12th INTERNATIONAL CONFERENCE ON GENERAL RELATIVITY AND GRAVITATION

Under the auspices of the International Society on General Relativity and Gravitation

Cosponsored by
the University of Colorado at Boulder and
the National Institute of Standards and Technology

Boulder, Colorado

July 2-8, 1989

Abstracts of Contributed Papers

A UNIFIED REPRESENTATION FOR THE SHEARFREE NORMAL MODELS

Andrzej Krasiński, Department of Mathematics, University of Colorado, Boulder, Colorado 80309–0426 (permanent address: N. Copernicus Astronomical Center, Bartycka 18,00 716 Warszawa, Poland)

Shearfree normal models are perfect fluid solutions of the Einstein's equations in which rotation and shear vanish while expansion is different from zero. All shear- and rotation-free perfect fluid solutions were found by Barnes [1]. These with nonzero expansion contain the FLRW models in the limiting case when spatial gradients of pressure vanish. Hence, they are capable of describing (exactly) the evolution of a certain class of inhomogeneities in the universe. There are 3 classes of them, each with a different symmetry group: the spherically symmetric models, the plane symmetric models and the hyperbolically symmetric models. As presented by Barnes, the 3 classes are disjoint and require separate formulae. However, it is possible to represent the whole collection in the form of a single 2-parameter family of spacetimes. The metric of the family is:

$$ds^2 = D^2dt^2 - (R^2/V^2)(dx^2 + dy^2 + dz^2) \tag{1}$$

where $D = F(R/V)(V/R)_{,t}$, $F(t)$ and $R(t)$ are arbitrary functions,

$$V(t, x, y, z) = (z + b)S(t, Z), \tag{2}$$

where b is an arbitrary constant, and $S(t, Z)$ is determined by the equation:

$$R(t)S_{,ZZ}/S^2 = f(Z), \tag{3}$$

where $f(Z)$ is an arbitrary function and the variable Z is defined by:

$$Z = \frac{1}{2}(a - x^2 - y^2 - z^2)/(z + b), \tag{4}$$

a being another arbitrary constant.

The equation (3) is of the same form as the one introduced by Kustaanheimo and Qvist [2]. In general, its solutions cannot be expressed in terms of elementary functions, and a completely general solution is unknown, but the body of literature dealing with particular subclasses of solutions is very large (see Refs. 3–5 for partial reviews).

When $a < b^2$, the symmetry group of the model is isomorphic to $O(3)$, when $a = b^2$ it is isomorphic to the symmetry group of the Euclidean plane, when $a > b^2$, it is isomorphic to $O(2, 1)$—the symmetry group of a surface with negative constant curvature. The coordinate transformations that relate (3) to the 3 classes of Barnes involve subcases of conformal symmetries of the 3-dimensional Euclidean plane and are rather complicated (see Ref. 4 for details).

REFERENCES

[1] A. Barnes, *Gen. Rel. Grav.* **4** (1973), 105.
[2] P. Kustaanheimo, B. Qvist, *Societas Scientiarum Fennica Commentationes Physico-Mathematicae* **XIII**, No. 16 (1948), 1.
[3] R. Sussman, *J. Math. Phys.* **29** (1988), 945.
[4] A. Krasiński, *J. Math. Phys.* **30** (1989), 433.
[5] A. Krasiński, Cosmological exact solutions, this volume.