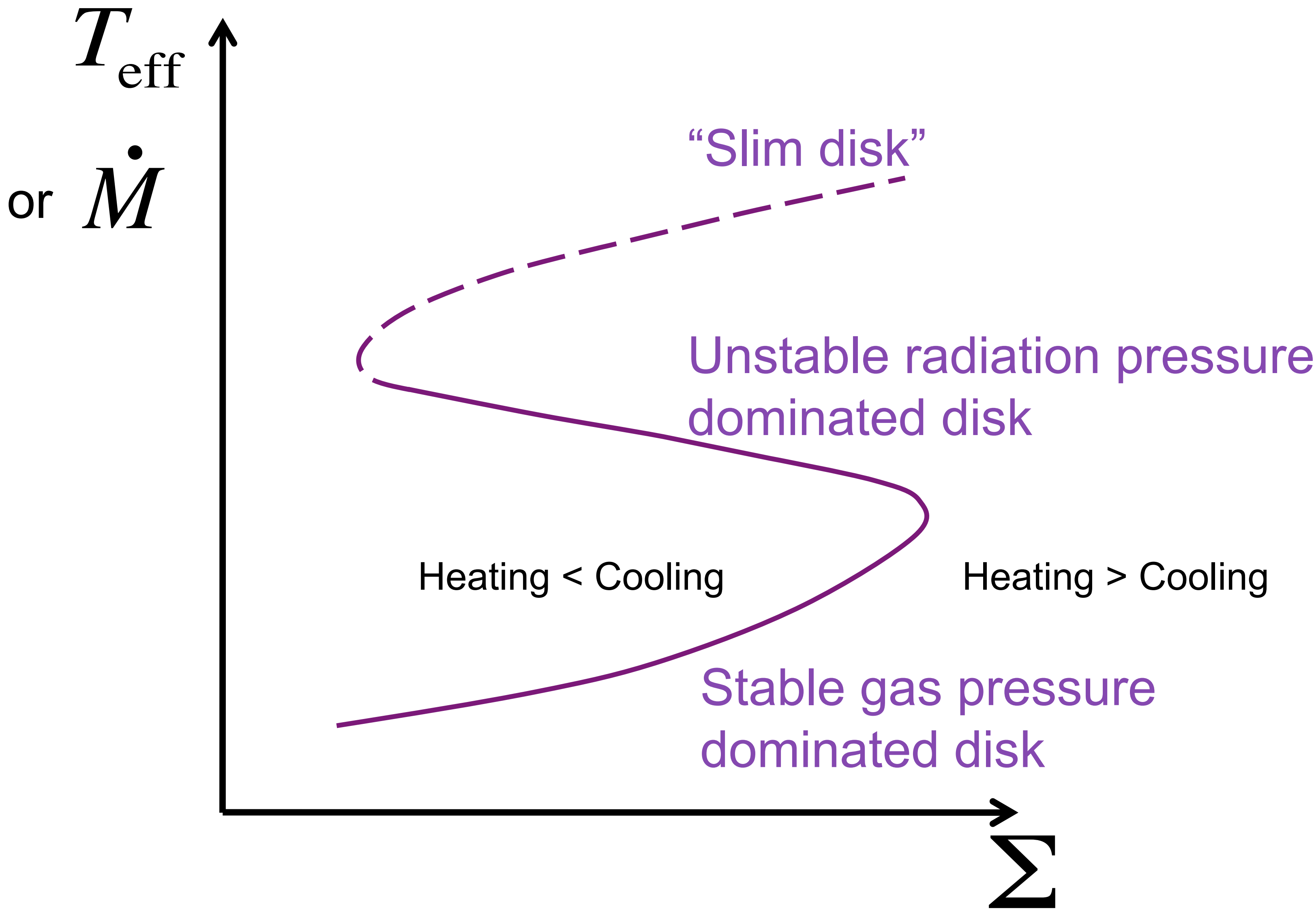
Radiation MHD simulations of MRI turbulence (Ohsuga et al. 2009, Hirose et al. 2009) are consistent with P_{thermal} being the *total* (gas plus radiation) thermal pressure, as Shakura & Sunyaev (1973) originally assumed.

Alternative Stress Prescriptions Are Ruled Out



-updated from Hirose et al. (2009)

Thermal Equilibrium of an Annulus in Standard Alpha Disk Theory



Thermal Instability

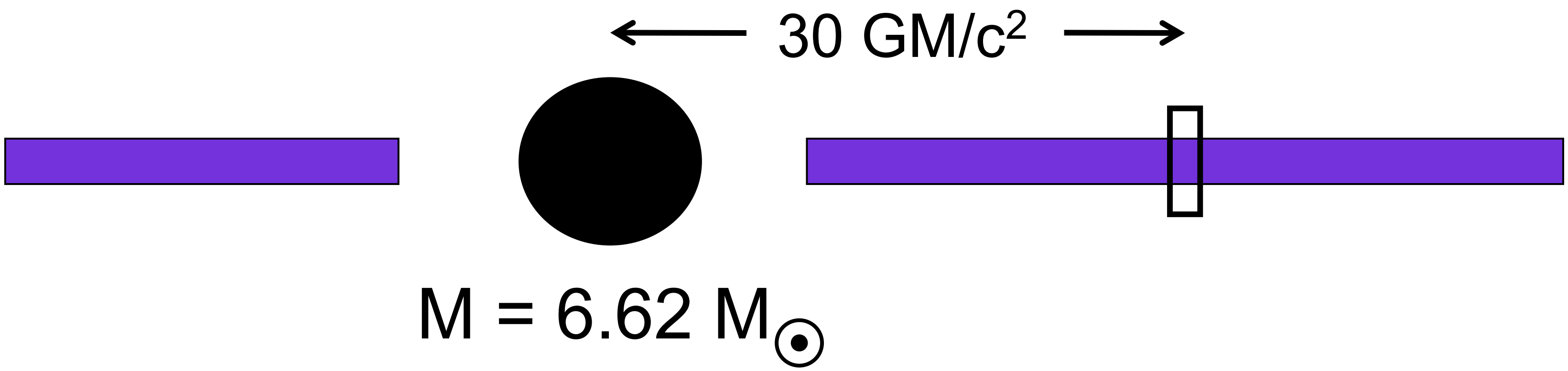
$$\text{Cooling} \approx \frac{4caT^4}{3\kappa\Sigma} \propto \frac{T^4}{\Sigma}$$

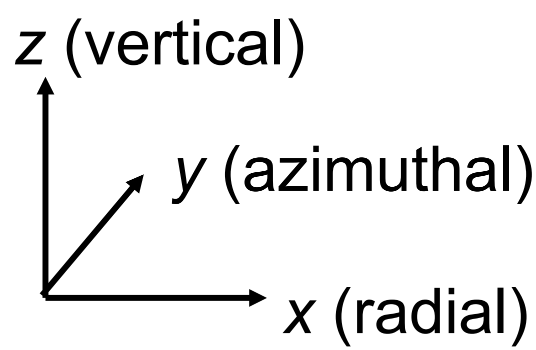
If $\tau_{r\phi} = \alpha P_{\text{tot}}$, then in the radiation pressure dominated regime,

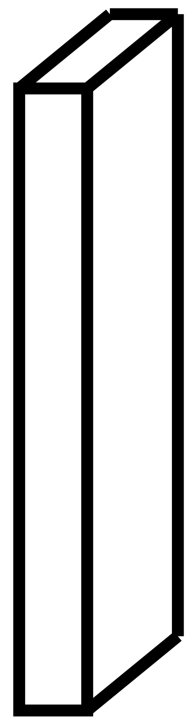
$$\text{Heating} \approx 2H\tau_{r\phi} r \left| \frac{d\Omega}{dr} \right| \propto \frac{T^8}{\Sigma}$$

