

ORTOCARTAN – A NEW COMPUTER PROGRAM FOR ALGEBRAIC CALCULATIONS

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The paper is intended to announce the appearance of a new computer program for algebraic calculations named ORTOCARTAN. It is readily applicable for calculating the curvature tensors in general relativity, and, with more effort from the user, can be applied to many other kinds of calculation. The paper describes shortly the main abilities of the program, its availability, and its speed in comparison with other systems. An example of a simple calculation is given.

1. Introduction – why ORTOCARTAN was created

Most computer applications in physics are connected with numerical calculations. It is not as generally known, however, that computers can also perform broad classes of so-called algebraic or analytic calculations where symbols with no numerical value are processed. Though several programs or systems for algebraic calculations are available in the world (see surveys in e.g. refs. [1,2]), their widespread distribution is restricted by two main obstacles:

1. Large and very general systems, like e.g. REDUCE, CAMAL or SYMBAL, are usually available on one type of computer only, or even on a single copy of a computer, like MACSYMA, and so are difficult to move to new places.

2. Less general systems might be less computer-bound, but at the same time more specialized in their abilities, and so of more limited interest to the physics community.

In this situation it is not without sense to attempt to produce new programs, both general and specialized. The aim of this paper is to announce that a new specialized program, named ORTOCARTAN, has appeared, and to discuss its potential applications.

2. What ORTOCARTAN can do

The program is directed towards applications in general relativity (see a relativity-oriented review in ref. [3]), and for the purpose of calculating the curvature tensors from a given metric tensor it can be used by an uninitiated user nearly without any preparatory learning. However, its potential applications are more general, and with the easy input syntax and readable output we devised we hope it could be useful to a larger group of users, especially those without previous experience in computing.

ORTOCARTAN can automatically simplify algebraic expressions (with limitations discussed below), differentiate functions of an arbitrary number of arguments, including composite functions whose arguments are functions themselves, invert arrays of functional expressions, and print the results obtained in the standard mathematical coupled-lines format, i.e. with exponents and superscripts raised, and subscripts lowered. The input format follows as closely as possible in the FORTRAN infix notation.

ORTOCARTAN cannot calculate integrals or simplify rational expressions of the form: polynomial divided by polynomial. In the course of calculation

all the products in which sums appear as factors are automatically fully developed with the use of the distributive rule. However, there is the possibility offered to the user to define substitutions of his own, to be performed at an arbitrary selected stage of the calculation. With this facility, the user can define a single symbol for any sum, and thus prevent the product involving the sum from being developed. Such substitutions can be used to help the program simplify rational expressions. The problem of simplifying rational functions is in fact common to all the symbol-processing programs because it was not satisfactorily solved even in theory.

3. What is needed to run ORTOCARTAN

ORTOCARTAN is written in the LISP programming language (UT version 4.1), and implemented on a CDC Cyber 73 computer under the operating system SCOPE 3.4.4. Consequently, it should run on any CDC which uses the SCOPE system, and, with minor modifications, on any computer on which LISP is available. Small-size calculations can be processed in 30 000 words of core memory, of which 20 480 are needed to load the LISP system. Moderately complicated calculations required 40 960 words (120 000 octal), and with 53 248 words (150 000 octal) the computer could handle the largest expressions we needed. If a larger memory should appear necessary in a particular case, then possibly the result would be a print too huge to be comprehended by the user. The program runs only in the batch mode and no interactive version is at present available.

Copies of ORTOCARTAN may be obtained from the authors as magnetic tape records or boxes of punched cards, together with a listing in the LISP-interpreter code, a detailed description of the code [4] and a user-manual [5] which is intended to assist only those users who apply ORTOCARTAN to calculating curvature tensors.

