COSMIC MAGNETIC FIELDS

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Dynamo

DYNAMO

- Dynamo means amplification (re-generation) of magnetic fields due to plasma motions (MHD).

- Generation of magnetic fields from zero is termed the battery (non-MHD).

- Dynamos are inferred to operate in:

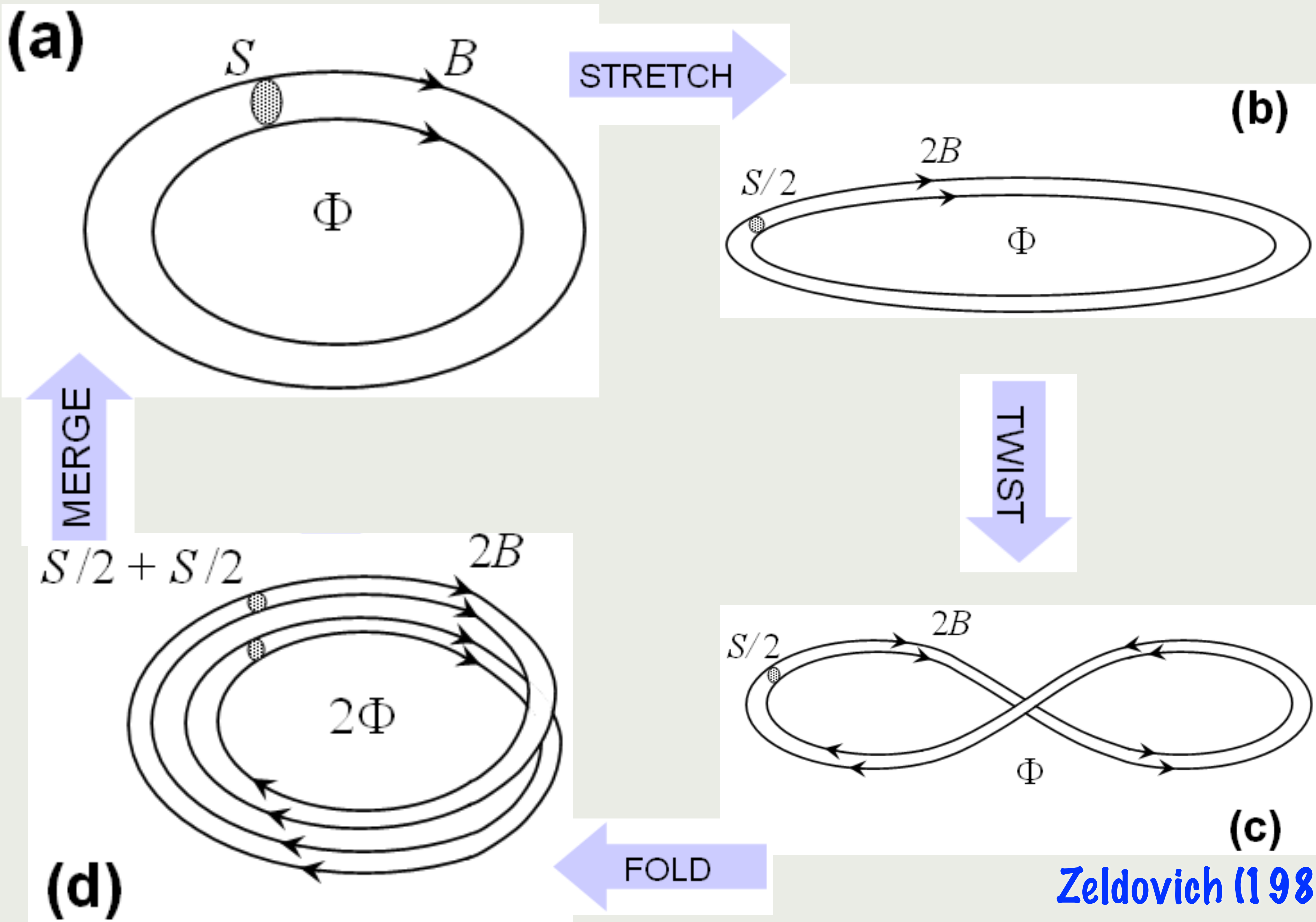
  - planets (inc. Earth), supporting their magnetospheres against diffusion, polarity reversals;

  - low-mass stars (inc. Sun), explaining their activity cycles with polarity reversals;

  - spiral galaxies (inc. Milky Way), explaining their globally ordered fields;

  - (possibly) accretion disks, a hypothesis for the origin of poloidal fields that launch relativistic jets;

  - (possibly) proto-neutron stars, a hypothesis for the origin of magnetar fields.



