

~~THE UNIVERSE THAT CAN OPEN UP OR CLOSE~~

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S U M M E R

A generalization of the Friedman Universe model, found in 1967 by Stophand, is discussed. Each spatial section $t = \text{const}$ of the Stophand model is identical with a section of one of the Friedman models, but the sign of curvature of the section may be different at different times. A number of physical problems, which emerge if the Stophand solution is taken as the model of the actual Universe, is posed.

1. Introduction.

Astronomers are quite satisfied with the Friedman-Robertson-Walker (FRW) models of the Universe. The huge amount of literature [1] mostly confirms that the models describe the observed Universe with a good approximation. From a theoretic's point of view, however, the FRW models are obtained under simplifying assumptions which are so extreme that it is surprising to find that such a simple description works. What forces the Universe to be so regular in the large when its constituent parts (galaxies, clusters of galaxies) are so highly unsymmetric and so irregularly spaced in a small scale? Did the success of the FRW models result merely from the low precision of most astronomical tests which are indirect and involve poorly known intermediate data [2]? To what extent are these models unique among the cosmological solutions of the Einstein's equations?

This article is connected with the last question. It presents a class of generalized models in which the distinction between the open and the closed Universe becomes meaningless; a single Universe can be open at one time and closed at another.

The details of the derivation are published separately [3]. Here only the principles of reasoning and the conceptual novelties of the results will be described. The model considered further is a legitimate solution of the Einstein field equations (it was obtained previously by Stephani [4]), and no new physical concepts are needed to introduce it. Only the strong symmetry assumptions made about the Universe *a priori* have to be partly relaxed.

2. Spacetimes whose spatial hypersurfaces are spherically symmetric and homogeneous.

The astronomical observations are capable of investigating

the geometry of the 3-dimensional space which corresponds to our "now" in the spacetime. In fact, even this can be done only indirectly [5]: only the distribution of matter is observed, and its state now is reconstructed from that observed on our past light-cone. Let us however trust the astrophysicists that our space "now" is spherically symmetric and homogeneous.

The local geometry of such a 3-space can

be described by the following metric form:

$$ds_3^2 = \frac{dr^2}{(1 + \frac{1}{4} kr^2)^2} [dr^2 + r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2)] \quad (3.1)$$

where R and k are arbitrary constants. The result (3.1) is well known - the 3-spaces known of the FRW models have just this symmetry.

Let us now ask the question: what can be the spacetime in which the subspaces $t = \text{const}$ have the geometry given by (3.1)? Let us study this problem under two more assumptions which will simplify the calculations:

1. The t-lines are orthogonal to the spaces $t = \text{const}$.
2. The matter moves along the t-lines.

The technical question is now: (3.1) resulted from a 4-dimensional metric which depended on the time t, by substituting any definite value of t. What could that dependence have been? Assumption 1 implies that the components g_{tp} , $g_{t\vartheta}$, and $g_{t\varphi}$ of the metric tensor are zero. The component g_{00} is left unspecified, while in the others the only way to include the time-dependence is to change the constants R and k into functions of time. Hence the spacetime metric searched for will be:

$$ds^2 = \delta^2(t, r, \vartheta, \varphi) dt^2 - \frac{r^2(t)}{\left[1 + \frac{1}{4} k(t)r^2\right]^2} [dr^2 + r^2(d\vartheta^2 + \sin^2 \vartheta d\varphi^2)] \quad (3.2)$$

where Φ is an arbitrary function of 4 coordinates.

Already here we have got something more general than in the standard cosmology. There it is assumed that the whole spacetime has the same symmetry group as the space (3.1). The direct result is that D may be a function only of t , and that it must be a constant [3, 6, 7]. Consequently, either all the spaces $t = \text{const}$ are closed and finite, or all are open and infinite. Here, some of the spaces may be closed ($k > 0$) while some others may be open ($k < 0$).

So far this is only a mathematical possibility, but is it compatible with the Einstein's theory of gravitation? It appears to be the case. If we further assume that matter in the Universe is a perfect fluid, then the Einstein field equations can be solved for the metric (3.2). We shall deal with the solution in the next section.