→ Runaway heating or cooling

The Stratified Shearing Box





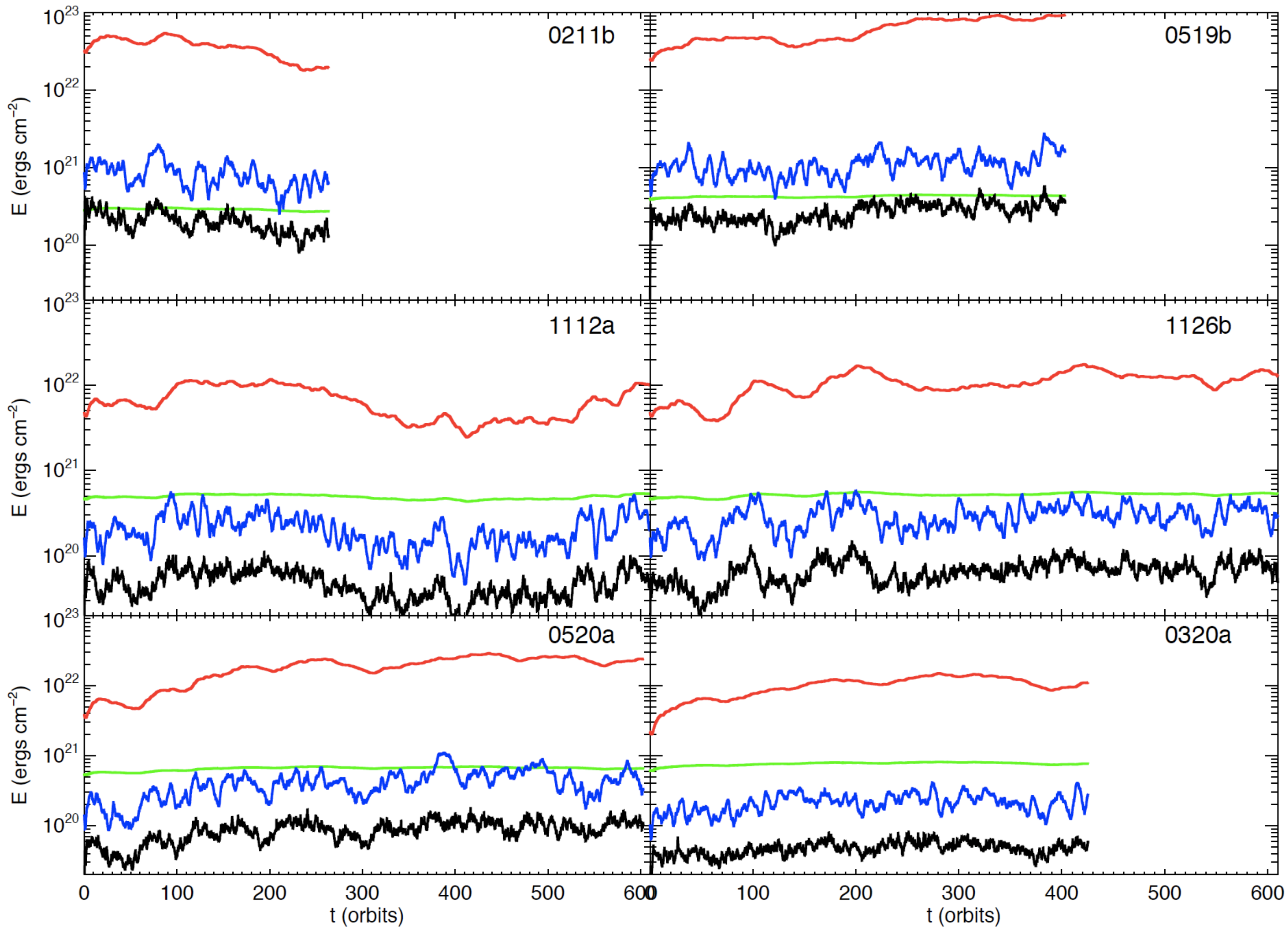


Dimensions:
~0.5X2X8 or 9.3

Resolution:
48X96X896
or 32X64X512

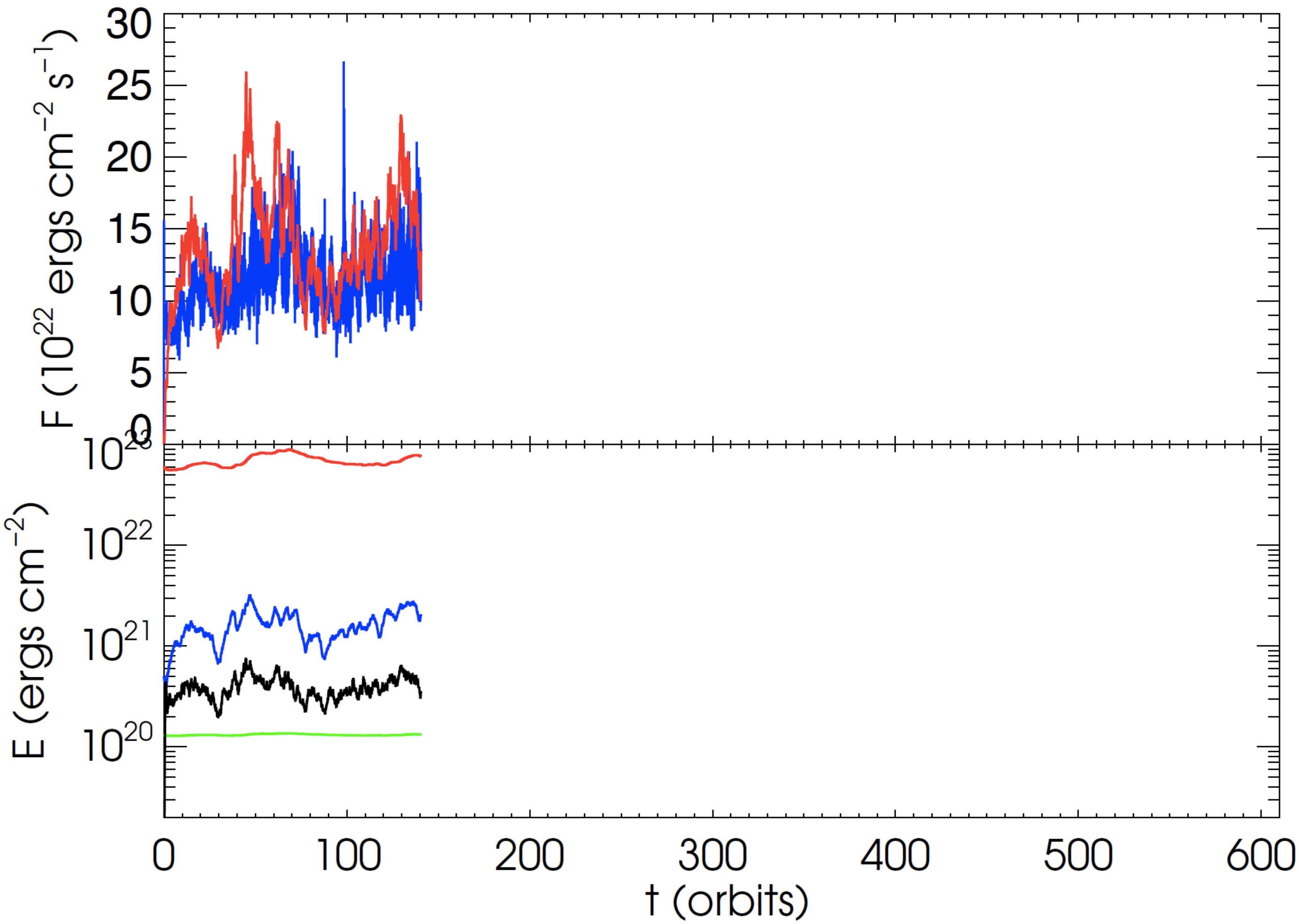
(Apologies to Arthur C. Clarke and Stanley Kubrick)

Radiation, Gas Internal, Magnetic, and Turbulent Kinetic Energies in Local, Radiation Pressure Dominated, Shearing Box Simulations