Zeldovich (1983)
Childress & Gilbert (1995)

LOCAL CHANGE OF MAGNETIC FIELD

- Recall the induction equation in ideal MHD: $\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{\mathrm{v}} \times \vec{B} \right)$

- Kinematic dynamo: $\vec{\mathrm{v}}$ affects $\vec{B}$ (via induction equation), but $\vec{B}$ does not affect $\vec{\mathrm{v}}$.

- Expanding the curl of vector cross product:
$$\frac{\partial \vec{B}}{\partial t} = \left(\vec{B} \cdot \vec{\nabla} \right) \vec{\mathrm{v}} - \left(\vec{\mathrm{v}} \cdot \vec{\nabla} \right) \vec{B} - \vec{B} \left(\vec{\nabla} \cdot \vec{\mathrm{v}} \right)$$

- $\left(\vec{B} \cdot \vec{\nabla} \right) \vec{\mathrm{v}}$ means velocity shear along $\vec{B}$;

- $-\left(\vec{\mathrm{v}} \cdot \vec{\nabla} \right) \vec{B}$ means transport (advection) of $\vec{B}$ along $\vec{\mathrm{v}}$;

- $-\vec{B} \left(\vec{\nabla} \cdot \vec{\mathrm{v}} \right)$ means compression due to velocity convergence.

LOCAL CHANGE OF MAGNETIC ENERGY DENSITY

- $$\frac{\partial \vec{B}}{\partial t} = \left(\vec{B} \cdot \vec{\nabla}\right) \vec{\mathrm{v}} - \left(\vec{\mathrm{v}} \cdot \vec{\nabla}\right) \vec{B} - \vec{B} \left(\vec{\nabla} \cdot \vec{\mathrm{v}}\right)$$

- recall the magnetic energy density:

  $$u_{\mathrm{B}} = \frac{B^2}{8\pi} \equiv \frac{\vec{B} \cdot \vec{B}}{8\pi}$$

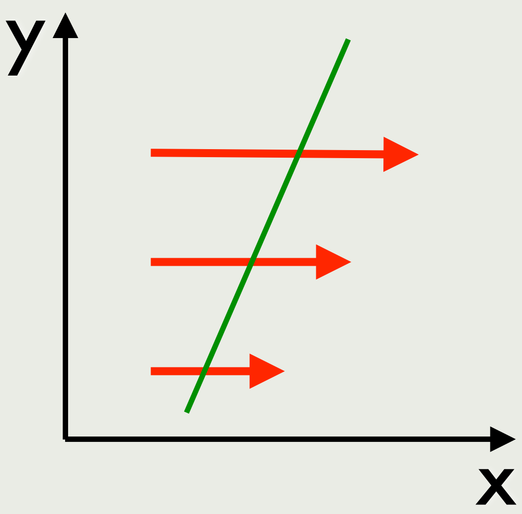
- evolution of magnetic energy density:

  $$\frac{\partial u_{\mathrm{B}}}{\partial t} = \frac{\vec{B}}{4\pi} \cdot \frac{\partial \vec{B}}{\partial t} = \frac{1}{4\pi} \left[\vec{B} \cdot \left(\vec{B} \cdot \vec{\nabla}\right) \vec{\mathrm{v}} - \left(\vec{\mathrm{v}} \cdot \vec{\nabla}\right) \frac{B^2}{2} - B^2 \left(\vec{\nabla} \cdot \vec{\mathrm{v}}\right) \right]$$

  with contributions from shear, transport and compression, respectively.

EXAMPLE: AMPLIFICATION BY SHEAR

- Consider a sheared velocity field $\vec{\mathrm{v}} = \mathrm{v}_x(y)\hat{x}$
- Velocity gradient $\vec{\nabla}\mathrm{v}_x$ has one non-zero component: $\partial_y \mathrm{v}_x$
- The shear term of the induction equation:
  $$\left(\vec{B}\cdot\vec{\nabla}\right)\vec{\mathrm{v}} = B_y \partial_y \mathrm{v}_x \hat{x}$$
  The transport and compression terms vanish.
- Magnetic energy changes at the rate:
  $$\frac{\vec{B}}{4\pi}\cdot\left(\vec{B}\cdot\vec{\nabla}\right)\vec{\mathrm{v}} = \frac{B_x B_y}{4\pi}\partial_y \mathrm{v}_x$$
- In this illustration $\partial_y \mathrm{v}_x > 0$ and $B_x B_y > 0$. Magnetic energy grows since the field line is stretched by velocity shear.



