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GOLDEN OLDIE EDITORIAL

Editorial note to: Hans Thirring, On the formal analogy between the basic electromagnetic equations and Einstein's gravity equations in first approximation

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1 Technical comments on the Thirring paper

The paper contains some inconsistencies and errors, and an undefined quantity, which, however, does not invalidate the final equations for gravitomagnetism.

Since in a realistic rotating body (angular velocity ω) there arise centrifugal stresses of order ω^2, it is inconsistent to incorporate the velocities $\mathbf{v}'$ of the field-generating body up to second order but to treat this body as incoherent matter (dust). The same inconsistency appeared in Thirring's model of a rotating mass shell [1], which therefore did not correctly solve the Einstein equations (in the shell). For this case the inconsistency was observed and corrected by Lanczos [2]. In the present paper the inconsistency has no severe consequences because the second order terms in $\mathbf{v}'$ anyhow are quite unimportant.

An error in Thirring's paper appears in the integration volume dV_0 which has to be substituted by $dV = dV_0/(dx_4/ds)$. The same error appeared in Thirring's paper [1] on the rotating mass shell, was there observed by M. Laue and W. Pauli, and corrected by Thirring in [3]. But, as with the inconsistency with the incoherent

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matter, the corrections are of second order in $\mathbf{v}'$, and do not really change the decisive gravitomagnetic field equations.

The undefined quantity $\ddot{\mathbf{s}}$ in equation (2a) is, according to equation (2), obviously identical to $d^2\mathbf{x}/dt^2$.

2 The importance of the Thirring paper as the first source of the correct equations of (linear) gravitomagnetism

It is widely unknown that the basic equations of (linear) gravitomagnetism have been correctly derived already in 1918 by Hans Thirring, and the appertaining paper is seldom quoted. Even the extensive presentation of gravitomagnetism in chap.6 of the textbook [4], filling 58 pages, and having 160 references to it, does not quote the Thirring paper.

The idea of a new gravitomagnetic "force" appears vaguely already in 1896 in a booklet by the brothers Friedlaender [5], and then in the title of an Einstein paper from 1912 [6]. More concretely, relativistic field equations for gravitomagnetism appear first in Einstein's speech at the Naturforscher congress 1913 in Vienna [7], based on the preliminary Entwurf theory, where Einstein explicitly says: "The [gravitational] equations correspond largely to those of electrodynamics, ... up to the sign, and ... to a factor 1/2." Thirring surely was decisively stimulated by this Einstein speech which he quotes already in the introduction of his paper. Nevertheless, the straightforward and essentially correct derivation of the fundamental equations of gravitomagnetism, now based on Einstein's final theory general relativity, is an original and important work by Hans Thirring. One may even argue that this paper has more originality than the "famous" papers by Thirring (and Lense) on the rotating mass shell [1] and on the "Lense–Thirring effect" [8] because it is now known (from a Thirring notebook from 1917 [9], evaluated in [10]) that the latter research project started with rudimentary, partly wrong, and physically unrealistic calculations of centrifugal effects (of order ω^2 !) for rotating bodies and mass shells, before an eye-opening letter by Einstein [11] guided Thirring on the right path to consider the gravitomagnetic effects of first order in ω. And still, Thirring was able to derive the "Lense–Thirring effect" in [8] only for points very far off the rotating body, which is not true for the successful modern satellite experiments (see Sect. 3 below). In the present paper Thirring does not evaluate the integrals in his equations (6) and (6a) for concrete matter models; therefore there is no danger that his gravitomagnetic equations (7a) and (2a) would be valid only for special relations between the size of the source and the distance R to the position of the measuring instrument.

Thirring explicitly mentions the different sign and a factor 4 in his gravitomagnetic equations in comparison to electromagnetism (as already Einstein did in his speech [7]). Although he presents no physical interpretation of these differences, it surely was clear to him that the different sign comes from the fact that all (positive!) masses attract each other, whereas charges of equal sign repel each other. Not so evident is whether he was aware of the fact that the factor 4 results from the tensorial character (spin 2) of general relativity, in contrast to the vectorial theory electrodynamics (spin 1 of the photons).

