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Editors

A. Molina & J. M. M. Senovilla

Departament de Física Fonamental,
Universitat de Barcelona, Spain

World Scientific
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PHYSICS IN AN INHOMOGENEOUS UNIVERSE

ANDRZEJ KRASIŃSKI
N. Copernicus Astronomical Center, Polish Academy of Sciences
ul. Bartycka 18, 00 716 Warszawa, Poland

ABSTRACT

The paper presents selected applications of the Lemaître–Tolman cosmological model to describing observed or observable phenomena in the real Universe. The following effects are described: 1. Formation of voids; 2. The spatial variability of the curvature index; 3. Formation of black holes in a nonempty expanding background; 4. Formation of naked singularities; 5. The influence of cosmic expansion on planetary orbits; 6. The influence of electric charge on the expansion and collapse; 7. The influence of inhomogeneities in the matter distribution on the cosmic microwave background radiation; 8. Formation of galaxies. In addition, the problem of deriving field equations appropriate for cosmology by averaging the Einstein equations is briefly discussed.

1. Introduction

Ever since the 1930-ies, students of physics and astronomy have been taught that our Universe is really like the Friedmann–Lemaître–Robertson–Walker (FLRW) models describe it to be: perfectly isotropic and homogeneous on some large (but never explicitly defined) scale. The truth is that such a model of the Universe was the simplest one that did not overly contradict observations. However, its assumed very high symmetry was not a reflection of man's knowledge about the Universe (even though it was repeatedly claimed to be), but on the contrary —the assumption was acceptable because of a nearly total lack of knowledge. The only observations to support it were the very sparse and inexact observations by Hubble which implied that other galaxies tend to recede from our Galaxy. The "Hubble law" was a very far-reaching extrapolation thereof, and in fact the value of the Hubble parameter is still a subject of controversy.

There were physicists who could see already in the 1930-ies that the real Universe just cannot be so simple. They were not taken seriously by the majority of the astronomical community, but their ideas were picked up and developed further by a small group of dedicated free thinkers. The papers written by them, still little known and underappreciated, contain a wealth of knowledge that in fact has already created a new branch of cosmology. This article presents a small selection of material from those papers. It is based on a broader review, just prepared by this author[17].

For brevity, only one class of models will be invoked here (see sec.2). It is a family of solutions of the Einstein equations found by Lemaître[19] which is better known under the name of the "Tolman–Bondi" model. It was used to describe the following processes, potentially or actually observable:

1. Formation of voids in the Universe.

2. Formation of black holes in a nonempty expanding background.
3. Formation of naked singularities.
4. The influence of cosmic expansion on planetary orbits.
5. The influence of electric charge on the expansion and collapse of a matter distribution.
6. The influence of inhomogeneities in the matter distribution on the cosmic microwave background radiation.
7. Formation of galaxies out of fluctuations of density in a homogeneous background.

These phenomena will be briefly described in the following sections.

In addition, the problem of deriving the field equations appropriate for cosmology by averaging the Einstein equations will be described in a separate section. This is a new, just emerging, area of investigation that is of great importance for cosmology.

The main point that this author hopes to make with this article is the following: a solid body of evidence already exists that our Universe is not homogeneous, and theoretical tools for cosmology in an inhomogeneous Universe, based on the exact Einstein theory, are already in place. It is time to stop repeating the old tale how wonderfully isotropic and homogeneous our Universe is and face the reality.

2. The Lemaître–Tolman model

Since the FLRW models did have some success in describing the evolution of the real Universe, and are generally believed to be the correct first approximation to relativistic cosmology, it is reasonable to consider such more general models that include the FLRW ones as a homogeneous limit. (Not all researchers in relativity agree with this, and the literature on models having no FLRW limit is in fact quite abundant). There are two natural ways of generalizing the isotropic and homogeneous FLRW models:

1. To abandon isotropy and consider spatially homogeneous anisotropic models.
2. To abandon homogeneity and consider models that are spherically symmetric with respect to a single observer in the Universe.

Generalizations of the first kind were considered in a very large number of papers, but they are of rather limited use in cosmology. Matter density and most other cosmologically relevant quantities still depend only on comoving time in such models. This does not allow for considering formation and evolution of the observed structures.

