Editor's Note: On the Curvature of Space.

by A. Friedman [1]
Zeitschrift für Physik **10, 377-386 (1922).**

On the Possibility of a World with Constant Negative Curvature of Space.

by A. Friedmann
Zeitschrift für Physik **21, 326-332 (1924).**

There is probably not a single relativist who does not know about these two papers, the first published papers on the idea of an expanding and evolving universe. Nevertheless, the number of those who know what the papers actually contain or do not contain is much smaller. Legends have established themselves about what part of the credit belongs to Alexander Friedmann, what part belongs to Georges Lemaître, etc., and false information is being multiplied through citations. Hence, the main purpose of reprinting these not-too-easily accessible papers is to make them more generally available to contemporary physicists and to put some of those persistent legends aside. Also, Friedmann's own presentation has pedagogical value and may be of use in modern courses on relativity.

Friedmann's derivation of the metrics of spatially homogeneous and isotropic spacetimes is not perfectly precise from the mathematical point of

[1] The two mutually inconsistent spellings of the author's name are copied from the original papers. The correct transcription from Russian would in fact be "Fridman", but the form "Friedmann" seems to have become the favourite in the English-language literature. Until experts give a different verdict, this habit will be followed here.

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view. The derivation was made more precise by H. P. Robertson [1,2] and by A. G. Walker [3]; they also discovered other properties of these metrics (e.g. their symmetry and imbedding properties). The model with $k > 0$ was generalized for nonzero pressure and placed in the context of cosmological observations by Lemaître [4,5].[2] Therefore, the general class of metrics is now often called Friedmann-Lemaître-Robertson-Walker (FLRW). The name "Friedmann models" or "Friedmann-Lemaître models" should be reserved for the explicit solutions derived by Friedmann and Lemaître.

Despite their originality and importance, Alexander Friedmann's two pioneering papers were largely ignored by the community of relativists and cosmologists at that time. On first reading, A. Einstein rejected the conclusion of the first paper. He published a note [6] in which he claimed that the result seemed suspicious ("verdächtig") to him and that the true result of the paper should be that the curvature radius of Friedmann's model is constant in time. With considerable trouble and effort, and with the help of his friend Yu. A. Krutkov, Friedmann managed to deliver an explanation to Einstein, a perennial traveler at that time. As a result, Einstein retracted his critique in another note [7]. The whole story is described in more detail in Ref. 8.

The same lack of recognition befell Georges Lemaître when he apparently independently developed these models in 1927 [4], and discussed their astronomical implications (see Ref. 9 for a discussion). It seems that at that time, cosmologists (theoreticians and observers alike) were convinced the universe must be static, and so were engaged in a debate as to whether the Einstein static universe [10] or the de Sitter stationary universe [11] in one of its guises [12] were better models of the universe (see e.g. Hubble, Ref. 13). Consequently they resisted the idea of an expanding universe for about a decade (see Ref. 14).

The situation changed dramatically because of a meeting of the Royal Astronomical Society in 1930 when A. S. Eddington and others started musing whether there could not be expanding solutions [15]. Eddington, the chief astronomical opinion-maker at that time, began to investigate the stability of the Einstein static universe, and Lemaître brought *his* paper [4] to Eddington's attention. Eddington proved the instability of the Einstein static universe [16] and started to publicise Lemaître's work [17] (which he

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2 Reference 5 is supposed to be the English translation of Ref. 4. However, the translation is not perfectly faithful to the original paper. Some of the original footnotes and references were omitted in Ref. 5, some others were added. These modifications may have made Lemaître's paper from 1927 more up-to-date in 1931, but they have harmed the value of the translation as a source for research on history of relativity and cosmology.

had been aware of before, but had forgotten). This resulted in the English translation [5] of Ref. 4 and in initially awarding the credit to Lemaître. Along with both notes by Einstein, Friedmann's first paper is mentioned (as "also discussed") in a footnote to Ref. 5 — the later version.

Thus, one of the major discoveries of twentieth century science, the expansion of the universe, was not acclaimed by the astronomical community of the time; this theoretical discovery was ignored and forgotten until the time became ripe for it to be welcomed [14],[3] even though the basic observational data that would vindicate it was already at hand, albeit in primitive form, since 1924 (see Eddington's tables of redshifts due to Slipher, Ref. 18).

In retrospect, despite the lack of immediate recognition, it is clear that Friedmann can be credited with developing first the now standard idea of the expanding universe, and presenting both the $k = +1$ (positively spatially curved) and $k = -1$ (negatively spatially curved) spatially homogeneous and isotropic dust FLRW universe models. Curiously, neither Friedmann nor Lemaître discovered the geometrically simpler spatially flat models; these were first found by Robertson in a paper [1] that determined all three classes of expanding universe models, but discarded them in favour of static universes.

The two papers reprinted here show that the oversimplified associations "positive spatial curvature $\Leftrightarrow$ closed universe $\Leftrightarrow$ finite lifetime of the universe; negative or zero spatial curvature $\Leftrightarrow$ open universe $\Leftrightarrow$ infinite lifetime", so very common even today, were known to be false already to Friedmann. They are false because other topologies than the obvious ones are possible and because the cosmological constant can strongly influence the dynamics.[4] However, the papers also show that (possibly because he considered only a non-zero cosmological constant λ, but apparently also because of a somewhat strange indifference to the signature of spacetime), Friedmann failed to clearly distinguish the dynamics of the $k = -1$ and $k = +1$ models — indeed his statements on this in the second paper are somewhat misleading; and above all he missed the golden opportunity of calculating the cosmic redshift that would eventually vindicate his expanding universe models. However, he did somewhat cautiously estimate the cycle period of an oscillating universe from a creditable estimate of the

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[3] This is somewhat reminiscent of the fate of Gamow's exploration of the hot big bang idea in collaboration with Alpher and Herman.