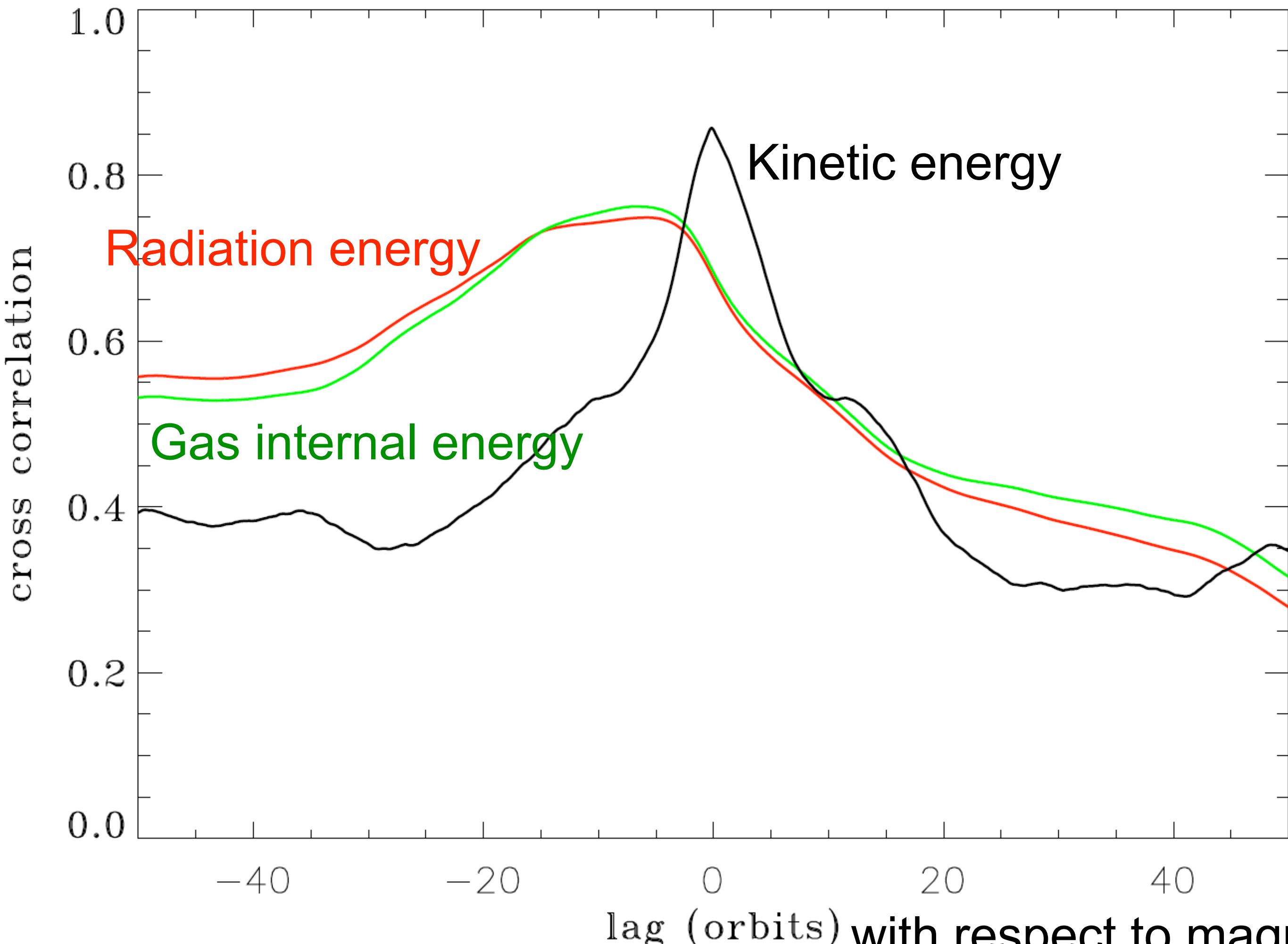


No thermal instability! (First shown by Turner 2004.)

Our Most Radiation Pressure Dominated Simulation: P_{rad}/P_{gas}~270



Fluctuations in thermal energy are correlated to fluctuations in turbulent magnetic and kinetic energies, but with a LAG



with respect to magnetic energy fluctuations

-Hirose et al. (2009)

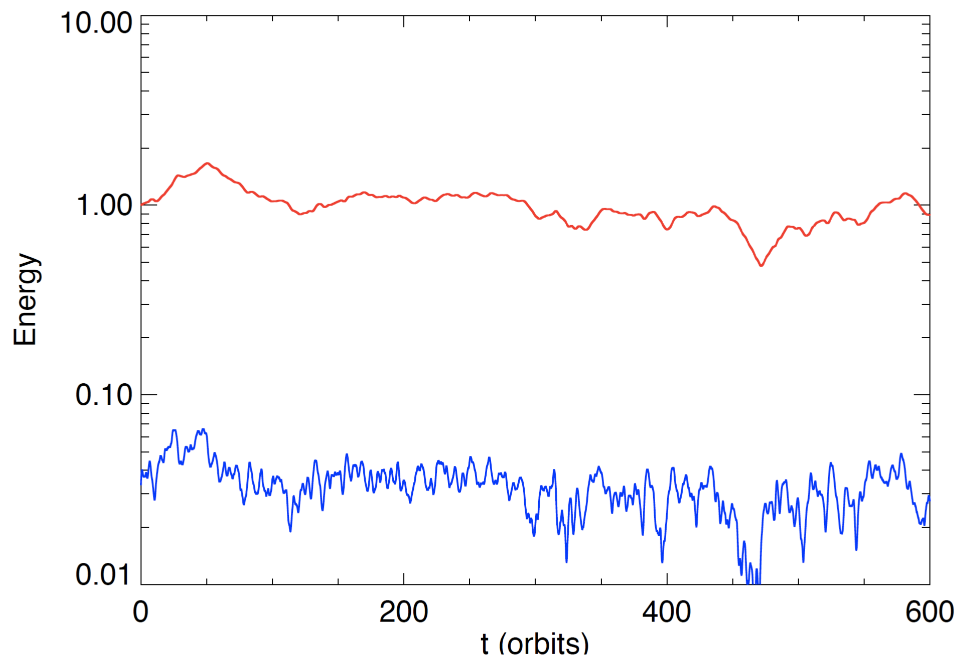
A Simple Toy Model Can Explain this Behavior

$$\frac{dE_{\mathrm{mag}}}{dt} = R(t)\frac{E_{\mathrm{mag},0}}{t_{\mathrm{growth}}} - \frac{E_{\mathrm{mag}}(t)}{t_{\mathrm{diss}}}$$

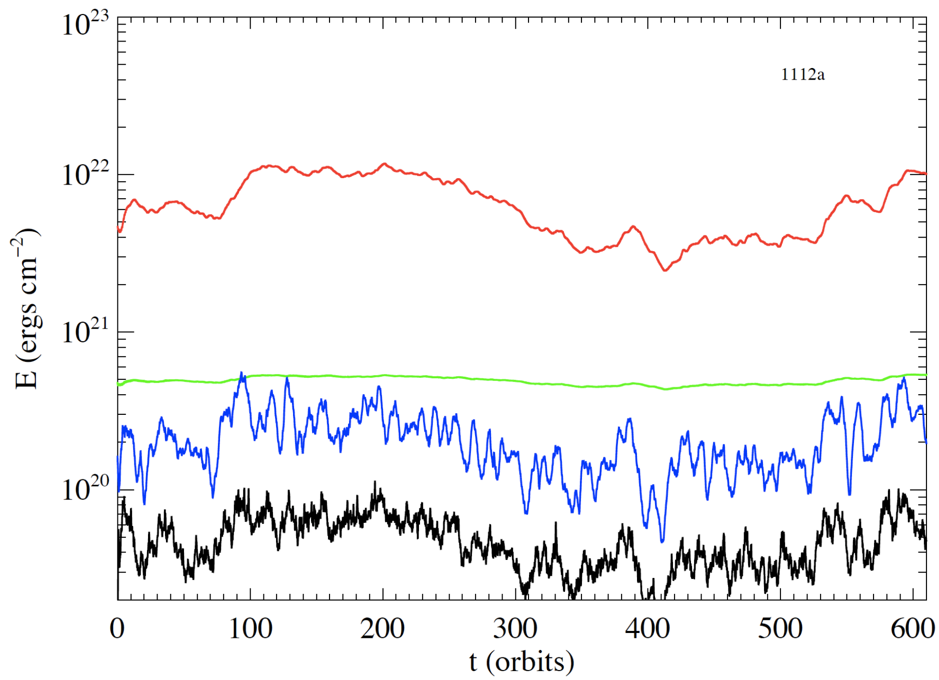
$$\frac{dE_{\mathrm{rad}}}{dt} = \frac{E_{\mathrm{mag}}(t)}{t_{\mathrm{diss}}} - \frac{E_{\mathrm{rad}}(t)}{t_{\mathrm{cool}}}$$

-where $R(t)$ is a stochastic function of time with mean of unity and with power spectrum chosen to match simulations.

