19737 vol. 1

ABSTRACTS OF CONTRIBUTED PAPERS

11TH INTERNATIONAL CONFERENCE ON GENERAL RELATIVITY AND GRAVITATION

July 6-12, 1986, Folkets Hus, Stockholm, Sweden

Under the auspices of the International Society on General Relativity and Gravitation

ORGANIZED BY
The University of Stockholm

MAIN SCIENTIFIC SPONSOR
The Royal Swedish Academy of Sciences

MAIN FINANCIAL SPONSOR
The Marcus Wallenberg Foundation for International Cooperation in Science

SPACETIMES WITH CONFORMALLY FLAT FLOW-ORTHOGONAL SECTIONS

Andrzej Krasiński
N. Copernicus Astronomical Center, Polish Academy of Sciences, Bartycka 18, 00 716 Warszawa, Poland

The aim of this work is to find inhomogeneous solutions of the Einstein field equations which contain the Robertson-Walker (RW) spacetimes as limitting cases. If successful, this program may lead to an exact (as opposed to perturbative) description of structures evolving on the RW background.

Investigated are spacetimes in which matter-flow is shearfree and hypersurface-orthogonal while the hypersurfaces thus defined are conformally flat. No definite equation of state for the perfect fluid source is assumed and acceleration of the fluid flow is allowed. The common subset of these models with the Szekeres-type models (Ref. 1) are the RW solutions only.

The field equations are easily integrated with the result

$$ds^2 = D^2 dt^2 - (R^2/V^2)(dx^2 + dy^2 + dz^2) \quad (1)$$

$$D = F(t)\ (R/V)\ (V/R)_{,t} \quad (2)$$

$$R(t)\ V_{,ij}/V^2 = F_k(x,y,z) \quad (3)$$

$$R(t)\ (V_{,ii} - V_{,jj})/V^2 = G_k(x,y,z) \quad \text{(no summation)} \quad (4)$$

where $R(t)$ and $F(t)$ are arbitrary functions of t, the indices i, j, k run cyclically through the values 1, 2, 3, and F_k, G_k are sets of arbitrary functions of x, y and z. The equations (3) and (4) require integrability conditions (IC). They admit nonstatic solutions different from RW only in case when at most two functions in the set (F_k, G_k) are independent (see Ref. 2 for more details). The IC solve differently in 2 cases: I. Any of F_k is zero, II. All F_k are nonzero. In case I, all solutions are given by (6) and (7) below. In case II, eqs. (8) and (9) give only a subclass of solutions, the most general class is still under investigation. The solutions found so far are all generated by the following master equation (known already from the investigations of spherically symmetric solutions, see Ref. 3)

$$R(t)\ w_{,gg}/w^2 = f(g) \quad (5)$$

where $f(g)$ is in each case an independent arbitrary function while g and w are given by one of the following formulae:

$$g = z,\ w = V \quad \text{(the plane-symmetric Universe)} \quad (6)$$

$$g = x/y,\ w = V/y \quad \text{(the line-homogeneous Universe)} \quad (7)$$

$$g = x^2 + y^2 + z^2,\ w = V \quad \text{(the spherically symmetric Universe)} \quad (8)$$

$$g = (C - x^2 - y^2 - z^2)/(2z),\ w = V/z \quad \text{(the axially symmetric Universe)} \quad (9)$$

Cases (6) and (7) contain only the flat RW-model, cases (8) and (9) generalize all the 3 RW-models. Eq. (5) shows that at an initial instant $t = t_o$ an arbitrary function $w(t_o, g)$ may be taken as a solution (thus defining $f(g)$), and then the time-development of w will be given through arbitrary functions of t which arise as the integration "constants" of (5). The behavior of these arbitrary functions may be limited or even specified through energy conditions or an equation of state. An interesting case is when $w(t_o, g)$ is periodic in g. Only in case (6) will this lead to the matter-density being periodic in space, and thus to the Universe which is inhomogeneous on a small scale, but homogeneous on a large scale (the homogeneity group being discrete). Unfortunately, this happens with respect to only one spatial dimension, and so (6) is not sufficiently general to be a model of the observed Universe (but is more general than the flat RW!). In the other 3 cases, even with $w(t_o, g)$ being periodic in g, the matter density ρ will only have maxima and minima distributed periodically in g, but the values of ρ at different extrema will be different. Also, owing to the peculiar symmetries, even the distribution of extrema cannot be called homogeneous.

References:

1. D. A. Szafron, J. Wainwright, J. Math. Phys. 18, 1668 and 1673 (1977).
2. A. Krasiński, in: Proceedings of the 4th Marcel Grossman Meeting. Academic Press, in press.
3. H. Stephani, J. Phys. A16, 3529 (1983).