A generalization of the second kind will be discussed in this article. However, readers must be aware that still more general models are known. The most remarkable among them are the solutions of Szekeres[33], they are neither homogeneous nor isotropic (in fact they have no symmetry at all). The Lemaître–Tolman model described below is the spherically symmetric limit of one of the classes found by Szekeres. The literature on the Szekeres models is already quite extended, and some of it is cosmologically relevant (see Ref.17 for a detailed account).

The Lemaître–Tolman (L-T) model emerges from the following assumptions:

1. The spacetime is spherically symmetric.

2. The source in the Einstein equations is dust.
The solutions of the Einstein equations obtained under these assumptions split into two families. One of them is a generalization of the Kantowski–Sachs[15] solutions; it was first found by Datt[5] in the case of zero cosmological constant and generalized for $\Lambda \neq 0$ by Ruban[27]. These solutions were discussed in a number of interesting papers, but they seem to be less important for cosmology, and therefore will not be described here (see Ref.17).

The other family was first discovered by Lemaître[19], and then interpreted physically by Tolman[34] and Bondi[3]. It has the metric:

$$ds^2 = dt^2 - \frac{R_{,r}^2\, dr^2}{1+2E(r)} - R^2(t,r)(d\vartheta^2 + \sin^2\vartheta d\phi^2), \tag{1}$$

where $R(t,r)$ is determined by:

$$R_{,t}^2 = 2E(r) + 2M(r)/R + \frac{1}{3}\Lambda R^2, \tag{2}$$

The functions $E(r)$ and $M(r)$ are arbitrary, they arise as "constants of integration" from the Einstein equtations, and Λ is the cosmological constant. The coordinates of (1) are comoving, i.e. the velocity field has the form $u^\alpha = \delta_0^\alpha$. By inspection of eq. (2) with $\Lambda = 0$ it may be recognized that $E(r)$ should be interpreted as the total energy of the spherical shell with comoving coordinate r, and $M(r)$ —as the active gravitational mass enclosed within $r =$ const. The mass-density ϵ corresponding to (1) is given by:

$$\frac{8\pi G\epsilon}{c^4} = 2M_{,r}/(R^2 R_{,r}). \tag{3}$$

(Note that the case $R_{,r} = 0$ is a singular limit of the solution (1)-(3). However, if $R_{,r} = 0$ is assumed from the beginning and substituted into the Einstein equations together with an arbitrary component $g_{rr}(t,r)$ of the metric, then a well-defined solution results —it is just the Datt–Ruban solution mentioned above).

Equation (2) has the same form as the equation defining the FLRW scale factor in the case of dust. Indeed, the FLRW dust models are contained in (1)–(3) as the special case:

$$\begin{aligned} 2E(r) &= -kr^2, \\ M(r) &= M_o r^3, \\ R(t,r) &= rS(t), \end{aligned} \tag{4}$$

where k is the FLRW curvature index, M_o is the FLRW mass integral for dust, and $S(t)$ is the FLRW scale factor.

Just as in the FLRW limit, with $\Lambda = 0$ the equation (2) can be explicitly solved in terms of elementary functions. When $E(r) < 0$ (i.e. with the limiting $k > 0$), the

solution is given in the parametric form as:

$$R(t,r) = -\frac{M(r)}{2E(r)}(1-\cos\eta),$$
$$\eta - \sin\eta = \frac{(-2E)^{3/2}}{M(r)}[t - t_o(r)], \tag{5}$$

where $t_o(r)$ is a new arbitrary function called the Bang time. When $E(r) \equiv 0$, the solution of (2) is:

$$R(t,r) = \{\frac{9}{2}M(r)[t-t_o(r)]^2\}^{1/3}, \tag{6}$$

and when $E(r) > 0$, it is:

$$R(t,r) = \frac{M}{2E}(\cosh\eta - 1),$$
$$\sinh\eta - \eta = \frac{(2E)^{3/2}}{M}[t - t_o(r)]. \tag{7}$$

It can be seen from (5)–(7) that the "age of the Universe" is not a global parameter in this model: in the synchronous comoving time, different parts of the Universe may have different ages. The initial singularity is in general not a single event in spacetime, but a process extended in time. This gives rise to the question: can we now see those parts of the Universe that lagged behind with the Big Bang as they emerge from it? This question was addressed in a few papers (see Ref.17), but since no conclusive answer was formulated, we shall not deal with it here.

Depending on the detailed shapes of the arbitrary functions $E(r)$, $M(r)$ and $t_o(r)$, a variety of phenomena may occur, and they will be discussed in the next sections.

