Astronomical distances from gravitational-wave observations

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The recent first direct detections of gravitational waves with the two LIGO detectors (Abbott et al. 2016a,b) create an unprecedented opportunity for studying the Universe through a novel, never before explored channel of spacetime fluctuations. Gravitational wave astronomy is often compared to 'listening to' rather than 'looking at' the skies. By design which is motivated by the choice of potential sources, the ground-based gravitational wave detectors of Advanced LIGO (Aasi et al. 2015) and Advanced Virgo (Acernese et al. 2015) are sensitive in the range of frequencies similar to the audible range of human ears - between 10 Hz and a few kHz. As in the case of an ear, a solitary laser interferometric detector is practically omnidirectional (has a poor angular resolution), and has no imaging capabilities. It registers a coherent signal emitted by a bulk movement of large, rapidly-moving masses. Once emitted, gravitational waves are weakly coupled to the surrounding matter and propagate freely without scattering. This has to be contrasted with the electromagnetic emission which originates at the microscopic level, is strongly coupled to the surroundings and often reprocessed; it carries a reliable information from the last scattering surface only. It seems therefore that gravitational wave detectors are the perfect counterpart to the electromagnetic observatories as they may provide us with information impossible to obtain by other means.

Very shortly after announcing the general theory of relativity in 1915, Albert Einstein realized that a linearised version of his equations resembles the wave equation (Einstein 1916). The solution is interpreted as a short-wavelength, time-varying curvature deformation propagating with the speed of light on an otherwise slowly-varying, large-scale curvature background (a gravitational-wave "ripple" propagating through the four-dimensional spacetime); from the point of view of a metric tensor, it represents a small perturbation of a stationary background metric. Linear approximation corresponds to the waves propagating in the far-field limit. By exploiting the gauge freedom of the theory one may show that the solution has features similar to electromagnetic waves: it is a transverse wave which may be polarized (has two independent polarizations). Over the next 40 years, during which Einstein changed his mind to argue against their genuineness, a controversy persisted over the true nature of gravitational waves. Only in the late 50s and early 60s the works of Felix Pirani (1956), Herman Bondi (1957), Ivor Robertson and Andrzej Trautman (1960) convincingly showed that gravitational waves are indeed physical phenomena that carry and deposit energy.

A realistic wave phenomenon (and not, say, a coordinate artifact) must be capable of transmitting energy from the source to infinity. If the amplitude of an exemplary isotropic field at a radial distance r from the source is $h(r)$, then the flux of energy over a spherical surface at r is $\mathcal{F}(r) \propto h^2(r)$, and the total emitted power (the luminosity) is $\mathcal{L}(r) \propto 4\pi r^2 h^2(r)$. Since the energy has to be conserved, the amplitude $h(r)$ falls like $1/r$, irrespectively of the multipole character of the source (the lowest radiating multipole in the gravity theory is the quadrupole distribution, because for an isolated system a time-changing monopole would correspond to the violation of the mass-energy conservation, and a time-changing dipole would violate the momentum conservation law).

Gravitational waves are related to the changes in the spacetime distance (the proper time interval), therefore they cannot be detected by a local measurement - one has to compare the spacetime positions of remote events (Pirani 1956). The detection principle in the case of the laser interferometric detector is to measure the difference of the relative change in its perpendicular arms' lengths L_x and L_y, $\delta L_x - \delta L_y = \Delta L/L$, by measuring the interference pattern in the output port located at the apex of the device. Due to the quadrupolar nature of a gravitational wave, shortening of one arm corresponds to elongation of the other. This change of lengths is reflected in varying paths that the laser light has to cross before the interference. The dimensionless gravitational-wave amplitude $h = \Delta L/L$ (the "strain") is proportional to the amount of outgoing laser light. The fact that the directly-measurable quantity is the amplitude $h \propto 1/r$, not the energy of the wave as in the electromagnetic antennæ, has a direct consequence for the reach of the observing device: one-order-of-magnitude sensitivity improvement corresponds to one-order-of-magnitude growth of distance reach r, as opposed to the factor of $\sqrt{10}$ in the electromagnetic observations (consequently, the volume of space grows like r^3 in case of gravitational-wave observations, encompassing hundreds of thousands of galaxies for a distance reach of the order of hundreds of Mpc, see Abbott et al. 2016d).

