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INHOMOGENEOUS COSMOLOGY – WORKSHOP REPORT

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This report is a compilation of contributions sent in by the speakers.

1 Does the Weyl curvature contribute to a classical gravitational entropy? (by Kayll Lake)

The *Weyl curvature hypothesis* [1] is fundamental to the concept of an *inhomogeneous cosmology*. The original formulation requires a zero Weyl tensor at the big bang [2]. Recent evidence suggests that this requirement is too strong. For example, in polytropic perfect fluid spacetimes [3] if the Weyl tensor is zero at the big bang, the spacetime must be *exactly* FRW in a neighbourhood of the big-bang in accord with the FRW conjecture [4]. Nonetheless, the motivation [1] for some form of the Weyl curvature hypothesis remains. We are interested in the hypothesis that there exists a scalar field Ψ , dominated in some sense by the Weyl tensor, such that Ψ is monotone along timelike trajectories. In general, only a restricted class of trajectories can be allowed since, for example, no development of Ψ can be expected along Killing trajectories. (The generalization to homothetic trajectories is known [5].)

The archetype for Ψ is $C_{\alpha\beta}{}^{\gamma\delta}C_{\gamma\delta}{}^{\alpha\beta} \equiv \Phi$ which fails to be monotone even for static perfect fluids. It is interesting to note, however, that this failure occurs above the horizon of a Reissner-Nordström black hole only close to degeneracy. It can be shown that up to order 7 (7 derivatives of the metric tensor) the *only* invariant (except for Φ) that is monotone in Schwarzschild is $\nabla_{\alpha}\nabla_{\beta}C_{\gamma\delta\epsilon\zeta}\nabla^{\alpha}\nabla^{\beta}C^{\gamma\delta\epsilon\zeta}$. Interestingly, this is the order we must *start* at in Petrov type III [6]. This invariant also fails to be monotone very near a Reissner-Nordström black hole only when it is almost degenerate. Indeed it could be used as a precursor to degeneracy.

In terms of the “electric” (E) and “magnetic” (H) components of the Weyl tensor, the failure of Φ ($= 8(E_{\alpha}^{\beta}E_{\beta}^{\alpha} - H_{\alpha}^{\beta}H_{\beta}^{\alpha})$) away from spherical symmetry can be traced to the $-$ sign in the sense that the Bel-Robinson tensor ($= 8(E_{\alpha}^{\beta}E_{\beta}^{\alpha} + H_{\alpha}^{\beta}H_{\beta}^{\alpha})$) is monotone in Kerr. This, of course, requires the inclusion of velocity fields in the definition of Ψ . A study in the Kerr-Newman geometry [7] suggests, in the terminology of Senovilla [8], the following conjecture: Except in the local neighbourhood of a naked singularity, one can always find a timelike trajectory $\ni \Psi > 0$ where

$$\Psi = (\text{super})^3 - \text{energy Weyl tensor.} \quad (1)$$

2 Singularities of inhomogeneous cosmological models (by Peter Szekeres)

The standard model of cosmology, based on the FRW line element has the physically comfortable feature of exhibiting an infinite redshift the closer the emitter is to the

initial "big bang" singularity. This means that although the initial singularity is "naked" in the sense that past directed null geodesics from an observer emanate from the singularity in the finite past, the singularity is in no way harmful to the observer. The most significant consequence of course is that the cosmic black body radiation cools off from its initial infinite temperature and creates no instabilities in space-time at later times.

It has been noted by several authors that this fortunate circumstance does not hold in other typical cosmologies, both homogeneous and inhomogeneous [9, 10, 11]. The singularities of most other cosmologies exhibit infinite blueshifts in at least one direction, a feature which is of some concern both for the possible effect on the subsequent evolution of the spacetime and the black body radiation.

This talk proposes a rigorous definition of what it means for a cosmological singularity to be "redshifted" or "blueshifted" by defining the concept of the *Hubble index* of a singularity. Firstly, given a unit timelike vector field u^μ , define its *Hubble parameter* with respect to the null direction field k^μ to be

$$H = \frac{u_{\mu;\nu} k^\mu k^\nu}{(u_\nu k^\nu)^2}.$$

Then the red-shift along any null geodesic tangent to k^μ is given by Hubble's law,

$$z = H \delta \ell$$

where $\delta \ell = -u_\mu k^\mu \delta s$ is the distance along the null geodesic from any timelike curve tangent to the time-like field.

