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COMMENT

Comment on the paper by J. T. Jebsen reprinted in Gen. Rel. Grav. 37, 2253 – 2259 (2005)

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Jebsen [1] anticipated Birkhoff [2] in claiming that spherically symmetric vacuum space-times, possibly with Λ, are static. That statement is wrong, counterexamples are the extension of the Schwarzschild solution into the region behind the horizon and analogous parts of DeSitter and Anti-DS. The mistake in the proof occurs where Jebsen claims that in a spherically symmetric metric

$$\mathrm{d}s^2 = F(r,l)\mathrm{d}r^2 + G(r,l)\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\,\mathrm{d}\varphi^2\right) + H(r,l)\mathrm{d}r\mathrm{d}l + D(r,l)\mathrm{d}l^2,$$

by transforming the (r,l) coordinates, one can achieve $H = 0$ and $G = r^2$. $H = 0$ can indeed be achieved, and in addition one can require $F > 0$, $G > 0$ and $D < 0$. Then, if the gradient of $\sqrt{G}$ is spacelike (respectively timelike), one can put $G = r^2$ (resp. $G = l^2$). If G is constant, no condition at all can be imposed on it; there exists an exact vacuum solution with $\Lambda > 0$ in this class, found by Nariai [3]. If $\sqrt{G}$ has a light-like gradient, no vacuum solution for any Λ exists.

In fact, all solutions of the required kind are known and can be continued to inextendable ones. A corrected version of the theorem states that all spherically symmetric solutions admit, besides the SO(3) generators, an additional hypersurface-orthogonal Killing vector field.

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