Among promising sources of gravitational radiation are all asymmetric collapses and explosions (supernovæ), rotating deformed stars (gravitational-wave 'pulsars' of continuous and transient nature), and tight binary systems of e.g., neutron stars and black holes. In the following we will focus on the binary systems, since their properties make them the analogues to standard candles of traditional astronomy. The fittingly descriptive term "standard siren" was first used in the work of Holz and Hughes (2005) in the context of gravitational waves from super-massive binary black holes as a target for the planned spaceborne LISA detector (Danzmann 1996). The idea of using well-understood signals to infer the distance and constrain the cosmological parameters was however proposed much earlier (Schutz 1986, 2002).

Magnitudes of the gravitational-wave strain h and the luminosity $\mathcal{L}$ may be estimated using dimensional analysis and Newtonian physics. As the waves are generated by the accelerated movement of masses and the mass distribution should be quadrupolar, one may assume that h is proportional to a second time derivative of the quadrupole moment $I_{ij} = \int \rho(\mathbf{x}) x_i x_j d^3x$ for some matter distribution $\rho(\mathbf{x})$. For a binary composed of masses m_1 and m_2, orbiting the center of mass at a separation a with the orbital angular velocity ω, h is proportional to the system's moment of inertia μa^2 and to ω^2, as well as inversely proportional to the distance, $h \propto \mu a^2 \omega^2 / r$, where $\mu = m_1 m_2 / M$ is the reduced mass, and $M = m_1 + m_2$ the total mass. In order to recover the dimensionless h, the characteristic constants of the problem, G and c, are used to obtain

$$h \simeq \frac{G}{c^4}\frac{1}{r}\mu a^2 \omega^2 = \frac{G^{5/3}}{c^4}\frac{1}{r}\mu M^{2/3}\omega^{2/3} \qquad \left(h_{ij} = \frac{2G}{c^4 r}\ddot{I}_{ij}\right), \tag{1}$$

with the use of Kepler's third law ($GM = a^3\omega^2$) in the second equation. The expression in brackets represents the strain tensor h_{ij} in the non-relativistic *quadrupole approximation* (Einstein 1918). Similarly, the luminosity $\mathcal{L}$ (the rate of energy loss in gravitational waves, integrated over a sphere at a distance r) should be proportional to $h^2 r^2$ and some power of ω. From dimensional analysis one has

$$\mathcal{L} = \frac{dE_{GW}}{dt} \propto \frac{G}{c^5} h^2 \omega^2 \propto \frac{G}{c^5}\mu^2 a^4 \omega^6 \qquad \left(\mathcal{L} = \frac{c^3}{16G\pi}\iint \langle \dot{h}_{ij}\dot{h}^{ij}\rangle \, dS = \frac{G}{5c^5}\langle \dddot{I}_{ij}\dddot{I}^{ij}\rangle\right), \tag{2}$$

with the proportionality factor of 32/5. Again, the expression in brackets refers to the quadrupole approximation; the angle brackets denote averaging over the orbital period. Waves leave the system at the expense of its orbital energy $E_{orb} = -Gm_1 m_2/(2a)$. Using the time derivative of the third Kepler's third law, $\dot{a} = -2a\dot{\omega}/(3\omega)$, one gets the evolution of the orbital frequency driven by the gravitational-wave emission:

$$\frac{dE_{orb}}{dt} \equiv \frac{Gm_1 m_2}{2a^2}\dot{a} = -\frac{dE_{GW}}{dt} \implies \dot{\omega} = \frac{96}{5}\frac{\omega^{11/3}}{c^5} G^{5/3}\mathcal{M}^{5/3}. \tag{3}$$

The system changes by increasing its orbital frequency; at the same time the strain amplitude h of emitted waves also grows. This characteristic frequency-amplitude evolution is called the *chirp*, by the similarity to birds' sounds, and the characteristic function of component masses $\mathcal{M} = \left(\mu^3 M^2\right)^{1/5}$ is called the *chirp mass*. Orbital frequency is related in a straightforward manner to the gravitational-wave frequency f_{GW}: from the geometry of the problem it is evident that the frequency of radiation is predominantly at twice the orbital frequency, $f_{GW} = \omega/\pi$. By combining the equations for $\dot{f}_{GW}$ and h, one recovers the distance to the source r. It is a function of the frequency and amplitude parameters, which are *directly* measured by the detector:

$$r = \frac{5}{96\pi^2}\frac{c}{h}\frac{\dot{f}_{GW}}{f_{GW}^3} = 512\frac{1}{h_{21}}\left(\frac{0.01\text{ s}}{\tau}\right)\left(\frac{100\text{ Hz}}{f_{GW}}\right)^2 \text{ Mpc}, \tag{4}$$

where h_{21} denotes the strain in the units of 10^{-21} and $\tau = f_{GW}/\dot{f}_{GW}$ denotes the rate of change of the gravitational-wave chirp frequency. Note that within the simple Newtonian approximation presented here (at the leading order of the post-Newtonian expansion) the product $h\tau$ is independent of the components' masses (Królak and Schutz 1987). The simplified analysis presented above does not take into account the full post-Newtonian waveform, polarization information, network of detectors analysis etc., but is intended to demonstrate that binary systems are indeed truly extraordinary "standard sirens".