H defines a real valued function on the sphere bundle of the space-time, whose restriction to the sphere at any point p is called the *Hubble index at p* . Setting $k^\mu(0) = A(u^\mu + e^\mu)$ its even part, called the "Doppler" part is given by $\theta_{\mu\nu} e^\mu e^\nu$ where $\theta_{\mu\nu} = \frac{1}{3}\theta h_{\mu\nu} + \sigma_{\mu\nu}$, while the odd part $\frac{1}{2}(H_p(e) - H_p(-e)) = a_\mu e^\mu$ is the "gravitational" part of the redshift. These concepts fit well with standard interpretations in familiar exact solutions.

Using the definition of a singularity given by Scott and Szekeres [12], a cosmological singularity p is said to be a *past point of u^μ* if there exists a timelike curve γ tangent to u^μ which approaches p with finite proper time parameter t in the past. We say p is an *infinite redshift point* or *benign singularity* of u^μ if

$$H_{\gamma(t)}(e) = v_{\mu;\nu} e^\mu e^\nu \rightarrow \infty$$

along every curve γ tangent to u^μ approaching p and every unit spacelike direction field e orthogonal to u^μ . Every infinite redshift point is an expansion point of u^μ in the sense that $\theta = v^\mu_\mu \rightarrow \infty$ for all $\gamma(t) \rightarrow p$ where $\gamma(t)$ is a tangent curve to u^μ . However the converse is not true in general. p is said to have an *infinite blueshift* (i.e. a "harmful singularity") if there exists a direction field e such that $H_{\gamma(t)}(e) \rightarrow -\infty$.

The singularities of Schwarzschild-Kruskal-Szekeres, Kasner, Heckmann-Schücking, Bondi-Tolman-Szekeres, Belinskii-Khalatnikov-Lifschitz, Eardley-Liang-Sachs, and Diagonal Dust [10] can all be analyzed using this concept.

3 Observations in inhomogeneous universes (by Charles Hellaby)

Can we prove the universe is homogeneous? Homogeneity is normally assumed under the guise of the 'Cosmological Principle'. While we surely don't occupy a special place in the universe, is our region exactly like all other regions? Actually, each deeper survey sooner or later reveals new structures on larger scales. What's on the next scale? One should not assume something that can be proved, and it should soon be possible to demonstrate homogeneity (or lack of it) from observations. There is strong inhomogeneity on all scales up to at least void size, but we have yet to determine the scale on which true homogeneity is found. The next step is to map the inhomogeneity, but source evolution is a serious complication.

Standard Observations: Observations are made on the past null cone — a 3-d slice through 4-d spacetime, so that spatial and temporal variations are merged into a single variation with redshift. The basic observations are: (a) apparent luminosity $\ell(z)$ or angular diameter $\delta(z)$ of sources against redshift z ; (b) number counts of sources (galaxies) at each redshift $n(z)$ (number density in redshift space). From these, we get the diameter distance d_D , luminosity distance d_L , and the density ρ , *IF* we know the true diameters, luminosities and masses of the galaxies. For FLRW models the functions $d_D(z)$, $\rho(z)$ are well known.

Observations in Inhomogeneous Models: MBHE [13] studied these observational relations in the Lemaître-Tolman model. They first specified the inhomogeneity, then located the past null cone, and then calculated expected $d_D(z)$ and $\rho(z)$. Calculations showed that mild inhomogeneities can seriously distort the FLRW functions, and even put loops in $d_D(z)$ & $\rho(z)$ near the maximum in $d_D(z)$.

Source Evolution and Fitting Theorems: Like many comparisons of observations with model predictions, the above assumed the true diameters, luminosities and masses are known and unevolving. But the evolution of the masses, luminosities, and diameters of sources is not well known, even for relatively low z values. In MHE [14] two theorems were proved: (A) Given any observations and any source evolution, an LT model can be found to fit; (B) Given any observations and any LT model, source evolution functions can be found to make them fit. Thus there's no way to separate inhomogeneity and source evolution. Time variation and spatial variation have very similar appearances on the past null cone. A particular example of this for the supernova observations is provided in C [15].

