COSMIC MAGNETIC FIELDS

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Rayleigh-Taylor instability (MHD)

MHD EQUILIBRIUM UNDER GRAVITY

- Consider the problem of a static background $\vec{v}_0 = 0$ under uniform gravitational acceleration $\vec{g} = -g\hat{z}$.

- We add background magnetic field $\vec{B}_0 = B_0(z)\hat{x}$.
  This implies a background current density $\vec{j}_0 = \frac{c}{4\pi}B_0'\hat{y}$
  and a background Lorentz force density $\vec{f}_{\mathrm{L},0} = \frac{1}{c}\left(\vec{j}_0 \times \vec{B}_0\right) = -\frac{B_0 B_0'}{4\pi}\hat{z} = -\frac{(B_0^2)'}{8\pi}\hat{z}$.

- Magnetohydrostatic equilibrium: $-\vec{\nabla}P_0 + \rho_0\vec{g} + \vec{f}_{\mathrm{L},0} = 0$
  or $\frac{\mathrm{d}}{\mathrm{d}z}\left(P_0 + \frac{B_0^2}{8\pi}\right) = -g\rho_0$.

- One can assume independent profiles of $\rho_0(z)$, $B_0(z)$ and compute $P_0(z)$. We choose exponential profiles such that $\rho_0' = \rho_0/\lambda_\rho$ and $B_0' = B_0/\lambda_B$.

PERTURBED MAGNETIC FIELD

- $$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{\mathrm{v}} \times \vec{B} \right) = \left(\vec{B} \cdot \vec{\nabla} \right) \vec{\mathrm{v}} - \left(\vec{\mathrm{v}} \cdot \vec{\nabla} \right) \vec{B} - \vec{B} \left(\vec{\nabla} \cdot \vec{\mathrm{v}} \right)$$

- $$\frac{\partial \vec{B}_1}{\partial t} = \left(\vec{B}_0 \cdot \vec{\nabla} \right) \vec{\mathrm{v}}_1 - \left(\vec{\mathrm{v}}_1 \cdot \vec{\nabla} \right) \vec{B}_0 - \vec{B}_0 \left(\vec{\nabla} \cdot \vec{\mathrm{v}}_1 \right)$$

- $$i\omega \vec{B}_1 = ik_x B_0 \vec{\mathrm{v}}_1 - \vec{B}'_0 \mathrm{v}_{1,z} - \vec{B}_0 \left(i\vec{k} \cdot \vec{\mathrm{v}}_1 \right)$$

- $$i\omega B_{1,x} = -ik_y B_0 \mathrm{v}_{1,y} - (B'_0 + ik_z B_0)\mathrm{v}_{1,z}$$
  $$i\omega B_{1,y} = ik_x B_0 \mathrm{v}_{1,y}$$
  $$i\omega B_{1,z} = ik_x B_0 \mathrm{v}_{1,z}$$

PERTURBED CURRENT DENSITY

- $i\omega B_{1,x} = -ik_y B_0 \mathrm{v}_{1,y} - \left(\frac{1}{\lambda_B} + ik_z\right) B_0 \mathrm{v}_{1,z}$

  $i\omega B_{1,y} = ik_x B_0 \mathrm{v}_{1,y}$

  $i\omega B_{1,z} = ik_x B_0 \mathrm{v}_{1,z}$

- $j_{1,x} = \frac{c}{4\pi}(ik_y B_{1,z} - B'_{1,y})$

  $j_{1,y} = \frac{c}{4\pi}(B'_{1,x} - ik_x B_{1,z})$

  $j_{1,z} = \frac{ic}{4\pi}(k_x B_{1,y} - k_y B_{1,x})$

- $j_{1,x} = \frac{ck_x B_0}{4\pi\omega}\left(-\frac{\mathrm{v}_{1,y}}{\lambda_B} - \mathrm{v}'_{1,y} + ik_y \mathrm{v}_{1,z}\right)$

  $j_{1,y} = \frac{cB_0}{4\pi\omega}\left[-\frac{k_y}{\lambda_B}\mathrm{v}_{1,y} - k_y \mathrm{v}'_{1,y} + \left(\frac{i}{\lambda_B^2} - \frac{2k_z}{\lambda_B} - ik_{xz}^2\right)\mathrm{v}_{1,z}\right]$ where $k_{xz}^2 \equiv k_x^2 + k_z^2$

  $j_{1,z} = \frac{cB_0}{4\pi\omega}\left[ik_{xy}^2 \mathrm{v}_{1,y} + k_y\left(\frac{1}{\lambda_B} + ik_z\right)\mathrm{v}_{1,z}\right]$ where $k_{xy}^2 \equiv k_x^2 + k_y^2$

PERTURBED LORENTZ FORCE DENSITY

- $$j_{1,x} = \frac{ck_x B_0}{4\pi\omega}\left(-\frac{\mathrm{v}_{1,y}}{\lambda_B} - \mathrm{v}'_{1,y} + ik_y \mathrm{v}_{1,z}\right)$$
$$j_{1,y} = \frac{cB_0}{4\pi\omega}\left[-\frac{k_y}{\lambda_B}\mathrm{v}_{1,y} - k_y \mathrm{v}'_{1,y} + \left(\frac{i}{\lambda_B^2} - \frac{2k_z}{\lambda_B} - ik_{xz}^2\right)\mathrm{v}_{1,z}\right]$$
$$j_{1,z} = \frac{cB_0}{4\pi\omega}\left[ik_{xy}^2 \mathrm{v}_{1,y} + k_y\left(\frac{1}{\lambda_B} + ik_z\right)\mathrm{v}_{1,z}\right]$$