Their observations provide absolute, physical distances directly, without the need for a calibration or a 'distance ladder'. Among them, binary black hole systems occupy a special position. In the framework of general relativity, binary black holes waveforms are independent from astrophysical assumptions about the systems' intrinsic parameters and their environment. At cosmological scales the distance r has a true meaning of the *luminosity distance*. However, since the vacuum (black-hole) solutions in general relativity are scale-free, the measurements of their waveforms alone cannot determine the source's redshift. The parameters measured by the detector are related to the rest-frame parameters by the redshift z: $f_{GW} = f^{rf}_{GW}/(1+z)$, $\tau = \tau^{rf}(1+z)$, $\mathcal{M} = \mathcal{M}^{rf}(1+z)$. Independent measurements of the redshift which would facilitate the cosmographic studies of the large-scale Universe requires the collaboration with the electromagnetic observers i.e., the *multi-messenger astronomy*. This may be obtained by assessing the redshift of the galaxy hosting the binary by detecting the electromagnetic counterpart of the event (Bloom et al. 2009; Singer et al. 2016) or by performing statistical study for galaxies' redshifts correlated with the position of the signal's host galaxy (MacLeod and Hogan 2008). The omnidirectional nature of a solitary detector is mitigated by simultaneous data analysis from at least three ground-based detectors in order to perform the triangulation of the source position, hence the crucial need for the LIGO-Virgo collaboration, which will be in the future enhanced by the LIGO-India detector, and the KAGRA detector in Japan (Aso et al. 2013).

An equally exciting type of source of gravitational signals are the neutron star binaries (or neutron star-black hole binaries), most-probably related to the short gamma-ray bursts (Paczynski 1986; see Berger 2014 for a recent review). Although the complete waveform which includes the merger requires the knowledge of the material properties of the components (the presently poorly-known dense-matter equation of state physics), the chirp waveforms are understood well enough to facilitate a firm detection and a distance measurement. Binaries involving neutron stars are the ideal "standard sirens" as they naturally provide both loud gravitational-wave and bright electromagnetic emission. Short gamma-ray bursts occur frequently within the reach of ground-based detectors, at redshifts $z < 0.2$. Nissanke et al. (2013) shows that observing a population of the order of 10 of gravitational-wave events related to short gamma-ray bursts would allow to measure the Hubble constant with 5% precision using a network of detectors that includes advanced LIGO and Virgo (30 beamed events could constrain the Hubble constant to better than 1%). In both cases of double black-hole binaries and those involving neutron stars, rapid electromagnetic counterpart observations and precise catalogs of galaxies are needed (Abbott et al. 2016d; Singer et al. 2016).

The main source of error in the distance measurement is the limited sensitivity of the detectors (finite signal-to-noise), which translates into the limited knowledge of the source's direction and orientation (see e.g., Nissanke et al. 2010 for a short gamma-ray burst related study). This may be improved with the measurements of gravitational-wave polarizations with a network of three or more detectors. Second limiting factor is the detectors' calibration uncertainties. Recent first direct detections by two Advanced LIGO detectors established the distances with rather large error bars mostly due to these factors (see e.g., Abbott et al. 2016c). For redshifts larger than $z = 1$ weak gravitational lensing will contribute to the distortion of the luminosity distance measurements at the order of 10% (Bartelmann and Schneider 2001; Dalal et al. 2006). For a detailed discussion of limiting factors in the case of a network of detectors see Schutz (2011) and references therein.

To conclude, observations of gravitational-waves from cosmological distances with current and planned detectors (e.g., spaceborne LISA, sensitive to low frequencies corresponding to chirping supermassive black hole binaries, Danzmann 1996, or a third-generation cryogenic underground Einstein Telescope, with a frequency range similar to Advanced LIGO and Advanced Virgo, Abernathy et al. 2011) promise a wealth of new astrophysical information. Future detectors will reach cosmological distances and redshifts of a few, being sensitive to practically all the chirping binaries in their sensitivity band in the Universe and providing precise distance measurements (see e.g., Lang and Hughes 2008). In addition to precisely measuring the Hubble constant, cosmological observations would help determine the distances to galaxies, thus contributing to building the standard 'distance ladder' (calibrating electromagnetic standard candles), establish the distribution of galaxies and voids, characterize the evolution of the dark energy and mass density of the Universe, mass distribution through the gravitational lensing, as well as the chemical evolution effects i.e., establishing the onset of star formation (Królak and Schutz 1987; Sathyaprakash and Schutz 2009).

