COSMIC MAGNETIC FIELDS

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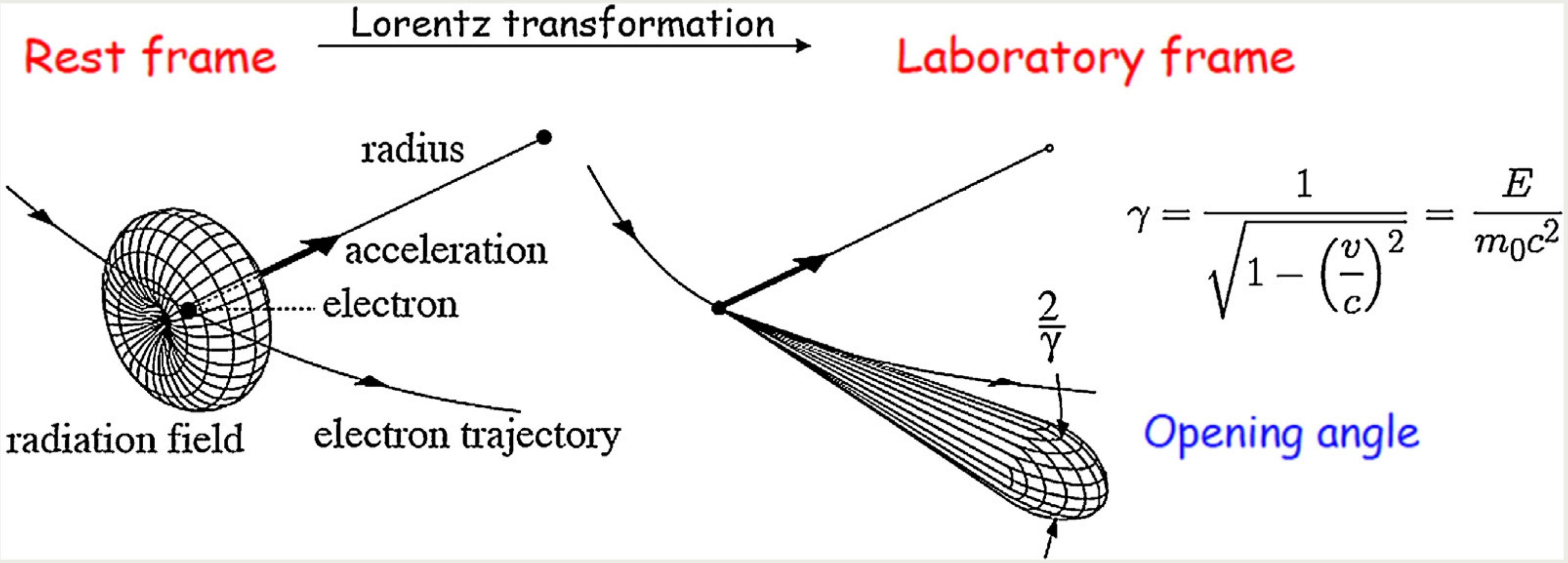
Radiation: synchrotron, curvature

ACCELERATION RADIATION

- Accelerating charged particles emit radiation.
- Magnetic field exerts Lorentz force on electrons, accelerating them and forcing to radiate energy.
- This may produce cyclotron (non-relativistic), synchrotron (relativistic, straight field lines), curvature (relativistic, curved field lines) or jitter (relativistic, chaotic field lines) radiation.
- The local magnetic field line and the line of sight define a unique coordinate system, which naturally leads to polarization.
- The magnetic vector of (high-frequency) synchrotron radiation is aligned with the source magnetic field.