-Hirose et al. (2009)

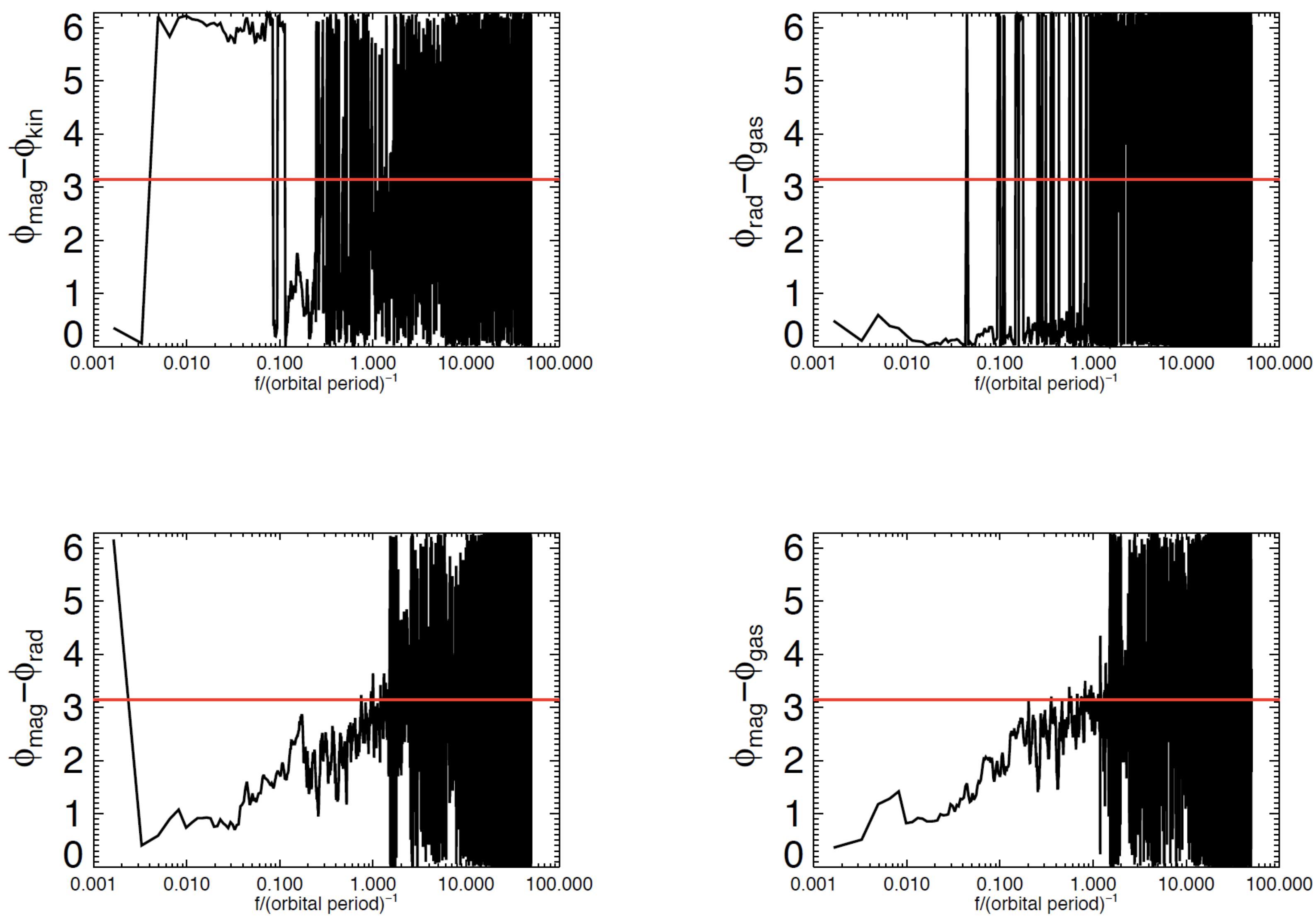


Stochastic differential equations.



Simulation data.

Fourier phase analysis reveals a more complex behavior…



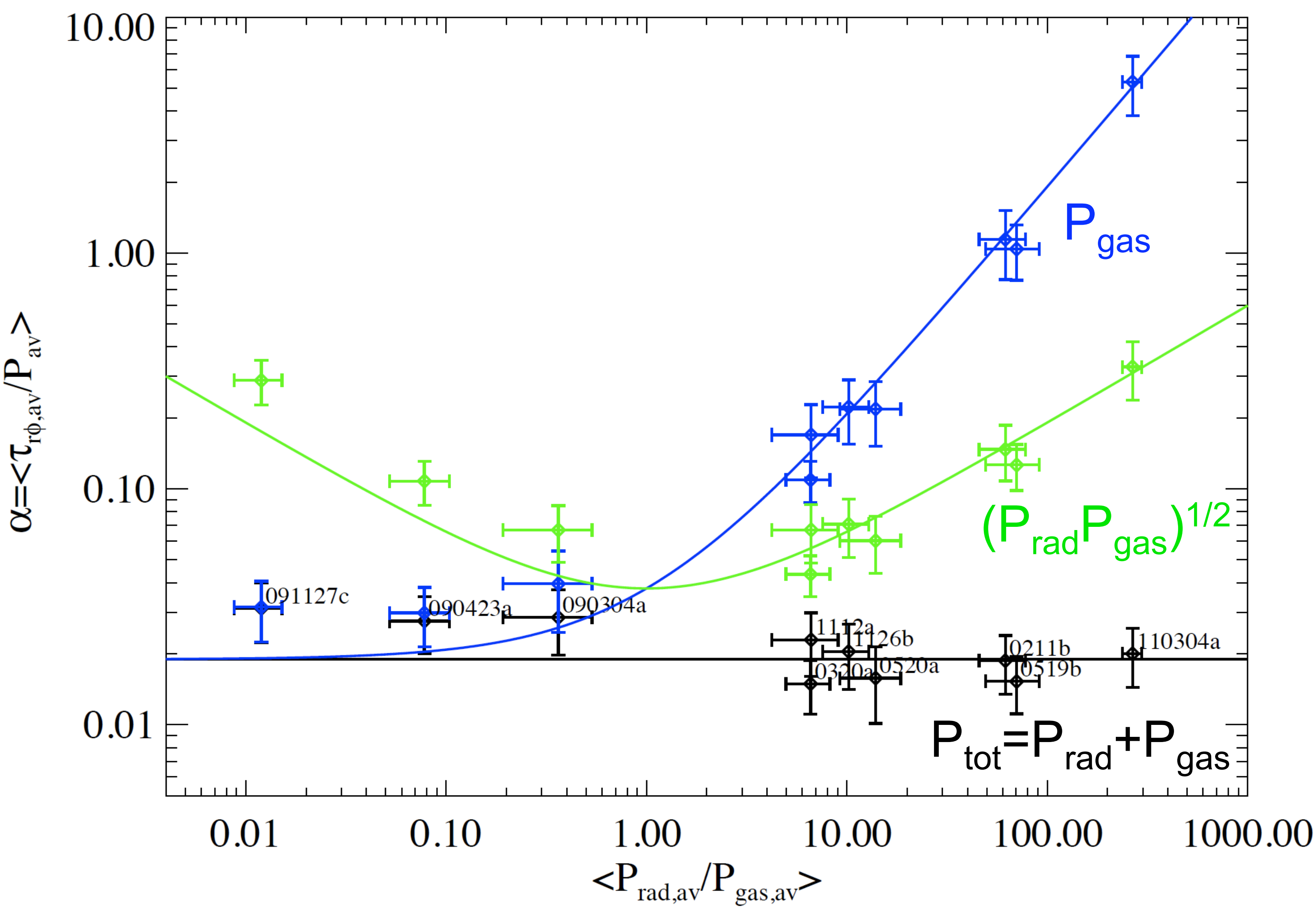
$$\tau_{r\phi} \rightarrow P_{\text{Total!!!}}$$