- $$f_{\mathrm{L,1,x}} = \frac{j_{0,y}B_{1,z}}{c} = \frac{k_x}{\lambda_B}\frac{B_0^2}{4\pi\omega}\mathrm{v}_{1,z}$$
$$f_{\mathrm{L,1,y}} = \frac{j_{1,z}B_{0,x}}{c} = ik_{xy}^2\frac{B_0^2}{4\pi\omega}\mathrm{v}_{1,y} + k_y\left(\frac{1}{\lambda_B} + ik_z\right)\frac{B_0^2}{4\pi\omega}\mathrm{v}_{1,z}$$
$$f_{\mathrm{L,1,z}} = -\frac{j_{0,y}B_{1,x}}{c} - \frac{j_{1,y}B_{0,x}}{c} = k_y\frac{B_0^2}{4\pi\omega}\left(\frac{2}{\lambda_B}\mathrm{v}_{1,y} + \mathrm{v}'_{1,y}\right) + \left(-\frac{2i}{\lambda_B^2} + \frac{3k_z}{\lambda_B} + ik_{xz}^2\right)\frac{B_0^2}{4\pi\omega}\mathrm{v}_{1,z}$$

LINEARIZED EULER EQUATION

- $$f_{\mathrm{L},1,\mathrm{x}} = \frac{j_{0,y}B_{1,z}}{c} = \frac{k_x}{\lambda_B}\frac{B_0^2}{4\pi\omega}\mathrm{v}_{1,z}$$
  $$f_{\mathrm{L},1,\mathrm{y}} = \frac{j_{1,z}B_{0,x}}{c} = ik_{xy}^2\frac{B_0^2}{4\pi\omega}\mathrm{v}_{1,y} + k_y\left(\frac{1}{\lambda_B} + ik_z\right)\frac{B_0^2}{4\pi\omega}\mathrm{v}_{1,z}$$
  $$f_{\mathrm{L},1,\mathrm{z}} = -\frac{j_{0,y}B_{1,x}}{c} - \frac{j_{1,y}B_{0,x}}{c} = k_y\frac{B_0^2}{4\pi\omega}\left(\frac{2}{\lambda_B}\mathrm{v}_{1,y} + \mathrm{v}'_{1,y}\right) + \left(-\frac{2i}{\lambda_B^2} + \frac{3k_z}{\lambda_B} + ik_{xz}^2\right)\frac{B_0^2}{4\pi\omega}\mathrm{v}_{1,z}$$

- Euler equation, introducing Alfvén velocity $\mathrm{v}_{\mathrm{A},0}^2 = \dfrac{B_0^2}{4\pi\rho_0}$:

  $$\omega^2\mathrm{v}_{1,x} = -k_x\omega\frac{P_1}{\rho_0} - ik_x\frac{\mathrm{v}_{\mathrm{A},0}^2}{\lambda_B}\mathrm{v}_{1,z}$$
  $$\omega^2\mathrm{v}_{1,y} = -k_y\omega\frac{P_1}{\rho_0} + k_{xy}^2\mathrm{v}_{\mathrm{A},0}^2\mathrm{v}_{1,y} + k_y\left(k_z - \frac{i}{\lambda_B}\right)\mathrm{v}_{\mathrm{A},0}^2\mathrm{v}_{1,z}$$
  $$\omega^2\mathrm{v}_{1,z} = i\omega\frac{P'_1}{\rho_0} + i\omega g\frac{\rho_1}{\rho_0} - ik_y\mathrm{v}_{\mathrm{A},0}^2\left(\frac{2}{\lambda_B}\mathrm{v}_{1,y} + \mathrm{v}'_{1,y}\right) + \left(k_{xz}^2 - \frac{3ik_z}{\lambda_B} - \frac{2}{\lambda_B^2}\right)\mathrm{v}_{\mathrm{A},0}^2\mathrm{v}_{1,z}$$

COMPLETE LINEARIZED EQUATIONS

- Continuity: $\omega \frac{\rho_1}{\rho_0} = -\left(\vec{k} \cdot \vec{\mathrm{v}}_1\right) + \frac{i\mathrm{v}_{1,z}}{\lambda_\rho}$

- Pressure: $i\omega P_1 + \mathrm{v}_{1,z} P_0' + \kappa P_0 \left(i \vec{k} \cdot \vec{\mathrm{v}}_1\right) = 0$

  $$\omega \frac{P_1}{\rho_0} = -\mathrm{v}_{\mathrm{s},0}^2 \left(\vec{k} \cdot \vec{\mathrm{v}}_1\right) - \left(g + \frac{\mathrm{v}_{\mathrm{A},0}^2}{\lambda_B}\right) i\mathrm{v}_{1,z}$$