Multi-Colour Observations: Can multi-colour observations help? The problem is that colours can also evolve, i.e. for every new observable there is a source evolution function, and spatial inhomogeneity in intrinsic colours can always create the same effect on the null cone. However, there is one rather simple point that makes a real difference, as presented in H [16]. Gravity affects all colours the same way. The luminosity distance of a source in red must be the same as the blue luminosity distance of the same source. If there is colour evolution of sources, then the ratio of the blue and red absolute luminosities, L_B/L_R , is z dependent, and equal to the ratio of the apparent luminosities, ℓ_B/ℓ_R , because both are divided by the same (unknown) luminosity distance d_L . In practice there are plenty of complications. It is much more realistic to use spectral line intensities for "colours", rather than colour filter magnitudes, as the filter pass bands select different sources

at different redshifts. Different galaxy types may evolve differently, and one must be able to identify the same galaxy type at different stages of evolution. Selection effects may play a role, but because it is the ratio of luminosities that matters here, this is not so serious. The use of galaxy number counts in different colours does not help to separate source evolution from inhomogeneity. See H [16] for a fuller discussion.

CONCLUSIONS Measurements of the ratios of apparent luminosities against redshift put strong constraints on the evolution of absolute luminosities, fixing the ratios, but not the values. This method can provide clear evidence for source colour evolution, even if unknown inhomogeneity is present, and is a useful complement to other methods of studying galaxy evolution. The required observations are suitable for large scale sky surveys — along the lines of Sloan, or for the new generation of large telescopes. Unfortunately we still can't get the inhomogeneity for sure until the source evolution is well known.

4 Lemaitre-Tolman Universes and CMB anisotropies (by Diego Saez)

This contribution is a critical review of a series of papers. In these papers, the Lemaitre-Tolman (LT) solution of Einstein's equations was used to estimate the Cosmic Microwave Background anisotropies produced by an *isolated spherical pressureless nonlinear cosmological inhomogeneity*. In particular, the LT solution was applied to compute the gravitational anisotropy produced by Great Attractor-Like (GAL) structures. The most interesting result [17, 19] was that, *in sufficiently open universes*, some GAL structures located at $Z \sim 5$ produce anisotropies of the order of 10^{-5} with angular scales of a few degrees. These angular scales are those subtended by the gravitational potential (not by the density profile) of the structure. Such scales are of a few degrees when the compensation radius is of a few hundreds of Megaparsec. Although these radii are greater than the compensation radii of the observed Great Attractor, they are not forbidden either by theoretical arguments or by observational data.

Recent observations of the CMB (BOOMERANG and MAXIMA) support a flat universe and, furthermore, the $m = m(Z)$ curves obtained from I_a supernovae strongly suggest a dominant cosmological constant with $\Omega_\Lambda \sim 0.7$; therefore, the main conclusion described above (valid in a sufficiently open universe, with $\Omega \sim 0.2 - 0.3$), reduces to a theoretical result without practical application. In spite of this fact, many valuable conclusions have been obtained in the mentioned series of papers; for example, the so-called Potential Approximation (PA) was used [18] to compute the anisotropies produced by GAL structures having compensation radius of a few hundred Megaparsecs and evolving in very open universes; afterwards, the results were compared with those given by the exact LT solution; this comparison showed that the PA — although marginally applicable to the chosen case — led to significant deviations with respect to LT results. These deviations appeared because, in the open case, the PA only applies in the case of sufficiently small compensated structures.

In the future, the PA seems to be preferable over the use of the LT solution. It is due to the fact that the Universe seems to be a flat one with a significant

cosmological constant and, for $\Omega_\lambda \neq 0$, the LT solution involves complicated Weierstrass functions, while the PA works very well because this approximation was designed to apply in any flat universe.

4.1 Discussion

M. N. Célérier: 1. The published Boomerang and Maxima results are currently analysed assuming that our universe is a pure FLRW solution, and we know from a number of works on large scale inhomogeneities and averaging that this assumption can lead to wrong conclusions. 2. Even with this oversimplifying FLRW assumption, the error bars of the published data are such that the value of Ω has been claimed to be unity with only a 12% precision (see e.g. P. de Bernardis et al., *Nature* 2000, 404, 955), which does not definitely rule out Saez' assumption of an open background.