This then gives the alpha prescription ***if***

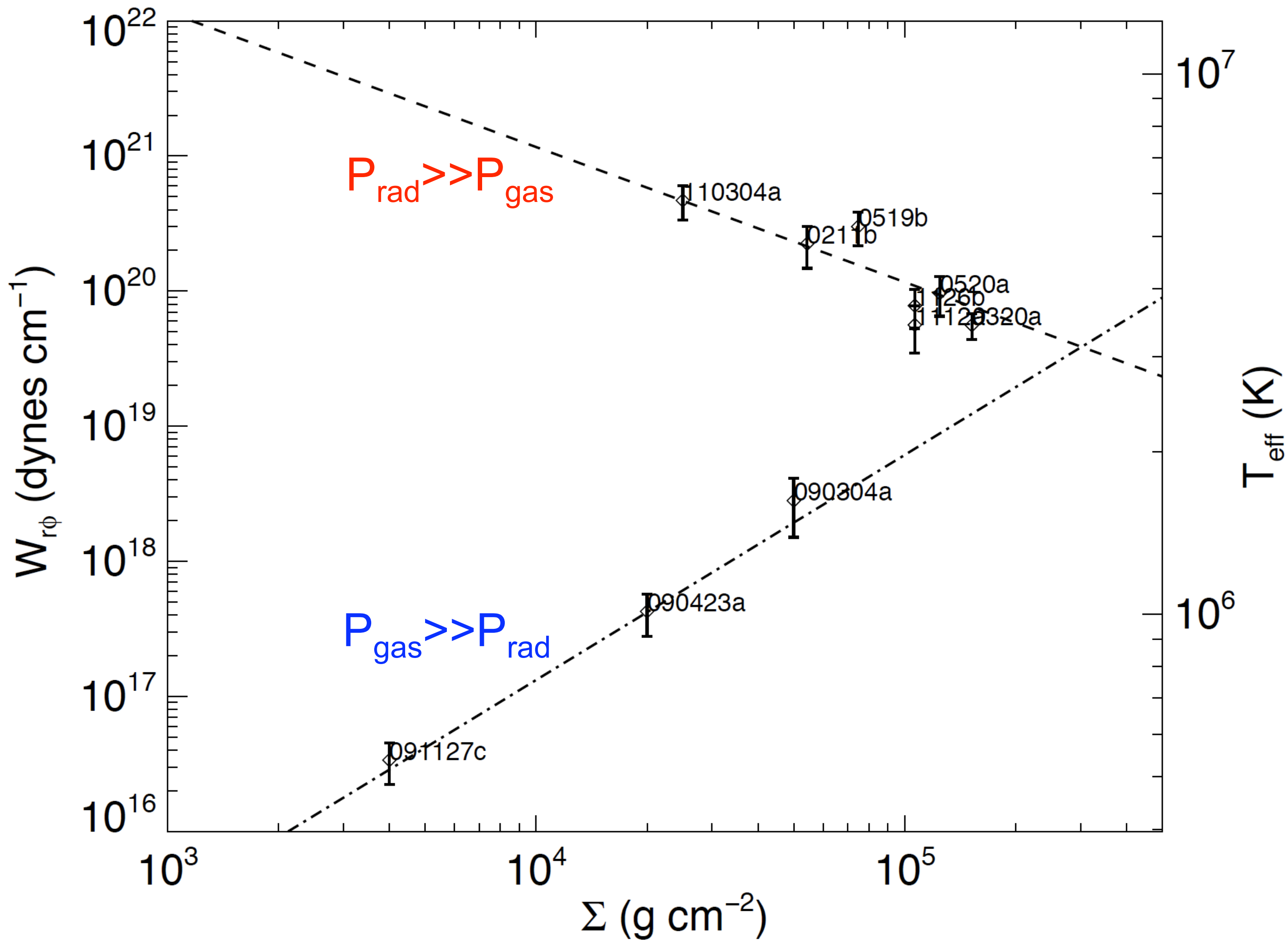
$$\frac{1}{\Omega t_{\text{cool}}} \sim \frac{c}{\Omega \kappa \Sigma H} \sim \text{constant} \equiv \alpha$$

We find this to be approximately true in the simulations, but why?!?

