

THE SIXTH
MARCEL GROSSMANN MEETING

On recent developments in theoretical and experimental
general relativity, gravitation and relativistic field theories

Proceedings of the Meeting held at
Kyoto International Conference Hall
Kyoto, Japan
23 – 29 June 1991

PART A

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A SURVEY OF COSMOLOGICAL EXACT SOLUTIONS

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ABSTRACT

This is a preliminary presentation of a review of those exact solutions of Einstein's equations that can be interpreted as inhomogeneous cosmological models.

This note is a report on an ongoing project. The purpose of the project is to assemble and classify those exact solutions of Einstein's equations that contain the FLRW models as a limiting case, and so can be interpreted as inhomogeneous cosmological models. It is hoped that this will allow to prevent duplication of work already done, and to identify those problems of mathematical cosmology that are not yet solved. The review omits Bianchi models because sufficient information on them is already available. The solutions classified so far fall into 5 groups:

The Szekeres - Szafron family. These are perfect fluid solutions with zero rotation and acceleration (a unique geometrical definition of this family exists, but is rather extended⁽⁺⁾). The most general case ($p \neq 0$) was worked out by Szafron². It divides into class I, generalizing FLRW only, and class II, being a different generalization of FLRW and at the same time a generalization of the Kantowski - Sachs³ solutions. The more important subcases are the solutions found by Szekeres⁴⁻⁵ ($p = 0$), Ellis⁶ ($p = -\Lambda = \text{const} +$ a 3-dimensional symmetry group with 2-dimensional orbits), Lemaitre⁷ (the spherically symmetric limit of class I with $p = -\Lambda$, known under the name "Tolman-Bondi"), Datt⁸ (the spherically symmetric limit of class II with $p = 0$). For some of the solutions in this family generalizations were found: by Goode⁹ of the whole class II to nonzero heat flow, by Vickers¹⁰ and Ori¹¹ of the Lemaitre solution to the charged dust case, by De and Ray¹² - the plane symmetric counterpart of Vickers, and by Stephani¹³ - a rotating inhomogeneous generalization of Kantowski and Sachs³. In all, 64 papers⁽⁺⁾ were published with derivations of the various solutions. Only 33 of them⁽⁻⁾ can claim to have been new at the time of publication. In addition, more than 50 papers⁽⁺⁾ were published that discussed geometrical and physical properties of the solutions. The whole family, together with the generalizations, can be generated (by taking limits) from just 8 parent solutions^(*); in the strict perfect fluid case there are just 3 parent solutions: the 2 classes of Szafron² and the Stephani¹³ solution.

The Stephani - Barnes family. These are perfect fluid solutions with zero rotation and shear and nonzero expansion - it is the complete invariant definition

(+) All these numbers are likely to be revised upwards.

(-) These numbers are likely to be revised downwards.

(*) The most general solutions are defined by an ordinary differential equation to be solved.

of this family. It was worked out in this generality by Barnes¹⁴, but special cases were solved by Dingle¹⁵ and Kustaanheimo - Qvist¹⁶ (the spherically symmetric case), Stephani¹⁷ (the conformally flat case) and McVittie¹⁸ (a superposition of the Schwarzschild solution with a FLRW background). A generalization of the spherical case to charged perfect fluid source was first found by Shah and Vaidya¹⁹. The author of this note showed²⁰ that the 3 type D cases of Barnes can be unified into a single 2-parameter family. In all, 81 papers⁽⁺⁾ were published with derivations of the solutions, of which 18⁽⁻⁾ can lay claim to have discovered a new solution or an important subclass, or to have introduced a new method of treating several solutions in a unified way (among the latter ones, the paper by Sussman²¹ made an outstanding contribution). This family holds the record of repeated discoveries: so far 16 for the Kustaanheimo - Qvist class and 12 for the spherical limit of the Stephani¹⁷ solution. About 15 other papers⁽⁺⁾ were devoted to studying geometrical and physical properties of the solutions of this family. Several of the 81 papers discussed physical properties of the spherically symmetric solutions from the point of view of stellar collapse. The family, together with generalizations, can be generated from 9 parent solutions^(*), and from just two in the strict perfect fluid case: the Stephani¹⁷ solution and the one from Ref. 20.