Thirring makes clear (already in the title of his paper) that he considers only the first (weak field) approximation of general relativity, and he further restricts himself to a quasi-stationary gravity field, and to slowly moving source masses and test masses (in the measuring device), which seems to be realized for all foreseeable experiments measuring gravitomagnetism (see Sect. 3 below). It should, however, be mentioned that, like in electromagnetism, the strength of the gravitomagnetic field in relation to the gravitoelectric (quasi-Newtonian) field depends strongly on the velocity of the observer, and that in the literature there are some misunderstandings about which effects are real gravitomagnetic effects, and which are not. In this connection it is reassuring that general relativity provides an invariant expression for gravitomagnetism, namely (in analogy to the "magnetic invariant" $\mathbf{E} \cdot \mathbf{B}$ in electromagnetism) the contraction of the Riemann tensor with its dual: ${}^{*}R \cdot R = R^{\mu\nu\lambda\kappa}\epsilon_{\mu\nu\alpha\beta}R^{\alpha\beta}_{\lambda\kappa}$ (see also [4]), and the characterization ${}^{*}R \cdot R \neq 0$ for real gravitomagnetic effects is of course valid also for strong gravity fields.

The most original and far-sighted idea in the Thirring paper is to be found in its last paragraph. Here Thirring presents the conjecture that electromagnetism and gravity should have analogies not only in the weak field approximation, but that in electromagnetism—like in full, non-linear general relativity—there should show up non-linear effects in the case of extremely strong fields as they exist inside atoms. Thirring mentions earlier such "sketches" by Hilbert and Mie. Surely he refers to the three quite concrete and detailed papers "Foundations of a theory of matter" [12–14] by Gustav Mie where Mie tries to solve "the problem of the electron" by a non-linear extension of the Maxwell theory, conjectures about an analogy between Planck's constant and the basic unit of charge, and treats also gravity. Hilbert's paper "The foundation of physics" [15], famous for its Einstein–Hilbert action of general relativity, contains also some sort of derivation of the generalized equations of electrodynamics (in the sense of Mie) from relativistic gravity. However, these "sketches" by Mie and Hilbert had no lasting significance for the quantum theory of atomic physics. In contrast, the attempt of Born and Infeld [16] to generalize the classical Maxwell theory to a non-linear, relativistic, and gauge invariant theory (the main goal here being a finite self-energy) is of relevance until today, e.g., in string theory. Shortly later, Heisenberg and Euler [17] derived that the phenomenon of vacuum polarization in relativistic quantum electrodynamics necessarily leads to a non-linear extension of Maxwell's theory. Much theoretical progress, and many proposals and calculations for such experimental effects have been performed meanwhile, and the availability of extremely strong laser fields opens now the possibility for experimental tests of some of these non-linear effects. (For an informative recent review see, e.g., [18].)

3 Comments on successful and newly proposed experiments to measure gravitomagnetism

Nowadays the Thirring paper deserves interest and knowledge in the physics community also because finally (roughly 90 years after its theoretical conception!) gravitomagnetism is on the way to be studied and confirmed experimentally.

Before proposing or even setting up an experiment for a new effect, it is of course mandatory to estimate the order of magnitude of this effect. (Thirring obviously did not do this when he originally intended to measure centrifugal effects inside a rotating hollow cylinder [10].) For measuring gravitomagnetism in earth laboratories or with earth satellites, we have on one hand (in units with $G = c = 1$) a factor $M_{earth}/R_{earth} \approx 10^{-9}$ for any deviations from Newtonian gravity. (If the source is not the whole earth but a rotating heavy fly-wheel as in an experiment of the brothers Friedlaender [5], or a rotating hollow cylinder in the experimental project of Thirring, the ratio M/R is ridiculously small.) For rotational effects there comes another factor $\omega_{earth} R_{earth}/c \approx 10^{-6}$, therefore a factor 10^{-15} for any gravitomagnetic field, in comparison to Newtonian gravity. Since, in contrast to the electromagnets, there exist no gravitomagnetic materials in nature, there comes typically another factor $v/c \leq 10^{-5}$ from the velocity v of the rotating parts of the measuring device (except where these are photons or neutrinos). The resulting demand of a total precision of 10^{-20} can presumably not be fulfilled by any laboratory experiment in the foreseeable future, although some groups make attempts in this direction, e.g., with an underground multi-ring-laser gyroscope [19], or with Bose–Einstein condensates in a drop tower [20]. For neutron stars, pulsars, and black holes the numbers for M/R and v/c are of course much more favourable, but there exist usually competing, poorly understood processes near these systems. Furthermore, near neutron stars there exist of course no laboratories or other precisely measurable test systems, and for greater distances from the star the gravitomagnetic field falls off like r^{-3}, as any dipole field does. Therefore also for these astrophysical systems there is not much hope for an unambiguous measurement of gravitomagnetism in the near future [21]. Some time ago it was claimed [22] that the Lense–Thirring effect has already been measured with 0.1 % precision by lunar laser ranging. Meanwhile, however, several authors have convincingly shown (see, e.g., [23], and literature quoted there) that the measured effect is essentially a coordinate effect, and that the gravitomagnetic invariant $^*R \cdot R$ is zero in this case.

