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Editor's Note: Observable Relations in Relativistic Cosmology. II.†

by W. H. McCrea, *Zeitschrift für Astrophysik* **18 (1939),** 98–115

It is one thing to construct a cosmological model as a solution of Einstein's equations, and quite another thing to express its various properties in terms of relations that can be tested observationally. McCrea was among the first physicists who understood that in order to make itself credible observational cosmology has to go beyond the Robertson - Walker geometry and to begin searching for departures of the real Universe from these highly idealized models. His arguments for considering inhomogeneous models are interesting in themselves. However, the most valuable contribution of this paper is the first-order formula connecting redshift to distance in an arbitrary geometry, eq. (17). The conceptual apparatus and the notation that are commonly used today had not yet been invented in 1939, so eq. (17) is written in a notation that is a bit obscure. In the modern notation it should be written as

$$z = -(\Theta_{\alpha\beta}k^\alpha k^\beta + \dot{u}_\alpha k^\alpha)_o \delta l,$$

where z is the redshift, $\Theta_{\alpha\beta} = \frac{1}{2}(u_{\alpha;\beta} + u_{\beta;\alpha})$, u_α is the velocity field of the cosmic medium, $\dot{u}_\alpha$ is the acceleration of the flow, k^α is the tangent vector to the light-ray normalized so that $k^\rho u_\rho = 1$, δl is the instantaneous spatial distance between the source of light and the observer, and the subscript "o"

† This paper is a sequel to another one, as the first sentence of the text explains. Since the results of the first paper are rather well-known today (even if not in connection with this author's name), we decided to reprint only the second part. It is readable independently.

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denotes evaluation at the observer's position. This equation is equivalent to the better-known formula of Ehlers [1],

$$z = (-\sigma_{\alpha\beta}n^{\alpha}n^{\beta} + \tfrac{1}{3}\theta + n^{\alpha}\dot{u}_{\alpha})_o \delta l,$$

in which $\sigma_{\alpha\beta}$ and θ are the shear and expansion, respectively, of the flow of cosmic medium, and n^{α} is the projection of the vector k^{α} on the instantaneous rest space of the observer, normalized to unit length. Moreover, McCrea calculated the second-order correction to eq. (17) for rotation-free dust[1] [eq. (36)], and indicated how this result may be used (in principle) to estimate the cosmic distribution of matter-density on the basis of observations; this latter result does not seem to have found its way into later literature.

— *Andrzej Krasiński, Associate Editor*

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McCrea: a brief biography

Sir William Hunter McCrea: born 1904 December 13, Dublin. Mathematics undergraduate 1923–26 Cambridge. Mathematical physics postgraduate 1926–28 Cambridge, 1928–29 Göttingen. Lecturer in mathematics 1930–32 Edinburgh, 1932–36 London. Professor of mathematics 1936–44 Belfast, 1944–66 London (Royal Holloway College). Professor of Astronomy 1966–72 Sussex. Since 1972 Emeritus Professor, Sussex. Leaves of absence: 1943–44, London, operations research at the Admiralty, 1952–53, Cambridge, sabbatical year at Gonville and Caius College.

He experienced the early days of quantum mechanics, modern astrophysics, radio-astronomy and relativistic cosmology, being privileged to share in pioneering researches in some of these developments. Some of

[1] The assumption $g_{\alpha 4} = 0$ introduced after eq. (21) implies zero rotation.

his research students up to about 1965 have gone on to become professors in the U.K. and several other countries. From 1965 up to retirement in 1972 he shared in developing the Astronomy Centre in Sussex, where astronomers from many parts of the world have been coming to work for some intervals in their careers.

Further reference in *Who's Who*, published by A.+C. Black.

— Dictated by Sir W. H. McCrea to Isabella Stevens, his daughter.

Acknowledgement

The editor is grateful to J. D. Barrow for his help in contacting Sir William and for other valuable information. More information on the biography of Sir W. H. McCrea can be found in Refs. 1 and 2.

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1. McNally, D. (1995). *Quart. J. Roy. Astr. Soc.* 36, 181.
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