D. Saez: Even if an accelerated expansion is not confirmed because the weak luminosity of the observed supernovae admits some alternative interpretation, the position of the Doppler peak – rather well established – corresponds to an almost flat universe and my opinion is that this fact strongly reduces the interest of the LT solution.^a

5 A Delayed Big-Bang LTB model solving the horizon problem without inflation (by Marie-Noëlle Célérier)

Lemaître-Tolman-Bondi (LTB) models are solutions of Einstein's equations for a spatially spherically symmetric space-time with dust (pressureless perfect gas) as the source of gravitational energy. A flat LTB subclass with a non simultaneous Bang time of "delayed" type is shown to be an interesting candidate to account for the matter dominated area of our universe, as it solves the horizon problem without need for any inflationary phase [20, 21, 22].

The shell-crossing surface, which can be considered as the physical singularity of the model, exhibits a null character which forces in effect all matter to have been causally connected at the singularity, and any finite region to have been correlated at some positive "cosmic" time after the singularity.

Contrary to inflation, which solves the horizon problem only temporarily on the observer's world line, the Delayed Big-Bang (DBB) model yields a solution for any location of the observer in the whole space-time region situated "above" the last-scattering surface.

The parameters of the model can be chosen to be the comoving shell where the observer is located and the shape of the monotonically increasing Big-Bang function. The large scale anisotropies measured in the microwave background radiation temperature can be reproduced in this model and used to select allowed regions in its parameter space. The farther the observer is from the symmetry center, the closer our neighbouring universe is from a homogeneous pattern (corresponding to

^aM. N. Celerier stands by her opinion. Lack of time in Rome and lack of space in this volume have not allowed for an exhausting discussion of this point.

a flat Big-Bang function). It is interesting to note that these properties hold for any DBB universe arbitrarily locally close to a standard FLRW universe.

Even if some of the assumptions retained for simplification sake can be discussed (the dust approximation, the spatial spherical symmetry, ...), these preliminary results emphasize that a “non simultaneous Bang time” is a fruitful idea worth to be further explored.

6 Cosmological singularities (by Jose Maria Martin Senovilla)

Do we really know what is a cosmological singularity? Which are the properties defining it? The simplest definition would be to call “cosmological” any singularity appearing in a *cosmological model*, but then again there is no generally accepted definition of cosmological model. Some attempts to give such a definition can be found in [23]. Furthermore, perhaps this is not the concept one wishes to capture.

Thus, we usually rely on “intuition”, using for inspiration the Friedman-Lemaître-Robertson-Walker (FLRW) models and some of their spatially homogeneous generalizations. In this sense, apart from the traditional schemes (pancakes, cigars, point, mixmaster...) for singularities, which are valid for *any* type of singularity not necessarily cosmological, some important tries involved the *Weyl tensor hypothesis* (see Lake’s contribution) such as the so-called *isotropic singularities*. Unfortunately, it has usually been required that these singularities be *spacelike*, which misses several important simple cases, including some FLRW models themselves. Many FLRW models have *null* big-bang singularities [23] (by the way, this shows that to avoid the horizon problem one does not need to resort to the null singularities of inhomogeneous models, see Celerier’s contribution). Some examples are given by the spatially flat FLRW models with $p = \gamma\rho$ and $-1 < \gamma \leq -1/3$, see [23]. Notice that all of them satisfy the Dominant Energy Condition, and that the extreme case $\gamma = -1/3$ satisfies the Strong Energy Condition too.

Several tries to capture the notion of cosmological singularity have been recently put forward. One is due to Szekeres, see his contribution to this workshop. Another one was explained in [23] and will be analyzed now. The intuitive idea one wishes to keep for a cosmological singularity is that it gives birth to *everything*, so that it must be a kind of *singular* Cauchy hypersurface for the spacetime. Therefore:

Definition [23]. A singularity set S relative to a singular extension is called a cosmological, (also initial, or big-bang) singularity if every past-endless causal curve approaches S at a finite generalized affine parameter. A similar definition can be used for a big-crunch.

