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Abstracts of Contributed Papers

COSMOLOGICAL EXACT SOLUTIONS

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Many more papers were devoted to deriving or discussing exact solutions of cosmological significance than most researchers would expect. This note is a report on an ongoing research aimed at clarifying the current state of knowledge of the subject.

A cosmological model is defined as an exact solution of Einstein's equations which contains the FLRW models as a limiting case. At least 80 papers were published in which solutions having this property were derived. However, on closer inspection, this wealth of results can be organized into just 5 hierarchies, 4 of them having a single solution-class at the top, from which other solutions follow by various limiting procedures. Hence, it is enough to study just 5 to 10 papers to have a good overview of the field.

Hierarchy no. 1 has the class of solutions found by Szafron [1] at the top. They are defined by the following properties: 1. The source is a perfect fluid, 2. Rotation and acceleration of the flow vanish, 3. Hypersurfaces orthogonal to the flow are conformally flat, 4. The 3-dimensional Ricci tensor and the shear tensor have each a double eigenvalue. Among the better known special cases, this family includes the solutions of Szekeres [2], Lemaitre [3] (known as the "Tolman–Bondi" model) and Kantowski–Sachs (obtained earlier, and in a greater generality, by Kompaneets and Chernov [4]). At the moment of writing this note, 24 independently derived solutions are in this class, several of them duplicate the earlier ones.

Hierarchies 2 and 3 are partly interlocked. No. 2 has the solution of Stephani [5] at the top. It is the most general conformally flat perfect fluid solution with nonzero expansion. It has no symmetry in general. Hierarchy 3 has has at the top the papers by Kustaanheimo and Qvist [6], Barnes [7] and this author [8]. The models are perfect fluid solutions with zero rotation, zero shear and nonzero expansion. These properties force a symmetry on the spacetime, which is either spherical, plane or hyperbolic. The full class was first found in Ref. 7, the spherical case was found in Ref. 6, the 3 cases were represented as a single 2-parameter family in Ref. 8. The better known subcases are the solutions by Wyman [9] and McVittie [10], the spherically symmetric case was generalized by Faulkes [11] to include the electromagnetic field. Currently, 53 independently derived solutions are in the classes 2 and 3, with an amazing number of repetitions (so far, 10 in the subset where the solutions are conformally flat and spherically symmetric).

Hierarchy 4 contains just one paper, by Oleson [12], where all Petrov type N perfect fluid solutions were found in which the principal congruence of the Weyl tensor is geodesic. No other attempts along this line were detected so far.

Finally, family 5 contains spherically symmetric solutions in which both shear and acceleration are nonzero. Here, no general "master" solution seems to have been found what makes it difficult to systematise the results. Two solutions are surely in this family, a few suspected candidates await analysis. In spite of their sophistication, the models discussed here are still too simple in many ways. For example, those in family 1 have spatially homogeneous pressure, those in family 2 have spatially homogeneous density, and those in family 3 have symmetries which allow for only 1-dimensional structures. However, before serious work on generalizations is undertaken, it would be useful to have a complete overview of the work already done, in order to avoid wasteful repetitions that were so frequent and still continue. It is hoped that the research reported here will help to achieve this goal. Readers are kindly asked to contribute to the review which is still not closed.

REFERENCES

[1] D. A. Szafron, *J. Math. Phys.* **18** (1977), 1673.
[2] P. Szekeres, *Commun. Math. Phys.* **41**, (1975), 55.
[3] G. Lemaitre, *Ann. Soc. Sci. Bruxelles* **53A** (1933), 51.
[4] A. S. Kompaneets, A. S. Chernov, *Sov. Phys. JETP* **20** (1965), 1303.
[5] H. Stephani, *Commun. Math. Phys.* **4** (1967), 137.
[6] P. Kustaanheimo and B. Qvist, *Societas Scientiarum Fennica Commentationes Physico-Mathematicae* **XIII**, No. 16 (1948), 1.
[7] A. Barnes, *Gen. Rel. Grav.* **4** (1973), 105.
[8] A. Krasiński, *J. Math. Phys.* **30** (1989), 433.
[9] M. Wyman, *Phys. Rev.* **70** (1946), 396.
[10] G. C. McVittie, *Mon. Not. Roy. Astr. Soc.* **93** (1933), 325.
[11] M. Faulkes, *Can. J. Phys.* **47** (1969), 1989.
[12] M. Oleson, *J. Math. Phys.* **12** (1971), 666.