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Editor's Note

R- and T-Regions in a Spacetime with a Spherically Symmetric Space

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Once there was a time when scientists were not pushed to publish instantly whatever they could in the leading journals. The ISI citation index (and the ISI itself) did not yet exist, so authors were free to choose where to submit their papers. As a result, brilliant papers were occasionally published in inconspicuous local journals where the authors assumed they naturally belonged. Still, somehow the most important results were able to find their way to public knowledge.

Novikov's paper reprinted in this issue is an example. Few people have had the chance to see it (which is one good reason to republish it), and yet most researchers in relativity have heard about the Novikov coordinates for the Schwarzschild solution. These coordinates were defined and discussed in the paper reprinted here.

Some statements and results in the paper need to be related to the remaining literature. The starting point of the paper is the observation that for a general spherically symmetric metric:

$$ds^2 = a(t,r)dt^2 + b(t,r)dtdr + c(t,r)dr^2 + d(t,r)(d\vartheta^2 + \sin^2\vartheta d\phi^2) \quad (1)$$

the "standard" spherical coordinates, in which:

$$b = 0, \qquad d = -r^2, \quad (2)$$

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exist only if the gradient of $\sqrt{-d}$ is a spacelike vector. The points and regions of the spacetime in which this can be done are called R-points and R-regions. If the gradient of $\sqrt{-d}$ is timelike, then $\sqrt{-d}$ may be chosen as the time coordinate. The corresponding points and regions are called T-points and T-regions. These notions were defined in an earlier paper by the same author [1], and Novikov's papers were apparently the first effort to systematically explore the geometry of the R- and T-regions. However, related problems appeared, without being explicitly named, in at least three earlier papers. Datt [2] first found a solution of Einstein's equations with a dust source in which the underlying spacetime is globally a T-region. Datt's solution is uniquely determined by the properties that it is a spherically symmetric dust solution in which the coordinates defined by $t = \sqrt{-d}$ are at the same time comoving. The well-known Kantowski-Sachs solution [3] is a spatially homogeneous limit of the Datt solution. Kantowski and Sachs are rightly credited for exploring the geometry of such spatially homogeneous spacetimes in which the complete symmetry group is 4-dimensional, multiply transitive and has no simply transitive 3-dimensional subgroups (see also Ref. 4). However, the solution of Einstein's equations that they found is a subcase not only of Datt's solution, but also of the Kompaneets–Chernov solution [5], in which the symmetry was the same as in the Kantowski–Sachs model, but the source was a general perfect fluid. The Datt solution was rediscovered, discussed in much detail and generalized (for the cosmological constant and for the electric charge) in a series of papers by Ruban [6–9]. The so far most elaborate generalization of the Datt solution is the Szekeres dust solution [10] that has no symmetry.

The other two papers in which the problem of the T- and R-regions implicitly appeared are those of Nariai [11]. Nariai discussed the vacuum solution with a Λ-term for the case when d is constant, which is an invariant property under all the admissible coordinate transformations in eq. (1). Hence, the Nariai solution is neither an R- nor a T-region. The existence of the Nariai solution does not contradict the remark in Novikov's paper that follows his eq. (2.7). Novikov referred there to the case when the gradient of $\sqrt{-d}$ is a nonzero null vector, and in the Nariai solution this gradient is identically zero. Also, the Nariai solution does not contradict what Novikov says in his sec. 3 because it has no limit $\Lambda \to 0$. As far as this editor is aware, the case of $(\sqrt{-d})_{,\alpha}$ being null but nonzero in an open 4-dimensional region has been investigated only in Ref. 17 [info from M. MacCallum].

Apart from the Schwarzschild solution, discussed in Novikov's paper, and the Datt solution which is a T-region globally, a natural area of application of the definitions of R- and T-regions is the Lemaître–Tolman (L–T) cosmological model that was discussed in many papers (Novikov's Ref. 15, see Ref. 12 below for an overview). In a few other papers, Novikov proved several theorems that apply to the L–T model (see Ref. 12 again). In particular, his result (stated after his eq. (2.3)) that the boundary between the R- and T-regions in the L–T model is given by the equation $R = F$ fits in well with the result by Barnes [13] that the hypersurface $R = F$ is the outer boundary of the region of trapped surfaces.

The "throat" that Novikov discussed at the end of his sec. 5 was also identified and discussed by other authors a few years later, see Refs. 13 and 14 (where the "throat" was named a "neck" and discussed for the L–T model) and Ref. 15 (where it was named "bottleneck" and discussed for a spherically symmetric perfect fluid model).

As Novikov stated in his sec. 9, the coordinates that are initially used there were first introduced by Lemaître [16]. Lemaître found that the Schwarzschild solution is the subcase of the L–T solution corresponding to $F = \text{const}$ and $f = 0$ (in Novikov's notation) and that this limit taken in the L–T model defines such coordinates in which the singularity at the Schwarzschild horizon just disappears. In fact, the condition $f = 0$ is superfluous, the Schwarzschild solution results with $F = m = \text{const}$ with arbitrary form of f. Novikov extended the discussion to the cases $f \neq 0$ and provided a geometrical interpretation in each case. The Lemaître coordinates are comoving coordinates of such freely falling observers in the Schwarzschild spacetime whose velocity at infinity is zero. The other two cases correspond to freely falling observers who never reach infinity (when $f < 0$) and those whose velocity at infinity is still nonzero (when $f > 0$).

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by Andrzej Krasiński

Short Biography

Igor D. Novikov

I was born on November 10, 1935 in Moscow. I got my M.Sc. degree from the Moscow State University in 1959 and Ph.D. degree in 1963.

I worked at the Institute of Applied Mathematics, Moscow; Space Research Institute, Moscow; Lebedev Physical Institute, Moscow as a Research Fellow, a Senior Research Fellow and head of the Department. Simultaneously I worked as a Professor of the Moscow State University and Moscow Pedagogical University. Some of my students have gone on to become Professors in Russia and many other countries.

I visited and gave lectures in many universities around the globe.

In 1991 I left Moscow to take up a Professorship at the Copenhagen University where I continue to research and teach. In 1994 I became Director of the Theoretical Astrophysics Center, Copenhagen.

My researches are mainly devoted to the theory of gravity, physics and astrophysics of black holes, manifestations of relativistic objects in the Universe, and different aspects of Cosmology.

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A more extended scientific biography can be found in ref. 1 below.

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