Eberhardt (2015)

SYNCHROTRON RADIATION

radiation of accelerating charged particle

$$\boldsymbol{E}_{\rm rad} = \frac{q}{(1-\boldsymbol{n}\cdot\boldsymbol{\beta})^3 rc}\left\{\boldsymbol{n}\times\left[(\boldsymbol{n}-\boldsymbol{\beta})\times\frac{\partial\boldsymbol{\beta}}{\partial t}\right]\right\} \qquad \frac{\partial\boldsymbol{\beta}}{\partial t} = \frac{q}{\gamma m_{\rm e} c}(\boldsymbol{\beta}\times\boldsymbol{B}_0)$$

$$\boldsymbol{B}_{\rm rad} = \boldsymbol{n}\times\boldsymbol{E}_{\rm rad}$$

line of sight — strong beaming — particle velocity, acceleration — source magnetic field

particle velocity close to the line of sight

$$\boldsymbol{\beta} = \boldsymbol{n} - \boldsymbol{\epsilon}_\beta, \qquad \epsilon_\beta \ll 1$$

$$\boldsymbol{E}_{\rm rad} = \frac{q^2}{(\boldsymbol{n}\cdot\boldsymbol{\epsilon}_\beta)^3 \gamma m_{\rm e} c^2 r}\left\{\left[-(\boldsymbol{n}\cdot\boldsymbol{\epsilon}_\beta)+\epsilon_\beta^2\right](\boldsymbol{n}\times\boldsymbol{B}_0) - (\boldsymbol{B}_0\cdot\boldsymbol{\epsilon}_\beta)(\boldsymbol{n}\times\boldsymbol{\epsilon}_\beta)\right\}$$

linear term — quadratic term

integrated emissivity

$$j_\perp(\nu,\gamma) = \frac{\sqrt{3}\,e^3 B\sin\alpha}{2m_{\rm e}c^2}\left[F\left(\frac{\nu}{\nu_{\rm c}}\right)+G\left(\frac{\nu}{\nu_{\rm c}}\right)\right]$$

$$j_\parallel(\nu,\gamma) = \frac{\sqrt{3}\,e^3 B\sin\alpha}{2m_{\rm e}c^2}\left[F\left(\frac{\nu}{\nu_{\rm c}}\right)-G\left(\frac{\nu}{\nu_{\rm c}}\right)\right]$$

$$F(x) = x\int_x^\infty K_{5/3}(\xi)\,{\rm d}\xi$$

$$G(x) = xK_{2/3}(x)$$

$$\nu_{\rm c} = \frac{3e\gamma^2 B\sin\alpha}{4\pi m_{\rm e}c}$$

polarization of synchrotron radiation

$$\Pi(\nu) = \frac{G(\nu/\nu_{\rm c})}{F(\nu/\nu_{\rm c})}$$

Rybicki & Lightmann (1979)

TOTAL ENERGY LOSSES

- $j(\nu,\gamma,\alpha) = \frac{\sqrt{3}e^3 B\sin\alpha}{m_e c^2} F\left(\frac{\nu}{\nu_c}\right)$ spectral emissivity in all directions

  $\int_0^\infty dx\, F(x) = \frac{8\sqrt{3}}{27}\pi$ $\quad \alpha = \angle(\vec{v}, \vec{B})$ pitch angle

- $\nu_c = \frac{3}{2}\gamma^2 \nu_B \sin\alpha = \frac{3eB\sin\alpha}{4\pi m_e c}\gamma^2$ characteristic synchrotron frequency

  $\nu_B = \frac{eB}{2\pi m_e c}$ cyclotron frequency

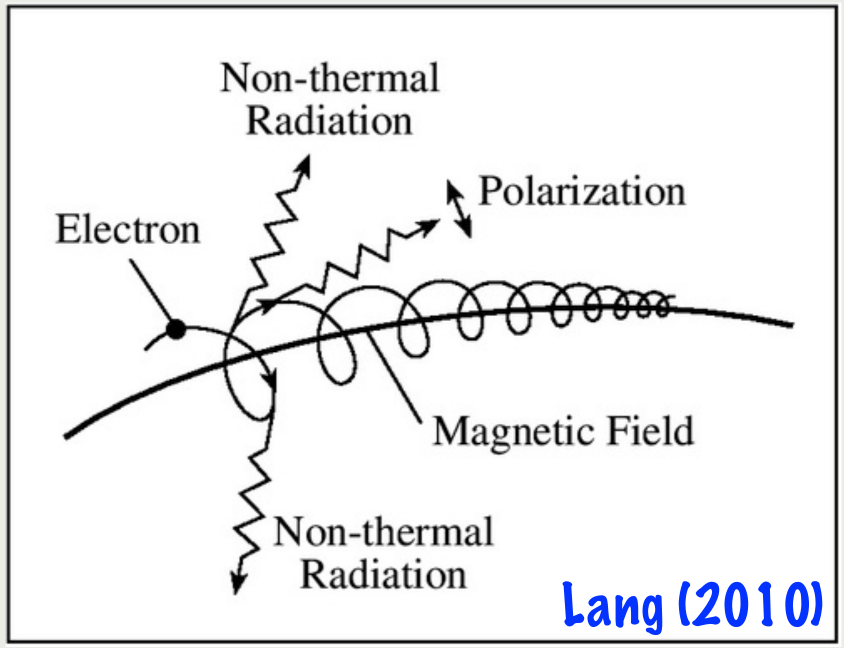
- $j(\gamma,\alpha) = \int d\nu\, j(\nu,\gamma,\alpha) = \frac{\sqrt{3}e^3 B\sin\alpha}{m_e c^2}\nu_c \int dx\, F(x) = 2c\sigma_T u_B \sin^2\alpha\,\gamma^2$ total energy losses