The Vaidya - Patel - Koppar family. It contains 8 papers⁽⁺⁾. These solutions are superpositions of the FLRW background with the Kerr, Kerr-Newman and Demianski solutions. The source is in each case a mixture of a perfect fluid, null radiation and electromagnetic field. The 3 components of the source are, unfortunately, coupled together in such a way that, for example, vanishing radiation density automatically implies zero electromagnetic field and zero shear, rotation and acceleration, leaving only either vacuum or the FLRW spacetime as a limit. In spite of this drawback, the family is an interesting experiment in generating new solutions from old ones. This activity was initiated by Vaidya²² who found a superposition of the Kerr solution with a FLRW background (it does not reproduce the McVittie¹⁸ solution in the limit $a \rightarrow 0$!). Five of the 8 are parent solutions, with the highest sophistication achieved in the papers by Patel and Koppar²³⁻²⁵.

The Tabensky - Taub - Letelier - Tomita family. The papers of this family, 15 so far, elaborate the field equations for a nonrotating "stiff fluid" source ($\rho = p$), with a 2-dimensional Abelian symmetry group G_2 acting on spacelike orbits. Only 5 of the papers⁽⁺⁾ provide explicit examples of solutions, the other ones are prescriptions for solving the Einstein's equations and form a partly ordered sequence of progressing generality, from Tabensky - Taub²⁶ (plane symmetry) through Letelier²⁷ and Ray²⁸ (orthogonal generators of G_2) to Tomita²⁹ (arbitrary G_2). A subcase of the Ray - Letelier spacetime was generalized by Charach and Malin³⁰ to include electromagnetic field of the same symmetry. Worth a separate mention is the paper by Belinskii³¹ discussing solitonic distortions of a FLRW background with such beautiful physical insight that one begins to regret that matter with $\rho = p$ probably never existed. The sets of equations elaborated by Tomita²⁹ and Charach - Malin³⁰ are parent sets for the whole family; in the case

of pure perfect fluid all the solutions are subcases of Tomita's equations.

Miscellaneous experiments. This is a loose collection of 14 papers⁽⁺⁾ whose authors found generalizations of the FLRW models by trial and error in various directions. The families of solutions found by Wils³² and Oleson³³ are most elaborate. The first one is a collection of solutions with an abelian G_2 having no definite equation of state, the second one is a complete collection of perfect fluid solutions of Petrov type N where the principal null congruence of the Weyl tensor is geodesic. Both can reproduce all 3 types of FLRW limits. Thirteen of the 14 are parent solutions, not derivable as limits of the others.

The review allows to draw the following preliminary conclusions:

1. Very interesting solutions were found already in the 1930ies (Lemaitre⁷, McVittie¹⁸, Datt⁸) and 1940ies (Kustaanheimo-Qvist¹⁶), and their importance was immediately recognized by a few authors (Dingle¹⁵, Tolman³⁴, McVittie¹⁸, Sen³⁵). Unfortunately, those papers were ignored by the opinion-forming elite who just knew³⁶ that the Universe we live in is homogeneous. The papers by Tolman³⁴ and Sen³⁵ contain proposals of paradigms that would be attractive still today. In particular, Sen showed that the Lemaitre⁷ solution predicts a behavior of density distribution that today would be called formation of voids.

2. No rotating generalization of the FLRW models is known. Several solutions exist for which $\theta \cdot \omega \neq 0$, but they become static in the limit $\omega \rightarrow 0$. The only exception is the Stephani¹³ solution that reduces to an inhomogeneous generalization of Kantowski - Sachs when $\omega = 0$.

3. Those spherically symmetric perfect fluid solutions for which $\theta \cdot \sigma \cdot \dot{u}^\alpha \neq 0$ were studied without solving the Einstein's equations^{37,38}. The examples of solutions found by Narlikar-Moghe^{39,40} and McVittie-Wiltshire⁴¹ are rather special and no invariant definition was provided. The effort spent at investigating the special cases $\sigma = 0$ and $\dot{u}^\alpha = 0$ produced multiple repetitions of the same results.

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