All past singularities of the FLRW models are cosmological according to this definition. Nevertheless: 1) many singularities may look like cosmological but they are not [23]. 2) The cosmological singularities need not be in the past of the whole universe and, in fact, they can be partly to the *future of the universe*, see e.g. [24]. 3) They can be certainly null, and they do not even need to be achronal [23, 24]. 4) Some physical particles may have its future end in the past region of the cosmological singularities [24]. 5) Big-crunches may be partly naked [24].

In my opinion, all these as well as many other surprising facts are inherent to cosmological singularities and they cannot be avoided by a better definition unless many truly cosmological ones are missed.

7 Locally discretely symmetric space-times (by Filipe C. Mena)

Schmidt showed that if R_{abcd} and its first three covariant derivatives are invariant under the discrete groups arising in class A Bianchi metrics, then there is a local group of isometries G_3 acting on 3-surfaces orthogonal to the (perfect) fluid vector flow. The discrete symmetries can be observable, since they leave invariant the null-geodesics equation, and therefore can be used to constrain cosmological models.

In a joint work with Malcolm A. H. MacCallum [25], we have generalised Schmidt's result by considering other discrete isotropy groups. We do not assume the dynamics of general relativity or any particular source-field, therefore our arguments are purely geometric. The discrete symmetries considered act at each point of a space-time neighbourhood and form groups of reflections with respect to an orthonormal tetrad. We call spacetimes with such symmetries *locally discretely isotropic* or *LDI*.

In this talk, we have considered only the case where the discrete symmetries act along each pair of the three spatial directions and form a group called H . The first derivative of the Riemann tensor was assumed to be invariant under H . We showed that continuous groups of isometries will be induced by the discrete symmetry groups. The proposition below is a generalisation of the old result of Robertson and Walker:

Proposition: Consider a space-time with a metric of class C^4 . If R_{abcd} and $R_{abcd;e}$ are invariant under the *LDI* group H then there is a local group G_3 of isometries acting transitively on the space-like hypersurfaces orthogonal to the time-like vector preserved by H .

8 General properties of Bel currents (by Ruth Lazkoz, Jose M. M. Senovilla and Raul Vera, speaker: R. Vera)

A consequence of the Principle of Equivalence is that there is no proper definition of *local gravitational energy-momentum tensor* constructed from the metric and its first derivatives. Nevertheless, there exist *local* tensors describing the *strength* of gravity. The outstanding example is the Bel–Robinson (B-R) tensor, a four-index tensor constructed for vacuum spacetimes whose properties are similar to that of energy-momentum (e-m) tensors, including the zero divergence. This has led to some work on B-R-like—also called super-energy (s-e)—tensors with the idea of finding, for the rest of the physical fields, objects analogous to, and related with, the B-R tensor. A purely algebraic construction of these s-e tensors for arbitrary fields was presented in [8], and includes the usual Bel tensor, which generalizes the B-R tensor for non-vacuum spacetimes. Contracting with Killing vectors one constructs some ‘Bel currents’ which are not divergence-free in general (the matter acts as source), so that they may lead to the interchange of “s-e” between the gravitational and other physical fields. This possibility is analyzed in [8] using as

inspiration the mixed divergence-free currents (which are not conserved separately) traditionally found by adding the e-m tensors describing fields in interaction. The outcome is that there are indeed interchange and conservation of s-e quantities in the cases of minimal coupling with scalar or electromagnetic fields. This s-e interchange has been explicitly found in spacetimes admitting a non-orthogonally transitive G_2 group of motions [26]. In simpler cases, the Bel currents are shown to be conserved automatically depending on some geometrical properties of the Killing vectors used in their construction, independently of the matter content [26]. These properties are analogous to some very well-known statements concerning the e-m or Ricci tensors.

9 Conformal factors of generalized cosmologies (by Alicia Herrero and J. A. Morales, speaker: A. Herrero)

We analyzed the kinematic properties of timelike *radial conformal Killing fields* (RCKF) in a conformally flat space-time and used them to obtain the form of some conformal factors [27, 28].