  $\sigma_T = \frac{8\pi}{3} r_e^2 = \frac{8\pi e^4}{3 m_e^2 c^4}$ Thomson cross section

- $\left\langle \sin^2\alpha \right\rangle_\alpha = \frac{2}{3}$, hence $j(\gamma) = \langle j(\gamma,\alpha)\rangle_\alpha = \frac{4}{3} c\sigma_T u_B \gamma^2$ pitch angle averaged energy losses

- $j = \langle j(\gamma)\rangle_\gamma = \frac{4}{3} c\sigma_T u_B \left\langle \gamma^2 \right\rangle$ particle energy averaged energy losses

POLARIZATION

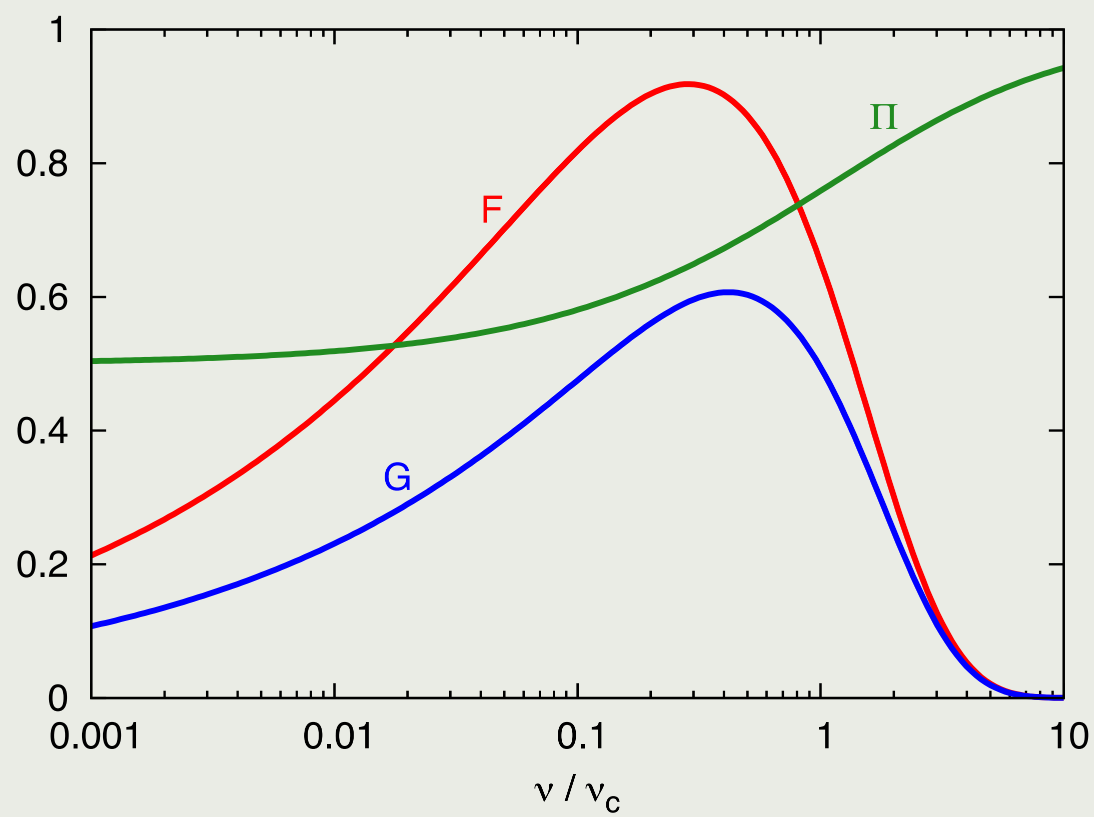


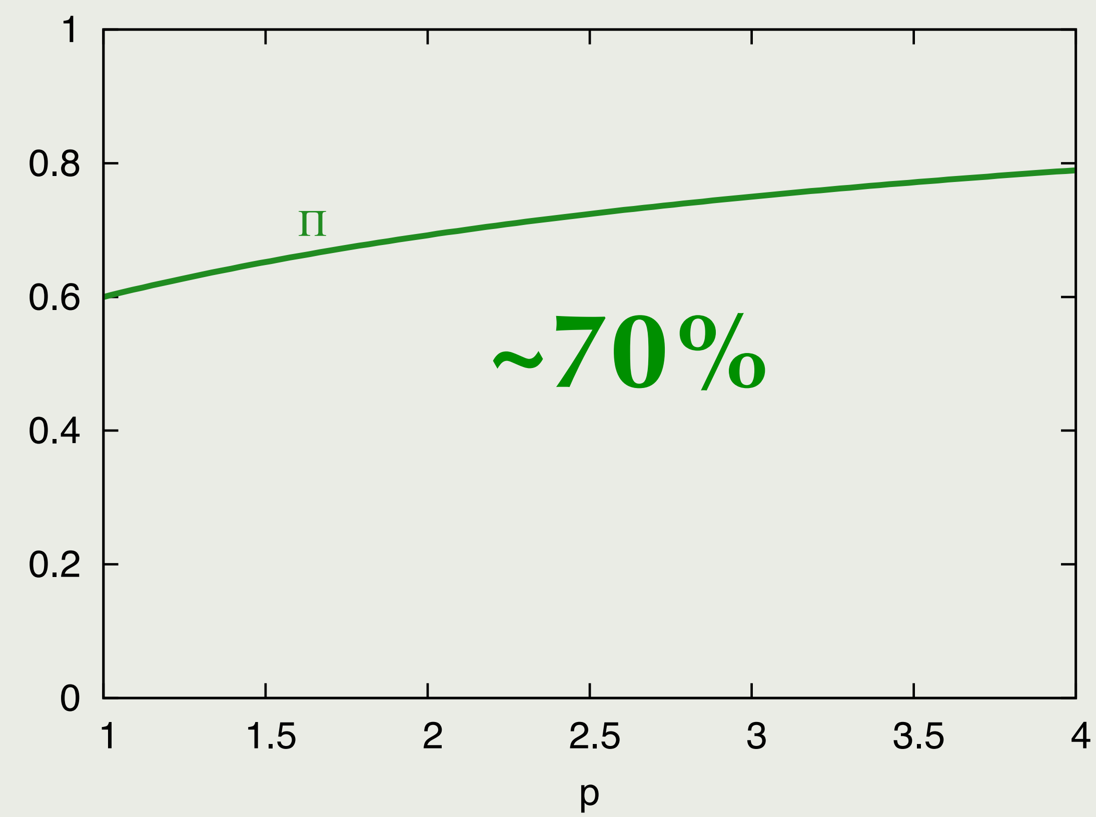
power-law energy distribution of particles

$$
\begin{aligned}
n(\gamma) &= K\gamma^{-p} \\
j_\perp(\nu) + j_\parallel(\nu) &= \Gamma\left(\frac{3p-1}{12}\right)\Gamma\left(\frac{3p+19}{12}\right)\frac{2\pi e^2 K}{\sqrt{3}\,c}\left(\frac{3eB\sin\alpha}{2\pi m_e c}\right)^{(p+1)/2}\frac{\nu^{-(p-1)/2}}{(p+1)} \\
j_\perp(\nu) - j_\parallel(\nu) &= \Gamma\left(\frac{3p-1}{12}\right)\Gamma\left(\frac{3p+7}{12}\right)\frac{\pi e^2 K}{2\sqrt{3}\,c}\left(\frac{3eB\sin\alpha}{2\pi m_e c}\right)^{(p+1)/2}\nu^{-(p-1)/2}
\end{aligned}
$$