- Euler equations:

  $$\omega^2 \mathrm{v}_{1,x} = -k_x \omega \frac{P_1}{\rho_0} - ik_x \frac{\mathrm{v}_{\mathrm{A},0}^2}{\lambda_B} \mathrm{v}_{1,z}$$

  $$\omega^2 \mathrm{v}_{1,y} = -k_y \omega \frac{P_1}{\rho_0} + k_{xy}^2 \mathrm{v}_{\mathrm{A},0}^2 \mathrm{v}_{1,y} + k_y \left(k_z - \frac{i}{\lambda_B}\right) \mathrm{v}_{\mathrm{A},0}^2 \mathrm{v}_{1,z}$$

  $$\omega^2 \mathrm{v}_{1,z} = i\omega \frac{P_1'}{\rho_0} + i\omega g \frac{\rho_1}{\rho_0} - ik_y \mathrm{v}_{\mathrm{A},0}^2 \left(\frac{2}{\lambda_B} \mathrm{v}_{1,y} + \mathrm{v}_{1,y}'\right) + \left(k_{xz}^2 - \frac{3ik_z}{\lambda_B} - \frac{2}{\lambda_B^2}\right) \mathrm{v}_{\mathrm{A},0}^2 \mathrm{v}_{1,z}$$

INTERCHANGE MODE

- Let $k_x = 0$ and $k_y \neq 0$ (transverse mode with $\overrightarrow{k} \perp \overrightarrow{B}_0$).

- The x component of Euler equations implies that $\mathrm{v}_{1,x} = 0$.

- $$\omega \frac{\rho_1}{\rho_0} = -\left(\overrightarrow{k} \cdot \overrightarrow{\mathrm{v}}_1\right) + \frac{i\mathrm{v}_{1,z}}{\lambda_\rho}$$
  $$\omega \frac{P_1}{\rho_0} = -\mathrm{v}_{\mathrm{s},0}^2 \left(\overrightarrow{k} \cdot \overrightarrow{\mathrm{v}}_1\right) - \left(g + \frac{\mathrm{v}_{\mathrm{A},0}^2}{\lambda_B}\right) i\mathrm{v}_{1,z}$$
  $$\omega^2 \mathrm{v}_{1,y} = -k_y \omega \frac{P_1}{\rho_0} + k_y^2 \mathrm{v}_{\mathrm{A},0}^2 \mathrm{v}_{1,y} + k_y \left(k_z - \frac{i}{\lambda_B}\right) \mathrm{v}_{\mathrm{A},0}^2 \mathrm{v}_{1,z}$$
  $$\omega^2 \mathrm{v}_{1,z} = i\omega \frac{P_1'}{\rho_0} + i\omega g \frac{\rho_1}{\rho_0} - \frac{2ik_y}{\lambda_B} \mathrm{v}_{\mathrm{A},0}^2 \mathrm{v}_{1,y} - ik_y \mathrm{v}_{\mathrm{A},0}^2 \mathrm{v}_{1,y}' + \left(k_z^2 - \frac{3ik_z}{\lambda_B} - \frac{2}{\lambda_B^2}\right) \mathrm{v}_{\mathrm{A},0}^2 \mathrm{v}_{1,z}$$

- Lengthy calculations in the $k_y^2 v_{\mathrm{FM},0}^2 \gg \omega^2$ limit result in the dispersion relation:
  $$\left(1 + \frac{k_z^2}{k_y^2} \frac{\mathrm{v}_{\mathrm{s},0}^2}{\mathrm{v}_{\mathrm{FM},0}^2}\right) \omega^2 \simeq -\frac{g}{\lambda_\rho} - \frac{g^2}{\mathrm{v}_{\mathrm{FM},0}^2}$$