3. Formation of voids

For simplicity, let us take $E(r) = 0 = \Lambda$ so that (6) applies, and then let us ask the question: where are the maxima and minima of the mass-density (3) at a constant t? The positions of those extrema (if they exist) are solutions of the equation:

$$\epsilon_{,r}\,|_{t=\text{const}} = 0, \tag{8}$$

and in the case assumed this is equivalent to:

$$(-MM_{,rr}\,t_{o,r} + 2M_{,r}^2\,t_{o,r} + MM_{,r}\,t_{o,rr})(t-t_o) - MM_{,r}\,t_{o,r}^2 = 0. \tag{9}$$

As is easily seen, a generic solution of this equation, $r = r_m(t)$, will depend on time. Since the coordinate r is comoving, this means that maxima and minima of density are in general not comoving —they travel across the flow-lines of matter as density waves. Solutions of (9) are time-independent only in two cases:

1. When $t_{o,r} = 0$, i.e. when the initial singularity is simultaneous.

2. When $M_{,r} = 0$, in which case the L-T model degenerates to a vacuum solution. This is of course the Schwarzschild solution, represented in coordinates defined by freely falling observers (these coordinates were, by the way, first introduced by Lemaître in the same paper[19]).

Such density waves are present also when $E(r) \neq 0$. They are responsible for the formation of structures that can be interpreted as voids or galaxy clusters. That voids should be expected to form was predicted in casual remarks by Tolman[34] and Bondi[3], and in an almost explicit form by Sen[29,30]. However, astronomers knew better until voids were actually observed in late 1970-ies and announced as a surprising discovery. The L-T model was used to describe the evolution of voids in a number of papers (see Ref.16 for a complete bibliography and Ref.17 for a summary of their contents). The most complete such investigation was done by Sato and coworkers and summarized by Sato[28]. The models they used were evolved from initial density distributions in which a homogeneous region around the center of symmetry was surrounded by an L-T transition zone with the density slowly growing with r, and this in turn was surrounded by another homogeneous region of somewhat higher density. The whole configuration is in fact a single L-T model with suitably chosen $M(r), E(r)$ and $t_o(r)$. The main results of the investigation were these:

1. A void existing in the initial moment will expand faster than the homogeneous matter surrounding it. It will form and push outwards an envelope of higher (and growing with time) mass-density. In an L-T model, because of absence of pressure gradients, the envelope will become a shell-crossing singularity (see further) in a finite time. Since the background is homogeneous, one may consider several voids in it simultaneously. This model applies only until the envelopes of the voids begin to collide. However, one can conjecture that the colliding envelopes will flatten against each other and form the "cellular structure" that seems to be observed.

2. In the Newtonian limit, the equation of motion of the void's edge obeys the Sedov equation of propagation of explosive waves. Hence, the expansion of a void can be compared to an explosion.

3. Perturbative studies in the Newtonian limit imply that, unlike condensations, nonspherical voids tend to become spherical during the evolution. Hence, spherical symmetry of a void is a stable property, and so the L-T model is appropriate to describe voids.

4. The spatial variability of the curvature index

In the FLRW dust models with $\Lambda = 0$, the sign of k determines the future fate of the whole Universe: when $k > 0$, the Universe will recollapse after a finite time; when $k \leq 0$, it will keep expanding forever. In the L-T model, the sign of $(-E)$ has the same consequences, but $k = -2E/r^2$ need not have the same sign over the whole space. This means that shells with differing values of the comoving coordinate r can have different futures: some of them may recollapse while others will keep expanding. Moreover, the time between the Big Bang and the Big Crunch will in general be different for each recollapsing shell.

One of the possible consequences of this is the following: suppose $E(r)$ is nearly constant and negative in region I of the L-T spacetime, and nearly constant and positive in region II. An orthodox observer (believing firmly in the FLRW models) in the region I will then claim that the Universe has positive spatial curvature and will recollapse, while a second orthodox observer in the region II will claim that the Universe has negative spatial curvature and will exist eternally. Both observers will be right. The spatial curvature index is a local characteristic of space. It is global in the FLRW class only because of the very high symmetry of the latter. The sign of k is thus a characteristic of the FLRW models, but not necessarily of our real Universe.