References

Aasi, J., B. P. Abbott, R. Abbott, T. Abbott, et al. (2015). Class. Quant. Grav. 32.7, 074001, p. 074001. arXiv: 1411.4547.

Abbott, B. P., R. Abbott, T. D. Abbott, M. R. Abernathy, et al. (2016a). Phys. Rev. Lett. 116.24, 241103, p. 241103. arXiv: 1606.04855.

— (2016b). Phys. Rev. Lett. 116.6, 061102, p. 061102. arXiv: 1602.03837.

Abbott, B. P. et al. (2016c). *Physical Review Letters* 116.24, 241102, p. 241102. arXiv: 1602.03840 [gr-qc].

— (2016d). *Living Reviews in Relativity* 19. arXiv: 1304.0670 [gr-qc].

Abernathy, M., F. Acernese, B. Ajith P. Allen, and et al. (2011). URL: https://tds.ego-gw.it/ql/?c=7954.

Acernese, F., M. Agathos, K. Agatsuma, D. Aisa, et al. (2015). Class. Quant. Grav. 32.2, 024001, p. 024001. arXiv: 1408.3978.

Aso, Y., Y. Michimura, K. Somiya, M. Ando, et al. (2013). Phys. Rev. D 88.4, 043007, p. 043007. arXiv: 1306.6747.

Bartelmann, M. and P. Schneider (2001). Phys. Rep. 340, pp. 291–472. eprint: astro-ph/9912508.

Berger, E. (2014). ARA&A 52, pp. 43–105. arXiv: 1311.2603 [astro-ph.HE].

Bloom, J. S. et al. (2009). *ArXiv e-prints.* arXiv: 0902.1527 [astro-ph.CO].

Bondi, H. (1957). Nature 179, pp. 1072–1073.

Dalal, N., D. E. Holz, S. A. Hughes, and B. Jain (2006). *Phys. Rev. D* 74, 063006, p. 063006. eprint: astro-ph/0601275.

Danzmann, K. et al. (1996). *Class. Quantum Grav.* 13, A247–A250.

Einstein, A. (1915). *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin), Seite 778-786.*

— (1916). *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin), Seite 688-696.*

— (1918). *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin), Seite 154-167.*

Holz, D. E. and S. A. Hughes (2005). ApJ 629, pp. 15–22. eprint: astro-ph/0504616.

Królak, A. and B. F. Schutz (1987). *Gen. Relativ. Gravit.* 19, pp. 1163–1171.

Lang, R. N. and S. A. Hughes (2008). *Astrophys. J.* 677, pp. 1184–1200. eprint: arXiv:0710.3795.

MacLeod, C. L. and C. J. Hogan (2008). Phys. Rev. D 77.4, 043512, p. 043512. arXiv: 0712.0618.

Nissanke, S., D. E. Holz, S. A. Hughes, N. Dalal, and J. L. Sievers (2010). ApJ 725, pp. 496–514. arXiv: 0904.1017 [astro-ph.CO].

Nissanke, S. et al. (2013). *ArXiv e-prints.* arXiv: 1307.2638 [astro-ph.CO].

Paczynski, B. (1986). ApJ 308, pp. L43–L46.

Pirani, F. A. E. (1956). *Acta Physica Polonica* 15, pp. 389–405.

Robinson, I. and A. Trautman (1960). *Physical Review Letters* 4, pp. 431–432.

Sathyaprakash, B.S. and Bernard F. Schutz (2009). *Physics, Astrophysics and Cosmology with Gravitational Waves. Living Reviews in Relativity* 12.2.

Schutz, B. F. (1986). Nature 323, p. 310.

— (2002). In: *Lighthouses of the Universe: The Most Luminous Celestial Objects and Their Use for Cosmology.* Ed. by M. Gilfanov, R. Sunyeav, and E. Churazov, p. 207. eprint: gr-qc/0111095.

— (2011). *Classical and Quantum Gravity* 28.12, 125023, p. 125023. arXiv: 1102.5421 [astro-ph.IM].

Singer, L. P. et al. (2016). ApJ 829, L15, p. L15. arXiv: 1603.07333 [astro-ph.HE].