EXAMPLE: AMPLIFICATION BY TRANSPORT

- Consider a uniform velocity shear $\overrightarrow{\mathrm{v}} = \mathrm{v}_x\,\hat{x}$

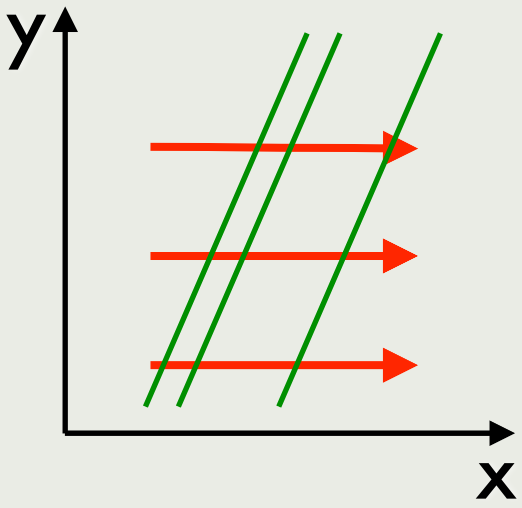
- The transport term of the induction equation:

  $$-\left(\overrightarrow{\mathrm{v}} \cdot \overrightarrow{\nabla}\right)\overrightarrow{B} = -\,\mathrm{v}_x \partial_x \overrightarrow{B}$$

  The shear and compression terms vanish.

- Magnetic energy changes at the rate:

  $$-\left(\overrightarrow{\mathrm{v}} \cdot \overrightarrow{\nabla}\right)\frac{B^2}{8\pi} = -\,\mathrm{v}_x \frac{\partial_x(B^2)}{8\pi}$$

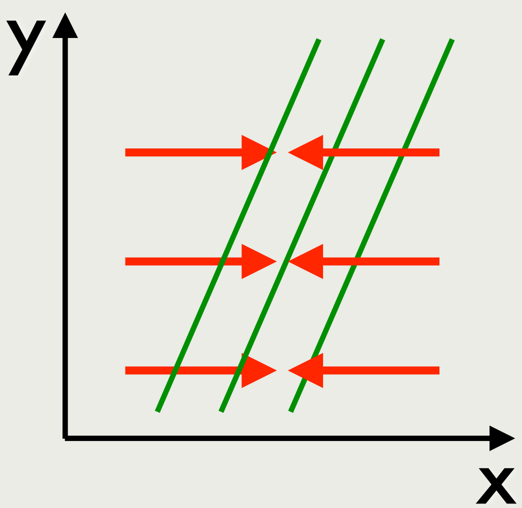


- In this illustration $\mathrm{v}_x > 0$ and $\partial_x B < 0$. Stronger magnetic field on the left is transported to the right.

EXAMPLE: AMPLIFICATION BY COMPRESSION

- Consider a converging velocity shear $\vec{\mathrm{v}} = \mathrm{v}_x(x)\,\hat{x}$ with uniform divergence $\partial_x \mathrm{v}_x < 0$, and uniform magnetic field $\vec{B} = [B_x, B_y, 0]$.