[4] Even today people are misled about these issues; one should note that in the inhomogeneous case, there is no relation between the curvature of space, its volume, and the lifetime of the model (see Hellaby and Lake, Ref. 19), while in the homogeneous case such relations hold only if $\lambda = 0$ and we exclude all but the simplest topologies.

mass of matter in the visible universe. It would be interesting to know where his mass estimate came from.

In spite of the dustup with Einstein, Friedmann's papers only slowly gained attention even after the expanding universe idea was accepted. After it was realised the expanding models proposed by Lemaître could explain Hubble's law of redshifts, made public in 1929 [20], hardly anybody remembered that Friedmann in fact first provided such universe models, and Friedmann himself was long dead. The recognition of the true importance of Friedmann's papers emerged gradually later (see Refs. 21 and 22 for more on this process).

However, the progress has not been smooth or universal. Hubble himself kept emphasizing [23] that interpreting redshifts as a consequence of recessional velocities of the "nebulae" is "generally adopted by theoretical investigators", but finds no direct observational confirmation. He was worried that "...rapidly receding nebulae should appear fainter than stationary nebulae at the same distances." (p.122 in Ref. 23), while this effect had not yet been confirmed observationally. In Ref. 24 (his last paper, published posthumously) Hubble's skepticism is somewhat toned down, but expressions like "red-shifts expressed on a scale of velocities" and "if red-shifts do measure the expansion of the Universe" show that even in 1953 he still refused to unequivocally embrace the expansion of the universe as a real phenomenon. R. W. Wilson, the co-discoverer of the relic radiation, admitted [25] that he liked the steady state idea all through his Nobel-winning research up to the moment of submitting the paper for publication. We scientists are supposed to be able to appreciate a clever new idea when we spot it (indeed, this is one of our tasks in society). So far, though, we have been less than perfect in fulfilling this expectation....

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Note added in proof: Russian translations of the Friedmann papers were published in *Uspekhi Fizicheskikh Nauk* **80**, issue no.3, p.439 and 447 (1963), along with Russian translations of the two notes by Einstein, Friedmann's biography and essays on Friedmann's work by V. A. Fok, Ya. B. Zel'dovich, E. M. Lifshitz and I. M. Khalatnikov. A Russian translation of the first paper by Friedmann was also published in 1924 in the *Journal of the Russian Physico-Chemical Society* (*Zhurnal Russkogo Fizichesko-Chimicheskogo Obshchestva*).

Brief biography

Aleksandr Aleksandrovich Fridman was born on 16 July 1888 in Sankt Peterburg.[5] In the years 1906-1910 he studied at the Department of Mathematics and Physics of the St. Petersburg University. He had a postgraduate stipend there in the years 1910-14, and was a collaborator of the geophysical observatory in Pavlovsk (1913-14). The latter position included flights in dirigibles to conduct meteorological experiments.

[5] All names in this sentence are given in the standard transcription from Russian. There is a confusion about Friedmann's birthdate; 16 July is based on the preserved church records, see Ref. 1.

During World War I he was a bomber pilot, compiled tables for aiming bombs and organized the aerial reconnaissance service (1914-16); then he was the commander of the Central Service of Air Navigation and a lecturer at the school of aerial reconnaissance in Kiev (1916-17), and an acting director of a factory of aviation instruments in Moscow (1917-18).

In the years 1918-20 he was a professor of mechanics at the newly organized State University in Perm, and then held several positions at the same time. He was employed at the University of Petrograd, as St. Petersburg was then called (1919-24), at the Institute of Transportation Engineering (1920-25, in the chair of applied aerodynamics), at the Petrograd Technical University (1920-25, at the department of physics and mechanics), at the Naval Academy (1920-25, in the chair of mechanics), at the Atomic Commission of the State Optical Institute (1920-24, calculating models of many-electron atoms) and in the Main Geophysical Observatory (1920-25, as a meteorologist, and later director). In addition, he was the editor of two journals (*Geophysics and meteorology*, 1923-25, and *Climate and Weather*, 1925), and in July 1925 he participated in a balloon flight to the then-(Soviet-)record-breaking altitude of 7400 m (together with P. F. Fedosenko; the flight included experiments on meteorology and biology and medical observations).

Friedmann was an author or co-author of papers on mathematics (his first, written while still a high-school student), hydrodynamics, elasticity theory, electrical and mechanical engineering, approximate calculations and relativity, of six books, two of them on relativity (all in Russian, see a list in Ref. 1), and the editor of the sections on geophysics and meteorology of the Great Soviet Encyclopaedia (1925).

Friedmann died of typhoid fever on 16 September 1925 in Leningrad (as the city of St. Petersburg/Petrograd was then called) and is buried there in the Smolensk cemetery.

Detailed and professionally compiled accounts of Friedmann's life and work can be found in Refs. 1 and 2.

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