INTERCHANGE MODE

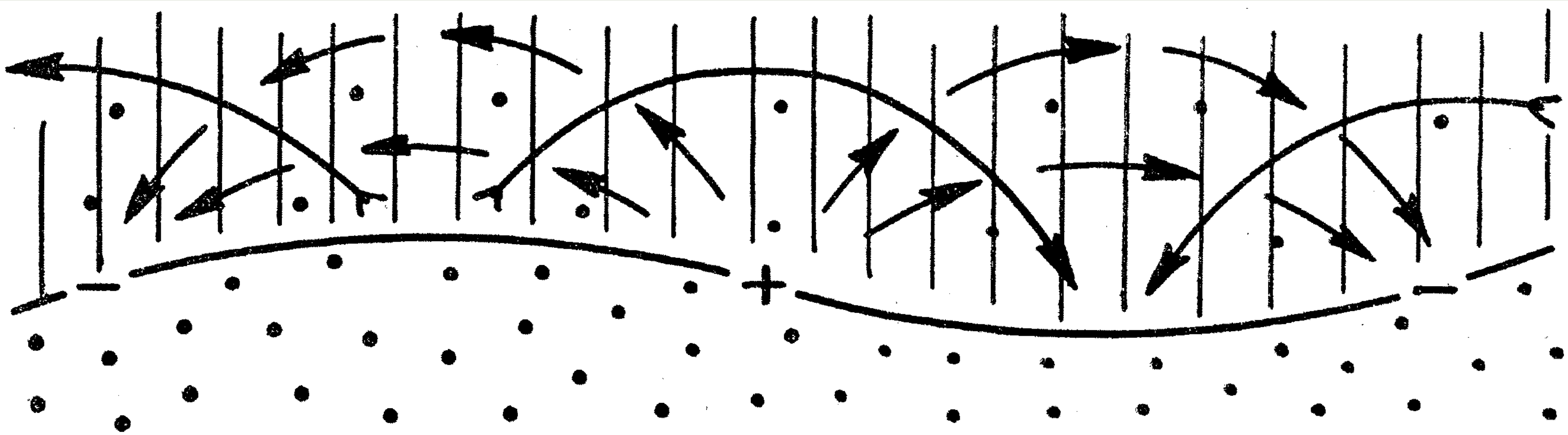


FIGURE 1. Instability of plasma supported against gravity by a magnetic field. ||| plasma, ∴ magnetic field; +, − electric charge; →→ electric field; ≻→ motion of plasma.

Kruskal & Schwarzschild (1954)

INTERCHANGE MODE

- Dispersion relation for the interchange mode:

$$\left(1+\frac{k_z^2}{k_y^2}\frac{\mathrm{v}_{\mathrm{s},0}^2}{\mathrm{v}_{\mathrm{FM},0}^2}\right)\omega^2 \simeq -\frac{g}{\lambda_\rho}-\frac{g^2}{\mathrm{v}_{\mathrm{FM},0}^2}$$

- In the hydro limit $\mathrm{v}_{\mathrm{FM},0}=\mathrm{v}_{\mathrm{s},0}$:

$$\left(1+\frac{k_z^2}{k_{xy}^2}\right)\omega^2 \simeq -\frac{g}{\lambda_\rho}-\frac{g^2}{\mathrm{v}_{\mathrm{s},0}^2}$$

- Suggests an additional feedback loop.

PHYSICAL PRINCIPLE

- Consider the short-wavelength limit $k_y \gg 1/|\lambda_\rho|$ with $k_x = k_z = 0$. Since λ_B does not contribute to the dispersion relation, consider $B_0' = B_0/\lambda_B = 0$.

- Recall the basic equations:

$$i\omega \frac{B_{1,x}}{B_0} = -ik_y \mathrm{v}_{1,y}$$

$$i\omega \frac{\rho_1}{\rho_0} = -ik_y \mathrm{v}_{1,y} - \frac{\mathrm{v}_{1,z}}{\lambda_\rho}$$

$$i\omega \frac{P_1}{\rho_0} = -ik_y \mathrm{v}_{\mathrm{s},0}^2 \mathrm{v}_{1,y} + g\mathrm{v}_{1,z}$$

$$i\omega \mathrm{v}_{1,y} = -ik_y \frac{P_1}{\rho_0} - ik_y \mathrm{v}_{\mathrm{A},0}^2 \frac{B_{1,x}}{B_0}$$

$$i\omega \mathrm{v}_{1,z} = -\frac{P_1'}{\rho_0} - g\frac{\rho_1}{\rho_0}$$

Approximations:

$$i\omega \frac{P_1'}{\rho_0} \simeq \left(\frac{g}{\lambda_\rho} + \frac{\kappa g^2}{\mathrm{v}_{\mathrm{FM},0}^2}\right)\frac{\mathrm{v}_{\mathrm{A},0}^2}{\mathrm{v}_{\mathrm{FM},0}^2}\mathrm{v}_{1,z}$$

$$\mathrm{v}_{\mathrm{FM},0}^2 ik_y \mathrm{v}_{1,y} \simeq g\mathrm{v}_{1,z}$$

$$i\omega \mathrm{v}_{1,z} \simeq -g\frac{\rho_1}{\rho_0}$$

$$i\omega \frac{\rho_1}{\rho_0} = -ik_x \mathrm{v}_{1,x} - \frac{\mathrm{v}_{1,z}}{\lambda_\rho}$$