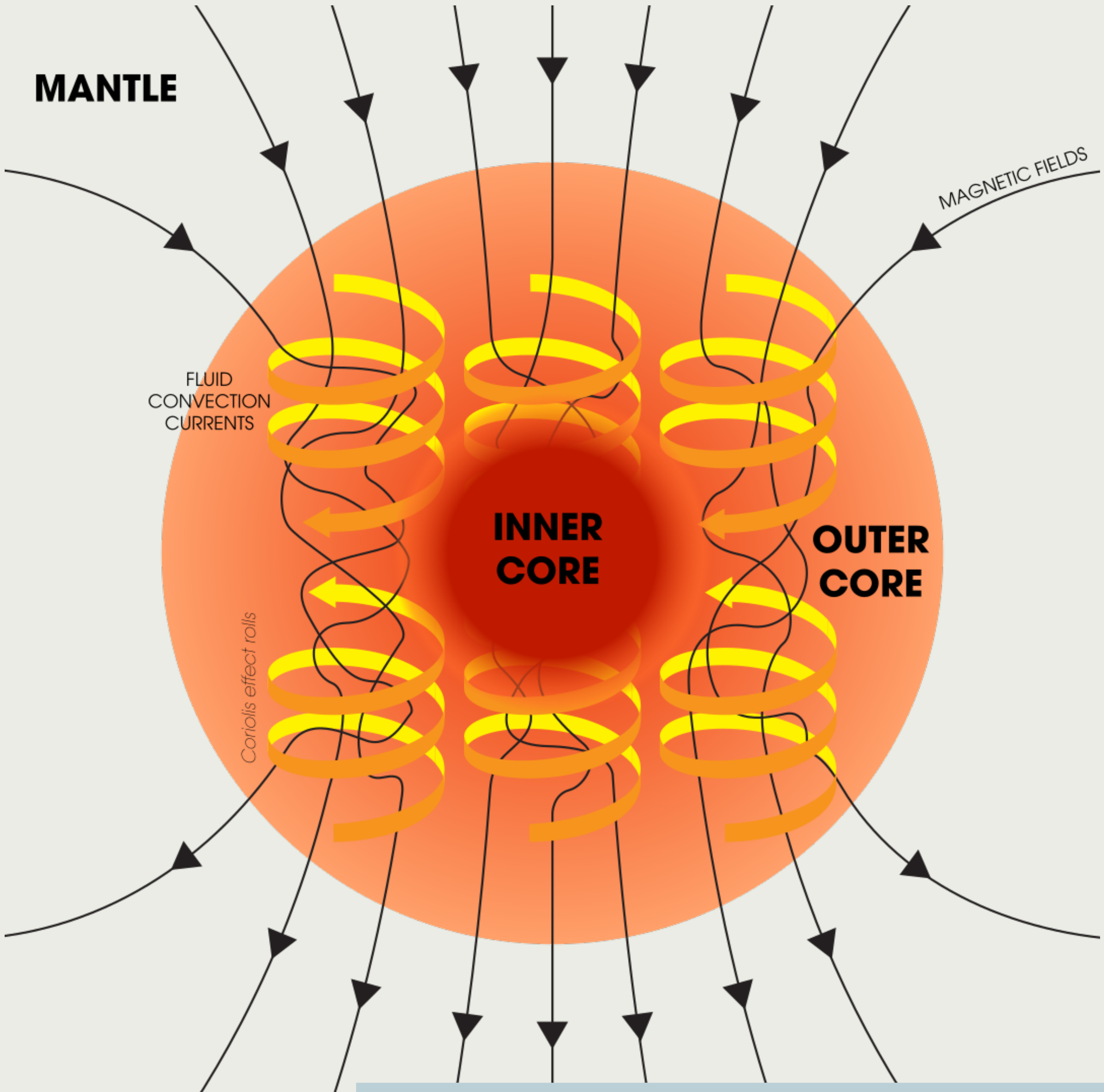
- The compression term of the induction equation:
  $$-\vec{B}\left(\vec{\nabla}\cdot\vec{\mathrm{v}}\right) = -\vec{B}\,\partial_x \mathrm{v}_x$$
  The shear term is $\left(\vec{B}\cdot\vec{\nabla}\right)\vec{\mathrm{v}} = B_x\,\partial_x \mathrm{v}_x$, canceling the x component of the compression term.
  The transport term vanishes.



- Magnetic energy changes at the rate:
  $$\frac{1}{4\pi}\left[\vec{B}\cdot\left(\vec{B}\cdot\vec{\nabla}\right)\vec{\mathrm{v}} - B^2\left(\vec{\nabla}\cdot\vec{\mathrm{v}}\right)\right] = -\frac{B_y^2}{4\pi}\,\partial_x \mathrm{v}_x$$