Besides these mainly negative prospects for measuring gravitomagnetism there exist recently also positive news from satellite experiments which, apart from the fact that they automatically are quite clean systems in high vacuum and at low temperature, have the advantage that they can run over years and very many revolutions of the satellite, in this way partly compensating the challenging precision demand of 10^{-20}. Ciufolini and coworkers have followed the orbits of the satellites LAGEOS I and LAGEOS II over 11 years, in particular measuring the motion of the nodes due to the gravitomagnetic field of the rotating earth. A special combination of the two data series' together with precise measurements by the satellites CHAMP and GRACE allowed subtraction of the 10^7 times greater node-motions due to the higher earth multipole moments, and, for the first time, confirmation of the Lense–Thirring effect with 10 % precision [24]. The recently launched satellite LARES presumably can improve this precision to 1 % [25]. (The original, ingenious proposal by Ciufolini [26] to launch the satellite LAGEOS II with orbital elements complementary to LAGEOS I, in this way cancelling the multipole contributions, unfortunately was never realized.) The technologically extremely challenging and expensive project Gravity Probe B for measuring the gravitomagnetic Schiff effect (precession of gyroscope axes) was developed over 40 years in Stanford. Due to problems with electric charges on the gyroscopes the originally intended measurement precision of 1 % unfortunately diminished to 19 % [27].

If gravitational waves can be analyzed in detail in the future, this will also be an indirect test for gravitomagnetism, because, similar to electromagnetism, gravitational waves have in equal parts gravitoelectric and gravitomagnetic contributions.

Hans Thirring: a brief biography

By Andrzej Krasiński, translated and abstracted from Ref. [28]

Hans Thirring was born on 23 March 1888 in Vienna. In 1907 he passed the examination for the secondary school certificate at the Sophiengymnasium and began to study mathematics, physics and physical exercise at the Vienna University. Fascinated by the lectures of Fritz Hasenöhrl he made physics his main subject, but managed, in his spare time, to pass the state examination for gym teachers in 1910.

Hasenöhrl employed Thirring as an assistant at the Institute of Theoretical Physics even before he obtained his PhD degree, in October 1910. He held this post unitl 1921. Thirring's promotion to PhD occurred in 1911, on the basis of the thesis entitled "On some thermodynamical relations in the neighbourhood of the critical point and the triple point". One year later he passed the state examination in mathematics and physics. In 1915 he completed his habilitation, on the basis of theoretical investigations of specific heat of crystals. He became interested in this subject under the influence of his friend Erwin Schrödinger. After Hasenöhrl was killed on the front on 7 October 1915, Thirring took over his mechanics lectures for that academic year.

Soon after, Thirring volunteered to work in the military technical committee on the development of electrical military devices. He became involved in this subject in 1914, with the work on selenium photo-cells.

In 1918, his two articles on the now-famous Thirring–Lense effect made him one of the champions of the theory of relativity. In consequence, in 1919 he was employed at the Technical University as a lecturer on physics. In 1920 he gave a course of lectures on mechanics at Vienna University. Then, in 1921 he became Associate Professor of Theoretical Physics at Vienna University and a member of the faculty at the Institute of Theoretical Physics. He became full Professor in 1927.

He had developed his anti-war views already during World War I, and in the 1930s he became increasingly involved in world peace movements. After Austria was incorporated into the German "Reich", Thirring was forcibly furloughed from the University in April 1938, because of "endangering the defence willingness of the German youth" ("Gefährdung des Wehrwillens der deutschen Jugend"), and also in consequence of his personal relations with Einstein and Freud. Shortly thereafter he left his Institute, and in November 1938, at his own request, was retired.