$$\mathrm{v}_{\mathrm{s},0}^2\, ik_x \mathrm{v}_{1,x} \simeq g\mathrm{v}_{1,z}$$

$$i\omega \mathrm{v}_{1,z} \simeq -g\frac{\rho_1}{\rho_0}$$

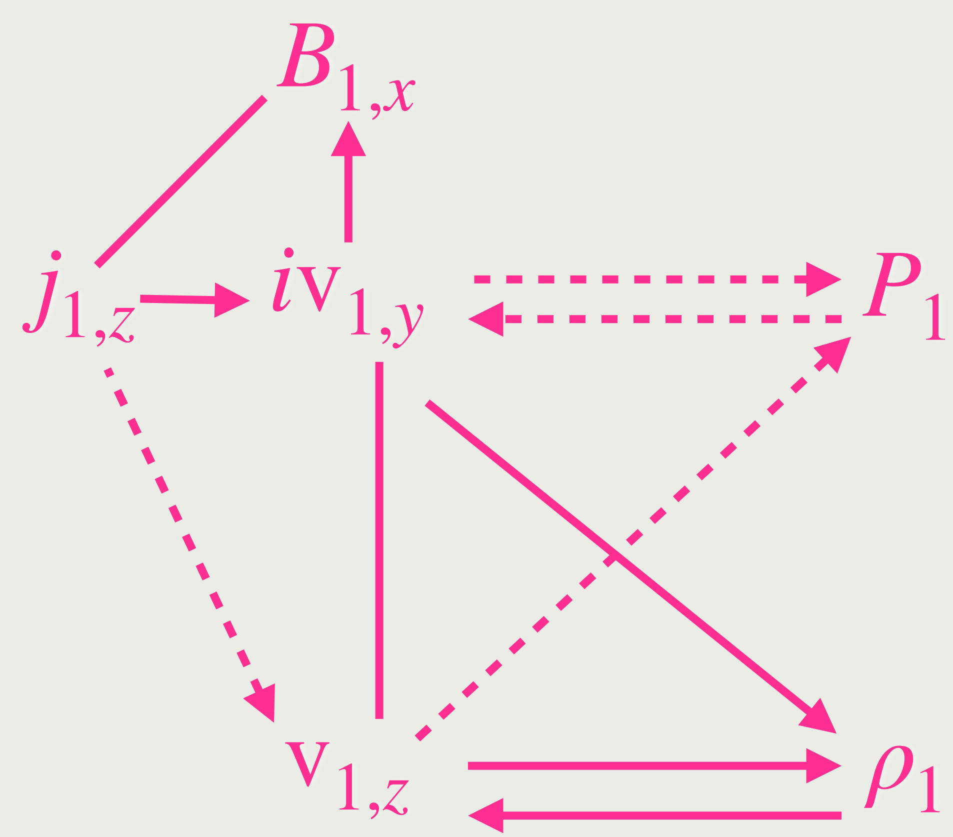
PHYSICAL PRINCIPLE

$$i\omega\frac{\rho_1}{\rho_0} = -ik_y \mathrm{v}_{1,y} - \frac{\mathrm{v}_{1,z}}{\lambda_\rho}$$

$$\mathrm{v}_{\mathrm{FM},0}^2\, ik_y \mathrm{v}_{1,y} \simeq g \mathrm{v}_{1,z}$$

$$i\omega \mathrm{v}_{1,z} \simeq -g\frac{\rho_1}{\rho_0}$$

- Fast-magnetosonic loop:
  $\mathrm{v}_{1,z} \rightarrow i\mathrm{v}_{1,y} \rightarrow \rho_1 \rightarrow \mathrm{v}_{1,z}$
  $v_{1,\mathrm{z}} > 0$ triggers a horizontal fast magnetosonic wave with $i\mathrm{v}_{1,y} > 0$, which triggers $\rho_1 < 0$. Perturbed gravitational force points upwards, increasing $\mathrm{v}_{1,z}$.



PARKER MODE

- Let $k_y = 0$ and $k_x \neq 0$ (longitudinal mode with $\overrightarrow{k} \cdot \overrightarrow{B}_0 \neq 0$).
- The y component of Euler equations implies that $\mathrm{v}_{1,y} = 0$.
- $$\frac{\mathrm{v}_{1,x}}{\mathrm{v}_{1,z}} = -k_x \frac{k_z \mathrm{v}_{\mathrm{s},0}^2 + ig}{k_x^2 \mathrm{v}_{\mathrm{s},0}^2 - \omega^2} \quad \text{(x component of Euler)}$$
  $$\frac{i\omega\rho_1}{\rho_0 \mathrm{v}_{1,z}} = -\frac{1}{\lambda_\rho} - \frac{k_x^2 g - ik_z\omega^2}{k_x^2 \mathrm{v}_{\mathrm{s},0}^2 - \omega^2} \quad \text{(continuity)}$$
  $$\frac{i\omega P_1}{\rho_0 \mathrm{v}_{1,z}} = g + \frac{\mathrm{v}_{\mathrm{A},0}^2}{\lambda_B} - \mathrm{v}_{\mathrm{s},0}^2 \frac{k_x^2 g - ik_z\omega^2}{k_x^2 \mathrm{v}_{\mathrm{s},0}^2 - \omega^2} \quad \text{(pressure)}$$
  $$\frac{i\omega P_1'}{\rho_0 \mathrm{v}_{1,z}} = \left(\frac{1}{\lambda_\rho} + ik_z\right) g + \left(\frac{2}{\lambda_B} + ik_z\right) \frac{\mathrm{v}_{\mathrm{A},0}^2}{\lambda_B} + \frac{k_x^2 g - ik_z\omega^2}{k_x^2 \mathrm{v}_{\mathrm{s},0}^2 - \omega^2} \left[-\left(\frac{1}{\lambda_\rho} + ik_z\right) \mathrm{v}_{\mathrm{s},0}^2 + \frac{\omega^2 (\mathrm{v}_{\mathrm{s},0}^2)'}{k_x^2 \mathrm{v}_{\mathrm{s},0}^2 - \omega^2} \right]$$
  $$-\omega^2 = -\frac{i\omega P_1'}{\rho_0 \mathrm{v}_{1,z}} - g\frac{i\omega\rho_1}{\rho_0 \mathrm{v}_{1,z}} + \left(\frac{2}{\lambda_B^2} + \frac{3ik_z}{\lambda_B} - k_{xz}^2\right) \mathrm{v}_{\mathrm{A},0}^2 \quad \text{(z component of Euler)}$$