Alternative Stress Prescriptions Are Ruled Out

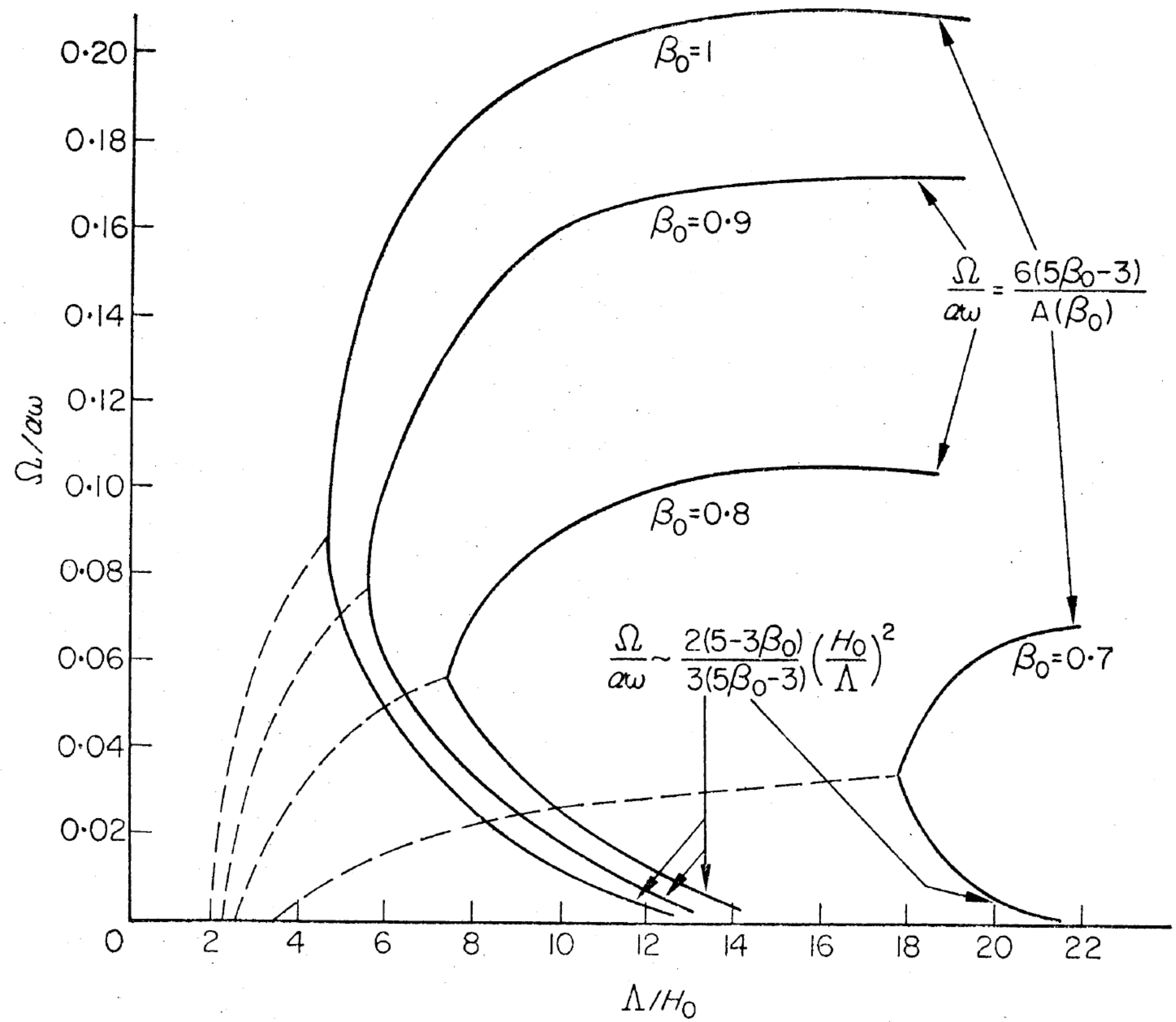


-updated from Hirose et al. (2009)



$$\frac{\partial \Sigma}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[\frac{1}{\ell'}\frac{\partial}{\partial r}\left(r^2 W_{r\phi}\right)\right]$$ so $$\frac{\partial W_{r\phi}}{\partial \Sigma} < 0$$ → "Viscous Instability" ?

Unfortunately, "viscous" instability has not been probed by MRI simulations as yet.



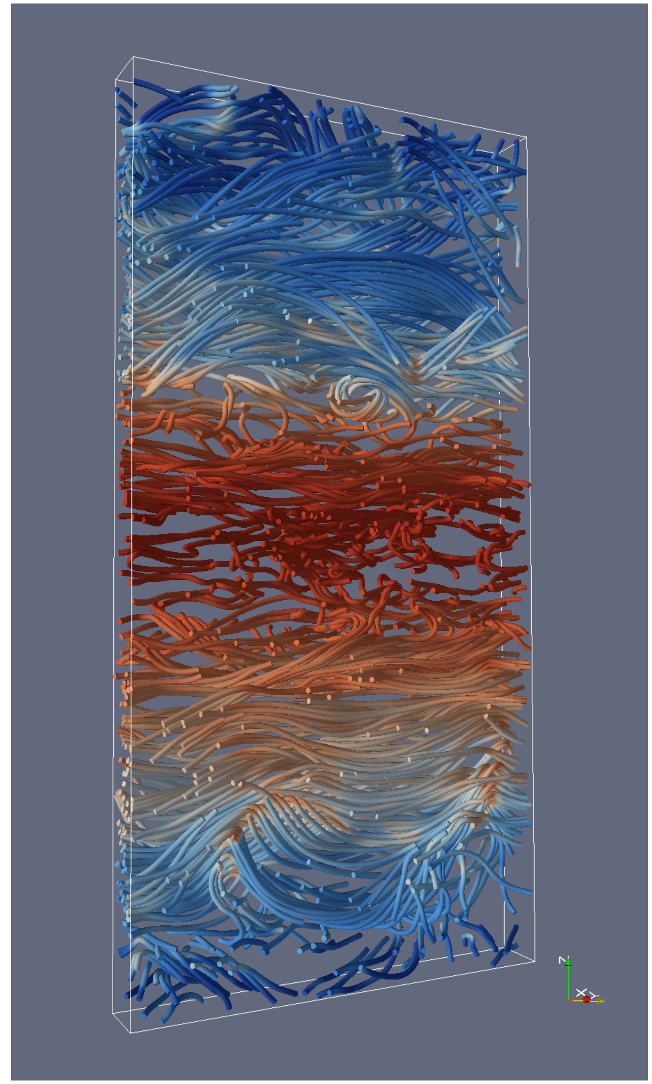
-Shakura & Sunyaev (1976)

Other Interesting Features: Surface Layers are Magnetically Supported and Parker Unstable