polarization degree

$$
\Pi = \frac{p+1}{p+7/3}
$$





CHAOTIC MAGNETIC FIELDS

polarization degree

spectral index

normal vector

1/compression

observer

$$\Pi = \frac{\alpha + 1}{\alpha + 5/3} \times \frac{(1-\kappa^2)\left[1-(\boldsymbol{k}'\cdot\boldsymbol{n}')^2\right]}{2-(1-\kappa^2)\left[1-(\boldsymbol{k}'\cdot\boldsymbol{n}')^2\right]},$$

Hughes, Aller & Aller (1985)

polarization angle

electric vector parallel to the plane normal

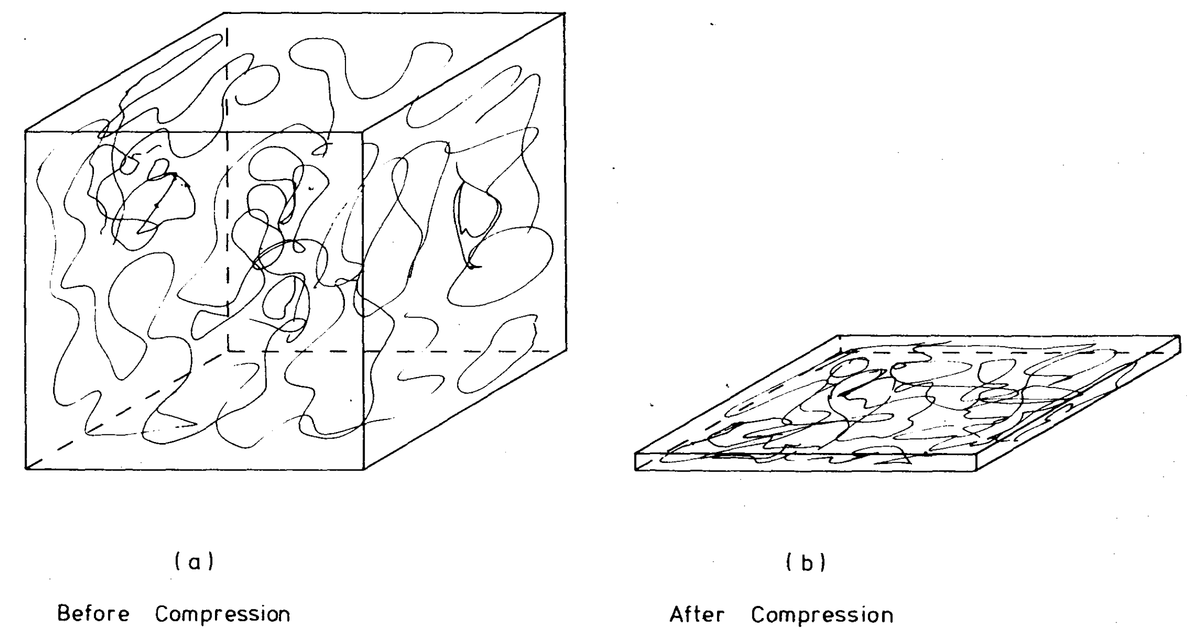
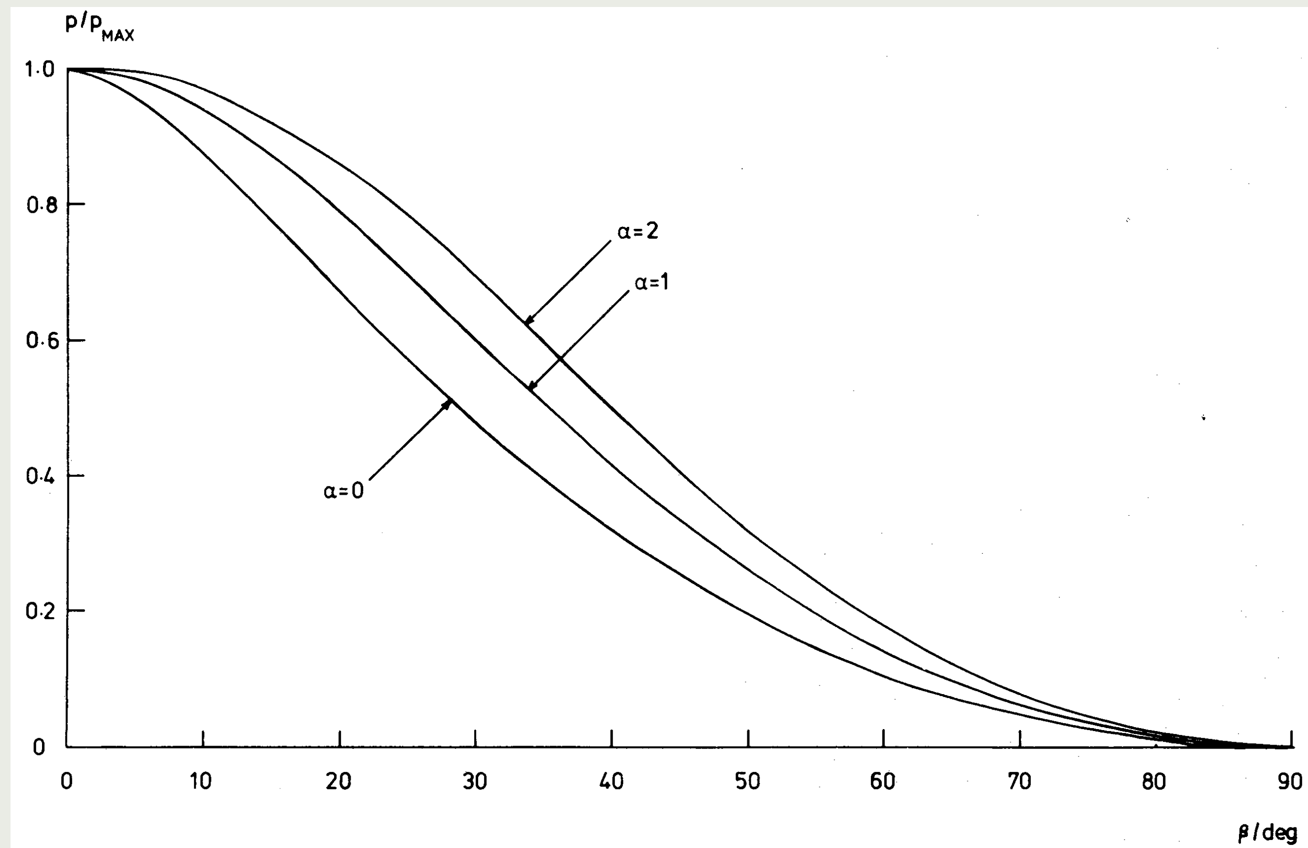