PARKER MODE

- short-wavelength limit $k_x \gg 1/|\lambda_\rho|$:

$$\left(1+\frac{k_z^2}{k_x^2}\right)\omega^2 \simeq -\frac{g^2}{\mathrm{v}_{\mathrm{s},0}^2}-\frac{g}{\lambda_\rho}+k_{xz}^2\mathrm{v}_{\mathrm{A},0}^2$$

- the RHS is dominated by the last term, which is stabilizing. The $k_x^2\mathrm{v}_{\mathrm{A},0}^2$ term can be traced to the $-j_{1,y}B_{0,x}$ term of the $f_{\mathrm{L},1,\mathrm{z}}$ Lorentz force density, where $j_{1,y}$ includes the tension term $-ik_x(c/4\pi)B_{1,z}$, and $B_{1,z}=(k_x/\omega)B_{0,x}\mathrm{v}_{1,z}$.

PARKER MODE

- Following Parker (1966), consider isothermal limit $(\mathrm{v}_{\mathrm{s},0}^2)' = 0$ and $(\mathrm{v}_{\mathrm{A},0}^2)' = 0$, hence $\lambda_B = 2\lambda_\rho$.

$$\omega^2 \mathrm{v}_{1,z} = -\frac{k_x^2 g}{k_x^2 \mathrm{v}_{\mathrm{s},0}^2 - \omega^2}\left(g + \frac{\mathrm{v}_{\mathrm{s},0}^2}{\lambda_\rho}\right)\mathrm{v}_{1,z} - \frac{1}{\lambda_\rho}\left(\mathrm{v}_{\mathrm{A},0}^2 - \frac{\omega^2 \mathrm{v}_{\mathrm{s},0}^2}{k_x^2 \mathrm{v}_{\mathrm{s},0}^2 - \omega^2}\right)\mathrm{v}_{1,z}' + \frac{\omega^2 \mathrm{v}_{\mathrm{s},0}^2}{k_x^2 \mathrm{v}_{\mathrm{s},0}^2 - \omega^2}\mathrm{v}_{1,z}'' + \mathrm{v}_{\mathrm{A},0}^2(k_x^2 \mathrm{v}_{1,z} - \mathrm{v}_{1,z}'')$$

- Eliminate the $\mathrm{v}_{1,z}'$ term by substituting $\mathrm{v}_{1,z} = \exp(ik_z z - z/2\lambda_\rho)\,\xi_1$:

$$\omega^2 = -\frac{k_x^2 g}{k_x^2 \mathrm{v}_{\mathrm{s},0}^2 - \omega^2}\left(g + \frac{\mathrm{v}_{\mathrm{s},0}^2}{\lambda_\rho}\right) + \left(\frac{1}{4\lambda_\rho^2} + k_{xz}^2\right)\mathrm{v}_{\mathrm{A},0}^2 - \left(\frac{1}{4\lambda_\rho^2} + k_z^2\right)\frac{\omega^2 \mathrm{v}_{\mathrm{s},0}^2}{k_x^2 \mathrm{v}_{\mathrm{s},0}^2 - \omega^2}$$

- Substituting $\lambda_\rho = -[\mathrm{v}_{\mathrm{s},0}^2 + (\kappa/2)\mathrm{v}_{\mathrm{A},0}^2]/(\kappa g) < 0$ and $\beta_{\mathrm{pl},0} = P_0/(B_{0,x}^2/8\pi)$:

$$\omega^4 - (1 + 4k_{xz}^2\lambda_\rho^2)\left(1 + \frac{2}{\kappa\beta_{\mathrm{pl},0}}\right)\frac{v_{\mathrm{s},0}^2}{4\lambda_\rho^2}\omega^2 + k_x^2 g^2\left[\frac{2\kappa\beta_{\mathrm{pl},0}}{(1+\beta_{\mathrm{pl},0})^2}k_{xz}^2\lambda_\rho^2 + \frac{\kappa\beta_{\mathrm{pl},0}}{2(1+\beta_{\mathrm{pl},0})^2} + \frac{\kappa\beta_{\mathrm{pl},0}}{1+\beta_{\mathrm{pl},0}} - 1\right] = 0$$

PARKER MODE

- $$\omega^4 - (1 + 4k_{xz}^2\lambda_\rho^2)\left(1 + \frac{2}{\kappa\beta_{\mathrm{pl},0}}\right)\frac{v_{\mathrm{s},0}^2}{4\lambda_\rho^2}\omega^2 + k_x^2 g^2 \left[\frac{2\kappa\beta_{\mathrm{pl},0}}{(1+\beta_{\mathrm{pl},0})^2}k_{xz}^2\lambda_\rho^2 + \frac{\kappa\beta_{\mathrm{pl},0}}{2(1+\beta_{\mathrm{pl},0})^2} + \frac{\kappa\beta_{\mathrm{pl},0}}{1+\beta_{\mathrm{pl},0}} - 1\right] = 0$$