4. Comparison with other systems

We compared the speed and memory requirements of ORTOCARTAN with the corresponding data on other systems, available from the literature [2,3]. A

kind of standard test for such programs in relativity has become the Bondi—Van der Burg—Metzner metric [6]. The Ricci tensor for this metric, when calculated by different systems or programs, required the following parameters:

System	Computer	Time (s)	Memory
LAM	IBM 360/375	104	400 kbytes
ALTRAN	IBM 360/375	255	350 kbytes
FORMAC	IBM 360/375	162	300 kbytes
REDUCE	IBM 360/375	856	500 kbytes
SAC-1	IBM 360/375	>3600	?
SYMBAL	CDC 6600	35	33 000 words
CLAM	CDC Cyber 73	33	40 960 words
ORTOCARTAN	CDC Cyber 73	536	53 248 words

The data given above concern ORTOCARTAN as written in the ordinary LISP interpreter code. We keep it in this form to have it ready for alterations which would otherwise be rather difficult to perform. However, sooner or later ORTOCARTAN will be compiled and further stored in the compiled form. This should make it at least five times faster, as judged from the already existing partly-compiled working version.

5. An example of work with ORTOCARTAN

To give the readers a grasp of work with ORTOCARTAN let us consider a simple example. The example will be simple for brevity only, as the program did successfully process calculations in which the final result consisted of 19 expressions, each occupying 2 or 3 pages of computer paper when printed.

Suppose you want to differentiate the expression $U \stackrel{\text{def}}{=} (F^2 + G^2)^{1/2}$ with respect to the variable X where F is a function of X while G is a function of $R = (X^2 + Y^2 + Z^2)^{1/2}$. To do so, essentially only the following two LISP commands are necessary:

```
(SETQ V (DERIV (QUOTE X) (SIMPLIFY (INTOPRE
  (QUOTE ((F ** 2 + G ** 2) ** (1 2)) ) T)))
```

 (1)

```
(MATHPRINT (QUOTE (DER X U)) NIL NIL (QUOTE =) V)
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 (2)

The meaning of the statements is: (QUOTE A) means

that the symbol A stands for itself and is not a representative of something else, (INTOPRE A) means: translate the expression represented by A from the infix format in which it is written into the internal prefix notation, (SIMPLIFY A T) means: perform the algebraic simplification on the expression represented by A (the simplification is performed on the input data to order all the sums and products in a unique way), (DERIV A B) means: calculate the derivative of the expression symbolized by B with respect to the variable represented by A, and (SETQ V ...) means: represent the result of the previous operations by the variable V. The command (2) will then result in printing the equation:

$$U_{,X} = X G R^{-1} (F^2 + G^2)^{-(1/2)} G_{,R} + F (F^2 + G^2)^{-(1/2)} F_{,X} \quad (3)$$

However, first the program must learn that X, Y and Z are primary variables which do not depend on anything else, that F and G are unknown functions of their respective arguments, and that R is a symbol for the expression $(X^2 + Y^2 + Z^2)^{1/2}$. This is done by the following set of commands:

(SETQ COORD (QUOTE (X Y Z))) (4)

(SETQ FUNNAMES (QUOTE (F G))) (5)

(SETQ SYMBOLS (QUOTE (R))) (6)

(PUT (QUOTE F) (QUOTE DEPS) (QUOTE (X))) (7)

(PUT (QUOTE G) (QUOTE DEPS) (QUOTE (R))) (8)

(PUT (QUOTE R) (QUOTE CONTENT) (SIMPLIFY (INTOPRE (QUOTE ((X ** 2 + Y ** 2 + Z ** 2) ** (1 2)))) T)) (9)

To make the print shorter one could change the command (6) to:

(SETQ SYMBOLS (QUOTE (R U))) (10)

and add the command:

(PUT(QUOTE U) (QUOTE CONTENT) (SIMPLIFY (INTOPRE (QUOTE ((F ** 2 + G ** 2) ** (1 2)))) T)) (11)

Then, if $((F^2 + G^2) ** (1 2))$ is replaced by U in the command (1), all occurrences of $(F^2 + G^2)^{-1/2}$ will be replaced by U^{-1} in the print (3).

Needless to say, this chain of operations can be coded as a LISP program, to be later called by a single command with different data, but to describe a sufficiently general program of this kind would require too much space here.

References

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