

## Editor's Note:

### T-Models of "Sphere" in General Relativity Theory

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### Spherically Symmetric T-Models in the General Theory of Relativity

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The two papers by Ruban contain the discussion of physical and geometrical properties of the solution of Einstein's equations first found by Datt [1]. The solution is, in view of the so-called current knowledge, of rather academic interest and is known mainly because it keeps coming up as a degenerate or limiting case in considerations of spherically symmetric inhomogeneous cosmological models. Nevertheless, the Ruban papers are a remarkable example that a careful and creative investigation of a geometry that seems to be remote from the physical reality can produce illuminating results.

The geometry considered by Ruban is defined by the metric form (eq. (1) in the first paper):

$$ds^2 = dt^2 - e^{\alpha(t,r)} dr^2 - R^2(t)(d\vartheta^2 + \sin^2 \vartheta d\varphi^2), \quad (1)$$

where  $e^{\alpha(t,r)}$  and  $R(t)$  are functions to be found from the Einstein equations (with a dust source in this case,  $t$  is a comoving time coordinate). The unusual properties of this geometry result from the function  $R(t)$  being independent of  $r$ . This case appears as one requiring a separate consideration every time when the more ordinary case, with  $R$  being a function of both  $t$  and  $r$ , is discussed (for the ordinary case see Refs. 2–4 and 5 for a review; it leads to the rather well-known Lemaître–Tolman model. The L-T model is discussed in the second section of Ruban’s second paper).

The most surprising result of the two Ruban’s papers is that the solution of Einstein’s equations resulting from the metric (1) can be interpreted as the spacetime of such an object whose active gravitational mass remains constant independently of the amount of matter that it has accreted. The mass added is exactly cancelled by the gravitational mass defect. Details are presented in the papers with sufficient clarity; here the relation of Ruban’s results to other ones found in the literature will be explained.

The solution first appeared in the paper by Datt [1] in 1938, but was instantly dismissed by that author as being “of little physical significance”. It was generalized to nonzero cosmological constant in the second paper reprinted here, then by Korkina and Martinenko [6] to the case of nonconstant pressure, and by Szekeres [7] to a dust solution with no symmetry (see Ref. 5 for more detailed descriptions). Remarkably, the simpler subcase when  $\alpha$  can be made independent of  $r$  (i.e. when  $\alpha(t,r) = f(t) \cdot g(r)$ ) was first identified and discussed only in 1966, it is the now-familiar Kantowski–Sachs [8] dust model. Later, Ruban has generalized the Datt solution to a solution of the Einstein–Maxwell equations with a charged dust source [9], and still later he placed it, also in an illuminating way, in the collection of all spherically symmetric perfect fluid models [10]. This last discussion is usually credited to Misner and Sharp [11], although its basic points were first presented by Lemaître in Ref. 2, and then independently rediscovered by Podurets [12].

The first of the papers reprinted here is basically a short communication that presents the results only. The details are filled in the second paper, and some of the results are generalized and extended. Also the bibliography, somewhat incomplete in the first paper, is extended in the second one. In addition, the second paper (second section) contains a derivation of the dust solutions for the metric (1), with  $R$  being a function of both  $t$  and  $r$ , carried out in such a manner that both the Lemaître–Tolman and the Datt–Ruban solutions emerge and can be compared. The review of properties of the L–T models given there is very insightful and would make a good textbook entry, but, unfortunately, it does not seem to have been appreciated in later literature.

Some of Ruban’s points and terminology may have to be explained:

1. The “T-models” (or “T-regions”) is a name assigned to such spherically

symmetric metrics for which the curvature coordinates (i.e.  $ds^2 = r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)$  on the surfaces  $\{t = \text{const}, r = \text{const}\}$ ) cannot be introduced. Examples are the Schwarzschild solution inside the horizon, the Kantowski–Sachs models [8], and the Nariai solution [13]. Those metrics for which the curvature coordinates do exist were called “R-models” or “R-regions”. The names were quite common in Russian literature of the 1960s.