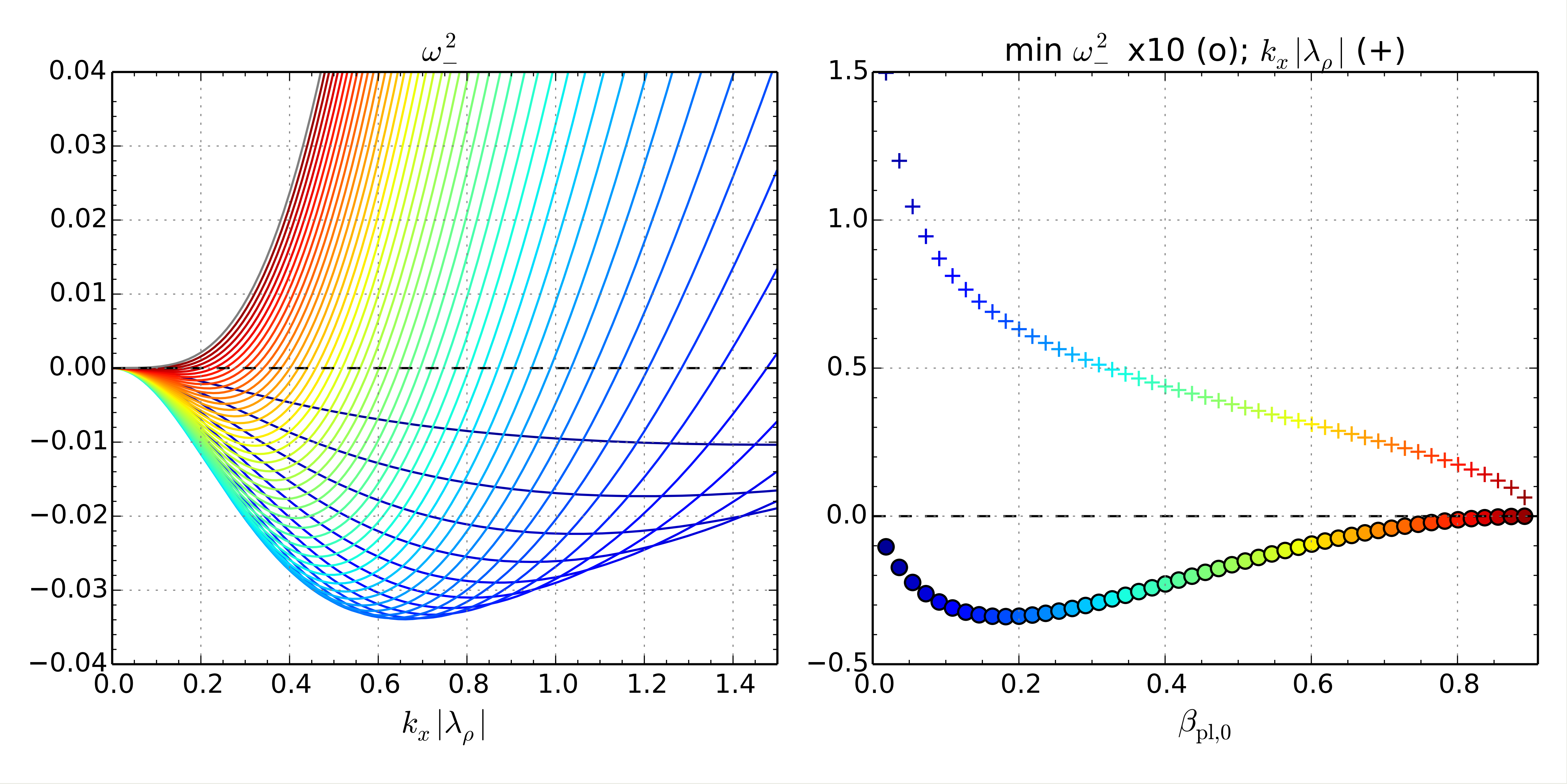
- For unstable solutions ($\omega^2 < 0$), the first two terms would be positive,
  hence the third term needs to be negative:
  $2(\kappa - 1)\beta_{\mathrm{pl},0}^2 + (3\kappa - 4)\beta_{\mathrm{pl},0} - 2 < 0$
  For $\kappa = 5/3$ this means $\beta_{\mathrm{pl},0} < 0.91$.

- In the hydrodynamic limit ($\beta_{\mathrm{pl},0} \to \infty$):
  $$\omega^4 - \left(\frac{1}{4\lambda_\rho^2} + k_{xz}^2\right)\mathrm{v}_{\mathrm{s},0}^2\omega^2 + (\kappa - 1)k_x^2 g^2 = 0$$: stable for $\kappa > 1$.

