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Editor's Note: The Expanding Universe

by the Abbé Georges Lemaître, *Annales de la Société Scientifique de Bruxelles* A53 **(1933), 51–85**

The paper by Lemaître printed in this issue has suffered a miserable fate on several accounts. Its main result is the derivation of a spherically symmetric dust solution of Einstein's equations that is today known under the name of the "Tolman model" [sometimes "Tolman–Bondi", eqs. (8.1)–(8.3)]. The origin of this misnomer is hard to understand; Tolman [1] made no secret of the fact that the solution he discussed was Lemaître's and quoted Lemaître. (Tolman's paper will be reprinted as the next entry in our series.) We propose naming this solution the "Lemaître–Tolman" (LT) model, to avoid possible confusion with the Friedmann–Lemaître models.

In addition, Lemaître's paper contains a few other results that have not been properly understood in the 1930s.

(i) The formulation of Einstein's equations for spherically symmetric perfect fluids that allows one to define the mass (Section 3). This formulation and the definition of mass were rediscovered much later by Podurets [2] and Misner and Sharp [3] and are usually credited to the latter. Lemaître's approach is in fact more general because he allowed for anisotropic pressure.

(ii) An inspiring discussion, based on the LT model, of the possible mechanism of formation of clusters of galaxies (then called "nebulae", Section 10). Lemaître's proposed explanation (the interplay between the cosmological repulsion and the gravitational attraction) would not be found correct today, but the idea of describing the process within the exact theory was at least 20 years ahead of its time (see the next attempt by Bonnor [4], by a different method, but also on the basis of the LT model; we plan to reprint Bonnor's paper in this series as well).

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(iii) The proof that the Schwarzschild horizon at $r = 2m$ is not a singularity, done by transforming the Schwarzschild solution to other coordinates (Section 11). (The new coordinates are defined by a congruence of freely falling observers, but this interpretation was provided much later by Novikov [5].) Lemaître's idea of removing the spurious singularity by a coordinate transformation was reinvented only at the end of 1950s and finally perfected by Kruskal in 1960 [6]. The Schwarzschild solution came up in this paper because it is a subcase (the vacuum limit) of the LT model, and Lemaître's coordinates emerge automatically in this limit.

(iv) Probably the historically earliest proof that the cosmological singularity persists if the high symmetry assumed in the Friedmann models is somewhat relaxed (Section 12). The model used by Lemaître is a Bianchi type I perfect fluid model in today's terminology.

Readers may discover more little pearls amid the details of this unusual paper.

The various influences of Lemaître's ideas and their interplay with other results were discussed by Eisenstaedt [7]. The importance of Lemaître's results shows up also in the review of inhomogeneous cosmological models [8].

The same paper was also published in a series of internal reports of the University of Louvain [9].[1] The translation printed here was made from the *Ann. Soc. Sci. Bruxelles* version, as indicated.

In view of results found later, the following must be explained:

The Lemaître solution does not exhaust the collection of spherically symmetric dust models. The whole collection splits into two subsets:

(i) The subset found by Lemaître, in which (in Lemaître's notation) necessarily $r,_\chi \neq 0$ (eqs. (3.1) and (8.1)–(8.3) in the paper).

(ii) The subset in which $r,_\chi = 0$. This subset emerges if $r,_\chi = 0$ is assumed from the beginning in the Einstein equations; this limit cannot be taken in the Lemaître solution because then it leads to a globally singular metric. The dust solution with zero cosmological constant in this subset was found by Datt [10] and is an inhomogeneous generalization of the Kantowski–Sachs solution [11]; see also Ref. 8.

The spherically symmetric solution from Section 5 was later generalized by Stephani [12] to a solution with no symmetry.

The Lemaître–Tolman solution (8.1)–(8.3) with $\lambda = 0$ is the spherically symmetric limit of the $\beta' \neq 0$ subfamily of Szekeres' solutions [13]. When $\lambda \neq 0$, it is the spherically symmetric limit with $p = \lambda$ of the $\beta' \neq 0$

[1] Ref. 9 is a report for 1932, but it was published in 1933 and refers to the *Ann. Soc. Sci. Bruxelles* version. Hence, the two versions are approximately simultaneous.

subfamily of the Szafron solutions [14]; see Ref. 8 for details. The solutions of Szekeres and Szafron have in general no symmetry at all.

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— *Andrzej Krasiński, Associate Editor*

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Lemaître: a brief biography

Georges Lemaître was born on 17 July 1894 in Charleroi, Belgium. He obtained two B.A. degrees (in mathematics and in philosophy) from the University of Louvain in 1919, and his Ph.D. degree in mathematics at the same University in 1920. He was ordained a priest in 1923. In the years 1923–1925 he was a research student in Cambridge and at Harvard College Observatory, during which time he submitted his second Ph.D. Thesis in mathematics at M.I.T.

From 1925 to the end of his life Lemaître was based at the Department of Mathematics of the University of Louvain. During his numerous foreign visits (most often to the USA) he collaborated or discussed with most of the leading physicists and astronomers of the period 1930–1960. In 1936 he became a member of the Pontifical Academy of Sciences in the Vatican, and served as its president in the years 1960–1964.

Lemaître is best known for his rediscovery of the Friedmann models, several contributions to relativistic cosmology based on these models, and for the theory of the "primaeval atom" — the first overall theoretical account of the origin and evolution of the Universe (now outdated). However, he was also active in research on cosmic rays (which he assumed to be the radiation from what is today called the Big Bang), mathematical physics, celestial mechanics and automated computing (first mechanical, then electronic). During his lifetime he was successful and highly respected in all of these fields. His complete bibliography includes 101 scientific papers and 11 books.

G. Lemaître died on 20 June 1966 in Louvain and is buried in Charleroi. An extended account of his life and work can be found in Ref. 1.

— *Andrzej Krasiński, Associate Editor*
based on A. Deprit, Ref. 1, p. 363
and O. Godart, Ref. 1, p. 393

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