Firstly, Robertson-Walker metrics are characterized as those conformally flat space-times that admit a *geodesic* timelike RCKF. Their conformal factor, already obtained in 1945 by Infeld and Schild, is recovered using a different method. Next, the condition of homogeneous expansion was also analyzed, showing that it is equivalent to consider that the acceleration of the RCKF is Fermi-Walker propagated along the field. And finally, we discussed the case of a RCKF with orthogonal surfaces of constant sectional curvature and the conformally flat space-times obtained from this condition, which belong to the Stephani universes for non-vanishing but homogeneous expansion.

So, we presented a method to obtain conformal factors and interpret them from a kinematic point of view. These results could be useful when considering physical and geometrical interpretations of generalized conformally flat cosmologies.

10 On the inhomogeneous cosmologies in presence of a massless scalar field (by Giovanni Montani)

We analyse some dynamical aspects concerning the asymptotic evolution toward the cosmological singularity characterizing inhomogeneous universes containing a massless scalar field. More precisely, we show how for the case of a "quasi-isotropic solution" containing a massless scalar field, the ultrarelativistic matter and the electromagnetic field [29, 30] play an equivalent dynamical role, characterized by an arbitrary spatial distribution of their energy densities. Indeed, when the presence of a massless scalar field is implemented in the framework of a pre-inflationary scenario, the possibility to have a quasi-isotropic space containing an inhomogeneous distribution of energy can lead to interesting cosmological implications on the very large scale structure of the universe. We also develop a detailed analysis, up to the first order in a perturbative approach, concerning the asymptotic behaviour of the generic cosmological solution in presence of a massless scalar field and ultrarelativistic matter [31].

We show that, during a Kasner-like regime the energy density retains the same form as in the absence of a scalar field, so outlining the weak coupling existing, in the generic inhomogeneous cosmologies, between these two physical fields. Even this result finds its natural implementation in a generic pre-inflationary scenario.

11 Inhomogeneity and nonlinear preheating (by Matthew Parry)

This is a summary of work carried out with Richard Easther (Brown University).

Figuratively speaking, the inflation supermarket generally only sells frozen universes! It is therefore crucial to understand how the universe is heated up after inflation. In some models this can occur in a period called **preheating**.

We investigated the possibility that nonlinear gravitational effects influence the preheating era. Our work is based on numerical solutions of the inhomogeneous Einstein equations, and is free of perturbative approximations. The restriction we imposed is to limit the inhomogeneity to a single spatial direction. We compared our results to perturbative calculations and to solutions of the nonlinear field equations in a rigid (unperturbed) spacetime, in order to isolate gravitational phenomena.

We confirmed the broad picture of preheating obtained from the nonlinear field equations in a rigid background, but found gravitational effects have a measurable impact on the dynamics. The longest modes in the simulation grow much more rapidly in the relativistic calculation than with a rigid background. Indeed this inverse cascade of power to long wavelengths may rule out some models of preheating, and highlights the limitations of the perturbative approximation for describing the long-term behaviour in these models. We used the Weyl tensor to quantify the departure from homogeneity in the universe.

Finally, we saw no evidence for the sort of gravitational collapse associated with the formation of primordial black holes.

12 Embedding a black hole in an isotropic universe (by Brien Nolan)

We address the problem of finding a solution of Einstein's field equations which represents the embedding of the Schwarzschild field into the isotropic Robertson-Walker space-times, and of studying the global structure of the resulting solution. This problem is motivated by the question of whether the cosmic expansion has an effect on local systems. McVittie [32] found a solution of the field equations with shear-free perfect fluid source which for (R-W curvature index) $k = 0$ is the desired solution. However a study of the asymptotic behaviour of the $k \neq 0$ solutions shows that they do not represent the embedding of the Schwarzschild field [33]. We approach this problem by giving *a priori* a set of conditions which, if satisfied by a space-time, we claim represents the embedding of the Schwarzschild field in an isotropic background. We provide an existence and uniqueness theorem for solutions of the field equations subject to these conditions (spherical symmetry, shear-free perfect fluid source, a covariant condition on the fall-off of the metric at spatial infinity and a condition on the Misner-Sharp mass of the space-time) [33]. The Penrose-Carter conformal diagram for the $k = 0$ solution is derived in [34]. One of the most interesting aspects of this space-time is that the event horizon $r = 2m$

of the Schwarzschild geometry develops a curvature singularity when embedded in the R-W geometry; this singularity is shown to be non-destructive [35].

