THE PROGRAM ORTOCARTAN FOR ALGEBRAIC CALCULATIONS - NEW DEVELOPMENTS.(*)

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This note describes the developments made in the program after 1980. It completes eralier publications on the subject [1 - 3].

1. Extending the abilities of the program.

Originally, given the tetrad components of the metric tensor in an orthonormal tetrad of Cartan forms, the program could calculate the tetrad and the coordinate components of the affine connection and of the Riemann, Ricci, Einstein and Weyl tensors. For this purpose it was equipped with powerful subroutines for algebraic simplification, differentiation, inverting matrices of symbolic elements, substitutions, reading the user's input (written in an easy FORTRAN-like format) and printing the results in a readable form. It was obvious from the beginning that these facilities could be combined into several other specialized programs, and this has happened. The author extended ORTOCARTAN for:

1. Inverting symbolic matrices of arbitrary dimension.
2. Calculating the Lie derivative of the Riemann tensor $R^{\alpha\beta}{}_{\gamma\delta}$.
3. Performing algebraic and differential operations on arbitrary symbolic expressions on a single user-command (with no LISP programming involved).

J. Richer extended ORTOCARTAN for:

4. Calculating algebraic invariants of the Riemann and Weyl tensors.
5. Performing the calculation in an arbitrary signature of the metric.

G. Üçoluk and E. Karabudak added to that:

6. Saving overlays at intermediate stages of the calculation (in order to restart the program at the same stage with modified data).

On request of F. Hehl the author has also written the subprogram for:

7. Calculating the left-hand side of the field equations in the Poincaré gauge-type theory of gravitation.

These extensions were, however, made at four different places independently, and no-one has access to all of them. The author has only 1, 2 and 3 at his disposal.

The program still lacks some important abilities, e.g. 1. Calculating indefinite integrals, 2. Automatic simplification of rational functions, 3. Noncommutative algebra, 4. Matrix algebra, 5. Automatic processing of complex numbers and functions, 6. The possibility to write programs in ORTOCARTAN as a language in itself, without resorting to its base language LISP.

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The first two were omitted deliberately as they still pose fundamental problems of principle [4-5] and find a better place in a large multi-purpose system maintained within a major project. The others were left out because of lack of sufficient demand on the user's side. The last two will nearly certainly be included at some point.

This note does not mention smaller improvements in the algorithm made by the author and idependently by some other users.

3. Making the program portable.

ORTOCARTAN is written in the LISP landuage. It was first implemented on a CDC Cyber 73 computer with University of Texas LISP 4.1. This version of LISP is a highly developed one and beautifully simple in use. Unfortunately, there are significant differences between implementations of LISP on different machines, and this situation is not likely to change soon (the "Standard LISP" exists so far only as a proposal on paper, and some implementations which ignore it do so in fact for the benefit of the user). Thus each reworking of ORTOCARTAN onto a new type of computer required much labor. There exist so far 4 versions of the program:

1. Cambridge LISP version for IBM/360 computers (accessible in Cambridge, England, and in Garching/Munich, W. Germany - produced by J. Richer and A. Norman in Cambridge).

2. U. T. LISP 4.1 version for CDC Cyber computers (accessible in Warsaw and Cracow, Poland, and in Cologne, W. Germany - the original version produced by M. Wardęcki, M. Perkowski, the author of this note and Z. Otwinowski in Warsaw).

3. SLISP/360 version for IBM/360 computers (accessible on a Siemens4004 computer in Konstanz, W. Germany - produced by the author).

4. A version for UNIVAC computers (accessible in Istanbul, Turkey - produced by G. Üçoluk and E. Karabudak).

4. Improving the efficiency of the program.

Very little work has been done on this subject directly. The gain in speed by the factor 15 over the original version (see [1]) was achieved simply by rewriting ORTOCARTAN into Cambridge LISP which has a very efficient (and bug-free) compiler. The relative speeds of the four versions of ORTOCARTAN are, roughly, in the ratio 1:15:30:180 respectively. These differences result mainly from the different degrees of usability of the corresponding LISP compilers. Version 1 was fully compiled, version 2 - compiled only in the parts crucial for the speed (because of lack of documentation on the compiler), version 3 - compiled in about 1/6 of its length (because of bugs in the compiler), version 4 was not compiled at all.

5. Actual applications of ORTOCARTAN.

The instances where ORTOCARTAN was applied include the other two notes by this author in this volume as well as several calculations by Hehl [6] and his group. In Warsaw a program was generated from ORTOCARTAN to handle sets of ordinary differential equations [7], and by that a minor bug in the rarely used procedure EXPAND has been detected and corrected. No other reports of bugs reached the author.

Meanwhile, ORTOCARTAN deserved one more praise in addition to those mentioned in [1]: its version 1 showed that the Kerr metric has its Ricci tensor equal to zero in 33 seconds (at 700 kbytes of core). In this way it became one of the few programs which were able to digest the Kerr metric, and it did so without any special adjustments to this calculation. The Kerr metric is long known for the serious difficulties it poses to any program as it requires very non-trivial simplifications of rational functions which can be handled only through user-generated substitutions. The success in processing this metric certifies the flexibility of the user-program interface as well as the power of the algorithm. The form of the Kerr metric was taken from [8], the list of user-generated substitutions in the Riemann tensor contained 77 items (partly with repetitions, but no attempt was made to optimize the number of substitutions).

Acknowledgments

This note reports summarily the results of efforts of several individuals working in a loose team. Their names are mentioned in the text where appropriate. They are credited here for their valuable contributions.

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