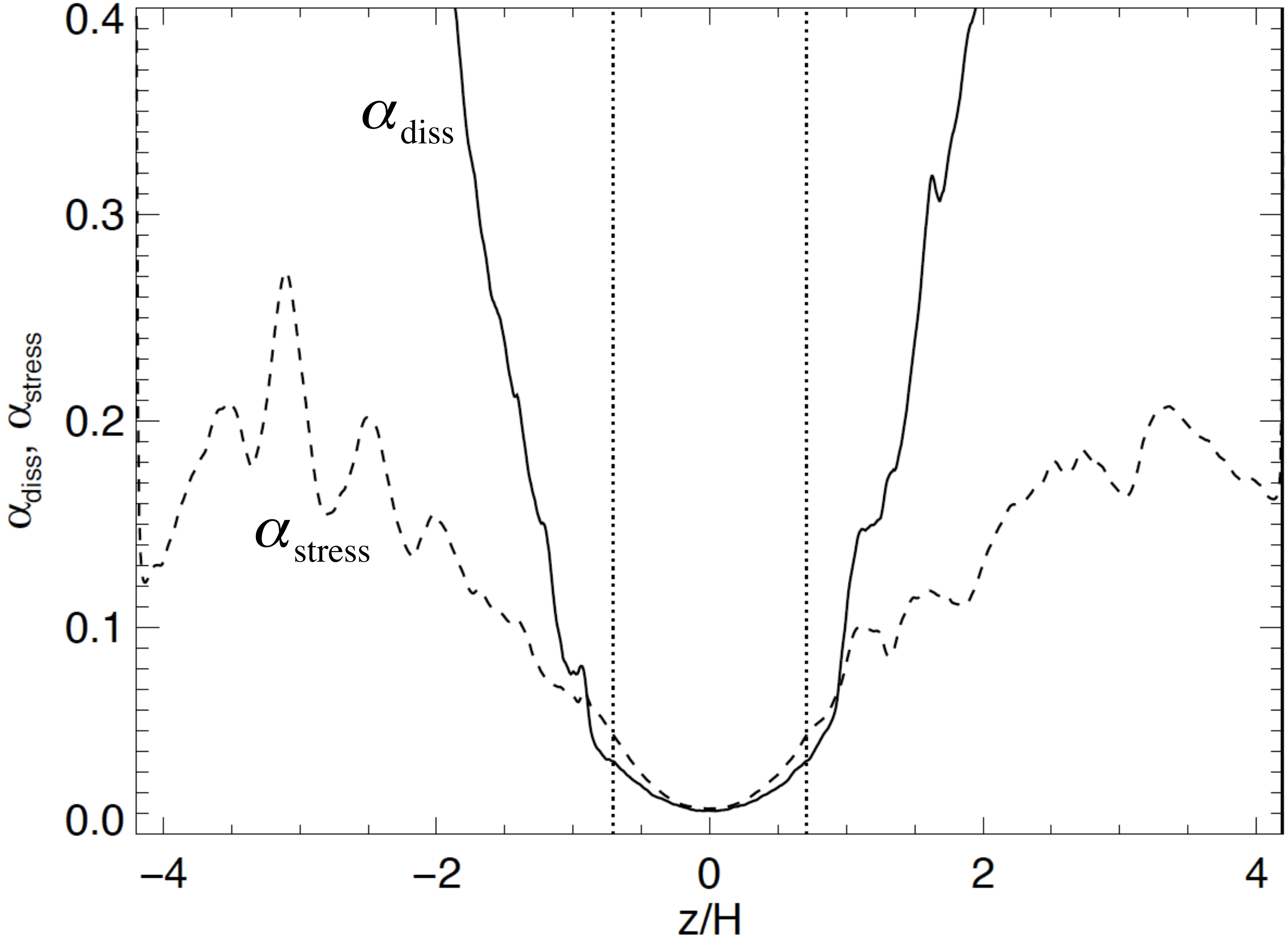
Parker

MRI turbulence in midplane regions

Parker

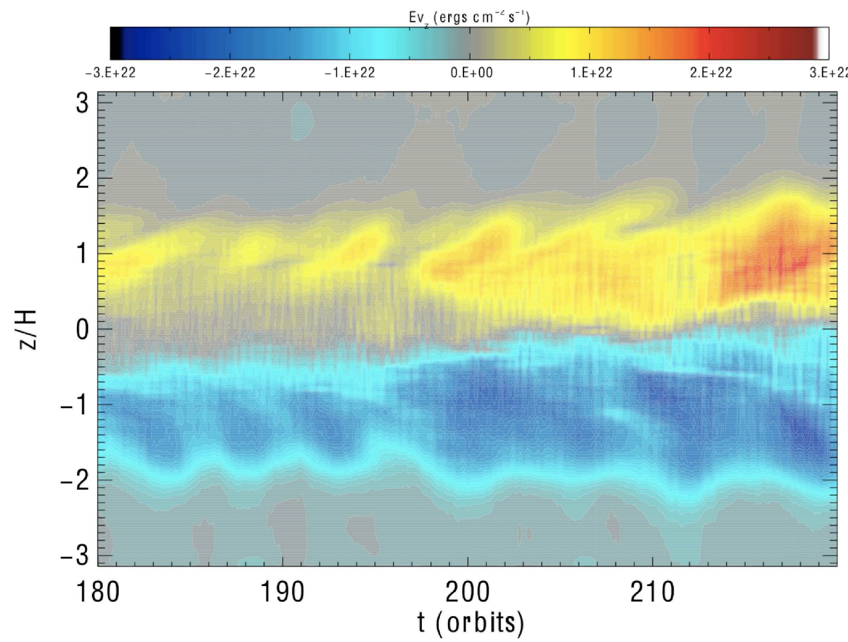


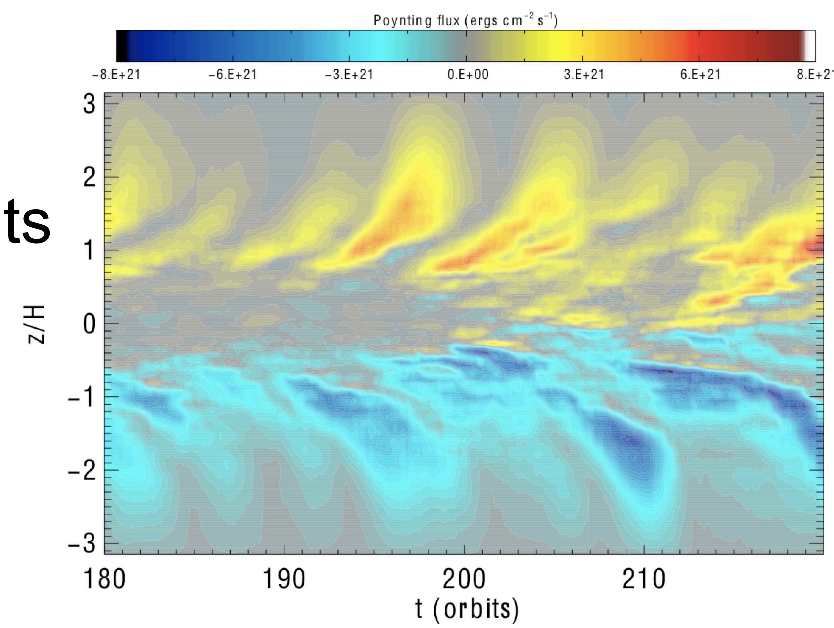
Alpha varies with height!



Vertical structure is very stably stratified, but nonlinear buoyant magnetic structures nevertheless advect radiation outward, transporting heat at a comparable rate to radiative diffusion.

Also, radiative damping of compressible fluctuations accounts for tens of percent of the total turbulent dissipation, and this is fully resolved numerically!





-Blaes et al. (2011)

Radiation-dominated regime is now also being simulated by Jiang, Stone, & Davis using the new radiation module of Athena (Jiang, Stone, & Davis 2012; Davis, Stone, & Jiang 2012)

$$\frac{\partial}{\partial t}(\rho\mathbf{v}) + \nabla\cdot(\rho\mathbf{v}\mathbf{v}) = -\nabla p + \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B} + \frac{\bar{\kappa}^{\mathrm{R}}\rho}{c}\mathbf{F} - \bar{\kappa}^{\mathrm{P}}\rho(aT^4 - E)\frac{\mathbf{v}}{c} \tag{1}$$

$$\frac{\partial e}{\partial t} + \nabla\cdot(e\mathbf{v}) = -p\nabla\cdot\mathbf{v} - (aT^4 - E)c\bar{\kappa}^{\mathrm{P}}\rho \tag{2}$$

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial t}\left(\frac{2\mathbf{v}}{c^2}\cdot\mathbf{F}\right) + \nabla\cdot(E\mathbf{v}) = -P_{ij}\nabla^i v^j + (aT^4 - E)c\bar{\kappa}^{\mathrm{P}}\rho - \nabla\cdot\mathbf{F} - v^i\nabla^j P_{ij} - \bar{\kappa}^{\mathrm{R}}\rho\frac{\mathbf{v}}{c}\cdot\mathbf{F} \tag{3}$$

$$\boxed{\frac{1}{c^2}\frac{\partial F^i}{\partial t} + \frac{1}{c^2}\frac{\partial}{\partial t}(v^i E + v_j P^{ij}) + \frac{1}{c^2}\nabla_j(v^j F^i + v^i F^j) = -\nabla_j P^{ij} - \frac{\bar{\kappa}^{\mathrm{R}}\rho}{c}F^i + \frac{v^i}{c}\bar{\kappa}^{\mathrm{P}}\rho(aT^4 - E) \qquad (4)}$$

Primary advance is to replace the flux-limited diffusion treatment in ZEUS with the full radiation momentum equation. Radiation pressure tensor is computed from a variable Eddington tensor computed from a full (grey) solution of the radiative transfer equation – but this is not yet working in the radiation and scattering dominated regime…

What do they find?

Ask me privately!

But what about the hydrogen ionization thermal instability???

Local Shearing Boxes with NO Vertical Structure (Latter & Papaloizou 2012)

- Fully incorporate MRI turbulence, with zero net flux, net vertical flux, and net toroidal flux.

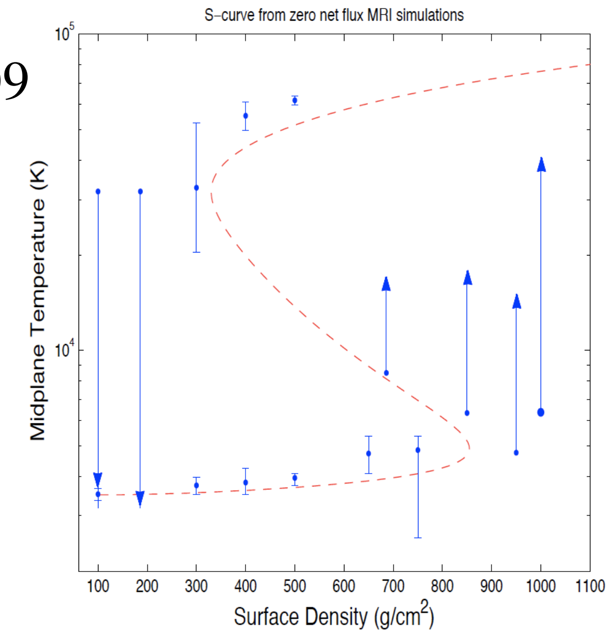
- Incorporate dissipation through fixed viscosity and resistivity, with $Pm=\nu/\eta=4$.