The recollapsing shells may be situated inside or outside of the ever-expanding shells. The first situation occurs e.g. during a black hole formation, the second one will lead to a shell-crossing. We shall come back to both these phenomena further on.

5. Formation of black holes in the expanding Universe

Take the solution (1)–(3) with $\Lambda = 0$. The radial null rays will be automatically geodesic, and their tangent vector fields will be:

$$k^\alpha = [1, (1+2E)^{1/2}/R_{,r}\,, 0, 0] \tag{10}$$

for the outgoing ray, $\alpha = 0, 1, 2, 3; t = x^o, r = x^1$; and:

$$l^\alpha = [1, -(1+2E)^{1/2}/R_{,r}\,, 0, 0] \tag{11}$$

for the ingoing ray. A black hole will appear when the "outgoing" ray will start falling toward the center. Let us assume that $R_{,r} > 0$, so that there are no shell crossings anywhere, and that $M_{,r} > 0$, so that adding new matter shells increases the active gravitational mass around $r = 0$. (Neither of these assumptions is a necessity, see Ref.17 for a short description of the unusual cases. We make those assumptions just for simplicity). Then, the derivative of the active mass $M(r)$ along the outgoing ray is:

$$M_{,\alpha}\, k^\alpha = (1+2E)^{1/2} M_{,r}\, / R_{,r} > 0, \tag{12}$$

which means that the ray moves outwards with respect to the shells. However, this may happen when the ray is already falling, but slower than the matter-shells. The true indicator of the ray's falling or flying away are the changes of the geometrical distance of the front of the ray from the center, i.e. the sign of $R_{,\alpha}\, k^\alpha$. From (10) we have:

$$R_{,\alpha}\, k^\alpha = R_{,t} + (1+2E)^{1/2}. \tag{13}$$

This may be negative only if:

$$R_{,t} < -(1+2E)^{1/2} < 0, \tag{14}$$

which means that the outgoing ray may start falling back only if the matter through which it propagates is collapsing sufficiently rapidly. Then, from (2) and (13) we

have:

$$R_{,\alpha}\, k^{\alpha} = -(2E + 2M/R)^{1/2} + (1 + 2E)^{1/2} = \frac{1 - 2M/R}{(2E + 2M/R)^{1/2} + (1 + 2E)^{1/2}}, \quad (15)$$

and this will be negative if:

$$R < 2M. \quad (16)$$

With (16) fulfilled, the ray is already trapped inside the black hole. Eq. (16) is analogous to the equation of the black hole interior in the Schwarzschild geometry, and was first derived by Bondi[3]. The process of forming a black hole in an L-T Universe was described in more detail by Barnes[2].

Since $R_{,t} < 0$ inside collapsing matter, once a given shell enters the region $R < 2M$, it will never leave it. The same is true for the ray: from (2) with $\Lambda = 0$ we find $R_{,tt} = -M/R^2 < 0$, i.e. once (14) holds for $t = t_o$, it will hold for all $t > t_o$, and so $R_{,\alpha}\, k^{\alpha}$ will remain negative.

6. Formation of naked singularities

The L-T model is an important device for testing the various formulations of the cosmic censorship hypothesis. Its subcase was the historically earliest example that the simplest formulation (no naked singularities shall ever form!) is not correct: Yodzis, Seifert and Müller zum Hagen[36] showed that a shell-crossing singularity (where $R_{,r} = 0$) can be globally naked. This is a relatively innocent kind of singularity: geodesics (and even the whole spacetime) can be prolonged through it, and objects sent into it are not crushed to zero volume. However, it was later shown by Eardley and Smarr[6] that a naked singularity may form at $R = 0$, and this one is strong. A number of papers were published about the implications of this for the cosmic censorship, see the book by Joshi[14] for a review. One of the rescues for the cosmic censorship hypothesis is that the L-T model is not generic enough. However, the reason for non-genericity is not just its symmetry; Królak et al.[18] (see also this volume) showed that in the Szekeres models that have no symmetry the problem survives: strong globally naked singularities still may form.

The subject is much more important and lively than this short note would imply, but it is rather difficult to describe in non-technical terms. Readers interested in it are advised to read the monograph by Joshi[14].