Subsequently, he became scientific advisor in the Elin AG company in Vienna, then from 1942 to 1945 he worked at Siemens & Halske in Vienna. Nevertheless, thanks to the friendly support of his colleague Prof. Ludwig Flamm, his office at the University was at his disposal throughout the Nazi period, and he could work there without being disturbed.

He returned to teaching in the fall of 1945 as a guest professor at the University of Innsbruck, with lectures on "World peace as a psychological problem". In the academic year 1946/47 he became the dean of the Philosophical Faculty at Vienna University.

He also took up his post at the Institute of Theoretical Physics. In 1946 he became a corresponding member of the Austrian Academy of Sciences.

With his real retirement approaching, in 1957, Thirring turned away from the usual bureaucratic routine in the search for his successor. Following the US model, he influenced the dean to survey the twelve most important theoreticians of the German-speaking countries (four Nobel laureates among them) to name the best-qualified candidate for this post among the younger Austrian physicists. Eleven of them named Thirring's son Walter.

From 1957 to 1963 Thirring was a member of the higher chamber of the Austrian Parliament (Bundesrat) as a representative of the Austrian Socialist Party (SPÖ), although he was not a member of it. While there, he formulated the plan of one-sided disarmament of Austria, which became known as the Thirring Plan. It included the dissolution of the army; the borders of the country were to be protected by UN troops. Throughout his life he felt connected to the Pugwash movement. He organised the first Pugwash Conference in Vienna in 1957. For his peace activities Thirring was twice nominated for the Nobel peace prize.

Apart from his professional duties, Thirring was engaged in several other activities. He was a member of, among others, the German-Austrian Alpine Club (D-ÖAV), the Ghibellineo Choir, the German Physical Society, the Chemical-Physical Society, the German Society for Technical Physics, the Paneuropa Union, the Union of German High School Teachers, the German Ski Club, the Biophysical Society for Shortwave Research, the Electrotechnical Union and the American Physical Society.

He died in Vienna on 22 March 1976, one day before his 88th birthday.

In addition to being a scientist, teacher and politician, Thirring was also an inventor. In 1927/28 he constructed a sound-recording movie camera and projector that worked with use of the "Thirring selenium cells". These apparatuses were used until 1938 to produce the Austrian weekly film chronicle and also short movies. He had several patents for his other inventions, 2 in Austria and one in each of Germany, Spain, France and Italy. In 1937 he organised the "First International Congress on Shortwave Research" in Vienna. Being a dedicated skier he constructed the "Thirring coat", famous in its time, a kind of sail spanned between the arms and legs of a skier that gave the illusion of gliding. To accompany this, in 1939 he wrote the book "Der Schwebelauf" ("The gliderun").

Thirring was active as a popularizer of the theory of relativity and of the quantum theory. He himself considered as his most important achievement the book "Homo sapiens. Psychology of human relations", written in 1938 – 1945 (published in 1947, and in short form in 1953, under the title "The art of human coexistence", by the Austrian UNESCO committee). Its subjects were the most common behavioural errors in private life and in politics, and the possibilities of avoiding or correcting them.

Selected publications by Hans Thirring (apart from those mentioned above):

- Über die Wirkung rotierender ferner Massen in der Einsteinschen Gravitationstheorie. *Physikalische Zeitschrift* **19**, 33 (1918).
- Josef Lense and Hans Thirring, Über den Einfluß der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen Gravitationstheorie. *Physikalische Zeitschrift* **19**, 156 (1918).

- Berichtigung zu meiner Arbeit: "Über die Wirkung rotierender Massen in der Einsteinschen Gravitationstheorie". *Physikalische Zeitschrift* **22**, 29 (1921).
- Die Idee der Relativitätstheorie (1921), 3rd edition, corrected and extended: Springer, Vienna 1948; translated among others to English, French, Japanese and Swedish.
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- Der Weltfriede als psychologisches Problem (1946).
- Atomkrieg und Weltpolitik (1948).
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- Der Weg der theoretischen Physik von Newton bis Schrödinger. Springer, Vienna 1962.
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The source for this text [28] credits Ref. [29] as the origin of all information reported here. Detailed biographies of Hans Thirring may be found in Refs. [30,31].

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