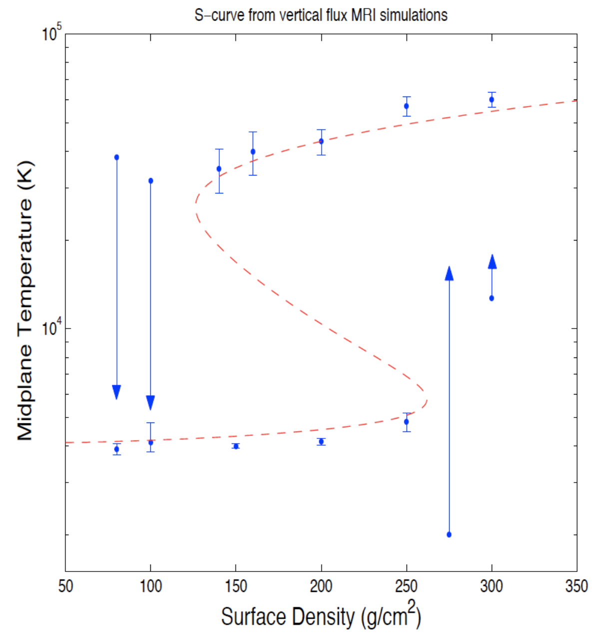
- Mock up cooling through effective cooling functions $\Lambda(T)$ based on prescriptions from Faulkner, Lin & Papaloizou (1983):

$$\Lambda = 2\sigma T_e^4 / H_0, \qquad T_e = \begin{cases} (4/3\tau_c)^{1/4}\, T_c, & \text{in Regime 1,} \\ \left(10^{36}\, E\rho_c^{-1/3}/\Sigma\right)^{1/10}, & \text{in Regime 2,} \\ (2\lambda\tau_c)^{1/4}\, T_c, & \text{in Regime 3,} \end{cases}$$

$$\tau_c = \kappa_c \Sigma \qquad \begin{aligned} \kappa &= 1.5 \times 10^{20} \rho T^{-2.5} && \text{(Regime 1)} \\ \kappa &= 10^{-36} \rho^{1/3} T^{10} && \text{(Regime 3)} \end{aligned}$$

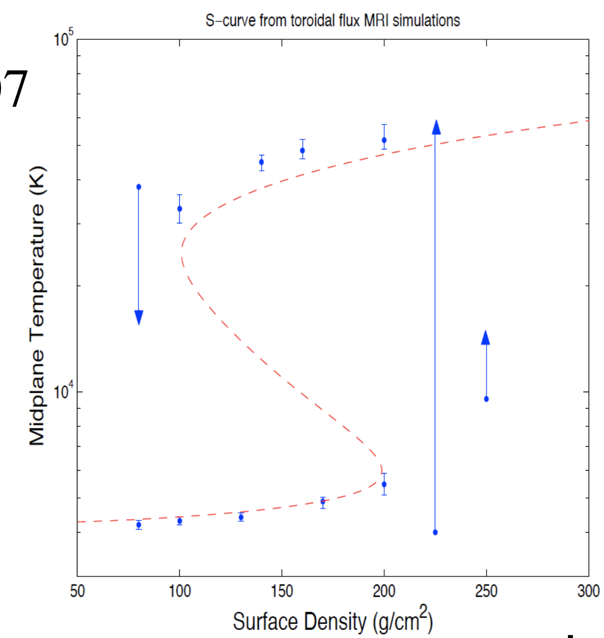
$\alpha_{ss} = 0.009$





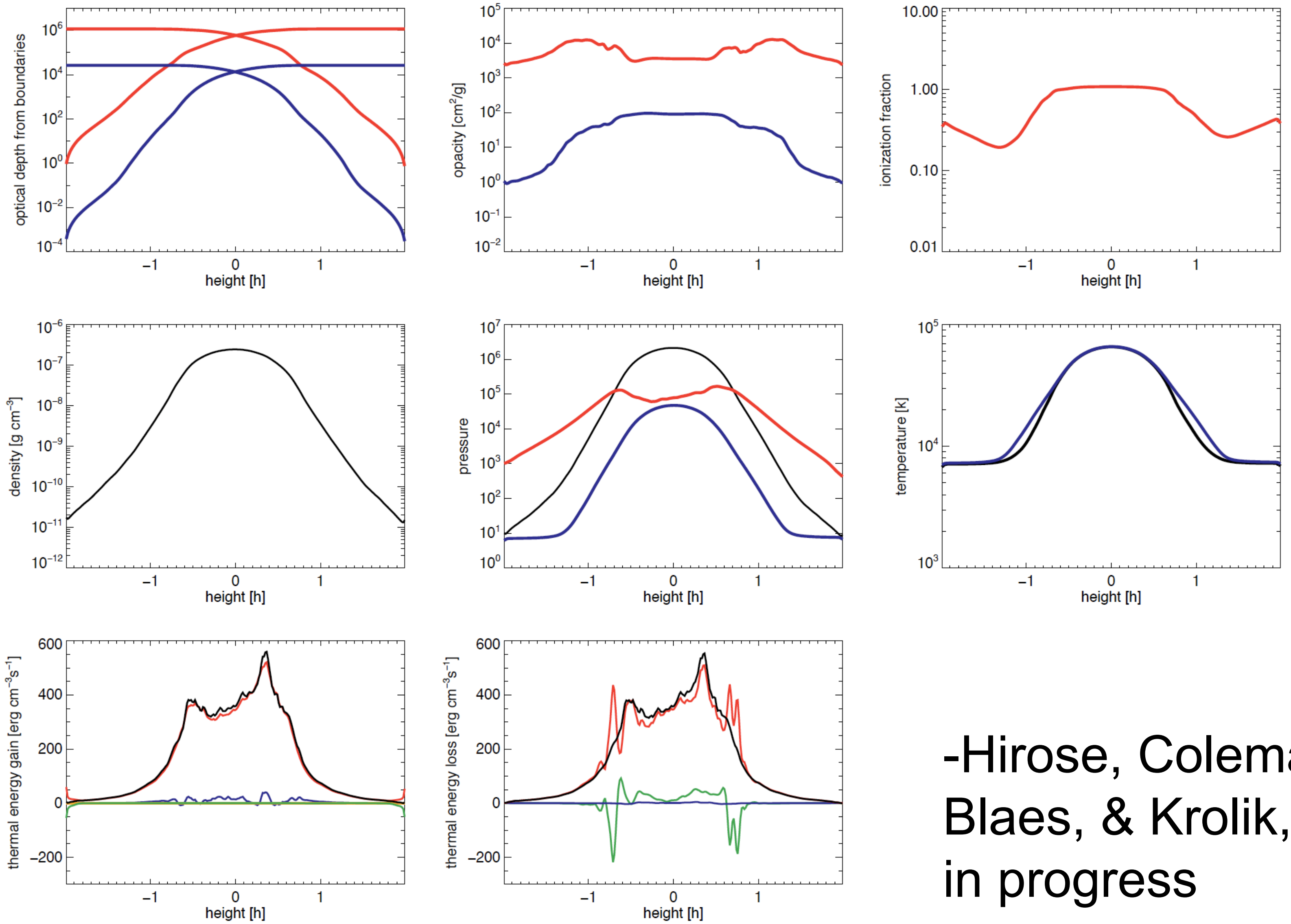
$\alpha_{ss} = 0.0475$

$\alpha_{ss} = 0.07$



-Latter & Papaloizou (2012)

A Vertically Stratified Shearing Box Simulation



-Hirose, Coleman, Blaes, & Krolik, in progress

Summary

- We still have a lot to learn about the nature and strength of angular momentum transport (never mind dissipation!) in MRI turbulence.

- Stress appears to scale with total thermal pressure in the radiation dominated regime, but why simulations appear to give a consistent value of alpha is a complete mystery.

- There is no evidence of a thermal instability in the radiation dominated regime. “Viscous” stability is still to be determined.

- Work is ongoing to incorporate the hydrogen ionization instability in thermodynamically consistent, vertically stratified simulations of MRI turbulence.