References

- [1] R. Penrose, *The Emperor's New Mind* (Oxford University Press, Oxford, 1990).
- [2] T. Rothman, *Gen. Rel. Grav.* **32**, 1185-1196 (2000).
- [3] K. Anguige and K. P. Tod *Ann. Phys.* **276**, 257-293 (1999).
- [4] S. M. Scott and G. Ericksson in *Proc. Int. Sem. Math. Cosmology, Potsdam*, ed. M. Rainer, H.-J. Schmidt, (World Scientific, Singapore, 1998).
- [5] N. Pelavas and K. Lake, *Phys. Rev. D* **62**, 044009 1-5 (2000).
- [6] V. Pravda, *Class. Quant. Grav.* **16**, 3321-3326 (1999).
- [7] N. Pelavas, N. Neary and K. Lake 2000 (in preparation).
- [8] J. Senovilla *Class. Quant. Grav.* **17**, 2799-2842 (2000).
- [9] D.M. Eardley, *Phys. Rev. Lett.* **33**, 442 (1974).
- [10] P. Szekeres, in *A Random Walk in Relativity and Cosmology: Essays in honour of P.C. Vaidya and A.K. Raychaudhuri*, ed. N. Dadhich, J. Krishna Rao, J.V. Narlikar, and C.V. Vishveshvara .(Wiley Eastern Ltd, New Delhi, 1985, p. 211).
- [11] C. Hellaby and K. Lake, *Astrophys. J.* **282**, 1 (1984).
- [12] S.M. Scott and P. Szekeres, *J. Geom. Phys.* **13**, 223 (1994).
- [13] N. Mustapha, B. Bassett, C. Hellaby and G. Ellis, *Class. Quant. Grav.* **15**, 2363-2379 (1998).
- [14] N. Mustapha, C. Hellaby and G. Ellis, *Mon. Not. Roy. Astr. Soc.* **292**, 817-830 (1997).
- [15] M. N. Celerier, astro-ph/99 07 206; *Astron. Astrophys.* **353**, 63 (2000).
- [16] C. Hellaby, *Astron. Astrophys.* (submitted, 2000).
- [17] J.V. Arnau, M.J. Fullana, and D. Sáez, *Mon. Not. Roy. Astr. Soc.* **268**, L17 (1994).
- [18] V. Quilis, and D. Sáez, *Mon. Not. Roy. Astr. Soc.* **293**, 306 (1998).
- [19] M.J. Fullana, J.V. Arnau, and D. Sáez, *Vistas in Astronomy* **41**, 467 (1998).
- [20] M.N. Célérrier and J. Schneider, *Phys. Lett. A* **249**, 37 (1998).
- [21] J. Schneider and M.N. Célérrier, *Astron. Astrophys.* **348**, 25 (1999).
- [22] M.N. Célérrier, *Astron. Astrophys.* (2000), in press, gr-qc/0009031.
- [23] J. M. M. Senovilla, *Gen. Rel. Grav.* **30**, 701 (1998).
- [24] J. M. M. Senovilla, R Vera, *Class. Quant. Grav.* **17**, 2843 (2000).
- [25] F. C. Mena and M. A. H. MacCallum, *Locally discretely isotropic space-times*, in preparation.
- [26] R. Lazkoz, J. M. M. Senovilla and R. Vera, *Preprint in preparation* (2000).
- [27] A. Herrero and J.A. Morales, *J. Math. Phys.* **40**, 3499 (1999).
- [28] A. Herrero and J.A. Morales, *J. Math. Phys.* **41**, 4765 (2000).
- [29] G. Montani, *Class. Quant. Grav.* **16**, 723 (1999).
- [30] G. Montani, *Class. Quant. Grav.* **17**, 2197 (2000).
- [31] G. Montani, *Class. Quant. Grav.* **17**, 2205 (2000).
- [32] G. C. McVittie, *Mon. Not. Roy. Astr. Soc.* **93**, 325 (1933).

- [33] B. C. Nolan *Phys. Rev. D* **58**, 064006 (1998).
- [34] B. C. Nolan, *Class. Quant. Grav.* **16**, 1227 (1999).
- [35] B. C. Nolan, *Class. Quant. Grav.* **16**, 3138 (1999).