7. The influence of cosmic expansion on planetary orbits

This problem was discussed in a well-known paper by Einstein and Straus[7,8]. (This was not the historically earliest paper on this subject, the first one seems to have been the one by McVittie[22]. See Ref.17 for a more detailed account of the history of the problem and of the results obtained). The model assumed by Einstein and Straus (a Schwarzschild vacuole in an FLRW background) is in fact unstable[17]. Gautreau[10] proposed a more realistic description: the average cosmic density of mass was assumed

to be nonzero throughout the planetary system. Then, because of the expansion of the Universe, this mass keeps flowing outward past the planets and causes a decrease of the effective gravitational force acting on the planets. As a result, the planets spiral out. Gautreau used a subcase of the L-T model to show that circular orbits are impossible, and then calculated the rate of expansion of the orbits in the Newtonian approximation. The result was:

$$\frac{dR}{dt} = \kappa\epsilon R^4 H/(2\mu), \tag{17}$$

where $\kappa = 8\pi G/c^4$, ϵ is the average cosmic mass density, R is the initial radius of the orbit, H is the Hubble parameter and μ is the mass of the star. The effect is thus larger for large orbits; for Saturn it equals $6 \cdot 10^{-18}$m/year, i.e. one proton diameter in 100 000 years. This is of course much smaller than the precision of any imaginable measurement, but it is important to know that in principle the effect is nonzero.

8. The influence of electric charge on expansion and collapse

The generalization of the L-T model to the case of spherically symmetric charged dust was considered by several authors. The first to find such a generalization with zero cosmological constant were Markov and Frolov[20], the generalization with $\Lambda \neq 0$ was found by Vickers[35]. We shall not discuss their results because in fact they are not solutions: the Einstein–Maxwell equations were reduced to a coupled set of two partial differential equations. Ori[23] found such coordinates in which the equations decouple and can be explicitly integrated, but the integrals are rather complicated, and, so far, did not provide a better physical insight into the problem. Therefore, we shall describe here only the simpler case that is solvable explicitly in comoving coordinates: that of neutral dust moving in the exterior electric field of a charge filling a sphere at the center of symmetry (or of a point charge residing in the center). This solution was first found by Hamoui[12]. The metric has then still the form (1), but $R(t,r)$ is given by:

$$R_{,t}^2 = 2E(r) + 2M(r)/R - Q^2/R^2 + \frac{1}{3}\Lambda R^2, \tag{18}$$

where the arbitrary constant Q is the total electric charge in the central region. The matter density is still given by (3).

It is now easy to see that in this case the dust moves along geodesics, but in a different geometry, modified by the term $(-Q^2/R^2)$ in (18). This term represents a repulsive gravitational force induced by the electric charge. The effect of gravitational repulsion between electrically charged and neutral particles seems to have been first noticed by Gorelik[11] in the Reissner–Nordstrom solution. In the cosmological context, it was first noted by Shikin[31], and can be stated as follows:

An electric charge at (or around) the center of symmetry, no matter how small or large, will prevent the Big Crunch singularity in the collapsing matter.

The corresponding result for charged dust was first found by Vickers[35], and by Ivanenko, Krechet and Lapchinskii[13]: the Big Crunch is prevented for the shell with the comoving coordinate r if the electric charge density $\rho(t,r)$ is sufficiently small compared to the mass density $\epsilon(t,r)$, i.e. when:

$$\rho < G^{1/2}\epsilon/c^2. \tag{19}$$

This observation opened up a line of speculation that was followed in a few papers: that a collapsing charged dust ball, observed from an exterior Reissner–Nordstrom region, might disappear from view under the exterior horizon, bounce under the interior horizon and re-expand into another asymptotically flat sheet of the maximally extended Reissner–Nordstrom manifold. However, Ori[24] destroyed that illusion: exactly in those cases when the Big Crunch is avoided, shell-crossings will block the passage through the Reissner–Nordstrom throat. The bounce and re-expansion might be conceivable in charged matter with nonzero pressure gradients, but no such generalizations of the L-T model were so far found, and the other solutions with charged perfect fluid source that are known[17] were not discussed from this point of view.