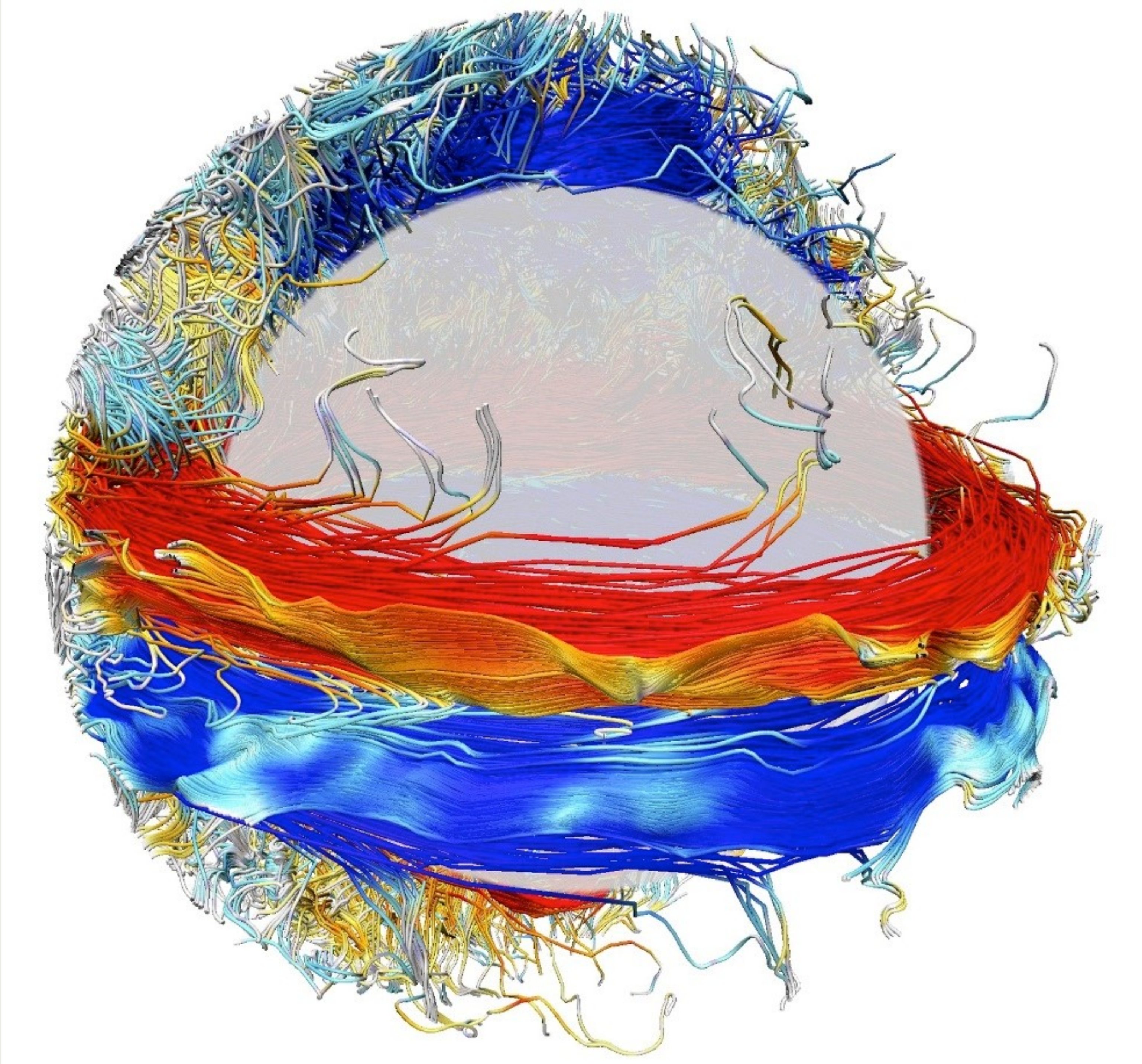
GEODYNAMO

- Earth's magnetic field decays on the time scale of ~50 kyr.
- A regeneration mechanism is necessary - the geodynamo.
- The geodynamo is supported by circulation of conducting matter, which is possible in the fluid outer core due to convection.
- Convection is enabled by a net heat flow from the core to the much cooler mantle.
- The Earth's rotation and Coriolis forces are important for shaping the core convection.