4. Generalized FRW models with no symmetry.

By writing (3.2) we have made a tacit assumption. Namely, although the space (3.1) is homogeneous and so has no center of spherical symmetry defined geometrically, one point ($x = 0$) is distinguished by the fact that it is the origin of coordinates. In proceeding to the spacetime metric (3.2) we then assumed that all the origins were placed on a single t -line. This assumption is not necessary: the origins may be shifted with respect to the t -lines arbitrarily and independently in each 3-space. Let us change to a new coordinate system in (3.1) which exhibits this arbitrariness:

$$ds_3^2 = (R^2/v^2) (dx^2 + dy^2 + dz^2) \quad (4.1)$$

where

$$v = 1 + \frac{1}{k} \ln [(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2] \quad (4.2)$$

a, b, x_0, y_0, s_0 being arbitrary constants. Consequently, in going over to the spacetime we have now five constants to change into functions of time. The spacetime metric becomes:

$$ds^2 = V^2(t, x, y, z) dt^2 - (R^2/V^2) (dx^2 + dy^2 + dz^2) \quad (4.3)$$

where V is given by (4.2), with a, b, x_0, y_0, s_0 being now unknown functions of t .

After the Einstein field equations are solved we obtain [3, 4]:

$$a = P(t) \sqrt{\frac{C}{V}} \quad (4.4)$$

$$V = 3 C^2(t) \quad (4.5)$$

$$p = -3 C^2(t) + 2 C \frac{dC}{dt} \frac{V}{R} / \sqrt{\frac{C}{V}} \quad (4.6)$$

$$b = (C^2 - 1/R^2) R^2 \quad (4.7)$$

$$= \frac{B+C}{C^2} \quad (4.8)$$

where P and C are arbitrary functions. The functions $x_0(t)$, $y_0(t)$, and $s_0(t)$ survive the field equations intact.

The spacetime given by (4.1) - (4.4) has no symmetry until in the previous section, where we assumed $x_0 = y_0 = s_0 = 0$.
the
spacetime as a whole was spherically symmetric about the single
line $r = 0$.

4. Did our real Universe ever open up or close?

It is instructive to know that the gravitation theory does not require the Universe to be closed or open forever, but there remains the question: Did our Universe ever change the sign of its spatial curvature, or, if not, is it going to change it in the future?

The prospects to answer the second part of the question do not seem to exist. The presence and the temperature of the cosmic

background radiation, together with the observed abundance of elements [1] put rather strong limits on the initial state of the Universe. On the other hand, the present state is also known with some precision from current observations. If no reversal of the sign of spatial curvature occurred in the past, then the evolution according to the Steady model is hardly closer to the predictions of the corresponding Einstein model, and the observations will not be able to distinguish these two models.

The first part of the question, however, immediately raises further problems:

1. Does the change of sign of spatial curvature leave any physical traces in the matter content of the Universe?
 - a. If so, are these traces, in principle, observable?
 - b. If so, can we observe them now?
2. If so, did the traces show up in any of the observations already made, as a side effect?
3. If the answer to question 1 is "yes", can we count on observations to actually detect the traces?
4. If the answer to question 3 is "no", what are the prospects to observe the traces in the future?
5. The change from an open Universe to a closed one or vice versa requires a discontinuous change of topology of the space $t = \text{const}$. How does this actually happen?
6. The t-lines establish a global 1-1 mapping between each pair of $t = \text{const}$ hypersurfaces. However, no obvious singularity in the t-congruence are seen: its divergence is equal to $3/R$ where R is the function from (4.4), its acceleration is $\dot{u}^2 + V_{R_1}/R^2$, $t = 1, 2, 3$ (coordinates of (4.3)), and its shear and rotation are zero. How can such a congruence map a closed and finite space of positive curvature into an open and infinite one of negative curvature?

9. The open Friedmann Universe corresponds, in the Newtonian limit, to a configuration of a mass-density smaller than a critical value (in a specific instant). The closed Universe has, in this limit, its mass-density above that value. In Newton's theory, the mass is conserved. What happens, then, with the mass in the Stephani model when the sign of curvature is changed? Does it have a Newtonian analog at all? (cf. L27).

10. In the PRW models the evolution of the Universe is fixed by the field equations plus the equation of state of matter. In principle, this is so, too, in the Stephani model, but no equation of state of the form $\epsilon = \epsilon(p)$ is compatible with it, or, if such an equation of state is forced into the model, we simply return to the PRW solutions. In the generic Stephani model, p depends not only on ϵ , but also on the point of space. In other words, the equation of state is different in each spatial location. What is the physics behind it?

Each of the answers to be given may lead to an interesting development or to a failure of the Stephani model. Whichever is the case, the model might be valuable as the first challenger of the PRW models (in the Einstein's theory) which cannot be readily discarded by simple arguments.

R D F B R D N C D G

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