9. Anisotropies in the cosmic microwave background radiation generated by inhomogeneities in matter distribution

The L-T model was successfully used for calculating the anisotropies in the temperature of the cosmic microwave background radiation caused by inhomogeneous structures encountered by radiation between the emission event and the observer. The method was invented by Raine and Tomas[26]. The rays are assumed to originate in a homogeneous region, where an FLRW model applies. The observer is situated also in an FLRW region. The inhomogeneity is a spherically symmetric deformation superposed on the FLRW background, and is situated around the center of symmetry of the space. The inhomogeneous region is modelled by an L-T solution with the functions $M(r)$ and $E(r)$ chosen so that they fit the observed parameters of a void, a galaxy cluster or the Great Attractor. The equations of null geodesics are integrated numerically for rays coming to the observer from different directions, i.e. passing through the inhomogeneity at different distances from its center. The result of the integration is then used to calculate the temperature of the incoming radiation for different directions at the observer's position. The dipole and quadrupole components of the anisotropy caused by local perturbations of the Hubble flow around the inhomogeneity are subtracted from the total anisotropy. The residue is the true gravitational anisotropy.

The so far most comprehensive studies of this effect were done by Panek[25] and by Arnau, Fullana and S
'aez[1]. Panek assumed the FLRW background to be flat and considered models of all three kinds of inhomogeneities. The conclusion was that maximal anisotropies (measured by $\Delta T/T$) to be expected from voids are of the order $4 \cdot 10^{-7}$, from the Great Attractor —of the order $2 \cdot 10^{-6}$, and from a cluster of galaxies— of the order

$6 \cdot 10^{-6}$. The angular scale of the anisotropy should be 15° in the first case and 10° in the other two cases.

Arnau, Fullana and S
'aez[1] considered models of the Great Attractor only, but allowed for different density parameters (corresponding to negative curvature) in the FLRW background. The maximal anisotropy to be expected turned out to be $3 \cdot 10^{-5}$ at the angular scale 10° for the density parameter $\Omega = 0.15$. Note that this is by an order of magnitude larger than the result of Panek found for the flat background.

These results should be compared to the anisotropies caused by upscattering of the microwave photons by hot gases in the galaxy clusters (10^{-4}) and to the anisotropies recently measured ($6 \cdot 10^{-6}$, see Ref.21). The anisotropies calculated by Arnau et al. seem to be on the verge of detectability, provided our Universe has a sufficiently low density.

The said results have one more important implication. The high degree of isotropy of the cosmic microwave background radiation was often invoked as an argument that our observed Universe is faithfully described by the FLRW models, i.e. that it really is homogeneous and isotropic. If it were not, the tale went, then the inhomogeneities would leave a trace in the background radiation and cause anisotropies in its temperature. The truth was that nobody before Panek actually calculated the magnitude of the anisotropies to be expected. This "argument" was just one more expression of a blind and uncritical faith in the cosmological principle. Now, after the papers by Panek and by Arnau et al., the conclusion is: there was no chance to observe any such trace before the precision with which the temperature is measured surpassed 10^{-6}. This has happened only recently, and the results are still inconclusive. There has been enough room for inhomogeneous cosmological models all the while.

10. Formation of galaxies

This result, found by Bonnor[4], was never given the attention that it deserved, and the main conclusion of the paper is now common knowledge, but it is credited to much later papers based on perturbative calculations.

The question that Bonnor wanted to answer was this: could galaxies have been formed out of statistical fluctuations of mass density in an initially homogeneous medium? He made the following assumptions:

1. Inside the condensation, an FLRW dust model applies.
2. The background Universe is also an FLRW dust, but has a somewhat lower density initially.
3. The two regions are matched together through an interpolating L-T region.
4. All of the $N = 3 \cdot 10^{67}$ nucleons that will constitute the galaxy take part in the initial fluctuation.
5. The fluctuation appears 1000 years after the Big Bang.

The conclusion was that the initial condensation would have to be at least 10^{29} times larger in amplitude than a statistical fluctuation in order to develop into a galaxy. An object of the same density contrast as a galaxy can evolve out of a

statistical fluctuation only if the fluctuation involves 10^{10} nucleons.

Moving the time of first appearance of the fluctuation towards the Big Bang makes the gap somewhat smaller, but cannot close it. This is still an unsolved problem of cosmology. It would disappear if the Universe were very much older than it is believed to be...