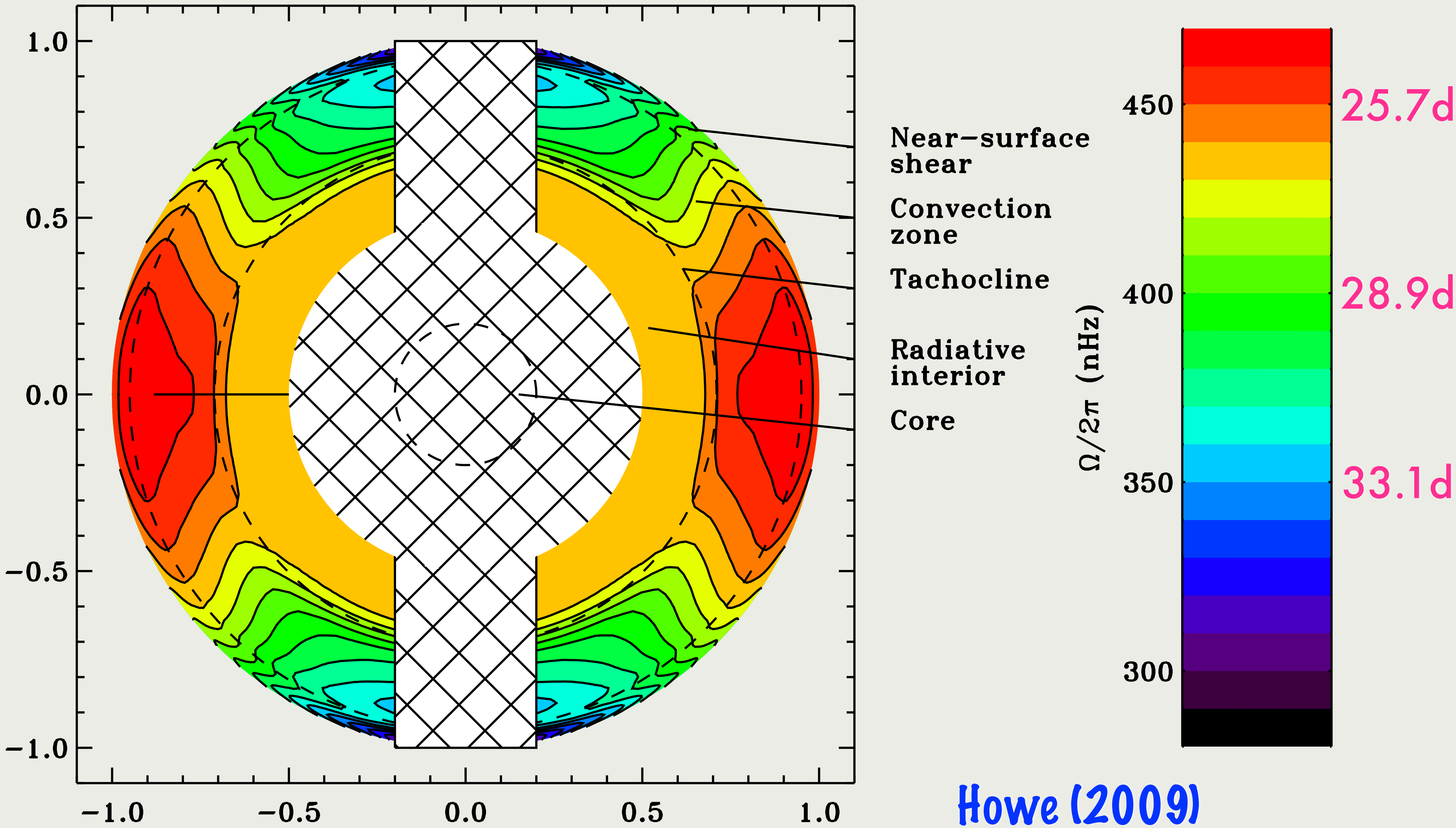
SOLAR DYNAMO: NUMERICAL SIMULATIONS

- global 3D MHD simulations of the solar convective layer
- bands of strong toroidal magnetic field are generated at the latitudes consistent with the occurence of sunspots



Brun & Browning (2017)

DIFFERENTIAL ROTATION OF THE SUN



Howe (2009)

AXISYMMETRIC FIELDS

- Consider a magnetic field structure with axial symmetry in cylindrical coordinates (r, ϕ, z), decomposed into toroidal and poloidal components: $\overrightarrow{B} = B_\phi \hat{\phi} + \overrightarrow{B}_{\mathrm{p}}$.
- The poloidal component $\overrightarrow{B}_{\mathrm{p}} \equiv B_r \hat{r} + B_z \hat{z}$ can be represented by toroidal component of magnetic vector potential $\overrightarrow{B}_{\mathrm{p}} = \overrightarrow{\nabla} \times (A_\phi \hat{\phi})$.
- If both B_ϕ and A_ϕ are independent of ϕ, $\overrightarrow{\nabla} \cdot \overrightarrow{B} = 0$ is satisfied automatically.

DIFFERENTIAL ROTATION

- Consider azimuthal velocity field $\vec{\mathrm{v}} = r\,\Omega(z)\,\hat{\phi}$, where $\Omega(z)$ is the angular velocity that allows for a vertically differential rotation $\mathrm{d}\Omega/\mathrm{d}z \neq 0$.

- The azimuthal induction equation includes the shear and transport terms (note that $\vec{\nabla} \cdot \vec{\mathrm{v}} = 0$):

$$\partial_t B_\phi = \left[(\vec{B} \cdot \vec{\nabla})\vec{\mathrm{v}}\right]_\phi - \left[(\vec{\mathrm{v}} \cdot \vec{\nabla})\vec{B}\right]_\phi$$

- Material derivatives in curved coordinates!

$$\partial_t B_\phi = \left[(\vec{B}_\mathrm{p} \cdot \vec{\nabla})\mathrm{v}_\phi \hat{\phi}\right]_\phi - \frac{B_r \mathrm{v}_\phi}{r}$$

- $\partial_t B_\phi = r\left[(\vec{B}_\mathrm{p} \cdot \vec{\nabla})\frac{\mathrm{v}_\phi}{r}\hat{\phi}\right]_\phi = rB_z\frac{\mathrm{d}\Omega}{\mathrm{d}z}$ the Ω effect!

differential rotation together with poloidal field make a source term for the growth of toroidal field.