- Instability is driven by the $-k_x^2 g^2$ term, which can be traced to the gravitational perturbation $-g\rho_1$,
  with the density perturbation ρ_1 including the term $-(k_x/\omega)\rho_0\mathrm{v}_{1,x}$,
  with the longitudinal velocity perturbation $\dfrac{\mathrm{v}_{1,x}}{\mathrm{v}_{1,z}} = -\dfrac{ik_x g + k_x k_z \mathrm{v}_{\mathrm{s},0}^2}{k_x^2\mathrm{v}_{\mathrm{s},0}^2 - \omega^2}$ contributing additional $k_x g$ factor.
  (while the $k_x k_z \mathrm{v}_{\mathrm{s},0}^2$ term cancels out)

PARKER MODE

$$\omega^4 - (1 + 4k_{xz}^2\lambda_\rho^2)\left(1 + \frac{2}{\kappa\beta_{\mathrm{pl},0}}\right)\frac{v_{\mathrm{s},0}^2}{4\lambda_\rho^2}\omega^2 + k_x^2 g^2\left[\frac{2\kappa\beta_{\mathrm{pl},0}}{(1+\beta_{\mathrm{pl},0})^2}k_{xz}^2\lambda_\rho^2 + \frac{\kappa\beta_{\mathrm{pl},0}}{2(1+\beta_{\mathrm{pl},0})^2} + \frac{\kappa\beta_{\mathrm{pl},0}}{1+\beta_{\mathrm{pl},0}} - 1\right] = 0$$



PARKER MODE

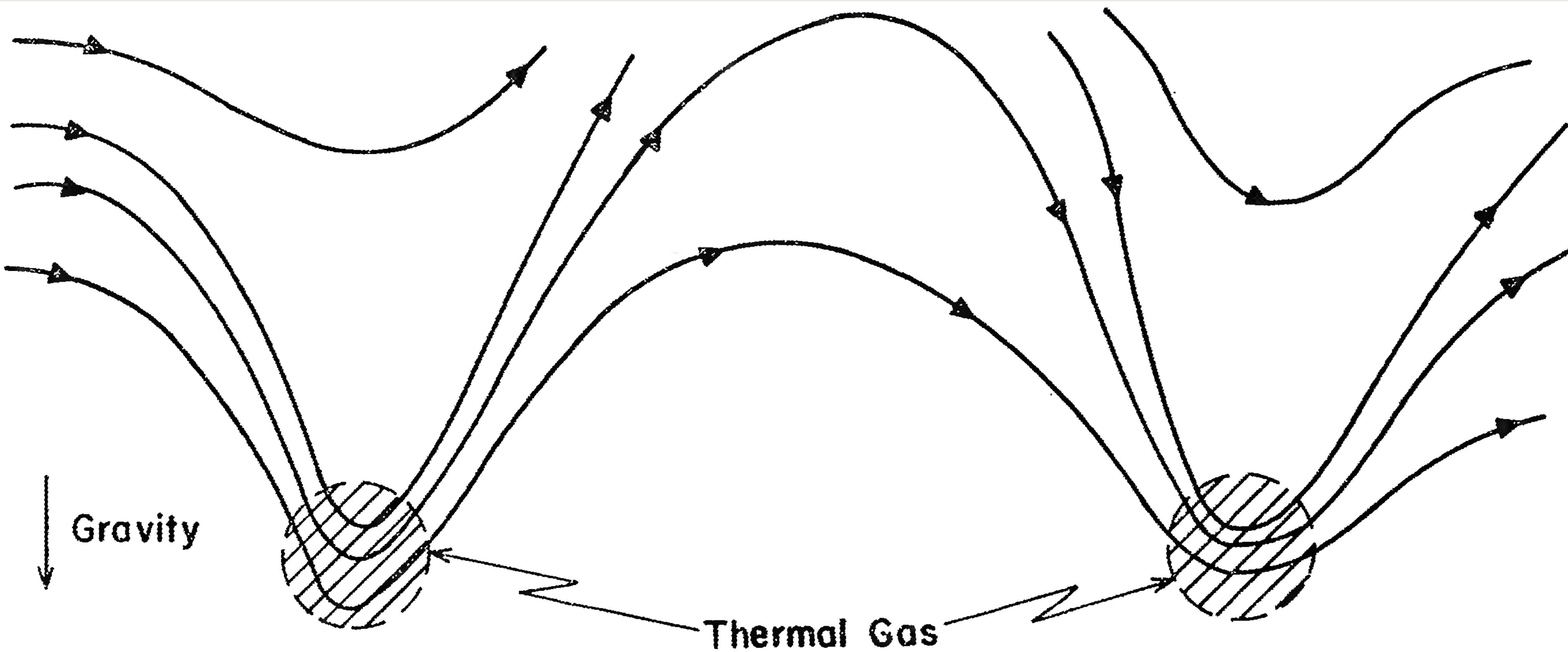


FIG 2.—Sketch of the local state of the lines of force of the interstellar magnetic field and interstellar gas-cloud configuration resulting from the intrinsic instability of a large-scale field along the galactic disk or arm when confined by the weight of the gas.

Parker (1966)

SUMMARY

- Rayleigh-Taylor instability with $\overrightarrow{g} = -g\hat{z}$ in the presence of horizontal magnetic field $\overrightarrow{B}_0 = B_0\hat{x}$ has two modes:

- interchange mode with transverse wave vector $k_y \neq 0$ mediated by fast magnetosonic waves;

- Parker mode with longitudinal wave vector $k_x \neq 0$ driven by gravitation of density perturbation (stable in HD) and stabilized on short wavelengths by magnetic tension.