11. Averaging the Einstein equations

The Einstein field equations cannot hold at all scales simultaneously. They are supposed to apply to the Universe in an averaged sense, but they are nonlinear in the metric tensor and averaging applied to nonlinear expressions changes their form. Since Einstein's theory was experimentally tested at the scale of planetary systems, it is reasonable to assume that the Einstein equations apply just at this scale:

$$G_{\mu\nu}(g) = \kappa T_{\mu\nu}, \tag{20}$$

where g is the metric taking into account the gravitational fields of single stars and planets, and $T_{\mu\nu}$ is the energy- momentum tensor of matter prescribed with the same accuracy. It is obviously impossible to construct a cosmological model describing our Universe down to such a detail. In cosmology, we are using an already-averaged metric $\bar{g}_{\mu\nu}$ that has to obey the averaged field equations:

$$\bar{G}_{\mu\nu} = \kappa \bar{T}_{\mu\nu}, \tag{21}$$

where now $\bar{g}_{\mu\nu}$, $\bar{G}_{\mu\nu}$ and $\bar{T}_{\mu\nu}$ take into account inhomogeneities of the size of galaxy clusters, for example. However, it is routinely being forgotten that $\bar{G}_{\mu\nu}(g) \neq G_{\mu\nu}(\bar{g})$, i.e. the Einstein tensor calculated from the fine-scale metric and then averaged is not equal to the Einstein tensor calculated in the ordinary way from a metric that was averaged before. In cosmology, different field equations should be used:

$$G_{\mu\nu}(\bar{g}) = \kappa \bar{T}_{\mu\nu} + \gamma_{\mu\nu}, \tag{22}$$

where $\gamma_{\mu\nu}$ is the "polarization curvature" that compensates for the difference between $G_{\mu\nu}(\bar{g})$ and $\bar{G}_{\mu\nu}(g)$. As can be seen, $\gamma_{\mu\nu}$ mimics a matter distribution and can influence the evolution of a model.

The problem was first formulated by Shirokov and Fisher[32] (the description above is very much borrowed from them), but their paper was, sure enough, ignored. It was only in 1984 that Ellis[9] seems to have gotten through with this message to the physics community.

There exists no generally accepted invariant definition of averaging, although several proposals were published (see Ref.17 for an overview). In most of the papers on the subject, starting with the one by Shirokov and Fisher, the Einstein equations were averaged one by one, the average being defined as an integral of the appropriate tensor component over 4-dimensional volume divided by that volume. This is not covariant and possibly makes no sense (in some papers, such averaging was applied

to the metric itself, but the metric tensor is needed to define volume, so what does it mean to average a metric component over a volume?). However, the consistency of results between different papers is rather remarkable. Averaging by volume was usually applied on top of an approximation scheme. The approximation schemes and the initial metric ansatzes were in general different in each paper. In spite of this, the conclusion was mostly the same: the correction term $\gamma_{\mu\nu}$ in (22) can be interpreted as negative pressure that pulls matter apart. With the present-day rate of expansion given (from observations), this means that in the past such a model was expanding slower than an FLRW model, and the origin of evolution was not a singularity, but a state with a finite maximal density.

The most recent and most promising contribution to the subject are the papers by Zalaletdinov[37,38] (see also this volume). The author proposed a self-consistent axiomatic theory which allows one, in principle, to deduce the form of $\gamma_{\mu\nu}$ in (22) at the macroscopic (already-averaged) level without explicitly invoking the microscopic metric $g_{\mu\nu}$. The theory has yet to be tested on exact solutions, but it has the great advantage of being fully covariant. It is presented in this volume by R. Zalaletdinov himself.

12. Conclusion

In this article, only a small selection from the material available in the literature was presented. Only those results were chosen for presentation here which were already shown to be applicable to the observed Universe. The complete material, rather carefully compiled by this author[17] from all journals and books published from 1915 onwards, consists of more than 800 papers written by about 400 authors. It is really high time to begin taking all this wealth of knowledge seriously.

By this opportunity, it must be mentioned that the forced belief in the cosmological principle and the FLRW models had a peculiar side-effect. Physicists trying to generalize those models were working in isolation, and their papers were in most cases denied any recognition. As a result, the same ideas were tried again and again by different authors who did not know about their predecessors. In particular, the derivations of the $\Lambda = 0$ subcase of the L-T model (see sec.2) were independently published in 20 papers and books, and already the first derivation (by Datt[5]) was not a new result —Lemaître[19] derived eq. (2) and discussed its solutions with $\Lambda \neq 0$ in terms of elliptic functions. This is not an extreme case, the number of rediscoveries exceeded 20 for two other classes of solutions (see Ref.17 for details). What progress could have been achieved if all this ingenuity were directed toward problems that were really not yet solved!

13. References

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