INDUCTION OF POLOIDAL FIELD

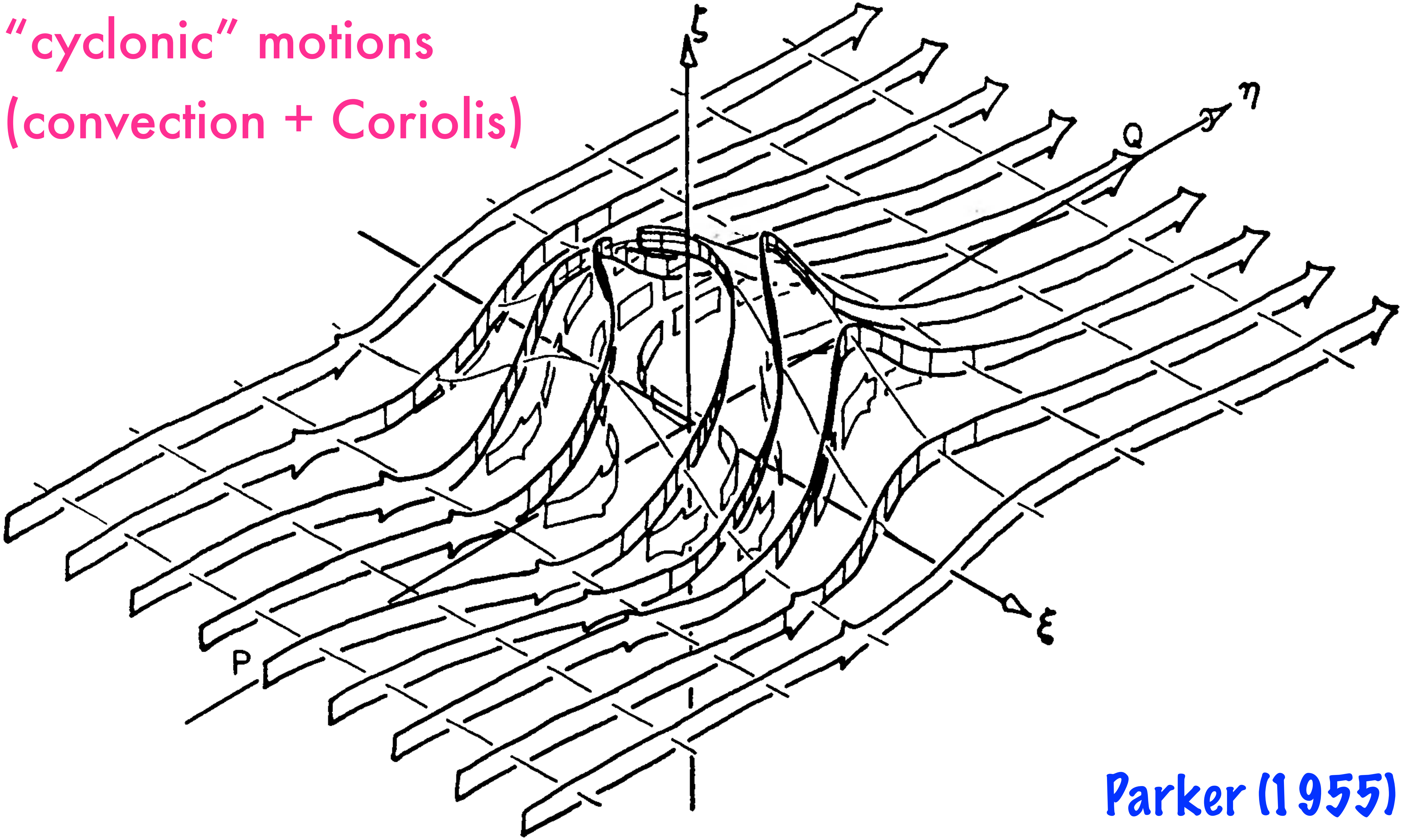
- Consider the axial induction equation:
$$\partial_t B_z = \left[(\vec{B} \cdot \vec{\nabla})\vec{\mathrm{v}}\right]_z - \left[(\vec{\mathrm{v}} \cdot \vec{\nabla})\vec{B}\right]_z$$

- For $\mathrm{v}_z = 0$ and $\partial_\phi B_z = 0$, both RHS terms vanish, hence the axial field cannot be induced in this case.

- The loop $B_z \to B_\phi \to B_z$ cannot be closed for axisymmetric velocity field. This is the simplest case of the Cowling's theorem.

BREAKING THE AXIAL SYMMETRY

"cyclonic" motions
(convection + Coriolis)



Parker (1955)

MEAN-FIELD ELECTRODYNAMICS

- In addition to globally symmetric mean magnetic and velocity fields $\overrightarrow{B}_0$, $\overrightarrow{\mathrm{v}}_0$, one can consider small-scale (turbulent) fluctuations $\overrightarrow{B}_1$, $\overrightarrow{\mathrm{v}}_1$:
$\overrightarrow{B} = \overrightarrow{B}_0 + \overrightarrow{B}_1$ and $\overrightarrow{\mathrm{v}} = \overrightarrow{\mathrm{v}}_0 + \overrightarrow{\mathrm{v}}_1$.

- Not a linearization, but averaging fluctuations over sufficiently large scales gives $\langle \overrightarrow{B} \rangle \simeq \overrightarrow{B}_0$ and $\langle \overrightarrow{\mathrm{v}} \rangle \simeq \overrightarrow{\mathrm{v}}_0$.