2. The “trial reference liquid” is the congruence of freely falling observers in the Schwarzschild manifold, used to define the coordinate system that is also known as “Lemaître coordinates” (after Ref. 2) or “Novikov coordinates” (after Ruban’s Ref. 4 in the first paper).
3. The “Tolman solution” that Ruban knew from second-hand citations, was first found by Lemaître in Ref. 2. Lemaître took  $\Lambda \neq 0$  into account, contrary to what the first sentence of the second paper suggests.
4. The “additional special solution” that Ruban mentioned in the second paper (in the paragraph containing his eq. (6)) is the Nariai solution [13]. Ruban credited it to Bondi [4] for reasons unknown; I was not able to find a trace of it in Ref. 4.
5. Contrary to Ruban’s suggestion in the paragraph after his eq. (7), it was Lemaître [2] again who first solved (7) in terms of the Weierstrass elliptic functions. Omer (Ruban’s Ref. 11) discussed the solutions in more detail than Lemaître.

The papers were reprinted from the published American translations, as indicated. Some obvious misprints were corrected without indication. In some places, this Editor found the translation not quite faithful to the original papers. In those places, the translation was corrected, and these corrections, along with less obvious misprints, were marked by footnotes.

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### Short Biography

Vladimir Afanasjevich Ruban was born on 2 December 1937 in the settlement Lozovaya Pavlovka of Kadijevka of Lugansk district, Ukraine, in the family of a miner. In 1955 he graduated from high school with the Silver Medal of the Ukrainian Ministry of Education, and tried to enter the Leningrad Polytechnical Institute, but was not accepted. He stayed in Leningrad (as the town St. Petersburg was then called) and worked as a stoker at the Polytechnical Institute till 1956 when he was accepted to the Physico-Mechanical Department there.

In 1962 Volodya Ruban graduated with honours from the LPI with the specialty “technology of separation and application of isotopes”, and was accepted as a postgraduate student to the Division of Solid State Theory of Theoretical Department of the A. F. Ioffe Institute. In 1965 he became a research assistant in the Division of Solid State Theory. His first scientific publication was about detecting nonstationary distributions of concentration of radioactive gases (1966). Soon after the two papers presented here appeared.

Since 1971 Volodya Ruban worked at the Leningrad Nuclear Physics Institute (created from the Gatchina Branch of the A. F. Ioffe Institute). He carried out research on relativity and on physics of slow neutrons, on scattering and depolarization of neutrons in magnetics, on dynamical diffraction of neutrons, he collaborated with experimentalists working on condensed states physics. The result were 12 publications and two patents—one of them is the neutron filter related to the neutron multilayers optics (supermirrors widely used now).

In 1979 Volodya presented his PhD thesis “Gravitational fields and cosmological models with 3-parametric groups of symmetry”. The Scientific Council of the Nuclear Physics Institute decided to appeal to the High Certifying Committee for permission to present this thesis for the higher degree (corresponding to Dr. *habilitatis*). The decision of the HCC was negative, and the only result of that appeal was a delay in getting his PhD.

In 1981 V. Ruban was promoted to the position of senior research worker. Since then on he concentrated on general relativity and gravitation, but he was still working in the Solid State Theory Division of Nuclear Physics Institute. At those times reduction of staff was frequently discussed and Volodya felt frustrated about doing research in relativity while working at the solid state theory department. This, along with difficult financial and family situation, resulted in a serious heart disease, followed by his death on 6 February 1984 when he was only 46. He was cremated in Leningrad and the ashes were buried in the Nikolskoye-Arkhangelskoye Cemetery in Moscow.

Today at the Solid State Theory Department of St. Petersburg Nuclear Physics Institute there is the big photo of Volodya Ruban, and his colleagues tell about him with great respect and sincere warmth.

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