Figure 1. The compression of a cube containing an initially random magnetic field.



Laing (1980)

CURVATURE RADIATION

- Characteristic frequency: $\nu_{\rm curv} = \frac{3}{4\pi}\gamma^3 \frac{c}{R_{\rm curv}}$

  where R_c is the curvature radius of the magnetic field line
  (Ruderman & Sutherland 1975)

- Emissivity: $j(\nu,\gamma) = \frac{\sqrt{3}e^2}{R_{\rm curv}}\gamma F\left(\frac{\nu}{\nu_c}\right)$

  Total energy losses: $\dot{\gamma}_{\rm curv} = -\frac{2}{3}\frac{c r_e}{R_{\rm curv}^2}\gamma^4$

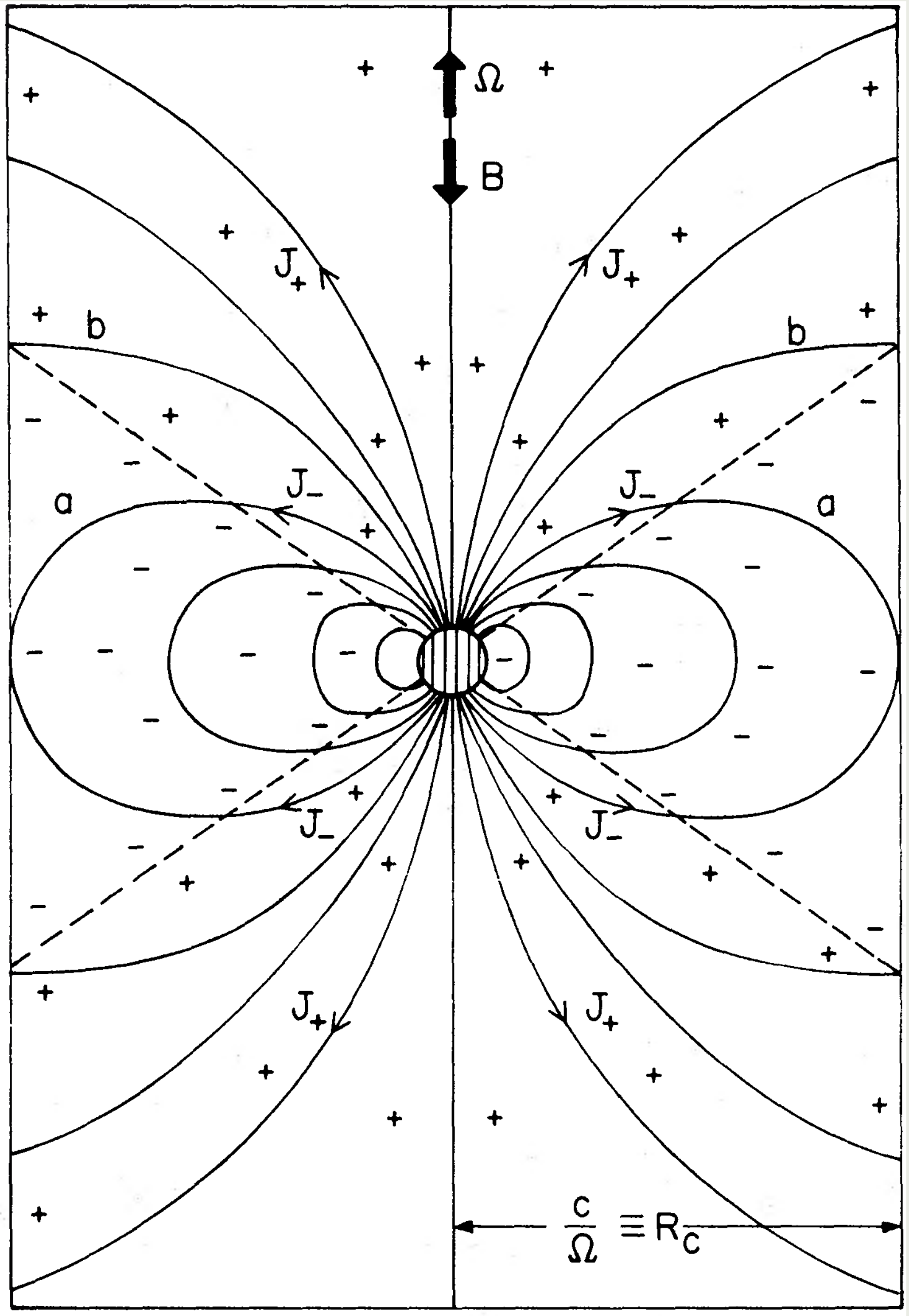
  where $r_e = \frac{e^2}{m_e c^2}$ is the classical electron radius
  (Harding & Muslimov 2001)

- Compare that with the synchrotron radiation:

  $\nu_c = \frac{3}{2}\gamma^2 \nu_B \sin\alpha = \frac{3}{4\pi}\gamma^3 \frac{c}{R_L}\sin^2\alpha$

  and $\dot{\gamma}_{\rm syn} = -\frac{4}{3}\frac{\sigma_T}{m_e c}u_B \gamma^2$

  where $R_L = \frac{\gamma m_e c^2}{eB}\sin\alpha = \frac{c}{2\pi\nu_B}\gamma\sin\alpha$ is the Larmor radius.



SUMMARY

- Magnetic fields induce non-thermal radiation from energetic charged particles by accelerating them.
- Synchrotron radiation is produced by relativistic particles in uniform magnetic field, its key signature is strong linear polarization.
- Curvature radiation is produced by relativistic particles propagating along curved magnetic field lines, it is particularly important for pulsars.