MEAN-FIELD INDUCTION EQUATION

- The induction equation:
  $\partial_t \overrightarrow{B} = \overrightarrow{\nabla} \times \left(\overrightarrow{\mathrm{v}} \times \overrightarrow{B} \right)$

- $\partial_t \langle \overrightarrow{B}_0 \rangle = \overrightarrow{\nabla} \times \langle (\overrightarrow{\mathrm{v}}_0 + \overrightarrow{\mathrm{v}}_1) \times (\overrightarrow{B}_0 + \overrightarrow{B}_1) \rangle$

- Assuming that $\langle \overrightarrow{\mathrm{v}}_0 \times \overrightarrow{B}_1 \rangle + \langle \overrightarrow{\mathrm{v}}_1 \times \overrightarrow{B}_0 \rangle = 0$:
  $\partial_t \langle \overrightarrow{B}_0 \rangle = \overrightarrow{\nabla} \times \left(\langle \overrightarrow{\mathrm{v}}_0 \rangle \times \langle \overrightarrow{B}_0 \rangle + \langle \overrightarrow{\mathrm{v}}_1 \times \overrightarrow{B}_1 \rangle \right)$

- $\overrightarrow{\mathcal{E}} \equiv \langle \overrightarrow{\mathrm{v}}_1 \times \overrightarrow{B}_1 \rangle$ mean turbulent electromotive force

MEAN TURBULENT ELECTROMOTIVE FORCE

- $\overrightarrow{\mathscr{E}} \equiv \langle \overrightarrow{\mathrm{v}}_1 \times \overrightarrow{B}_1 \rangle$

- $\mathscr{E}_i = \alpha_{ij} \langle B_0 \rangle_j$ where α_{ij} is a symmetric tensor.

- substituting to the mean-field induction equation:

$$\partial_t \langle \overrightarrow{B}_0 \rangle = \overrightarrow{\nabla} \times \left(\langle \overrightarrow{\mathrm{v}}_0 \rangle \times \langle \overrightarrow{B}_0 \rangle + \alpha \langle \overrightarrow{B}_0 \rangle \right)$$

the α effect!

MEAN-FIELD DYNAMO

- Induction of the mean axial field:

$$\partial_t \langle B_z \rangle = \left[\vec{\nabla} \times \left(\alpha \langle \vec{B}_0 \rangle \right) \right]_z = \frac{1}{r} \partial_r \left(r \alpha \langle B_\phi \rangle \right)$$

- Induction of the mean toroidal field:

$$\partial_t \langle B_\phi \rangle = r \langle B_z \rangle \frac{\mathrm{d}\Omega}{\mathrm{d}z} - \partial_r \left(\alpha \langle B_z \rangle \right)$$

- When the $r \left\langle B_z \right\rangle \dfrac{\mathrm{d}\Omega}{\mathrm{d}z}$ term dominates: $\alpha\Omega$ dynamo (stars)

  When the $\partial_r \left(\alpha \left\langle B_z \right\rangle \right)$ term dominates: α^2 dynamo (planets)

  When the terms are comparable: $\alpha^2\Omega$ dynamo (galaxies)

BIERMANN BATTERY

- Can magnetic field be created from scratch? Note that for $\overrightarrow{B} = 0$ resistive MHD predicts $\overrightarrow{E} = 0$, hence $\partial_t \overrightarrow{B} = 0$.

- More general regime: two-fluid plasma (electrons decoupled from ions).

- Consider electrons under a balance of Lorentz force and pressure gradient, $\vec{f}_e = -\overrightarrow{\nabla} P_e - e n_e \overrightarrow{E} = 0$, with $P_e = n_e k_\mathrm{B} T_e$.

- From the Maxwell-Faraday equation:

$$\partial_t \overrightarrow{B} = -c \overrightarrow{\nabla} \times \overrightarrow{E} = c \overrightarrow{\nabla} \times \left(\frac{\overrightarrow{\nabla} P_e}{e n_e} \right) = \frac{c k_\mathrm{B}}{e} \overrightarrow{\nabla} \times \left[\frac{\overrightarrow{\nabla} (n_e T_e)}{n_e} \right]$$

- $$\partial_t \overrightarrow{B} = \frac{c k_\mathrm{B}}{e} \left(\frac{\overrightarrow{\nabla} n_e}{n_e} \right) \times \left(\overrightarrow{\nabla} T_e \right)$$

  density gradient misaligned with the temperature gradient

SUMMARY

- Dynamo is a mechanism of amplifying magnetic fields using the kinetic energy of plasma motions.

- Local magnetic energy density can be amplified due to shear, transport or compression.

- Dynamo can operate as a positive-feedback cycle. Zeldovich cycle: stretch - twist - fold - merge.

- In axially symmetric systems (planets, stars, galaxies), toroidal fields can be amplified by shear due to differential rotation (Ω effect), poloidal fields can be amplified by turbulent electromotive force (α effect).